A New Method for Removing Random-Valued Impulse Noise

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Abstract. A new algorithm for removing random-valued impulse noise is proposed. We use a standardized version of the Rank Ordered Absolute Differences statistic of Garnett et al. [1] to attribute weights to noisy pixels. These weights are then incorporated into the Optimal Weights Filter approach from [2,3] to construct a new filter. Simulation results show that our method performs significantly better than a number of existing techniques.

Keywords: random-valued impulse noise, denoising, Optimal Weights Filter, Non-Local Means, Rank Ordered Absolute Difference.

1 Introduction

Random-valued impulse noise can be systematically introduced into digital images during acquisition and transmission [4]. Impulse noise is characterized by replacing a portion of an images pixel values with random values, leaving the remainder unchanged. In most applications, denoising is fundamental to subsequent image processing operations, such as edge detection, image segmentation, object recognition, etc. The goal of denoising is to effectively remove noise from an image while keeping its features intact. To this end, a variety of techniques have been proposed to remove impulse noise.

Recently, an edge-preserving regularization method has been proposed to remove impulse noise [5]. It uses a nonsmooth data-fitting term along with edge-preserving regularization. In order to improve this variational method in removing impulse noise, a two-stage method was proposed in [6] and [7]. It is efficient in dealing with high noise ratio, e.g., ratio as high as 90% for salt-and-pepper impulse noise and 50% for random-valued impulse noise. Its performance is impaired by the inaccuracy of the noise detector in the first phase. In order to find a better noise detector, especially for the random-valued impulse noise, Garnett et al. [1] introduced a new local image statistic called ROAD to identify the impulse noisy pixels. The result is a trilateral filter, which performs well for removing impulse noise. However, when the noise level is high, it blues the

C.K. Loo et al. (Eds.): ICONIP 2014, Part III, LNCS 8836, pp. 9–16, 2014.

images significantly. Dong et al. [8] amplified the differences between noisy pixels and noise-free pixels in ROAD, by introducing a new statistic called ROLD, so that the noise detection becomes more accurate. When the random-valued impulse noise ratio is as high as 60%, they still can remove most of the noise while preserving image details.

The ROAD statistic is efficient but turns out to be sensible to the proportion of the impulse noise, so that it is difficult to determine the parameters of the concerned filters. In this paper, we propose a standardized version of the ROAD statistic, called SROAD, to provide a more stable filter for which the determination of the parameters is simpler. We define new impulsive weights to magnify the difference of SROAD values between noisy pixels and noise free pixels. We then propose an efficient filter that combines the SROAD impulse noise detector with the optimal weights algorithm from [2,3] for removing random-valued impulse noise. Extensive experimental results show that our method performs significantly better than many known techniques.

2 Optimal Weights Filter for Random-Valued Impulse Noise

2.1 Impulse Noise Model

An image containing random-valued impulse noise can be described as follows:

$$y(x) = \begin{cases} u(x), \text{ with probability } 1-p; \\ n(x), \text{ with probability } p, \end{cases} \tag{1}$$

where $u(x)$, $x \in \mathbf{I} = \{1, 2, \cdots, M\} \times \{1, 2, \cdots, N\}$, is the original image, $n(x)$, $x \in \mathbf{I}$, are independent random variables uniformly distributed in $[s_{\min}, s_{\max}]$, $s_{\min}$ and $s_{\max}$ being respectively the lowest and the highest pixel luminance values within the dynamic range, and p denotes the proportion of noisy pixels. The goal is to recover the original image $u(x)$, $x \in \mathbf{I}$, from the observed image $y(x)$, $x \in \mathbf{I}$.

2.2 Standardized Rank Ordered Absolute Differences

The ROAD (Rank Ordered Absolute Differences) statistic introduced by Garnett et al. [1] is known to be efficient in removing impulse noise. However this statistic is too sensitive to the proportion p of noisy points. We find that the operability of the ROAD statistic can be improved by its standardization.

For any pixel $x_0 \in \mathbf{I}$, we define the square window of pixels (whose center is excluded) of size $(2d+1) \times (2d+1)$:

$$\Omega^0_{x_0,d} = \{x : 0 < \|x - x_0\|_\infty \leq d\}, \tag{2}$$

where d is a positive integer and $\|\cdot\|_\infty$ denotes the supremum norm: $\|y\|_\infty = \max\{|y_1|, |y_2|\}$ for $y = (y_1, y_2)$. We define the SROAD statistic by

$$\mathrm{SROAD}(x_0) = \frac{1}{K} \sum_{i=1}^{K} r_i(x_0), x_0 \in \mathbf{I}, \tag{3}$$

where $r_i(x_0)$ is the i-th smallest term in the set $\{|y(x) - y(x_0)| : x \in \Omega^0_{x_0,d}\}$ and $2 \leq K < \text{card}\, \Omega^0_{\mathrm{x_0,d}}$. Without the coefficient it is just the ROAD statistic introduced by Garnett et al. [1]. The factor $1/K$ makes the statistic less sensible to variations of the level of the noise p and to the choice of K.

We then define the impulsive weight as follows:

$$J(x, H) = e^{-\frac{(\mathrm{SROAD}(x)-b)^2_+}{2H^2}}, \tag{4}$$

where b is the hard threshold of SROAD values, H is a parameter and $(\cdot)_+$ is the positive part function: $(y)_+ = \max\{y, 0\}$. The impulsive weights measure the degree of contamination of a given pixel. With these weights we are able to construct a filter which is stable to the variations of the impulse noise levels: the constructed filter can remove most of the noise while preserving image details even when the impulse noise ratio is as high as 60%. Furthermore, we do not need the computationally expensive joint impulsivity weights introduced in Garnett et al. [1].

2.3 Construction of Optimal Weights Impulse Noise Filter

Now, we adapt the Optimal Weights Filter [2,3] to treat random-valued impulse noise. For any pixel $x_0 \in \mathbf{I}$ and a given $h \in \mathbb{N}_+$, the square window of pixels

$$\mathbf{U}_{x_0,h} = \{x \in \mathbf{I} : \|x - x_0\|_\infty \leq h\} \tag{5}$$

will be called *search window* at x_0. The size of the square search window $\mathbf{U}_{x_0,h}$ is the positive integer number $D = (2h+1)^2 = \text{card}\, \mathbf{U}_{x_0,h}$. For any pixel $x \in \mathbf{U}_{x_0,h}$ and a given $\eta \in \mathbb{N}_+$, a second square window of pixels $\mathbf{V}_{x,\eta} = \mathbf{U}_{x,\eta}$ will be called for short a *similarity patch* at x in order to be distinguished from the search window $\mathbf{U}_{x_0,h}$. The size of the similarity patch $\mathbf{V}_{x,\eta}$ is the positive integer $S = (2\eta+1)^2 = \text{card}\, \mathbf{V}_{x,\eta}$. The vector $\mathbf{Y}_{x,\eta} = (y(x))_{x\in\mathbf{V}_{x,\eta}}$ formed by the values of the observed noisy image at pixels in the patch $\mathbf{V}_{x,\eta}$ will be called simply *data patch* at $x \in \mathbf{U}_{x_0,h}$.

Consider the weighted patch distance

$$\|\mathbf{Y}_{x_0,\eta} - \mathbf{Y}_{x,\eta}\|_{J,\kappa} = \sqrt{\frac{\sum\limits_{x'\in\mathbf{V}_{x_0,\eta}} \kappa(x')J(x',H)J(T_x x',H)(y(T_x x') - y(x'))^2}{\sum\limits_{x'\in\mathbf{V}_{x_0,\eta}} \kappa(x')J(x',H)J(T_x x',H)}},$$

where T_x is the translation map defined by $T_x y = y + x - x_0$, and κ is the smoothing kernel defined by

$$\kappa(x) = \sum_{k=\max(1,j)}^{h} \frac{1}{(2k+1)^2} \tag{6}$$

if $\|x-x_0\|_\infty = j$ for some $j \in \{0, 1, \cdots, h\}$ and $x \in \mathbf{U}_{x_0,\eta}$. Introduce the impulse detection distance by

$$\widehat{\rho}_{J,\kappa,x_0}(x) = \left(\|\mathbf{Y}_{x_0,\eta} - \mathbf{Y}_{x,\eta}\|_{J,\kappa} - \mu\right)_+, \tag{7}$$

where μ is parameter which controls the robustness of the estimate.

We define our new filter, called *Optimal Weights Impulse Noise Filter* (OW-INF) by

$$\widetilde{u}_h(x_0) = \frac{\sum\limits_{x \in \mathbf{U}_{x_0,h}} J(x, H_2) K_{\mathrm{tr}}\left(\frac{\widehat{\rho}_{J,\kappa,x_0}(x)}{\widehat{a}_J}\right) y(x)}{\sum\limits_{x \in \mathbf{U}_{x_0,h}} J(x, H_2) K_{\mathrm{tr}}\left(\frac{\widehat{\rho}_{J,\kappa,x_0}(x)}{\widehat{a}_J}\right)}, \tag{8}$$

where the bandwidth $\widehat{a}_J > 0$ can be calculated as in Remark 1 of [2] (cf. the algorithm below), H_2 is a parameter and K_{tr} is the triangular kernel:

$$K_{\mathrm{tr}}(t) = (1 - t)_+.$$

Notice that H and H_2 may take different values. The weights defined by the triangular kernel appears as optimal in [2,3].

To give some insights on the filter (8), note that the function $J(x, H_2)$ acts as a filter on the points contaminated by the impulse noise. In fact, if x is an impulse noisy point, then $J(x, H_2) \approx 0$. So, in the new filter, the basic idea is to apply the Optimal Weights Filter [2] by giving nearly 0 weights to impulse noisy points. The computational algorithm is as follows.

- **Algorithm :** Optimal Weights Impulse Noise Filter
- **Step 1**
  For each $x \in \mathbf{I}$ compute:
  $\mathrm{ROADG}(x) = \frac{1}{K}\sum\limits_{i=1}^{K} r_i(x)$
  $J(x, H) = \exp\left(-\frac{(\mathrm{ROADG}(x)-b)_+^2}{H^2}\right)$
  $J(x, H_2) = \exp\left(-\frac{(\mathrm{ROADG}(x)-b)_+^2}{H_2^2}\right)$
- **Step 2** Repeat for each $x_0 \in \mathbf{I}$
  if $\mathrm{ROADG}(x) = 0$
  $\widetilde{u}_h(x_0) = u(x_0)$.
  else
  a) compute $\{\widehat{\rho}_{J,\kappa,x_0}(x) : x \in \mathbf{U}_{x_0,h}\}$ by (7);
  b) compute the bandwidth $\widehat{a}$ *at* x_0 :
  reorder $\{\widehat{\rho}_{J,\kappa,x_0}(x) : x \in \mathbf{U}_{x_0,h}\}$ as an increasing sequence, say
  $\widehat{\rho}_{J,\kappa,x_0}(x_1) \le \widehat{\rho}_{J,\kappa,x_0}(x_2) \le \cdots \le \widehat{\rho}_{J,\kappa,x_0}(x_M)$
  loop from $k = 1$ to M
  if $\sum\limits_{i=1}^{k} \widehat{\rho}_{J,\kappa,x_0}(x_i) > 0$

if $\frac{\sigma^2+\sum_{i=1}^{k}\widehat{\rho}^2_{J,\kappa,x_0}(x_i)}{\sum_{i=1}^{k}\widehat{\rho}_{J,\kappa,x_0}(x_i)} \geq \widehat{\rho}_{J,\kappa,x_0}(x_k)$

then $\widehat{a} = \frac{\sigma^2+\sum_{i=1}^{k}\widehat{\rho}^2_{J,\kappa,x_0}(x_i)}{\sum_{i=1}^{k}\widehat{\rho}_{J,\kappa,x_0}(x_i)}$

else quit loop
else continue loop
end loop;
c) compute the estimated weights: for $i = 1, ..., M$,
$\widehat{w}(x_i) = \frac{J(x_i,H_2)K_{\mathrm{tr}}\left(\frac{\widehat{\rho}_{x_0}(x_i)}{\widehat{a}}\right)}{\sum_{j=1}^{M} J(x_j,H_2)K_{\mathrm{tr}}\left(\frac{\widehat{\rho}_{x_0}(x_j)}{\widehat{a}}\right)}$;
d) compute the filter $\widetilde{u}_h$ *at* x_0 :
$\widetilde{u}_h(x_0) = \sum_{i=1}^{M} \widehat{w}(x_i)y(x_i)$.

To avoid the undesirable border effects in our simulations, we mirror the image outside the image limits. In more detail, we extend the image outside the image limits symmetrically with respect to the border. At the corners, the image is extended symmetrically with respect to the corner pixels.

Here the parameter σ acts as a smoothing factor for the restored image. The larger the value of σ, the more smooth the denoised image is. When computing SROAD values, we follow approximately the rules:

$$d = [4p + 1] \quad \text{and} \tag{9}$$
$$K = (2d + 1)^2 \times \min(0.5, -p/4 + 0.55). \tag{10}$$

For example when the noise ratio is 60% we use 7×7 windows and $K = 19$; when the noise ratio is 40% we use 5×5 windows and $K = 10$; when the noise ratio is 20% we use 3×3 windows and $K = 4$. The other parameters are chosen as follows:

$$S = (2[10p + 7] + 1)^2, \quad D = (2[4p + 1] + 1)^2,$$
$$H = 5 + \frac{30}{1 + 20p} \quad \text{and} \quad H_2 = 27 - 20p.$$

We point out that the values of parameters or the coefficients in the above formulae can vary within certain range. The dependence of the filter on the values of H and H_2 in a neighborhood of the suggested value given above is not very noticeable.

3 Simulation

In this section, the proposed algorithm is evaluated and compared with several other existing techniques for removing random-valued impulse noise. Extensive experiments are conducted on four standard 512×512 , 8-bit gray-level images

Table 1. Comparison of restoration in PSNR(db) for images corrupted with random-valued impulse noise

Images	Baboon			Bridge			Lena			Pentagon		
p%	20%	40%	60%	20%	40%	60%	20%	40%	60%	20%	40%	60%
Method	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR	PSNR
MF [9]	22.52	20.65	19.36	25.04	22.17	19.36	32.37	27.64	21.58	28.29	25.16	23.41
SS-I [10]	22.46	21.35	19.42	25.90	22.85	19.04	33.43	27.75	20.61	28.28	26.43	23.85
ACWM[11]	24.17	21.58	19.56	27.08	23.23	19.27	36.07	28.79	21.19	30.23	26.84	23.50
PWMAD[12]	23.78	21.56	19.68	26.90	23.83	20.83	36.50	31.41	24.30	30.11	27.33	24.46
IMF[13]	24.18	21.41	19.08	27.05	23.88	19.74	36.90	30.25	22.96	30.42	26.93	23.72
TriF [1]	24.18	21.60	19.52	27.60	24.01	20.84	36.70	31.12	26.08	30.33	27.14	24.60
ACWM-EPR [7]	23.97	21.62	19.87	27.31	24.60	20.89	36.57	32.21	24.62	30.03	27.35	24.59
ROLD-EPR [8]	24.49	21.92	20.38	**27.86**	24.79	**22.59**	37.45	32.76	29.03	30.73	27.73	25.70
FWNLM [14]	23.45	21.71	20.45	26.82	24.23	22.23	34.95	32.12	28.03	30.26	27.48	25.48
OWINF	**25.01**	**22.41**	**20.46**	**27.86**	**24.91**	22.49	**37.56**	**33.07**	**29.05**	**31.18**	**28.19**	**25.78**

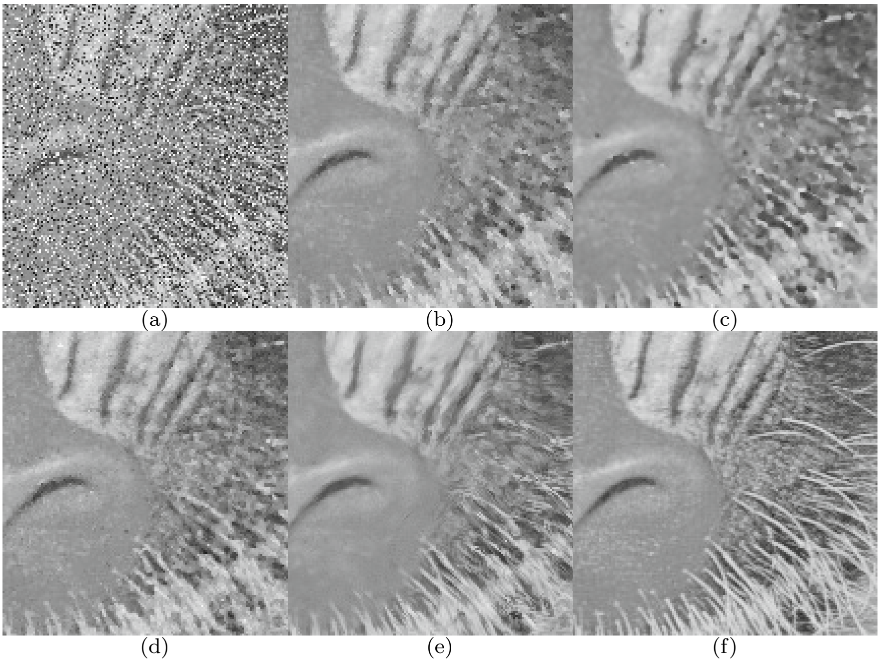

Fig. 1. Results of different methods in restoring 40% corrupted images "Baboon": (a)the noisy image; (b) results after the ACWM-EPR method [7]; (c) results after the ROAD-trilateral filter [1]; (d) results after the ROLD-EPR method [8]; (e) results after our OWINF; (f) the original image

with different features, including "Baboon," "Bridge", "Lena", and "Pentagon". Our experiments are done in the same way as in [8] in order to produce comparable results. The authors of [8] kindly provided us with their set of noisy images, restored images and PNSR values[1].

3.1 PSNR Comparison

We first concentrate on directly comparable and quantitative measures of image restoration. In particular, we evaluate the performance by using the peak signal-to-noise ratio (PSNR) [15]. If u is the original image and $\widetilde{u}$ is a restored image of u, the PSNR of $\widetilde{u}$ is given by

$$PSNR = 20 \log_{10} \frac{255}{\sqrt{MSE}},$$

$$MSE = \frac{1}{\operatorname{card} \mathbf{I}} \sum_{x \in \mathbf{I}} (u(x) - \widetilde{u}(x))^2.$$

Larger PSNR values signify better restoration.

In Table 1, we list the best PSNR values from all considered methods for the four images with $p \in \{20\%, 40\%, 60\%\}$. The best values are marked in bold. From Table 1, it is clear that OWINF proposed in this paper provides significant improvement over all other algorithms for the images "Baboon", "Lena" and "Pentagon". For the image "Bridge", ROLD-EPR and our algorithm all provide satisfactory denoising performance.

3.2 Visual Quality

Our main goal was to ensure that our approach provides improved denoising and visually pleasing results. To compare the results subjectively, we enlarge portion of the images restored by some methods listed in Table 1. Fig. 1 shows the results in restoring 40% corrupted images of "Baboon". In the images restored by ACWM-EPR [7] and ROAD-trilateral filter [1], we can see that there are still some loss of details in the hair around the mouth of the baboon. The visual qualities of images restored by ROLD-EPR [8] are improved obviously, but we can still find a few noise around the nose of baboon. Our restored images are quite good: they not only retain the abundance of image details, but also keep the continuity of the details.

4 Conclusions

In this paper, we use a standardized version of ROAD statistic [1] to define new impulsive weights in order to measure the degree of the contamination of a pixel by a random impulse noise. Then we combine it with the Optimal Weights Filter from [2,3] to get a new filter for removing impulse noise. Simulation results show that our method is competitive compared with a number of existing methods both quantitatively and visually.

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[1] All of them are in `www.math.cuhk.edu.hk/~rchan/paper/dcx/`

Acknowledgements. This research is partly supported by NSFC, China (No: 6127325831100672, 61375048,) China Postdoctoral Science Foundation (No. 2014M551412).

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