Chapter 4
Fashion Supply Chain Network Competition with Ecolabeling

Anna Nagurney, Min Yu and Jonas Floden

Abstract In this chapter we develop a competitive supply chain network model for fashion that incorporates ecolabeling. We capture the individual profit-maximizing behavior of the fashion firms which incur ecolabeling costs with information associated with the carbon footprints of their supply chains revealed to the consumers. Consumers, in turn, reflect their preferences for the branded products of the fashion firms through their demand price functions, which include the carbon emission information. We construct the underlying network structure of the fashion supply chains and provide alternative variational inequality formulations of the governing Nash equilibrium conditions. The model, as a special case, also captures carbon taxes. We discuss qualitative properties of the equilibrium product flow pattern and also propose an algorithm, which has elegant features for computational purposes. We provide both an illustrative example as well as a variant and then discuss a case study with several larger numerical examples.

4.1 Introduction

Apparel and fashion products, from fast fashion to luxury goods, are manufactured, stored, and distributed in global supply chains and, along with textiles, represent an immense industry with wide economic importance valued at US$ 3 trillion in terms of turnover in 2011 (cf. Martin 2013). At the same time, this industry utilizes extensive amounts of natural resources from water and grown cotton, energy, as well as chemicals. For example, it is estimated that cotton uses only 3 % of the world

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A. Nagurney (✉)
Isenberg School of Management, University of Massachusetts, Amherst, MA 01003, USA
e-mail: nagurney@isenberg.umass.edu

M. Yu
Pamplin School of Business Administration, University of Portland, Portland, OR 97203, USA
e-mail: yu@up.edu

J. Floden · A. Nagurney
School of Business, Economics and Law, University of Gothenburg, Gothenburg, Sweden
e-mail: jonas.floden@handels.gu.se

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farmlands but about 25 % of the world's pesticides (Chen and Burns 2006). Moreover, this industrial sector is a primary source of GHG (greenhouse gas) emissions, including CO_2, one of the principal sources of global warming. According to World Wildlife Fund (2013), because of the scope of the industrial sector's activities, it is a significant GHG emitter with apparel and textiles accounting for about 10 % of the total carbon emissions, and with textiles being the fifth largest contributor to CO_2 emissions in the USA. Ten of the total CO_2 emissions from a life cycle perspective can be attributed to transport (Allwood et al. 2006). Growth in this industry is expected, along with an expected increase in associated emissions, if appropriate environmental mitigation actions are not taken, with estimates of there being 9 billion people on our planet in 2050, all with a need to clothe themselves (see, Martin 2013). Also, as noted by CleanMetrics (2011), clothing and accessories are the consumer goods that, next to food and beverages, are purchased most often and also replaced most frequently.

The rapidly changing world of fashion pushes toward overconsumption of resources, as products no longer may be made to last, but, rather, to be replaced by the next trend. It is noteworthy that fashion trends that once lasted for years, if not centuries, are now replaced several times per season. The increasing competitive pressure on lower prices has led to production moving to low cost countries in the Far East with less stricter health and safety legislation (de Brito et al. 2008). The fashion supply chain is international and long distance. For example, only 20 % of the UK's annual consumption of clothing is manufactured there (Allwood et al. 2006). The long, complex, and fragmented fashion supply chain is characterized by low transparency and control, resulting in a divide between those who get the benefits from fashion on the customers' side and those who pay the social and environmental costs (Pedersen and Andersen 2013). Hence, there exists an immense opportunity in this sector to contribute to positive change in terms of sustainability.

A transformation of this industry, as noted by Martin (2013), should include transparency, as well as the "optimization" of environmental footprints. Changes in this sector will be driven by customer choice (Allwood et al. 2006). Preliminary efforts are underway with the establishment of the sustainable apparel coalition (SAC) and its creation of the Higg Index (cf. Westervelt 2012). SAC, according to its website, http://www.apparelcoalition.org/, is a trade organization consisting of over 100 leading brands, retailers, manufacturers, government, nongovernmental organizations, and academic experts, reflecting more than a third of the global apparel and footwear market, and focused on the reduction of both environmental and social impacts associated with apparel and footwear products. Members range from Coca-Cola, which licenses its brand name for apparel, to the retailer Target, and manufacturers such as REI, Levi's, and Nike. Nevertheless, much remains to be done especially in terms of the development of rigorous analytical tools that can capture the impact of environmental emission reductions on consumer choices as well as firms' profitability in a systemic and system-wide manner. Furthermore, any such quantitative tools must also be able to handle the reality of competition in this sector.

Indeed, many fashion firm brands are recognizing that green or eco-friendly apparel is a way of differentiating one's products and enhancing brand recognition (see also (Koszewska 2011)) with consumers also becoming increasingly aware of the negative environmental impacts of manufacturing apparel (cf. Infosys 2010). This is particularly evident in the market segment of children's clothing (Gam et al. 2010). Meyer (2001) argues that eco-friendly clothes are bought only if customers perceive the products as superior to competitors' offerings, thus looking at costs and benefits of the clothing. However, there are challenges, although a recent survey noted that 51 % identify environmental friendliness as being an important factor in their apparel purchasing decisions and only 26 % are willing to pay more for clothes that are identified as such (see (Cotton Incorporated 2013) or similar results in (Johannson 2008)). Consumers need a readily accessible and easily understandable mechanism to identify the environmental impact of the apparel that they purchase (cf. Rowe 2013). About 83 % of customers believe that the company selling the products should be responsible in informing the customers about the manufacturing conditions and 95 % of customers prefer to get this information through product labeling (Johannson 2008).

Ecolabels, in the form of carbon footprint labels, which reveal the product carbon footprint to consumers of a product, are a means of influencing consumer purchasing decisions in order to enhance supply chain sustainability (see, Craig et al. 2011; Vandenbergh et al. 2011). A study of the major Swedish clothing retailers found 12 different independent ecolabeling systems of different scope and complexity. In addition, several retailers offered their own labeling system (Holm 2010). Such labels entail a cost to producers, but provide valuable information to concerned consumers. Also, as noted by Mason (2011), some consumers may be willing to even pay a premium in order to "protect the environment" with possible other benefits attributed to a "warm glow" effect gained from adding to public welfare from one's benevolent activities (Andreoni 1989). There are currently 109 ecolabels related to textiles in the world (http://www.ecolabelindex.com). The environmental quality associated with an apparel or fashion product may, hence, be a positive attribute. For background on ecolabels, see the report by Global (2004).

In this paper, we contribute to the understanding of supply chain network sustainability through the development of a competitive fashion supply chain model with ecolabeling, consisting of multiple firms, each of which is distinguished by its brand. Unlike our previous research in fashion supply chain networks, in which the focus was on time and cost minimization (cf. Nagurney and Yu 2011) or emission reduction (Nagurney and Yu 2012), with various levels of concern, here we model supply chain network competition with environmental quality information shared with consumers via the ecolabeling of the firms' carbon footprints. Although recent research in competitive supply chain networks has explored issues of quality from product differentiation (cf. Nagurney and Li 2014a) to information asymmetry (cf. Nagurney and Li 2014b), as well as outsourcing issues in the context of a particular industry (Nagurney et al. 2013), in this chapter, for the first time, we focus on competition in a supply chain network framework where consumers, through ecolabeling, are provided, in a transparent way, the carbon footprints (and associated environmental quality or lack, thereof) attributed to fashion firms' supply chains. Consumers reflect their preferences through the demand price functions which depend both on the product quantities and the carbon emissions associated with the fashion firms' supply chain networks. For an excellent background on fashion supply chain management, we refer the reader to the edited volume by Choi (2011). For an overview of an edited collection of papers on green manufacturing and distribution in the fashion and apparel industry, see Choi et al. (2013). Also, see Chan and Wong (2012) for background and findings concerning the consumption side of sustainable fashion supply chains with managerial implications.

The literature on sustainable supply chains has been growing, with a recent edited volume by Boone et al. (2012) providing a scope of topics in both breadth and depth. Sustainable supply chain network design (cf. Nagurney and Nagurney 2011; Nagurney 2013) as well as the role of the frequency of supply chain network activities on sustainability (see (Nagurney et al. 2013a)) and integrated logistics for green supply chain management (Sheu et al. 2005) have also garnered attention from the academic community. For a literature review and conceptual background, see Seuring and Muller (2008).

This chapter is organized as follows. In Sect. 4.2, we develop the model and describe the firms' competitive behaviors. We state the governing Nash equilibrium conditions (Nash 1950, 1951), present alternative governing variational inequality formulations, and also provide an illustrative example and variant. We also note how a special case of our model captures carbon taxes. Qualitative properties of the solution pattern in terms of existence and uniqueness that further support the model are given in the Appendix. In Sect. 4.3, we discuss a computational procedure for the determination of the equilibrium pattern of product flows and the incurred environmental carbon emissions, as well as the firms' profits. We detail a case study that demonstrates how our modeling and computational framework can guide decision-makers in the fashion industry to enhance the sustainability of their supply chain networks. We summarize our results and present our conclusions in Sect. 4.4.

4.2 The Fashion Supply Chain Network Model with Ecolabeling

As mentioned in Sect. 4.1, the fashion and apparel industry is globalized with manufacturing plants often located geographically at great distances from the consumers. Moreover, many of such plants may be in regions of the world where the environmental regulations are not as stringent as in parts of the developed world. Furthermore, given the geographical distances, the selection of appropriate transportation modes may also make an impact on the overall supply chain network environmental sustainability. Such aspects of these important supply chains create both challenges and opportunities for sustainability in terms of carbon footprint reduction.

The model that we develop in this section captures the supply chain networks of individual fashion firms involved in the production, storage, and distribution of a fashion product, which is distinguished by the firm's brand. This is relevant to this unique industry whether we are dealing with fast fashion products of such major brands as H&M, Zara, etc., or even luxury brands such as Chanel, Hermes, Louis Vuitton, etc. In the model, there are I competing fashion firms, with a typical such firm denoted by i. The notation for the model is given in Table 4.1.

Table 4.1 Notation for the Fashion Supply Chain Model with Ecolabeling

Notation	Definition
L^i	the links comprising the supply chain network of fashion firm i; $i = 1, \ldots, I$ with a total of n_{L^i} elements.
L	the full set of links in the fashion supply chain network economy with $L = \cup_{i=1}^{I} L^i$ with a total of n_L elements.
P_k^i	the set of paths in fashion firm i's supply chain network terminating in demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$.
P^i	the set of all n_{P^i} paths of fashion firm i; $i = 1, \ldots, I$.
P	the set of all n_P paths in the fashion supply chain network economy.
x_p; $p \in P_k^i$	the nonnegative flow of firm i's fashion product to demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$. We group all the firms' product flows into the vector $x \in R_+^{n_P}$, where n_P denotes the number of paths.
f_a	the nonnegative flow of the fashion product on link a, $\forall a \in L$. We group the link flows into the vector $f \in R_+^{n_L}$.
d_{ik}	the demand for the product of fashion firm i at demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$. We group the $\{d_{ik}\}$ elements for firm i into the vector $d^i \in R_+^{n_R}$ and all the demands into the vector $d \in R_+^{I \times n_R}$.
$e_a(f_a)$	the carbon emissions generated on link a, $\forall a \in L$.
E_i	the emissions generated in the supply chain network of fashion firm i; $i = 1, \ldots, I$, where $E_i = \sum_{a \in L^i} e_a$.
E	We group the emissions generated by all the fashion firms into the vector $E \in R_+^I$.
$\hat{c}_a(f, e_a(f_a))$	the total cost associated with link a, $\forall a \in L$.
$l_i(\sum_{k=1}^{n_R} d_{ik})$	the ecolabeling cost of fashion firm i; $i = 1, \ldots, I$.
$\rho_{ik}(d, E)$	the demand price function for the product of fashion firm i at demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$.

The fashion supply chain network economy consists of the entirety of the firms' activities as depicted and labeled in Fig. 4.1. Each fashion firm i; $i = 1, \ldots, I$; is considering n_M^i manufacturing facilities/plants; n_D^i distribution centers, and serves the same n_R demand markets. Let $G = [N, L]$ denote the graph consisting of the set of nodes N and the set of links L in Fig. 4.1. According to Fig. 4.1, each fashion firm has, at its disposal, multiple transportation options from the manufacturing plants to the distribution centers and from the distribution centers to the demand markets.

Also, we include the option that a fashion firm may have its product transported directly from a manufacturing plant to a demand market, and avail itself of one or more transportation shipment modes. Having multiple transport options, including intermodal ones, enables greater flexibility, which may, in turn, depending on the firms' decisions, be good for consumers and also for the environment.

It is important to identify the supply chain network structure since the topology reveals different choices that may present themselves. Furthermore, the network topology may be different from industry to industry (cf. (Yu and Nagurney 2013;

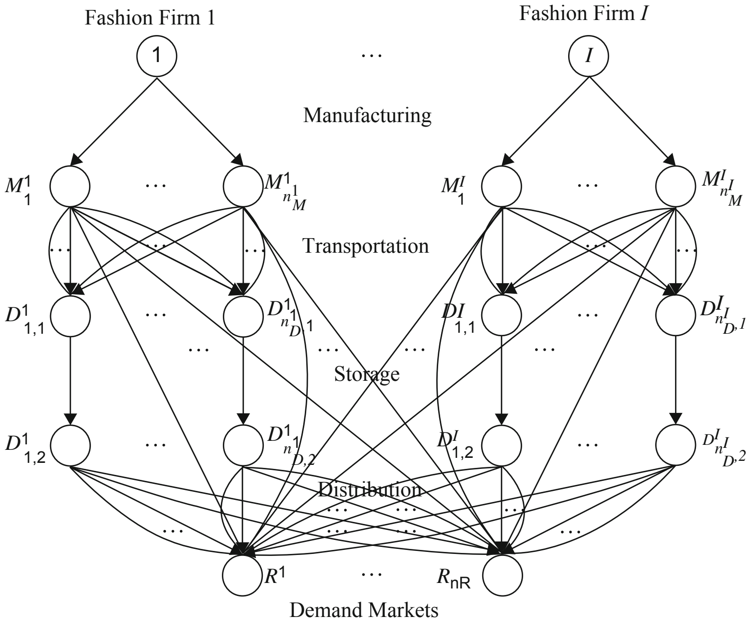

Fig. 4.1 The fashion supply chain network economy topology

Nagurney et al. 2013a, b) for several examples). In this chapter, we are interested in quantifying the effects of ecolabeling on fashion firms' profits as well as on their carbon footprints in the existing fashion supply chain network economy. Nevertheless, we emphasize that the framework constructed here may also be applied to other industries in which ecolabeling is being considered, with appropriate adaptation.

We first present the constraints in the form of the product conservation of flow equations. We then discuss the underlying supply chain network operational cost and emission functions, the ecolabeling cost functions, and the demand price functions.

The following conservation of flow equations must hold:

$$\sum_{p \in P^i_k} x_p = d_{ik}, \quad \forall i, \forall k, \tag{4.1}$$

that is, the demand for each firm's product at each demand market must be satisfied by the fashion product flows from the firm to that demand market.

Moreover, the path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P. \tag{4.2}$$

Furthermore, the expression that relates the link flows to the path flows is given by,

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L. \tag{4.3}$$

Hence, the flow on a link is equal to the sum of the flows on paths that contain that link.

The total cost on a link, be it a manufacturing/production link, a shipment/distribution link, or a storage link is assumed, in general, to be a function of the product flows on all the links as well as the emissions generated, that is,

$$\hat{c}_a = \hat{c}_a(f, e_a(f_a)), \quad \forall a \in L. \tag{4.4}$$

We emphasize that the manufacturing cost associated with manufacturing at different plants also includes the cost associated with sourcing and the corresponding emission function includes the emissions generated also through sourcing. The above link total cost functions capture competition on the supply side, since the total cost on a link may depend not only on the product flows of the particular firm but also on those on the other firms' links. Fashion firms may share common suppliers and compete for fabrics, adornments, and even human resources, etc.

It is well-known that one of the reasons for manufacturing in the less-developed parts of the world is that the environmental regulations there may be less stringent, which also may account for, in general, lower operational costs. The link emission functions are for carbon emissions and these can also include other GHG emissions when transformed into their carbon equivalents.

Here we assume that the fashion firms adopt ecolabeling due to peer pressure from organizations such as SAC, as noted in the Introduction, and/or environmental regulations and/or the possible consumer pressure. There is a cost associated with ecolabeling, which includes the extra labeling of the fashion product as well as the research cost associated with quantifying the emissions on the supply chain network links or paying a neutral party for this information. As noted in Table 4.1, the ecolabeling cost is assumed to be a function of the total amount of the product produced by a given fashion firm, that is,

$$l_i = l_i\left(\sum_{k=1}^{n_R} d_{ik}\right), \quad i = 1, \ldots, I. \tag{4.5}$$

In view of (4.1), we may reexpress the ecolabeling cost function, $l_i(\sum_{k=1}^{n_R} d_{ik})$, as follows:

$$\hat{l}_i = \hat{l}_i(x) \equiv l_i\left(\sum_{k=1}^{n_R} d_{ik}\right), \quad i = 1, \ldots, I. \tag{4.6}$$

According to Table 4.1, the demand price function ρ_{ik}; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$ depends not only on the firm's demand for its fashion product but also, in general, on the demands for the other firms' fashion products. Hence, we also capture competition on the demand side. In addition, because of ecolabeling, the consumers at the demand markets are now informed as to the total emissions generated by each of the fashion firms. Different demand markets may be more or less sensitive to the emissions generated and such functions provide enhanced modeling flexibility. Of course, we may expect that the price that the consumers are willing to pay for a fashion product will decrease if the overall emissions associated with that firm and product increase. Note that we consider the total emissions generated by firms' supply chain networks rather than the amount of emissions per product at the demand market since the negative environmental impact needs to be fully captured and accounted for. In view of (4.1) and (4.3), and the definition of the generated carbon emissions in Table 4.1, we may reexpress the demand price function, $\rho_{ik}(d, E)$, as follows:

$$\hat{\rho}_{ik} = \hat{\rho}_{ik}(x) \equiv \rho_{ik}(d, E), \quad \forall i, \forall k. \tag{4.7}$$

We assume that the operational cost functions, the emission functions, the demand price functions, and the ecolabeling cost functions are all continuous and continuously differentiable.

The profit of a fashion firm is the difference between its revenue and its total costs, where the total costs include the total operational cost and the ecolabeling cost, that is,

$$U_i = \sum_{k=1}^{n_R} \rho_{ik}(d, E) d_{ik} - \sum_{a \in L^i} \hat{c}_a(f, e_a(f_a)) - l_i\left(\sum_{k=1}^{n_R} d_{ik}\right). \tag{4.8}$$

Let X_i denote the vector of strategy variables associated with fashion firm i; $i = 1, \ldots, I$, where X_i is the vector of path flows associated with fashion firm i, that is,

$$X_i \equiv \{\{x_p\} | p \in P^i\} \in R_+^{n_{P^i}}. \tag{4.9}$$

X is then the vector of all fashion firms' strategies, that is, $X \equiv \{\{X_i\} | i = 1, \ldots, I\}$.

Through the use of the conservation of flow Eqs. (4.1) and (4.3), and the functions (4.6) and (4.7), and the definition of the generated carbon emissions in Table 4.1, we define $\hat{U}_i(X) \equiv U_i$; $i = 1 \ldots, I$. We group the profits of all the fashion firms into an I-dimensional vector $\hat{U}$, where

$$\hat{U} = \hat{U}(X). \tag{4.10}$$

In the competitive oligopolistic market framework, each fashion firm selects its product path flows in a noncooperative manner, seeking to maximize its own profit, until an equilibrium is achieved, according to the definition below.

Definition 1 Fashion Supply Chain Network Cournot–Nash Equilibrium with Ecolabeling
A path flow pattern $X^* \in K = \prod_{i=1}^{I} K_i$ constitutes a fashion supply chain network Cournot–Nash equilibrium with ecolabeling if for each firm i; $i = 1, \ldots, I$:

$$\hat{U}_i(X_i^*, \hat{X}_i^*) \geq \hat{U}_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i, \tag{4.11}$$

where $\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$ and $K_i \equiv \{X_i | X_i \in R_+^{n_{P^i}}\}$.

Hence, an equilibrium is established if no fashion firm can unilaterally improve its profit by changing its product flows throughout its supply chain network, given the product flow decisions of the other firms.

Next, we derive the variational inequality formulations of the Cournot–Nash equilibrium for the fashion supply chain network with ecolabeling satisfying Definition 1, in terms of path flows and link flows (see (Cournot 1838; Nash 1950, 1951; Gabay and Moulin 1980; Nagurney et al. 2013b)). For the details in the variational inequality theory, please refer to the book by (Nagurney 1999).

Theorem 1 ***Variational Inequality Formulations***
Assume that, for each fashion firm i; $i = 1, \ldots, I$, the profit function $\hat{U}_i(X)$ is concave with respect to the variables in X_i, and is continuously differentiable. Then $X^* \in K$ is a fashion supply chain network Cournot–Nash equilibrium with ecolabeling according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \tag{4.12}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{X_i} \hat{U}_i(X)$ denotes the gradient of $\hat{U}_i(X)$ with respect to X_i. Variational inequality (4.12), in turn, for our model, is equivalent to the variational inequality in path flows; determine the vector of equilibrium path flows $x^* \in K^1$ such that,

$$\sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \frac{\partial \hat{l}_i(x^*)}{\partial x_p} - \hat{\rho}_{ik}(x^*) - \sum_{j=1}^{n_R} \frac{\partial \hat{\rho}_{ij}(x^*)}{\partial x_p} \sum_{q \in P_j^i} x_q^* \right] \times [x_p - x_p^*] \geq 0, \quad \forall x \in K^1, \tag{4.13}$$

where $K^1 \equiv \{x | x \in R_+^{n_P}\}$, and for each path p; $p \in P_k^i$; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$, and

$$\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f, e_b(f_b))}{\partial f_a} \delta_{ap}; \tag{4.14}$$

$$\frac{\partial \hat{l}_i(x)}{\partial x_p} \equiv \frac{\partial l_i(\sum_{j=1}^{n_R} d_{ij})}{\partial d_{ik}}; \tag{4.15}$$

$$\frac{\partial \hat{\rho}_{ij}(x)}{\partial x_p} \equiv \frac{\rho_{ij}(d, E)}{\partial d_{ik}} + \frac{\partial \rho_{ij}(d, E)}{\partial E_i} \sum_{a \in L^i} \frac{\partial e_a(f_a)}{\partial f_a} \delta_{ap}. \tag{4.16}$$

In addition, (4.13) can be re-expressed in terms of link flows as: determine the vector of equilibrium link flows and the vector of equilibrium demands $(f^*, d^*) \in K^2$ such that,

$$\sum_{i=1}^{I} \sum_{a \in L^i} \left[\sum_{b \in L^i} \frac{\partial \hat{c}_b(f^*, e_b(f_b^*))}{\partial f_a} - \sum_{j=1}^{n_R} \frac{\partial \rho_{ij}(d^*, E)}{\partial E_i} d_{ij}^* \frac{e_a(f_a^*)}{\partial f_a} \right] \times [f_a - f_a^*]$$
$$+ \sum_{i=1}^{I} \sum_{k=1}^{n_R} \left[\frac{\partial l_i(\sum_{j=1}^{n_R} d_{ij}^*)}{\partial d_{ik}} - \rho_{ik}(d^*, E) - \sum_{j=1}^{n_R} \frac{\partial \rho_{ij}(d^*, E)}{\partial d_{ik}} d_{ij}^* \right] \times [d_{ik} - d_{ik}^*] \geq 0,$$
$$\forall (f, d) \in K^2, \tag{4.17}$$

where $K^2 \equiv \{(f,d) | \exists x \geq 0,$ *and* (*4.1*) *and* (*4.3*) *hold*$\}$.

Proof See the Appendix. □

Variational inequalities (4.13) and (4.17) can be put into standard form (see Nagurney 1999): determine $X^* \in \mathcal{K}$ such that,

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{4.18}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in n-dimensional Euclidean space. Let $X \equiv x$ and

$$F(X) \equiv \left[\frac{\partial \hat{C}_p(x)}{\partial x_p} + \frac{\partial \hat{l}_i(x)}{\partial x_p} - \hat{\rho}_{ik}(x) - \sum_{j=1}^{n_R} \frac{\partial \hat{\rho}_{ij}(x)}{\partial x_p} \sum_{q \in P^i_j} x_q; \right.$$
$$\left. p \in P^i_k; \; i = 1, \ldots, I; \; k = 1, \ldots, n_R \right], \tag{4.19}$$

and $\mathcal{K} \equiv K^1$, then (4.13) can be re-expressed as (4.18). If we define $X \equiv (f, d)$ and $F(X) \equiv (F_1(X), F_2(X))$, such that

$$F_1(X) = \left[\sum_{b \in L^i} \frac{\partial \hat{c}_b(f, e_b(f_b))}{\partial f_a} - \sum_{j=1}^{n_R} \frac{\partial \rho_{ij}(d, E)}{\partial E_i} d_{ij} \frac{e_a(f_a)}{\partial f_a}; \; a \in L^i; \; i = 1, \ldots, I \right], \tag{4.20}$$

$$F_2(X) = \left[\frac{\partial l_i(\sum_{j=1}^{n_R} d_{ij})}{\partial d_{ik}} - \rho_{ik}(d, E) - \sum_{j=1}^{n_R} \frac{\partial \rho_{ij}(d, E)}{\partial d_{ik}} d_{ij}; \right.$$
$$\left. i = 1, \ldots, I; \; k = 1, \ldots, n_R \right], \tag{4.21}$$

and $\mathcal{K} \equiv K^2$, then (4.17) can be re-expressed as (4.18).

For qualitative properties of the equilibrium solution, in particular, existence and uniqueness, please see the Appendix.

Game theory and variational inequalities were first applied to supply chain network equilibrium problems by (Nagurney et al. 2002) with precursors to such models lying in spatial oligopolistic market equilibrium problems (cf. (Dafermos and Nagurney 1987)) and in spatial price equilibrium problems (see (Dafermos and Nagurney 1987)). Various multitiered supply chain network equilibrium models, both static and dynamic, are synthesized in the book by (Nagurney 2006). Vertically integrated supply chain models, including competitive ones, in which perishability of the products is a feature, which includes, fast fashion, in a sense, are described in the book by (Nagurney et al. 2013b).

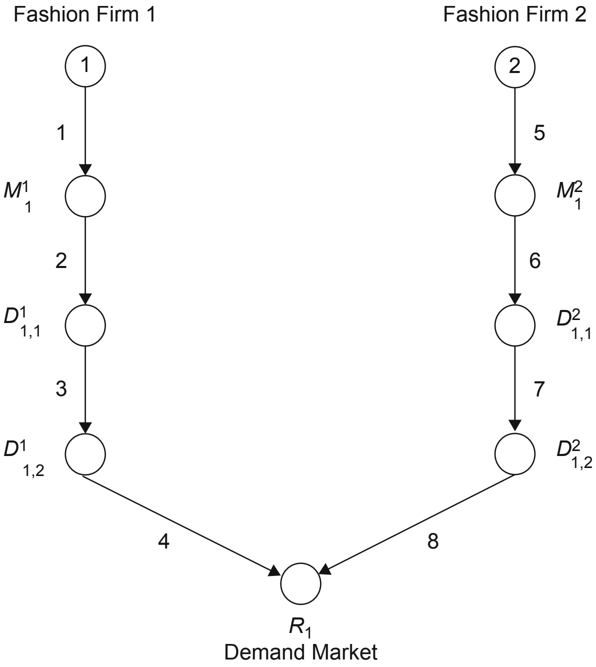

Fig. 4.2 Fashion supply chain network topology for the illustrative example

Remark

We emphasize that the above model contains, as a special case, a competitive supply chain network model in which the ecolabeling costs correspond to carbon taxes. In such a special case we remove the emissions terms in the demand price functions. We illustrate this feature with a variant example below.

4.2.1 An Illustrative Example and Variant

We now present a simple numerical example in order to illustrate the model.

In the example (cf. Fig. 4.2), two fashion firms compete in a single demand market R_1. Firm 1 is located in the USA and Firm 2 is in Bangladesh in Asia. The demand market R_1 is in Europe, specifically, in Germany. The product that they produce is a white ladies shirt.

Firm 1's distribution center is located in The Netherlands and Firm 2's in Germany. Firm 1 uses air transport to ship the product to The Netherlands to its distribution center and onward to the demand market in Germany. Firm 2 uses ship transport throughout.

Path p_1 corresponding to Firm 1 consists of the links: 1, 2, 3, and 4, whereas path p_2 corresponding to Firm 2 consists of the links: 5, 6, 7, and 8. Therefore, we have

$$x_{p_1} = d_{11}, \quad x_{p_2} = d_{21},$$

and

$$f_1 = f_2 = f_3 = f_4 = x_{p_1}, \quad f_5 = f_6 = f_7 = f_8 = x_{p_2}.$$

The emission functions reflect the total CO_2 generated on links, in kilograms, associated with this product. We utilized (Sarkar 2011), as a reference, in order to estimate the emission cost functions, which are given below:

$$e_1(f_1) = 5f_1, \quad e_2(f_2) = 2f_2, \quad e_3(f_3) = f_3, \quad e_4(f_4) = 2.5f_4,$$

$$e_5(f_5) = 6f_5, \quad e_6(f_6) = .1f_6, \quad e_7(f_7) = 2f_7, \quad e_8(f_8) = .07f_8.$$

Therefore, the respective total emissions generated by Firms 1 and 2 can be expressed in terms of path flows as

$$E_1 = 10.5x_{p_1}, \quad E_2 = 8.17x_{p_2}.$$

The total cost functions on the various links of manufacturing, shipment, storage, and distribution, in which we have embedded the emission functions are

$$\hat{c}_1(f_1, e_1(f_1)) = 5f_1^2 + 8f_1, \quad \hat{c}_2(f_2, e_2(f_2)) = 7f_2^2 + 3f_2,$$

$$\hat{c}_3(f_3, e_3(f_3)) = 2f_3^2 + f_3, \quad \hat{c}_4(f_4, e_4(f_4)) = 2f_4^2 + 2f_4,$$

$$\hat{c}_5(f_5, e_5(f_5)) = 3f_5^2 + 4f_5, \quad \hat{c}_6(f_6, e_6(f_6)) = 3.5f_6^2 + f_6,$$

$$\hat{c}_7(f_7, e_7(f_7)) = 2f_7^2 + 5f_7, \quad \hat{c}_8(f_8, e_8(f_8)) = 1.5f_8^2 + 4f_8.$$

We assume that both firms have quantified the per unit emissions on their supply chain network links associated with their fashion product. Hence, the ecolabeling cost function per firm only consists of the cost associated with marking the product with the emission information through a label. The ecolabeling cost functions are

$$l_1(d_{11}) = .02d_{11}, \quad l_2(d_{21}) = .01d_{21},$$

so that

$$\hat{l}_1(x) = .02x_{p_1}, \quad \hat{l}_2(x) = .01x_{p_2}.$$

The firms compete in the demand market R_1, and the consumers reveal their preferences for their products through the following demand price functions:

$$\rho_{11}(d, E) = -3d_{11} - d_{21} - .5E_1 + .2E_2 + 300,$$

$$\rho_{21}(d, E) = -4.5d_{21} - d_{11} - .5E_2 + .2E_1 + 300.$$

Hence,

$$\hat{\rho}_{11}(x) = -3x_{p_1} - x_{p_2} - .5(10.5x_{p_1}) + .2(8.17x_{p_2}) + 300 = -8.25x_{p_1} + .634x_{p_2} + 300,$$

and

$$\hat{\rho}_{21}(x) = -4.5x_{p_2} - x_{p_1} - .5(8.17x_{p_2}) + .2(10.5x_{p_1}) + 300 = -8.585x_{p_2} + 1.1x_{p_1} + 300.$$

Note that, in this example, the consumers at a demand market respond to the price of a fashion firm's product through the demands for both of the products, as well as the emissions generated by both firms.

Variational inequality (4.13) becomes, in the case of this example,

$$\left[\frac{\partial \hat{C}_{p_1}(x^*)}{\partial x_{p_1}} + \frac{\partial \hat{l}_1(x^*)}{\partial x_{p_1}} - \hat{\rho}_{11}(x^*) - \frac{\partial \hat{\rho}_{11}(x^*)}{\partial x_{p_1}} \times x^*_{p_1}\right] \times [x_{p_1} - x^*_{p_1}]$$

$$+\left[\frac{\partial \hat{C}_{p_2}(x^*)}{\partial x_{p_2}} + \frac{\partial \hat{l}_2(x^*)}{\partial x_{p_2}} - \hat{\rho}_{21}(x^*) - \frac{\partial \hat{\rho}_{21}(x^*)}{\partial x_{p_2}} \times x^*_{p_2}\right] \times [x_{p_2} - x^*_{p_2}] \geq 0, \quad \forall x \in R^2_+.$$

Under the assumption that $x^*_{p_1} > 0$ and $x^*_{p_2} > 0$, the two expressions on the left-hand side of the above inequality must be equal to zero, that is,

$$\left[\frac{\partial \hat{C}_{p_1}(x^*)}{\partial x_{p_1}} + \frac{\partial \hat{l}_1(x^*)}{\partial x_{p_1}} - \hat{\rho}_{11}(x^*) - \frac{\partial \hat{\rho}_{11}(x^*)}{\partial x_{p_1}} \times x^*_{p_1}\right] = 0,$$

and

$$\left[\frac{\partial \hat{C}_{p_2}(x^*)}{\partial x_{p_2}} + \frac{\partial \hat{l}_2(x^*)}{\partial x_{p_2}} - \hat{\rho}_{21}(x^*) - \frac{\partial \hat{\rho}_{21}(x^*)}{\partial x_{p_2}} \times x^*_{p_2}\right] = 0.$$

Simple arithmetic calculations, using the corresponding functions for the numerical example, yield the following system of equations:

$$\begin{cases} 48.5x^*_{p_1} - .634x^*_{p_2} = 285.98 \\ -1.1x^*_{p_1} + 37.17x^*_{p_2} = 285.99. \end{cases}$$

A solution of the above system of equations, yields the equilibrium path flows as

$$x^*_{p_1} = 6.00, \quad x^*_{p_2} = 7.87.$$

with the equilibrium demands being equal to

$$d^*_{11} = 6.00, \quad d^*_{21} = 7.87.$$

The equilibrium link flows are, hence

$$f^*_1 = 6.00, \quad f^*_2 = 6.00, \quad f^*_3 = 6.00, \quad f^*_4 = 6.00,$$

$$f^*_5 = 7.87, \quad f^*_6 = 7.87, \quad f^*_7 = 7.87, \quad f^*_8 = 7.87.$$

Finally, the equilibrium prices of the two white ladies shirts are

$$\rho_{11} = 255.50, \quad \rho_{21} = 239.02,$$

with the associated emissions being

$$E_1 = 62.99, \quad E_2 = 64.31.$$

The profits of the firms are

$$U_1 = 872.82, \quad U_2 = 1,151.58.$$

The result shows that Firm 2 emits more than Firm 1, delivers the fashion product at a lower price than Firm 1, and obtains a higher profit. Note that Firm 1 is the polluter with more emissions per unit. In order to maintain its total emissions within a competitive range, Firm 1 has to control its product quantity. Although the consumers are willing to pay more for the product from Firm 1, the profit of Firm 1 is still lower than that of Firm 2.

A Variant

We now consider the following variant of the above example. We remove the emission terms in both of the demand price functions so that the new demand price functions are:

$$\rho_{11}(d, E) = -3d_{11} - d_{21} + 300, \quad \rho_{21}(d, E) = -4.5d_{21} - d_{11} + 300.$$

A solution of the new system of equations, yields the equilibrium path flows

$$x^*_{p_1} = 7.27, \quad x^*_{p_2} = 9.61.$$

with the equilibrium demands being equal to

$$d^*_{11} = 7.27, \quad d^*_{21} = 9.61.$$

The equilibrium link flows are, hence

$$f^*_1 = 7.27, \quad f^*_2 = 7.27, \quad f^*_3 = 7.27, \quad f^*_4 = 7.27,$$

$$f^*_5 = 9.61, \quad f^*_6 = 9.61, \quad f^*_7 = 9.61, \quad f^*_8 = 9.61.$$

Now, the induced equilibrium prices of the two white ladies shirts are as follows:

$$\rho_{11} = 268.57, \quad \rho_{21} = 249.48,$$

with the associated emissions being

$$E_1 = 76.37, \quad E_2 = 78.52.$$

The profits of the firms are

$$U_1 = 1{,}005.00, \quad U_2 = 1{,}339.37.$$

We see from the two examples above, the value of information provided by ecolabeling, which results in lower emissions.

The variant example can also be interpreted, from a policy perspective, as an example in which the ecolabeling cost is actually a carbon tax. Producers would know how much they must pay out for their emissions in such a setting but consumers would be unaware since that information is not revealed to them.

Remark

Ecolabeling is a marketing tool, where the company carries a cost associated with the labeling and hopes to gain bigger returns through increased sales, just like in advertising or any other marketing activity. If successful, the ecolabeling increases the sale of eco-friendly clothes and reduces the environmental impact. The same effect is sought by governments and policymakers all around the world as a part of national and international efforts to reduce CO_2 emissions. However, national policy makers rely mainly on environmental taxes to reach this goal (Sterner and Köhlin 2003), which incurs a cost for the company but no direct effect on the customer. While ecolabeling in a positive way tries to influence the consumer to make an environmentally more informed decision, the tax is a market-based policy instrument that tries to reach the same goal by imposing a cost on the company side. The tax paid by the supply chain is often unknown for the end customer, particularly in an international supply chain where the tax might be paid by a third-tier supplier on the other side of the globe. Thus, from a policy maker's perspective, it is interesting to determine the difference in effect of the two approaches, particularly if the costs for the supply chain (cost of labeling and cost of tax) are at the same level.

4.3 The Algorithm and Case Study

The algorithm that we utilize for the computation of the equilibrium fashion product pattern satisfying variational inequality (4.13) is the Euler method (see, Dupuis and Nagurney 1993), which we have applied to solve several other competitive supply chain network models (cf. Nagurney and Yu 2012; Nagurney and Li 2014b, Nagurney et al. 2013a). For conditions of convergence, please refer to Dupuis and Nagurney (1993) and Nagurney and Zhang (1996).

The nice feature of the algorithm is that, in the context of our new model, the product flows can be determined explicitly, at each iteration, using a simple formula, because of the structure of the feasible set, which is the nonnegative orthant.

Explicit Formulae for the Euler Method Applied to the Fashion Supply Chain Network Variational Inequality (4.13)

At iteration $\tau + 1$, for all the product path flows x_p; $p \in P_k^i$; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$, compute

$$x_p^{\tau+1} = \max\left\{0, x_p^{\tau} + a_{\tau}(\hat{\rho}_{ik}(x^{\tau}) + \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x^{\tau})}{\partial x_p} \sum_{q \in P_l^i} x_q^{\tau} - \frac{\partial \hat{C}_p(x^{\tau})}{\partial x_p} - \frac{\partial \hat{l}_i(x^{\tau})}{\partial x_p})\right\}. \tag{4.22}$$

Once the equilibrium path flows are determined, according to the imposed convergence condition, the incurred link emissions and total emissions associated with each fashion firm and its profits can easily be determined.

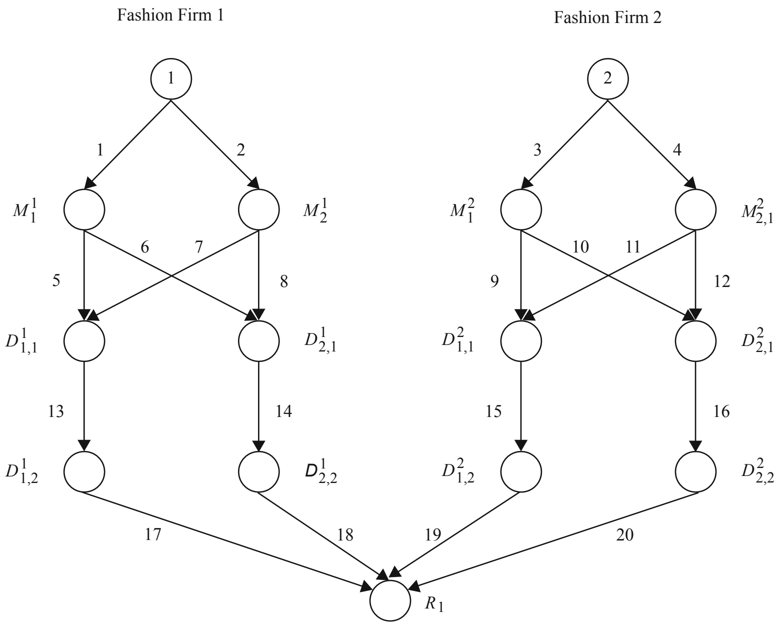

Fig. 4.3 The fashion supply chain network topology for the case study

We present a case study that builds upon our earlier work in sustainable fashion supply chain network competition (cf. Nagurney and Yu 2012). The supply chain network topology for this fashion economy is given in Fig.4.3. There are two fashion firms, Firm 1 and Firm 2, each of which has, at its disposal, two manufacturing plants, two distribution centers, and serves a single demand market R_1. The manufacturing plants M_1^1 and M_1^2 are located in the USA, whereas the manufacturing plants M_2^1 and M_2^2 are located off-shore with lower operational costs. The demand market is in the USA as are the distribution centers.

We implemented the Euler method, as described above, using MATLAB. The convergence tolerance was $\varepsilon = 10^{-6}$ and the sequence $a_\tau = .1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \dots)$. The algorithm was deemed to have converged when the absolute value of the difference between successive path flows differed by no more than ϵ. We initialized the Euler method by setting all the product path flows equal to 10.

Case Study Example 1

This example is inspired by Example 1 in (Nagurney and Yu 2012) but with a modification of the emission functions. Here we also add ecolabeling cost functions and consider more general demand price functions, which reveal the carbon emission information to the consumers through ecolabeling. The total cost and the emission functions for the links are given in Table 4.2, along with the computed equilibrium link flow solution. The product considered can represent a ladies short white nightgown. The carbon emissions are in kilograms.

Table 4.2 Total cost and emission functions with equilibrium link flow solution for case study Example 1

Link a	$\hat{c}_a(f, e_a(f_a))$	$e_a(f_a)$	f_a^*
1	$10f_1^2 + 10f_1$	$0.5f_1$	5.55
2	$f_2^2 + 7f_2$	$0.8f_2$	23.44
3	$10f_3^2 + 7f_3$	f_3	4.94
4	$f_4^2 + 5f_4$	$1.2f_4$	22.68
5	$f_5^2 + 4f_5$	f_5	2.33
6	$f_6^2 + 6f_6$	f_6	3.22
7	$2f_7^2 + 30f_7$	$1.2f_7$	9.63
8	$2f_8^2 + 20f_8$	f_8	13.81
9	$f_9^2 + 3f_9$	f_9	4.94
10	$f_{10}^2 + 4f_{10}$	$2f_{10}$	0.00
11	$1.5f_{11}^2 + 30f_{11}$	$1.5f_{11}$	9.55
12	$1.5f_{12}^2 + 20f_{12}$	f_{12}	13.13
13	$f_{13}^2 + 3f_{13}$	$0.1f_{13}$	11.96
14	$f_{14}^2 + 2f_{14}$	$0.15f_{14}$	17.03
15	$f_{15}^2 + 1.8f_{15}$	$0.3f_{15}$	14.49
16	$f_{16}^2 + 1.5f_{16}$	$0.5f_{16}$	13.13
17	$2f_{17}^2 + f_{17}$	f_{17}	11.96
18	$f_{18}^2 + 4f_{18}$	$0.8f_{18}$	17.03
19	$f_{19}^2 + 5f_{19}$	$1.2f_{19}$	14.49
20	$1.5f_{20}^2 + f_{20}$	$1.2f_{20}$	13.13

The ecolabeling cost functions are:

$$l_1(d_{11}) = .02d_{11}, \quad l_2(d_{21}) = .02d_{21}.$$

The demand price functions are:

$$\rho_{11}(d) = -3d_{11} - .5d_{21} - .5E_1 + .2E_2 + 450,$$

$$\rho_{21}(d) = -3d_{21} - .5d_{11} - .5E_2 + .2E_1 + 450.$$

We also provide the computed equilibrium path flows. There are four paths for each firm labeled as follows (cf. Fig. 4.3): for Fashion Firm 1

$$p_1 = (1, 5, 13, 17), \quad p_2 = (1, 6, 14, 18), \quad p_3 = (2, 7, 13, 17), \quad p_4 = (2, 8, 14, 18);$$

and for Fashion Firm 2

$$p_5 = (3, 9, 15, 19), \quad p_6 = (3, 10, 16, 20), \quad p_7 = (4, 11, 15, 19), \quad p_8 = (4, 12, 16, 20).$$

The computed equilibrium path flow pattern is:

$$x^*_{p_1} = 2.33, \quad x^*_{p_2} = 3.22, \quad x^*_{p_3} = 9.63, \quad x^*_{p_4} = 13.81,$$

$$x^*_{p_5} = 4.94, \quad x^*_{p_6} = 0.00, \quad x^*_{p_7} = 9.55, \quad x^*_{p_8} = 13.13.$$

The demand for Firm 1's fashion product is 28.99 and the price is 330.06, whereas the demand for Firm 2's fashion product is 27.62 and the price is 314.68.

Firm 1 generates 81.77 kg of carbon emissions and its profit is 6,155.01. Firm 2 generates 108.62 kg in carbon emissions and has a profit of 5,818.99.

Note that demand for Firm 1's fashion product is higher than that for Firm 2's product; while the price of Firm 1's product is also notably higher than that of Firm 2's product. Due to the effort of controlling its carbon emissions, Firm 1's product becomes more appealing in the demand market. It is interesting to observe that the shipment quantity between Firm 2's domestic manufacturing plant M_1^2 and its distribution center D_2^2 is zero, mainly because this transportation activity can cause serious pollution to the environment.

Case Study Example 2

Case Study Example 2 has the same data as Case Study Example 1 except that the consumers are more sensitive with respect to the carbon emissions generated by the fashion firms. The new demand price functions are given by

$$\rho_{11}(d) = -3d_{11} - .5d_{21} - E_1 + .2E_2 + 450,$$

$$\rho_{21}(d) = -3d_{21} - .5d_{11} - E_2 + .2E_1 + 450.$$

The new equilibrium path flow pattern is

$$x^*_{p_1} = 2.32, \quad x^*_{p_2} = 2.62, \quad x^*_{p_3} = 7.45, \quad x^*_{p_4} = 11.81,$$

$$x^*_{p_5} = 4.36, \quad x^*_{p_6} = 0.00, \quad x^*_{p_7} = 6.81, \quad x^*_{p_8} = 10.75.$$

The demand for the Firm 1's fashion product is 24.20 and the price is 315.59, whereas, the demand for Firm 2's fashion product is 21.92 and the price is 299.93.

Firm 1 generates 68.02 kg of carbon emissions and its profit is 5121.86. Firm 2 generates 85.80 kg in carbon emissions and has a profit of 4622.30.

The consumers' increasing environmental concerns lead to the decreas in the demands for the fashion products, as well as the prices of both products. Consequently, the profits of both firms drop dramatically, while the emissions generated by both firms reduce significantly.

Consumers' environmental consciousness has been an imperative motivation for Firm 2 to acquire and implement emission-reducing technologies. Firm 2 is now considering two options.

Table 4.3 Computed equilibrium demands, prices, profits, and total emissions for Examples 1, 2, 3, and 4

		Example 1	Example 2	Example 3	Example 4
Demands	Firm 1	28.99	24.20	24.17	24.13
	Firm 2	27.62	21.92	22.11	22.62
Prices	Firm 1	330.06	315.59	315.29	314.77
	Firm 2	314.68	299.93	301.15	302.31
Profits	Firm 1	6155.01	5121.86	5110.89	5091.95
	Firm 2	5818.99	4622.30	4658.51	4746.40
Emissions	Firm 1	81.77	68.02	67.94	67.82
	Firm 2	108.62	85.80	84.01	81.35

Case Study Example 3

Case Study Example 3 has identical data as in Case Study Example 2 except that Firm 2 now upgrades the manufacturing technologies at its domestic manufacturing plant M_1^2, resulting in new total cost and emission functions associated with the manufacturing link 3 as given below:

$$\hat{c}_3(f, e_3(f_3)) = 10f_3^2 + 10f_3, \quad e_3(f_3) = .5f_3.$$

Case Study Example 4

Case Study Example 4 has the same data as Case Study Example 2 except that Firm 2 implements advanced emission-reducing manufacturing technologies at its off-shore manufacturing plant M_2^2. The total cost and emission functions associated with the manufacturing link 4 are given by,

$$\hat{c}_4(f, e_4(f_4)) = f_4^2 + 7f_4, \quad e_4(f_4) = .8f_4.$$

The computed equilibrium demands, prices, profits, emissions, and utilities for Examples 1, 2, 3, and 4 are reported in Table 4.3.

Undoubtedly, the implementation of the advanced emission-reducing technologies could support Firm 2 to regain its competitive advantage. A comparison of the results in Examples 3 and 4 suggests that Firm 2 should first focus on its off-shore manufacturing plant, which will be more profitable.

4.4 Summary and Conclusions

Apparel and accessories are among the consumer products that are most frequently purchased as well as replaced. The globalization of these supply chains and their notable carbon emissions, ranked fifth among sectors in different countries, provide both challenges as well as opportunities for actions toward sustainability.

In this chapter, we develop a rigorous, computable fashion supply chain network model that captures such notable features as competition, brand differentiation, and ecolabeling. The ecolabeling has associated costs but provides valuable emission information to the consumers. We describe the competitive behavior of the fashion firms, along with their objective functions, and the constraints, define the governing equilibrium concept and derive alternative variational inequality formulations. We also provide qualitative results for the equilibrium pattern. We present an illustrative numerical example and a variant and also detail an algorithm for the computation of the fashion product flows on the supply chain network(s). The algorithm is easy to implement and, at each iteration, consists of explicit formulae for the determination of the path flows. We utilize the algorithm in a case study to solve larger numerical fashion supply chain network examples with ecolabeling.

The contributions in this chapter add to the growing literature on sustainable fashion supply chains, in particular, and to sustainable supply chains, in general. Importantly, the fashion supply chain network model with ecolabeling allows for the investigation of the impacts of ecolabeling on firms' and consumers' behavior and responses to such a policy. It also enables individual firms to assess investments in enhanced technologies that would reduce the emissions generated, the use of alternative modes of transportation, and even to assess the impacts of relocation of their manufacturing plants and distribution centers. Finally, a special case of our model captures carbon taxes.

Future research may entail investigating the trade-offs associated with ecolabeling vs. carbon taxes, among other environmental policy instruments in the fashion industry. Also, it would be interesting to evaluate the impacts of government encumbering some or all of the costs associated with ecolabeling. In addition, our model can be extended for fashion supply chain network design problems with the inclusion of different local environmental policies. Finally, it would be very interesting to conduct life cycle assessments of the fashion industry, for fast fashion and for luxury brands, in order to capture the impacts on the environment of consumers after they have purchased the fashion products.

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Appendix

Proof **Proof of Theorem 1**

Variational inequality (4.12) follows directly from Gabay and Moulin (1980); see also Dafermos and Nagurney (1987). We now observe that,

$$\nabla_{X_i}\hat{U}_i(X) = \left[\frac{\partial \hat{U}_i}{\partial x_p}; p \in P^i_k; k = 1, \ldots, n_R\right], \tag{A.1}$$

where for each path p; $p \in P_k^i$,

$$
\begin{aligned}
\frac{\partial \hat{U}_i}{\partial x_p} =& \frac{\partial \left[\sum_{j=1}^{n_R} \rho_{ij}(d,E)d_{ij} - \sum_{b\in L^i} \hat{c}_b(f, e_b(f_b)) - l_i(\sum_{j=1}^{n_R} d_{ij})\right]}{\partial x_p} \\
=& \sum_{j=1}^{n_R} \frac{\partial \left[\rho_{ij}(d,E)d_{ij}\right]}{\partial x_p} - \sum_{b\in L^i} \frac{\partial \hat{c}_b(f, e_b(f_b))}{\partial x_p} - \frac{\partial l_i(\sum_{j=1}^{n_R} d_{ij})}{\partial x_p} \\
=& \sum_{j=1}^{n_R} \sum_{l=1}^{n_R} \frac{\partial \left[\rho_{ij}(d,E)d_{ij}\right]}{\partial d_{il}} \frac{\partial d_{il}}{\partial x_p} + \sum_{j=1}^{n_R} \frac{\partial \left[\rho_{ij}(d,E)d_{ij}\right]}{\partial E_i} \frac{\partial E_i}{\partial x_p} \\
& - \sum_{a\in L^i} \sum_{b\in L^i} \frac{\partial \hat{c}_b(f, e_b(f_b))}{\partial f_a} \frac{\partial f_a}{\partial x_p} - \sum_{l=1}^{n_R} \frac{\partial l_i(\sum_{j=1}^{n_R} d_{ij})}{\partial d_{il}} \frac{d_{il}}{\partial x_p} \\
=& \sum_{j=1}^{n_R} \frac{\partial \left[\rho_{ij}(d,E)d_{ij}\right]}{\partial d_{ik}} + \sum_{j=1}^{n_R} \frac{\partial \left[\rho_{ij}(d,E)d_{ij}\right]}{\partial E_i} \frac{\partial \left[\sum_{a\in L^i} e_a(f_a)\right]}{\partial x_p} \\
& - \sum_{a\in L^i} \sum_{b\in L^i} \frac{\partial \hat{c}_b(f, e_b(f_b))}{\partial f_a} \delta_{ap} - \frac{\partial l_i(\sum_{j=1}^{n_R} d_{ij})}{\partial d_{ik}} \\
=& \rho_{ik}(d,E) + \sum_{j=1}^{n_R} \frac{\partial \rho_{ij}(d,E)}{\partial d_{ik}} d_{ij} + \sum_{j=1}^{n_R} \frac{\partial \rho_{ij}(d,E)}{\partial E_i} d_{ij} \sum_{a\in L^i} \frac{\partial e_a(f_a)}{\partial f_a} \frac{\partial f_a}{\partial x_p} \\
& - \sum_{a\in L^i} \sum_{b\in L^i} \frac{\partial \hat{c}_b(f, e_b(f_b))}{\partial f_a} \delta_{ap} - \frac{\partial l_i(\sum_{j=1}^{n_R} d_{ij})}{\partial d_{ik}} \\
=& \rho_{ik}(d,E) + \sum_{j=1}^{n_R} \left[\frac{\partial \rho_{ij}(d,E)}{\partial d_{ik}} + \frac{\partial \rho_{ij}(d,E)}{\partial E_i} \sum_{a\in L^i} \frac{\partial e_a(f_a)}{\partial f_a} \delta_{ap} \right] d_{ij} \\
& - \sum_{a\in L^i} \sum_{b\in L^i} \frac{\partial \hat{c}_b(f, e_b(f_b))}{\partial f_a} \delta_{ap} - \frac{\partial l_i(\sum_{j=1}^{n_R} d_{ij})}{\partial d_{ik}}.
\end{aligned}
\tag{A.2}
$$

By using the conservation of flow equation (4.1) and the definitions in (4.14), (4.15), and (4.16), variational inequality (4.13) is immediate. In addition, the equivalence between variational inequalities (4.13) and (4.17) can be proved with (4.1) and (4.3). □

We now provide some qualitative properties of the equilibrium solution. Since the feasible set K^1 is not compact, we cannot obtain the existence of a solution simply based on the assumption of the continuity of F. However, the demand d_{ik} for each fashion firm i's product, $i = 1, \ldots, I$ at every demand market R_k; $k = 1, \ldots, n_R$, may be assumed to be bounded by the market size. Consequently, in light of (4.1), we have,

$$\mathcal{K}_b \equiv \{x \,|\, 0 \le x \le b,\}, \tag{A.3}$$

where $b > 0$ and $x \leq b$ means that $x_p \leq b$ for all $p \in P_k^i$; $i = 1, \ldots, I$ and $k = 1, \ldots, n_R$. Then $\mathcal{K}_b$ is a bounded, closed, and convex subset of K^1. Thus, the following variational inequality

$$\langle F(X^b), X - X^b \rangle \geq 0, \quad \forall X \in \mathcal{K}_b, \tag{A.4}$$

admits at least one solution that $X^b \in \mathcal{K}_b$, since $\mathcal{K}_b$ is compact and F is continuous. Therefore, following Kinderlehrer and Stampacchia (1980; see also Nagurney 1999), we have Theorem 2.

Theorem 2 ***Existence***
There exists at least one solution to variational inequality (4.13) (equivalently, (4.17)), since there exists a $b > 0$, *such that variational inequality (A.4) admits a solution in* $\mathcal{K}_b$ *with*

$$x^b \leq b. \tag{A.5}$$

Furthermore, we study the uniqueness of the equilibrium solution in Theorem 3.

Theorem 3 ***Uniqueness***
With Theorem 2, variational inequality (A.4) and, hence, variational inequality (4.17) admits at least one solution. Moreover, if the function $F(X)$ *of variational inequality (4.17), as defined in (4.20) and (4.21), is strictly monotone on* $\mathcal{K} \equiv K^2$, *that is,*

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K},\ X^1 \neq X^2, \tag{A.6}$$

then the solution to variational inequality (4.17) is unique, that is, the equilibrium link flow pattern and the equilibrium demand pattern are unique.

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