Improved Robust Kalman Filtering for Uncertain Systems with Missing Measurements

Hossein Rezaei[1], Shady Mohamed[2], Reza Mahboobi Esfanjani[1], and Saeid Nahavandi[2]

[1] Electrical Engineering Department, Sahand University of Technology, Tabriz, Iran
{h_rezaei,mahboobi}@ sut.ac.ir
[2] Centre for Integrative Systems Research, Deakin University, Waurn Ponds VIC 3216, Australia
{shady.mohamed,saeid.nahavandi}@deakin.edu.au

Abstract. In this paper, a novel robust finite-horizon Kalman filter is developed for discrete linear time-varying systems with missing measurements and norm-bounded parameter uncertainties. The missing measurements are modelled by a Bernoulli distributed sequence and the system parameter uncertainties are in the state and output matrices. A two stage recursive structure is considered for the Kalman filter and its parameters are determined guaranteeing that the covariances of the state estimation errorsare not more than the known upper bound. Finally, simulation results are presented to illustrate the outperformance of the proposed robust estimator compared with the previous results in the literature.

Keywords: robust Kalman filter, miss measurement, state estimation, norm-bounded parameter uncertainties.

1 Introduction

The Kalman filter which is based on the minimization of the filtering error covariance is the popular tool for the state estimation through the noisy observations. The key assumptions in the standard Kalman filtering are that the perfect model of the underlying system is priory known and all the measurements are available[1]. However, in the many real-world applications, for instance in the networked control systems, unreliability of the communication channels together with modeling uncertainties imposes significant challenges in the optimal state estimation [2-4].

The initial work on the filter design problem with missing measurements can be traced back to [5], and [6], where a binary sequence specified by a probability distribution were utilized todescribe the missing data. On the other hand, robust Kalman filter with a guaranteed bound on the filtering error covariancefor systems with time-varying norm-bounded uncertainties in the state and output matrices were proposed in [7,8] and [9], for discreet and continues time systems; respectively.

Only a few papers have considered the common case wherein the problem of missing observations is combined with the norm-bounded modeling uncertainties. The infinite-horizon optimal filter was derived in [10], for discrete-time systems with

C.K. Loo et al. (Eds.): ICONIP 2014, Part III, LNCS 8836, pp. 509–518, 2014.

stochastic missing measurements and parameter uncertainties. However, finite-horizon filters leads to better transient performance if the noise inputs are notstationary. Then, for linear discrete time-varying systems with time-varying norm-bounded uncertainty in the state matrix and missing measurements a robust finite-horizon Kalman filter was introduced in [11_ENREF_15]. In[12], robust finite- horizon Kalman filter was developed for the more comprehensive system withnorm-bounded uncertainty in the state and output matricessuffering from missing measurements. In [13], within the different framework, robust state estimator was suggested for the systems with missing measurements based on minimizing the sensitivity of the estimation errors to the parameter variations.

In this paper, robust finite-horizon filtering problem is derived for uncertain time-varying linear system with intermittent measurements. The state and output matrices of the system model are subject to norm bounded uncertainty and missing data are described by Bernoulli distributed random sequence. Unlike [11] and [12], a two stage recursive structure is adopted for the robust Kalman filter and furthermore, a different augmented state space model is utilized to extract a procedure to determine filter parameters. Finally, simulation results are presented to illustrate that the introduced estimator leads to the remarkably improved performance compared to the previously developed approach in [12].

The rest of the paper is organized as follows: The estimation problem is formulated in the section II. In section III, the optimal estimator is derived for the uncertain system with missing observations. In section IV, numerical benchmark examples are presented to illustrate the outperformance of the proposed approach. Section V concludes this note.

Notations: $\Re$ denotes real numbers set. $\text{Prob}\{\}$ represents the probability of the stochastic variable. $\text{E}\{\}$ is the mathematical expectation. The superscript ^{T} stands for the matrix transposition.

2 Problem Setup

Consider the following class of the uncertain linear discrete-time stochastic systems:

$$x(t+1) = (A_t + \Delta A_t)x(t) + B_t w(t) \tag{1}$$

with the measurement equation

$$y(t) = \gamma_t (C_t + \Delta C_t)x(t) + v(t) \tag{2}$$

where, $x(t) \in \Re^n$ is the state vector, $y(t) \in \Re^m$ is the measured output, $w(t) \in \Re^n$ and $v(t) \in \Re^m$ are the process and measurement noise, respectively. It's assumed that $w(t)$ and $v(t)$ are uncorrelated white noises with zero means and variances Q and R. A_t, B_t and C_t are known real time-varying matrices with appropriate dimensions. ΔA_t is a real-valued uncertain matrix satisfying:

$$\begin{bmatrix} \Delta A_t \\ \Delta C_t \end{bmatrix} = \begin{bmatrix} H_{1,t} \\ H_{2,t} \end{bmatrix} F_t E_t, \quad F_t F_t^T \le I$$

Here, $H_{1,t}, H_{2,t}$ and E_t are known time-varying matrices of appropriate dimensions and F_t represents time-varying uncertainties. The output sequence is defined in (2). Some measurement data may be lost. The stochastic variable γ_t which takes the values 0 and 1, is a Bernoulli distributed variable with mean μ. It is assumed that γ_t is uncorrelatedwith $w(t)$, $v(t)$ and initial state x_0. From the properties of the Bernoulli distribution, thefollowingrelationshold:

$\text{Prob}\{\gamma_t = 1\} = E\{\gamma_t\} = \mu$, $\text{Prob}\{\gamma_t = 0\} = 1 - E\{\gamma_t\} = 1 - \mu$

$E\{(\gamma_t - \mu)^2\} = (1-\mu)\mu$. Also, it is assumed that: $E\{F_t\} = 0$ and $E\{F_t F_j\} = I\delta_{tj}$.

The aim of this note is to design finite-horizon robust Kalman filter for discrete-time systems with parameters uncertainty and missing observations. The structure of the proposed robust Kalman filter is given in (3) and (4). The estimation of the state is computed by the following recursive equations:

$$\hat{x}(t|t) = \hat{x}(t|t-1) + K_t(y(t) - \mu C_t \hat{x}(t|t-1)) \tag{3}$$

$$\hat{x}(t+1|t) = \hat{A}_t(t)\,\hat{x}(t|t-1) + L_t(y(t) - \mu C_t \hat{x}(t|t-1)) \tag{4}$$

where $\hat{x}(t)$ is the estimate of the state $x(t)$, and $\hat{A}_t, L_t$ and K_t are time-varying filter parameters are determined such that filtering error $e(t) = x(t) - \hat{x}(t|t)$, and prediction error $\tilde{e}(t) = x(t) - \hat{x}(t|t-1)$ variances be smaller than positive-definite matrices $\bar{\Theta}(t)$ and $\bar{\Sigma}(t), (0 < t \le N)$, respectively:

$$\mathrm{E}\{(x(t) - \hat{x}(t|t))(x(t) - \hat{x}(t|t))^T\} \le \bar{\Theta}(t) \tag{5}$$

$$\mathrm{E}\{(x(t) - \hat{x}(t|t-1))(x(t) - \hat{x}(t|t-1))^T\} \le \bar{\Sigma}(t) \tag{6}$$

3 Filer Design

In this section a procedure is developed to obtain the parameters of the two stage Kalman filter defined in (3) and (4). First, the upper bounds of the filtering and prediction covariance matrices presented in (5) and (6)are determined in the form of discrete time Riccarti-like difference equation.

3.1 Preliminaries

In this subsection, some preliminaries are introduced which will be used in derivation of the main results. First, new augmented state vectors and are defined as follows:

$$\zeta(t) = \begin{bmatrix} e(t) \\ \hat{x}(t|t) \end{bmatrix}, \tilde{\zeta}(t) = \begin{bmatrix} \tilde{e}(t) \\ \hat{x}(t|t-1) \end{bmatrix} \tag{7}$$

Then, by combination of (1)-(4), the augmented system equations can be written as follows:

$$\zeta(t+1) = (A_{c1} + H_{c1}F_k E_{c1})\tilde{\zeta}(t) + A_{e1}\tilde{\zeta}(t) + \tilde{A}_{e1}\tilde{\zeta}(t) + G_{v1}v(t) \tag{8}$$

$$\tilde{\zeta}(t+1) = (A_{c2} + H_{c2}F_k E_{c2})\tilde{\zeta}(t) + A_{e2}\tilde{\zeta}(t) + \tilde{A}_{e2}\tilde{\zeta}(t) + G_{v2}v(t) + G_{w2}w(t) \tag{9}$$

where

$$A_{c1} = \begin{bmatrix} I-\mu K_t C_t & 0 \\ \mu K_t C_t & \mathrm{I} \end{bmatrix}, \mathrm{A}_{c2} = \begin{bmatrix} A_t - \mu L_t C_t & \mathrm{A}_t\text{-}\hat{\mathrm{A}}_t \\ \mu L_t C_t & \hat{\mathrm{A}}_t \end{bmatrix}, G_{v1} = \begin{bmatrix} -K_t \\ K_t \end{bmatrix}, \mathrm{G}_{v2} = \begin{bmatrix} -L_t \\ L_t \end{bmatrix}$$

$$A_{e1} = \begin{bmatrix} -K_t\tilde{C}_t & -K_t\tilde{C}_t \\ K_t\tilde{C}_t & K_t\tilde{C}_t \end{bmatrix}, \mathrm{E}_{c1} = E_{c2} = \begin{bmatrix} E_t & \mathrm{E}_t \end{bmatrix}, \mathrm{G}_{w2} = \begin{bmatrix} B_t \\ 0 \end{bmatrix}$$

$$\mathrm{A}_{e2} = \begin{bmatrix} -L_t\tilde{C}_t & -L_t\tilde{C}_t \\ L_t\tilde{C}_t & L_t\tilde{C}_t \end{bmatrix}, \mathrm{H}_{c1} = \begin{bmatrix} -\mu K_t H_{2,t} \\ \mu K_t H_{2,t} \end{bmatrix}, H_{c2} = \begin{bmatrix} H_{1,t} - \mu K_t H_{2,t} \\ \mu K_t H_{2,t} \end{bmatrix}$$

$$\tilde{A}_{e1} = \begin{bmatrix} -\eta K_t \Delta C_t & -\eta K_t \Delta C_t \\ \eta K_t \Delta C & \eta K_t \Delta C \end{bmatrix}, \tilde{A}_{e2} = \begin{bmatrix} -\eta L_t \Delta C_t & -\eta L_t \Delta C_t \\ \eta L_t \Delta C & \eta L_t \Delta C \end{bmatrix}$$ In

which $\mathrm{A}_{e1}, \tilde{\mathrm{A}}_{e1}, \mathrm{A}_{e2}$ and $\tilde{\mathrm{A}}_{e2}$ are stochastic matrix sequences with the zero mean and $\eta = (\gamma_t - \mu), \tilde{C} = \eta C_t$. The covariance matrices of the augmented state vector in (8) and (9) are represented as:

$$\tilde{\Theta}(t+1) = E\left\{\zeta(t)\zeta^T(t)\right\} \tag{10}$$

$$\tilde{\Sigma}(t+1) = E\left\{\tilde{\zeta}(t)\tilde{\zeta}^T(t)\right\} \tag{11}$$

According to the equations (8) and (10) the Lyapunov equations for the filtering covariance matrix can be obtained as the following:

$$\tilde{\Theta}(t+1) = (A_{c1} + H_{c1}F_t E_{c1})\tilde{\Sigma}(t)(A_{c1} + H_{c1}F_t E_{c1})^T + G_{v1}RG_{v1}^T + \psi_t + \tilde{\psi}_t \tag{12}$$

Similarly, regarding to the equations (9) and (11) the Lyapunov equations for the prediction covariance matrix can be attained as follows:

$$\tilde{\Sigma}(t+1) = (A_{c2} + H_{c2}F_t E_{c2})\tilde{\Sigma}(t)(A_{c2} + H_{c2}F_t E_{c2})^T + G_{v2}RG_{v1}^T + \varphi_t + \tilde{\varphi}_t + G_{w2}QG_{w2}^T \tag{13}$$

where:

$$\psi_t = E\left\{A_{e1}\tilde{\Sigma}(t)A_{e1}^T\right\} = \delta \begin{bmatrix} -K_t C_t & -K_t C_t \\ K_t C_t & K_t C_t \end{bmatrix} \tilde{\Sigma}(t) \begin{bmatrix} -K_t C_t & -K_t C_t \\ K_t C_t & K_t C_t \end{bmatrix}^T$$

$$\tilde{\psi}_t = E\left\{\tilde{A}_{e1}\tilde{\Sigma}(t)\tilde{A}_{e1}^T\right\} = \delta \begin{bmatrix} -K_t H_{2,t} E_t & \text{-}K_t H_{2,t} E_t \\ K_t H_{2,t} E_t & K_t H_{2,t} E_t \end{bmatrix} \tilde{\Sigma}(t) \begin{bmatrix} -K_t H_{2,t} E_t & \text{-}K_t H_{2,t} E_t \\ K_t H_{2,t} E_t & K_t H_{2,t} E_t \end{bmatrix}^T$$

$$\varphi_t = E\left\{A_{e2}\tilde{\Sigma}(t)A_{e2}^T\right\} = \delta\begin{bmatrix} -L_tC_t & -L_tC_t \\ L_tC_t & L_tC_t \end{bmatrix}\tilde{\Sigma}(t)\begin{bmatrix} -L_tC_t & -L_tC_t \\ L_tC_t & L_tC_t \end{bmatrix}^T$$

$$\tilde{\varphi}_t = E\left\{\tilde{A}_{e2}\tilde{\Sigma}(t)\tilde{A}_{e2}^T\right\} = \delta\begin{bmatrix} -L_tH_{2,t}E_t & -L_tH_{2,t}E_t \\ L_tH_{2,t}E_t & L_tH_{2,t}E_t \end{bmatrix}\tilde{\Sigma}(t)\begin{bmatrix} -L_tH_{2,t}E_t & -L_tH_{2,t}E_t \\ L_tH_{2,t}E_t & L_tH_{2,t}E_t \end{bmatrix}^T$$

where $\delta = \mu(1-\mu)$. The following theorem which introduces two RDEs is obtained for equations (12) and (13).

Theorem 1: If there exist positive scalar a_t such that $a_t^{-1}I - E_{c2}\Sigma(t)E_{c2}^T > 0$, where $\Sigma(t)$ is symmetric positive-definite matrix, then

$$\begin{aligned}\Theta(t+1) = {} & A_{c1}\Sigma(t)A_{c1}^T + A_{c1}\Sigma(t)E_{c1}^T(a^{-1}I - E_{c1}\Sigma(t)E_{c1}^T)^{-1} \\ & \times E_{c1}\Sigma(t)A_{c1}^T + \psi_t + \tilde{\psi}_t + G_{v1}RG_{v1}^T + a^{-1}H_{c1}H_{c1}^T\end{aligned} \tag{14}$$

$$\begin{aligned}\Sigma(t+1) = {} & A_{c2}\Sigma(t)A_{c2}^T + A_{c2}\Sigma(t)E_{c2}^T(a^{-1}I - E_{c2}\Sigma(t)E_{c2}^T)^{-1} \\ & \times E_{c2}\Sigma(t)A_{c2}^T + \varphi_t + \tilde{\varphi}_t + a^{-1}H_{c2}H_{c2}^T + G_{v2}RG_{v2}^T + G_{w2}QG_{w2}^T\end{aligned} \tag{15}$$

and $\tilde{\Theta}(t) \le \Theta(t)$ and $\tilde{\Sigma}(t) \le \Sigma(t)$,where $\tilde{\Theta}(t)$ and $\tilde{\Sigma}(t)$ satisfy (12) and (13), respectively.

Proof: The proof can be done along the lines of [2] and [11].

Briefly, the upper bounds of the prediction and filtering covariance matrices are written as follows:

$$E\left\{\zeta(t)\zeta^T(t)\right\} \le [I\ \ 0]\Theta(t)[I\ \ 0]^T = \bar{\Theta}(t) \tag{16}$$

$$E\left\{\tilde{\zeta}(t)\tilde{\zeta}^T(t)\right\} \le [I\ \ 0]\Sigma(t)[I\ \ 0]^T = \bar{\Sigma}(t) \tag{17}$$

3.2 Design of Robust Kalman Filter Parameters

In this subsection,the upper bounds of the filtering and prediction covariances are computed in the form of Riccati-type equation. Then, the optimal values of the proposed Kalman filter parameters in (3) and (4), $\hat{A}_t$, L_t and K_t, are determined such that minimize $tr\left(\bar{\Sigma}(t)\right)$ and $tr(\bar{\Theta}(t))$.

Theorem 2: Suppose a_t be a sequence of positive scalars. Let $\bar{\Sigma}(t)$ and $P(t)$ are the positive-definite solutions of the following recursive equations**:**

$$\begin{aligned}\bar{\Sigma}(t+1) = {} & B_tQB_t^T + a_t^{-1}H_{1,t}H_{1,t}{}^T + A_t\bar{\Sigma}(t)\left(I + E_t^TM_t^{-1}E_t\bar{\Sigma}(t)\right)A_t^T \\ & -(\mu A_t\bar{\Sigma}(t)(I + E_t^TM_t^{-1}E_t\Sigma(t))C_t^T + a_t^{-1}\mu H_{1,t}H_{2,t}^T)\Lambda_t^{-1} \\ & \times(\mu A_t\bar{\Sigma}(t)(I + E_t^TM_t^{-1}E_t\Sigma(t))C_t^T + a_t^{-1}\mu H_{1,t}H_{2,t}^T)^T\end{aligned} \tag{18}$$

$$\Lambda_t = \mu^2 C_t \bar{\Sigma}(t)\left(I + E_t^T M_t^{-1} E_t \bar{\Sigma}(t)\right) C_t^T + (1-\mu)\mu H_{2,t} E_t P(t) E_t^T H_{2,t}^T)$$
$$+ (1-\mu)\mu C_t P_t C_t^T + R + a_t^{-1}\mu^2 H_{2,t}^T H_{2,t}^T$$
$$P(t+1) = A_t \left(P^{-1}(t) - a_t E_t^T E_t\right)^{-1} A_t^T + a_t H_{1,t} H_{1,t}^T + B_t Q B_t^T \qquad (19)$$

wherein $M_t = a_t^{-1} I - E_t \bar{\Sigma}(t) E_t^T > 0$ and $P^{-1}(t) - a_t E_t^T E_t > 0$. The Kalman filter parameters in (3) and (4) are as follows:

$$\hat{A}_t = A_t + a_t (A_t - \mu L_t C_t) \bar{\Sigma}(t) E_t^T M_t^{-1} E_t \qquad (20)$$
$$L_t = (\mu A_t \bar{\Sigma}(t)(I + E_t^T M_t^{-1} E_t \bar{\Sigma}(t)) C_t^T + a_t^{-1} \mu H_{1,t} H_{2,t}^T) \Lambda_t^{-1} \qquad (21)$$

where

$$K_t = \mu \bar{\Sigma}(t)\left(I + E_t^T \tilde{M}_t^{-1} E_t \bar{\Sigma}(t)\right) C_t^T \Xi_t^{-1} \qquad (22)$$

in which

$$\Xi_t = \mu C_t \left(I + \bar{\Sigma}(t) E_t^T \tilde{M}^{-1} E_t\right) \bar{\Sigma}(t) C_t^T + (1-\mu)\mu H_{2,t} E_t P(t) E_t^T H_{2,t}^T$$
$$+ (1-\mu)\mu C_t P_t C_t^T + R + a_t^{-1}\mu^2 H_{2,t}^T H_{2,t}^T$$
$$\tilde{M}_t = a_t^{-1} I - E_t P(t) E_t^T$$

Proof: Regarding (13) and (15), the $\Sigma(t)$ can be rewritten as follows [8]:

$$\Sigma(t) = \begin{bmatrix} \Sigma_{11}(\mathrm{t}) & \Sigma_{12}(t) \\ \Sigma_{21}(\mathrm{t}) & \Sigma_{22}(t) \end{bmatrix} = \begin{bmatrix} \bar{\Sigma}(t) & 0 \\ 0 & \mathrm{P(t)}-\bar{\Sigma}(\mathrm{t}) \end{bmatrix}$$

wherein $\bar{\Sigma}(t)$ and $P(t)$ are defined in (18) and (19), respectively. In order to determine K_t that minimizes $\bar{\Theta}(t)$, its first variation is computed as follows:

$$\frac{\partial \bar{\Theta}(t+1)}{\partial K_t} = (1\text{-}\mu)\mu(K_t C_t P_t C_t^T + K_t H_{2,t} E_t P(t) E_t^T H_{2,t}^T)$$
$$+ (I - \mu K_t C_t) \bar{\Sigma}(t) E_t^T \tilde{M}_t^{-1} E_t \bar{\Sigma}(t)(-\mu C_t)^T \qquad (23)$$
$$+ K_t R + (I - \mu K_t C_t) \bar{\Sigma}(t)(-\mu C_t)^T = 0$$

Then, the K_t in (22) is achieved by straightforward manipulation of (23).On the other hand, considering the equations (15), (17), we have:

$$\Pi(t+1) = [I \;\; 0] \Sigma(t) [I \;\; 0]^T$$
$$= (A_t - \hat{A}_t)(\bar{\Sigma}(t) - P(t))(A_t - \hat{A}_t)^T + (A_t - \mu L_t C) \bar{\Sigma}(t)(A_t - \mu L_t C_t)^T$$
$$+ (1\text{-}\mu)\mu(L_t C_t P(t)(L_t C_t)^T + L_t H_{2,t} E_t P(t)(L_t H_{2,t} E_t)^T) + L_t R L_t^T \qquad (24)$$
$$+ a_t^{-1}(H_{1,t} - \mu L_t H_{2,t})(H_{1,t} - \mu L_t H_{2,t})^T + B_t Q B_t^T$$
$$+ (A_t P(t) - \mu L_t C_t \bar{\Sigma}(t) + \hat{A}_t (\bar{\Sigma}(t) - P(t))) E_t^T \tilde{M}_t^{-1} E_t$$
$$\times (A_t P(t) - \mu L_t C_t \bar{\Sigma}(t) + \hat{A}_t (\bar{\Sigma}(t) - P(t)))^T$$

To determine the $\hat{A}_t$, the first variation of Π is set to be zero:

$$\frac{\partial \Pi(t+1)}{\partial \hat{A}_t} = (A_t - \hat{A}_t)(\bar{\Sigma}(t) - P(t))(I)$$
$$+ (A_t P(t) - \mu L_t C_t \bar{\Sigma}(t) + \hat{A}_t(\bar{\Sigma}(t) - P(t))) \qquad (25)$$
$$\times E_t^T \tilde{M}_t^{-1} E_t((\bar{\Sigma}(t) - P(t)))^T = 0$$

Rearranging the (25) leads to:

$$\hat{A}_t = (A_t(I + P(t)E_t^T \tilde{M}_t^{-1} E_t) - \mu L_t C_t \bar{\Sigma}(t) E_t^T \tilde{M}_t^{-1} E_t) \times (I - (\bar{\Sigma}(t) - P(t)) E_t^T \tilde{M}_t^{-1} E_t)^{-1} \qquad (26)$$

Adding and subtracting of $\bar{\Sigma}(t)E_t^T \tilde{M}_t^{-1} E_t$ in (26) yields:

$$\hat{A}_t = A_t + (A_t - \mu L_t C_t)\bar{\Sigma}(t) E_t^T \tilde{M}_t^{-1} E_t \times (I - (\bar{\Sigma}(t) - P(t)) E_t^T \tilde{M}_t^{-1} E_t)^{-1} \qquad (27)$$

On the other side, the following relation is true [8]:

$$E_t^T \tilde{M}_t^{-1} E_t = E_t^T M_t^{-1} E_t [I + (\bar{\Sigma}(t) - P(t)) E_t^T M_t^{-1} E_t]^{-1} = [I + E_t^T M_t^{-1} E_t(\bar{\Sigma}(t) - P(t))]^{-1} E_t^T M_t^{-1} E_t \qquad (28)$$

$$I - (\bar{\Sigma}(t) - P(t)) E_t^T \tilde{M}_t^{-1} E = [I + (\bar{\Sigma}(t) - P(t)) E_t^T M_t^{-1} E_t]^{-1} \qquad (29)$$

Combining (27)-(29) the equation (20) is obtained. Substituting (20) into (24), we have:

$$\tilde{\Pi}(t+1) = (A_t - \mu L_t C_t)\bar{\Sigma}(t)(I + E_t^T M_t^{-1} E_t \bar{\Sigma}(t))(A_t - \mu L_t C_t)^T$$
$$+ (1\text{-}\mu)\mu(L_t C_t P(t)(L_t C_t)^T + L_t H_{2,t} E_t P(t)(L_t H_{2,t} E_t)^T) \qquad (30)$$
$$+ a_t^{-1}(H_{1,t} - \mu L_t H_{2,t})(H_{1,t} - \mu L_t H_{2,t})^T + B_t Q B_t^T + L_t R L_t^T$$

The matrix L_t is computed by taking the first variation of $\tilde{\Pi}$ in (30) as follows:

$$\frac{\partial \tilde{\Pi}(k+1)}{\partial L_t} = L_t R_t + a_t^{-1}(H_{1,t} - \mu L_t H_{2,t})(\mu H_{2,t})^T$$
$$+ (A_t - L_t C_t)\bar{\Sigma}(t)(I + E_t^T M_t^{-1} E_t \bar{\Sigma}(t))(-\mu C_t)^T \qquad (31)$$
$$+ (1\text{-}\mu)\mu(L_t C_t P(t) C_t^T + L_t H_t E_t P(t)(H_t E_t)^T) = 0$$

The matrix L_t in (21) is easily derived from (31). Substituting (21) into (30) leads to (18).The covariance matrix of the state is as follows:

$$P(t+1) = E\left\{x(t+1)x^T(t+1)\right\}$$
$$= E\left\{((A_t + \Delta A_t)x(t) + B_t w_t)((A_t + \Delta A_t)x(t) + B_t w_t)^T\right\} \qquad (32)$$
$$= (A_t + H_t F_t E_t) P(t) (A_t + H_t F_t E_t)^T + B_t Q B_t^T$$

Relation (32) can be transformed to (19) [12].

4 Simulation Example

we consider the following uncertain discrete-time systems with missing measurements [12]:

$$x(t+1)=\left(\begin{bmatrix}0 & 0.1\sin(6t)\\ 0.2 & 0.3\end{bmatrix}+\begin{bmatrix}0.5\\ 1\end{bmatrix}F_t\begin{bmatrix}0.2 & 0.1\end{bmatrix}\right)x(t)+\begin{bmatrix}1\\ 0.5\end{bmatrix}w(t)$$

$y(t)=\gamma_t(\begin{bmatrix}0.5+0.3\sin(6t) & 1\end{bmatrix}+4F_t\begin{bmatrix}0.2 & 0.1\end{bmatrix})x(t)+v(t)$ The noise signals $F_t=\sin(0.6t)$

$w(t)$ and $v(t)$ are uncorrelated with zero-mean and unity covariances. The scalar binary stochastic variable γ_t isBernoulli distributed. Figures1 and 2compare the error variances of the results obtained by the proposed method and the one in [12]

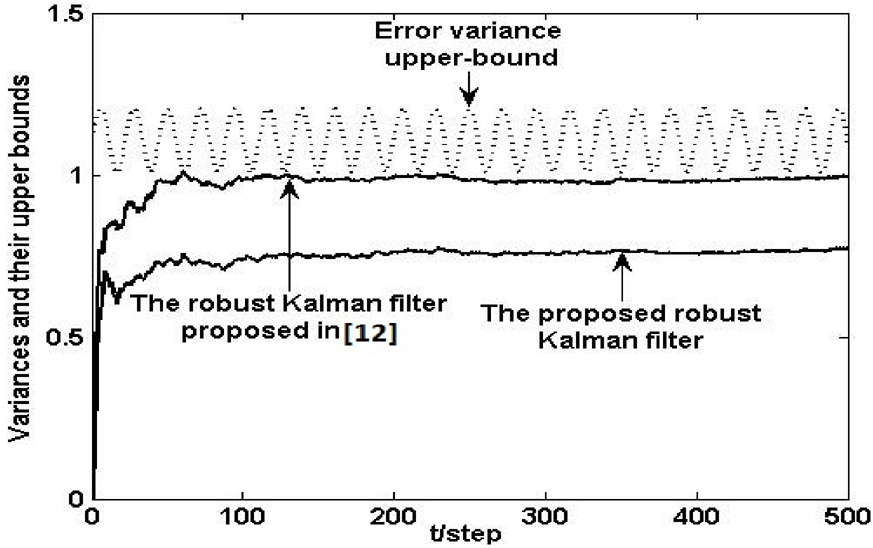

Fig. 1. Comparison of the error variances for the first state

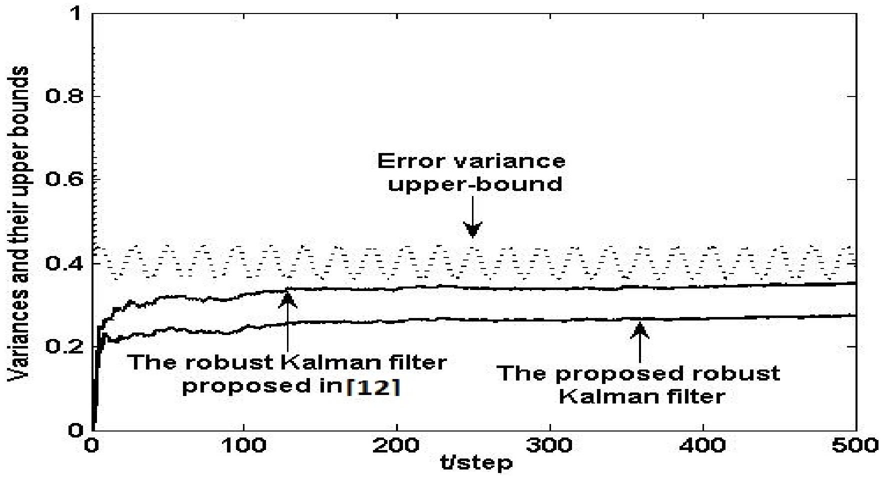

Fig. 2. Comparison of the error variances for the second state

by 100 times Monte-Carlo test, with $\mu = 0.8$ and $a_t = 3$. The outperformance of the introduced procedure is evident.

5 Conclusions

In this paper, a novel approach has been developed to design a finite-horizon robust Kalman filter for uncertain linear discrete time-varying systems subject to intermittentobservations and time-varying norm-bounded uncertainties in the state and output matrices. Filter parameters are determined such that the upper bound on the estimation error covariance matrix be minimal. The illustrative examples verified the advantages of the proposed filter.

References

1. Simon, D.: Optimal state estimation: Kalman, H infinity, and nonlinear approaches. Wiley.com (2006)
2. Yang, F., Wang, Z., Feng, G., Liu, X.: Robust filtering with randomly varying sensor delay: the finite-horizon case. IEEE Transactions on Circuits and Systems I: Regular Papers 56, 664–672 (2009)
3. Lu, X., Xie, L., Zhang, H., Wang, W.: Robust Kalman filtering for discrete-time systems with measurement delay. IEEE Transactions on Circuits and Systems II: Express Briefs 54, 522–526 (2007), Dyer, F.C.: The biology of the dance language. Annual review of Entomology 47(1), 917–949 (2002)
4. Wang, Z., Ho, D.W., Liu, X.: Robust filtering under randomly varying sensor delay with variance constraints. IEEE Transactions on Circuits and Systems II: Express Briefs 51, 320–326 (2004)
5. Nahi, N.E.: Optimal recursive estimation with uncertain observation. IEEE Transactions on Information Theory 15, 457–462 (1969)
6. Hadidi, M., Schwartz, S.: Linear recursive state estimators under uncertain observations. IEEE Transactions on Automatic Control 24, 944–948 (1979)
7. Souto, R.F., Ishihara, J.Y.: Comments on "Finite-Horizon Robust Kalman Filtering for Uncertain Discrete Time-Varying Systems With Uncertain-Covariance White Noises". IEEE Signal Processing Letters 17, 213–216 (2010), Giurfa, M.: Cognitive neuroethology: dissecting non-elemental learning in a honeybee brain. Current Opinion in Neurobiology 13(6), 726–735 (2003)
8. Zhu, X., Soh, Y.C., Xie, L.: *Design and analysis of discrete-time robust Kalman filters*. Automatica 38, 1069–1077 (2002)
9. Shaked, U., De Souza, C.E.: Robust minimum variance filtering. IEEE Transactions on Signal Processing 43, 2474–2483 (1995)
10. Wang, Z., Ho, D.W.C., Xiaohui, L.: Variance-constrained filtering for uncertain stochastic systems with missing measurements. IEEE Transactions on Automatic Control 48, 1254–1258 (2003)
11. Wang, Z., Fuwen, Y., Ho, D.W.C., Xiaohui, L.: Robust finite-horizon filtering for stochastic systems with missing measurements. IEEE Signal Processing Letters 12, 437–440 (2005)

12. Mohamed, S.M., Nahavandi, S.: Robust finite-horizon Kalman filtering for uncertain discrete-time systems. IEEE Transactions on Automatic Control 57, 1548–1552 (2012)
13. Liang, H., Zhou, T.: Robust state estimation for uncertain discrete-time stochastic systems with missing measurements. Automatica 47, 1520–1524 (2011)
14. Chu, J.-U., et al.: Spontaneous synchronized burst firing of subthalamic nucleus neurons in rat brain slices measured on multi-electrode arrays. Neuroscience Research 72(4), 324–340 (2012)