Exponential Synchronization of Coupled Stochastic and Switched Neural Networks with Impulsive Effects

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Abstract. In this paper, we investigate the exponential synchronization of coupled stochastic and switched neural networks (CSSNNs) with mixed time-varying delays. By exerting impulsive controller to the considered dynamical systems in each switching interval, and combining the multiple Lyapunov theory, we obtain a class of sufficient exponential synchronization criteria in terms of nonlinear equations and LMIs, which are easy to check. A simple example is presented to show the application of the criteria obtained in this paper.

Keywords: Coupled stochastic and switched neural networks · Exponential synchronization · Mixed time-varying delays · Impulsive effects

1 Introduction

Synchronization is an important and interesting collective behavior in coupled networks, and the study on synchronization of coupled neural networks can help us understand brain science and design coupled neural networks for real world applications. So synchronization of coupled neural networks has become a hot topic and been extensively investigated in recent years [1-8]. It is well known that time delays are unavoidable in the information processing of neurons due to various reasons, so most of the above-mentioned literatures are on synchronization of delayed neural networks.

It should be noted that because of link failures and the creation of new links in the information processing of neurons, the communication topology may change

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in time, thus it is more natural and important to design switching signal when modeling real-world networks. There have been some works on the synchronization of switched neural networks, see for example, [9-11]. On the other hand, impulsive control has been proved to be an important and economical control method, because it acts only at the discrete times and synchronize the coupled systems quickly. Recently, hybrid impulsive switched systems have received increasing attentions due to their wide applications in various fields, one can refer to [12,13]. Zhang et al. have investigated in [13] the exponential synchronization of coupled impulsive switched neural networks by using average dwell time approach and comparison principle, but the coupling is linear and coupling delay was not taken into account in the associated networks. As discussed in [14], sometimes state variables $x_i(t)$ may be unobservable, but $g(x_i(t))$ can be observed easily, so nonlinear coupling is more practical. Additionally, Haykin pointed out in [15] that synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. Practically, the stochastic phenomenon often appears in the electrical circuit design of neural networks. Hence, stochastic disturbances should also be considered in the dynamical behaviors of neural networks. However, the authors of [13] did't consider stochastic perturbation either.

Motivated by above discussions, this paper aims to analyze the exponential synchronization of nonlinearly coupled impulsive switched neural networks with stochastic perturbation and mixed time-varying delays. The rest of this paper is organized as follows: in Section 2, we first give the problem statement, and then present some definitions, lemmas and assumptions required throughout this paper; in Section 3, we will give a novel criterion to ensure the exponential synchronization for the considered neural networks in terms of LMIS and nonlinear equations; in Section 4, a simple example is provided to show the application of the theoretical results obtained in this paper.

2 Preliminaries

In this paper, we consider the following switched coupled neural networks with stochastic perturbations and impulsive effects:

$$\begin{cases} dx_i(t) = [-Cx_i(t) + B\tilde{f}(t, x_i(t)) + D\tilde{f}(t, x_i(t-\tau(t)))]dt \\ \quad +\tilde{g}(t, x_i(t), x_i(t-\rho(t)))dw(t) + \sum\limits_{j=1}^{N} a_{ij,\sigma(t)}\tilde{h}(x_j(t))dt, \quad t \neq t_{k,l_k} \\ \qquad x_i(t_{k,l_k}) = (1+\mu_{l_k})x_i(t^-_{k,l_k}), \qquad t = t_{k,l_k} \end{cases} \tag{1}$$

where $t \in [t_k, t_{k+1}), i = 1, \cdots, N,\ x_i(t) = [x_{i1}(t), \cdots, x_{in}(t)]^T \in \mathbb{R}^n$ is the state of the ith node at time t; $\tau(t), \rho(t)$ are the time-varying connected delay of neurons and coupled delay of nodes, respectively, and satisfying $0 < \tau(t) < \tau, 0 < \rho(t) < \rho$ with τ, ρ are positive constants; $\sigma(t) : [0, +\infty) \to \mathfrak{M} = \{1, 2, \cdots, m\}$ is a piecewise right-continuous function representing the switching signal. The switching time instants t_k satisfy $0 = t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots,\ \lim\limits_{k\to+\infty} t_k = +\infty$

and $\inf\limits_{0\leq k<\infty}\{t_{k+1}-t_k\} \geq \aleph$ where $\aleph = \max\{\tau, \rho\}$; $\{t_{k,l_k}, l_k \in \mathbb{N}^+\} \subset [t_k, t_{k+1})$ are impulsive instances satisfying $t_k < t_{k,1} < t_{k,2} < \cdots < t_{k,l_k} < \cdots < t_{k+1}$, and $x_i(t^+_{k,l_k}) = x_i(t_{k,l_k})$; μ_{l_k} is the impulsive strength satisfying $(1+\mu_{l_k})^2 \leq \mu < 1$; $C = diag\{c_1, \cdots, c_n\}, c_l > 0 (l = 1, \cdots, n)$ denotes the rate with which the l-th neuron $x_{il}(t)$ reset their potential to the resting state when disconnected from the networks and inputs, $B, D \in \mathbb{R}^{n\times n}$ denote the connection weight matrices of the neurons, $\tilde{f}(t, x_i(t)) = (\tilde{f}_1(t, x_i(t)), \cdots, \tilde{f}_n(t, x_i(t)))^T \in \mathbb{R}^n$ is the activation function of the neurons; $\tilde{g}(t, x_i(t), x_i(t-\rho(t))) \in \mathbb{R}^{n\times m}$ is the noise intensity function matrix; $w(t) = (w_1(t), w_2(t), \cdots, w_m(t))^T \in \mathbb{R}^m$ is a Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, P)$ with a nature filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying $E(w_j(t)) = 0, E(w_j^2(t)) = 1, E(w_j(t)w_k(t)) = 0$ $(j \neq k)$. The configuration coupling matrices $A_{\sigma(t)} = (a_{ij,\sigma(t)})_{N\times N}$ are defined as follows: if there is a directed edge from node j to node i, then $a_{ij,\sigma(t)} > 0$, otherwise, $a_{ij,\sigma(t)} = 0$, and $a_{ii,\sigma(t)} = -\sum\limits_{j=1,j\neq i}^{N} a_{ij,\sigma(t)}$ for $i = 1, \cdots, N$; $\tilde{h}(x_j(t)) = (\tilde{h}_1(x_j(t)), \cdots, \tilde{h}_n(x_j(t)))^T \in \mathbb{R}^n$ is the inner coupling vector function between two connected nodes i and j.

The initial condition of system (1) is given by $x_i(t) = \varphi_i(t) \in C([-\aleph, 0], \mathbb{R}^n)$, where $C([-\aleph, 0], \mathbb{R}^n)$ is the set of continuous functions from $[-\aleph, 0]$ to $\mathbb{R}^n$. Let $s(t)$ be a solution of the following stochastic delayed dynamical system of an isolate neural network:

$$ds(t) = [-Cs(t)+B\tilde{f}(t, s(t))+D\tilde{f}(t, s(t-\tau(t)))]dt+\tilde{g}(t, s(t), s(t-\rho(t)))dw(t), \quad (2)$$

which is the same as other neural networks. $s(t)$ can be any desired state: equilibrium point, a nontrivial periodic orbit, or even a chaotic orbit. In this paper, we hope to force the network (1) to globally exponentially synchronize with $s(t)$. The initial condition (2) is given by $s(t) = \phi(t) \in C([-\aleph, 0], \mathbb{R}^n)$.

Let $e_i(t) = x_i(t)-s(t), f(t, e_i(t)) = \tilde{f}(t, e_i(t)+s(t))-\tilde{f}(t, s(t)), g(t, e_i(t), e_i(t-\tau(t))) = \tilde{g}(t, e_i(t) + s(t), e_i(t-\tau(t) + s(t-\tau(t))) - \tilde{g}(t, s(t), s(t-\tau(t))), h(e_j(t-\rho(t))) = \tilde{h}(e_j(t-\rho(t)) + s(t-\rho(t))) - \tilde{h}(s(t-\rho(t))); e(t) = (e_1^T(t), \cdots, e_N^T(t))^T, C^N = I_N\otimes C, B^N = I_N\otimes B, D^N = I_N\otimes D, \mathbf{A} = A\otimes I_n,$ $F(t, e(t)) = (f^T(t, e_1(t)), \cdots, f^T(t, e_N(t)))^T, H(e(t-\rho(t))) = (h^T(e_1(t-\rho(t))), \cdots, h^T(e_N(t-\rho(t))))^T, G(t, e(t), e(t-\tau(t))) = diag\{g(t, e_1(t), e_1(t-\tau(t))), \cdots, g(t, e_N(t), e_N(t-\tau(t)))\}, dW(t) = 1_N \otimes dw(t)$, where $1_N = (1, 1, \cdots, 1)^T, \sigma(t) = r_k \in \mathfrak{M},\ t \in [t_k, t_{k+1})$. Then we can write the error system of the coupled neural networks (1) in the following compact form when $t \in [t_k, t_{k+1})$:

$$\begin{cases} de(t) = [-C^N e(t) + B^N F(t, e(t)) + D^N F(t, e(t-\tau(t)))dt \\ \qquad +G(t, e(t), e(t-\tau(t)))dW(t) + \mathbf{A}_{r_k} H(e(t-\rho(t)))]dt, & t \neq t_{k,l_k} \\ \qquad\qquad e(t_{k,l_k}) = (1+\mu_{l_k})e(t^-_{k,l_k}). & t = t_{k,l_k} \end{cases} \quad (3)$$

Definition 1. *The dynamical neural networks (1) is said to be globally exponentially synchronized to $s(t)$ in mean square if for any initial condition $x_i(t_0)$, there exist constants $\lambda > 0$ and $M > 1$ such that the following inequality is* *satisfied for $t \geq t_0$:*

$$E\Big(\sum_{i=1}^{N}\|x_i(t)-s(t)\|^2\Big) \leq M \sup_{t_0-\aleph\leq\iota\leq t_0} E\Big(\sum_{i=1}^{N}\|x_i(\iota)-s(\iota)\|^2\Big)e^{-\lambda(t-t_0)}.$$

Definition 2. [10]*: An impulsive sequence $\varsigma = \{t_1, t_2, \cdots\}$ is said to have average impulsive interval T_a if there exist positive integer δ and positive constant T_a such that*

$$\frac{T-t}{T_a} - \delta \leq N_\delta(T,t) \leq \frac{T-t}{T_a} + \delta, \qquad \forall T \geq t \geq 0,$$

where $N_\delta(T,t)$ denotes the number of impulsive times of the impulsive sequence $\{t_1, t_2, \cdots\}$ on the interval (t,T).

Assumption 1: Assume that there exist diagonal matrices L_1 and L_2 such that for $\forall x, y \in \mathbb{R}^n$, the function $\tilde{f}(t,\cdot)$ and $\tilde{h}(\cdot)$ satisfy the following Lipschitz conditions:

$$\|\tilde{f}(t,x)-\tilde{f}(t,y)\| \leq \|L_1(x-y)\|; \quad \|\tilde{h}(x)-\tilde{h}(y)\| \leq \|L_2(x-y)\|.$$

Assumption 2: Assume that there exist positive constants η_1, η_2 such that

$$\begin{aligned}&\mathrm{trace}\Big\{[\tilde{g}(t,x_1,y_1)-\tilde{g}(t,x_2,y_2)]^T \cdot [\tilde{g}(t,x_1,y_1)-\tilde{g}(t,x_2,y_2)]\Big\}\\ &\leq \eta_1\ \|x_1-y_1\|^2 + \eta_2\ \|x_2-y_2\|^2,\ \forall x_1,y_1,x_2,y_2 \in \mathbb{R}^n,\ t \in \mathbb{R}^+.\end{aligned}$$

Lemma 1. [13]*: Let $0 \leq \tau_i(t) \leq \tau, F(t,u,\bar{u}_1,\cdots,\bar{u}_m) : \mathbb{R}^+ \times \underbrace{\mathbb{R}\times\cdots\times\mathbb{R}}_{m+1}$ be nondecreasing in $\bar{u}_i$ for each fixed $(t,u,\bar{u}_1,\cdots,\bar{u}_{i-1},\bar{u}_{i+1},\cdots,\bar{u}_m)$, $i=1,\cdots,m$, and $I_k(u):\mathbb{R}\to\mathbb{R}$ be nondecreasing in u. Suppose that*

$$\begin{cases} D^+u(t) \leq F(t,u(t),u(t-\tau_1(t)),\cdots,u(t-\tau_m(t))) \\ \quad u(t_k^+) \leq I_k(u(t_k^-)),\ k\in\mathbb{N}_+ \end{cases}$$

and

$$\begin{cases} D^+v(t) > F(t,v(t),v(t-\tau_1(t)),\cdots,v(t-\tau_m(t))) \\ \quad v(t_k^+) \geq I_k(v(t_k^-)),\ k\in\mathbb{N}_+ \end{cases}$$

where the upper-right Dini derivative is defined as $D^+y(t) = \overline{lim}_{h\to0^+}\frac{y(t+h)-y(t)}{h}$. Then $u(t)\leq v(t)$ for $-\tau\leq t\leq 0$ implies that $u(t)\leq v(t)$ for $t\geq 0$.

Lemma 2. [17]*: For any real matrices X, Y and any positive matrix U, the following inequality holds:*

$$2X^TY \leq X^TUX + Y^TU^{-1}Y.$$

3 Exponential Synchronization Analysis

Theorem: Under Assumptions 1-2, the coupled neural networks (1) can be globally exponentially synchronized to $s(t)$, if there exist positive constants $\varepsilon_{1,r_k}, \varepsilon_{2,r_k}, \varepsilon_{3,r_k}$, positive matrices $P_{r_k} \in \mathbb{R}^{nN \times nN}$ satisfying $P_{r_k} \leq \theta_{r_k} I_{nN}$ with θ_{r_k} are positive constants, such that the following conditions are satisfied:

$$(\mathbf{H}_1) \quad \Phi_{r_k} = \begin{pmatrix} \Phi_{11,r_k} & P_{r_k}B^N & P_{r_k}D^N & P_{r_k}\mathbf{A}_{r_k} & 0 & 0 \\ (B^N)^T P_{r_k} & -\varepsilon_{1,r_k} I_{nN} & 0 & 0 & 0 & 0 \\ (D^N)^T P_{r_k} & 0 & -\varepsilon_{2,r_k} I_{nN} & 0 & 0 & 0 \\ \mathbf{A}_{r_k}^T P_{r_k} & 0 & 0 & -\varepsilon_{3,r_k} I_{nN} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Phi_{55,r_k} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Phi_{66,r_k} \end{pmatrix} < 0,$$

where $r_k \in \mathfrak{M}$, $\Phi_{11,r_k} = -2P_{r_k}C^N + \varepsilon_{1,r_k}(L_1^T L_1)^N + \eta_1\theta_{r_k} I_{nN} + \alpha_{r_k} P_{r_k}$ and $\Phi_{55,r_k} = \varepsilon_{2,r_k}(L_1^T L_1)^N + \eta_2\theta_{r_k} I_{nN} - \beta_{r_k} P_{r_k}$, $\Phi_{66,r_k} = \varepsilon_{3,r_k}(L_2^T L_2)^N - \gamma_{r_k} P_{r_k}$.

$$(\mathbf{H}_2) \qquad -\alpha + \tfrac{ln\mu}{T_a} + \mu^{-\delta}(\beta + \gamma) < 0,$$

where $\alpha = \min\limits_{r_k \in \mathfrak{M}}\{\alpha_{r_k}\}, \beta = \max\limits_{r_k \in \mathfrak{M}}\{\beta_{r_k}\}, \gamma = \max\limits_{r_k \in \mathfrak{M}}\{\gamma_{r_k}\}$.

$$(\mathbf{H}_3) \qquad \lambda - \tfrac{ln\Upsilon}{T_a} > 0,$$

where λ is the sole positive solution of the equation $-\alpha + \frac{ln\mu}{T_a} + \lambda + \mu^{-\delta}(\beta e^{\lambda\tau} + \gamma e^{\lambda\rho}) = 0$, $\Upsilon = \mu^{-\delta}\max\{\frac{\overline{p}}{\underline{p}}, e^{\lambda\aleph}\}, \overline{p} = \max\limits_{r_k \in \mathfrak{M}}\{\lambda_{max}(P_{r_k})\}, \underline{p} = \min\limits_{r_k \in \mathfrak{M}}\{\lambda_{min}(P_{r_k})\}$.

Proof: Define the following Lyapunov functions for system (3):

$$V(t) = e^T(t)P_{r_k}e(t), t \in [t_k, t_{k+1}), k \in \mathbb{N}^+.$$

Differentiating $V(t)$ along the trajectories of system (3) for $t \in [t_k, t_{k+1})$, we can obtain

$$dV(t) = \mathcal{L}V(t)dt + 2e^T(t)P_{r_k}G(t, e(t), e(t-\tau(t)))dW(t). \tag{4}$$

By applying the It$\hat{o}$'s formula to $\bar{V}(t)$ we can obtain

$$\begin{aligned}\mathcal{L}V(t) = {} & 2e^T(t)P_{r_k}[-C^N e(t) + B^N F(t, e(t)) + D^N F(t, e(t-\tau(t)))] \\ & + trace[G^T(t, e(t), e(t-\tau(t)))P_{r_k}G(t, e(t), e(t-\tau(t)))] \\ & + 2e^T(t)P_{r_k}\mathbf{A}_{r_k}H(e(t-\rho(t))).\end{aligned}$$

By using Lemma 2 and Assumption 1 we get

$$\begin{aligned} & 2e^T(t)P_{r_k}B^N F(t, e(t)) \\ \leq {} & \frac{1}{\varepsilon_{1,r_k}} e^T(t)P_{r_k}B^N(P_{r_k}B^N)^T e(t) + \varepsilon_{1,r_k}F^T(t, e(t))F(t, e(t)) \\ \leq {} & \frac{1}{\varepsilon_{1,r_k}} e^T(t)P_{r_k}B^N(P_{r_k}B^N)^T e(t) + \varepsilon_{1,r_k}e^T(t)(L_1^T L_1)^N e(t). \end{aligned} \tag{5}$$

Similar to (5), we can obtain the following inequalities:

$$\begin{aligned}&2e^T(t)P_{r_k}D^N F(t,e(t-\tau(t)))\\&\le \frac{1}{\varepsilon_{2,r_k}}e^T(t)P_{r_k}D^N(P_{r_k}D^N)^T e(t)+\varepsilon_{2,r_k}e^T(t-\tau(t))(L_1^T L_1)^N e(t-\tau(t)), \quad (6)\\&2e^T(t)P_{r_k}\mathbf{A}_{r_k}H(e(t-\rho(t)))\\&\le \frac{1}{\varepsilon_{3,r_k}}e^T(t)P_{r_k}\mathbf{A}_{r_k}\mathbf{A}_{r_k}^T P_{r_k}^T e(t)+\varepsilon_{3,r_k}e^T(t-\rho(t))(L_2^T L_2)^N e(t-\rho(t)). \quad (7)\end{aligned}$$

According to Assumption 2 we have

$$\begin{aligned}&trace[G^T(t,e(t),e(t-\tau(t)))P_{r_k}G(t,e(t),e(t-\tau(t)))]\\&\le \theta_{r_k}\sum_{i=1}^{N}\Big(\eta_1\ \|e_i(t)\|^2+\eta_2\ \|e_i(t-\tau(t))\|^2\Big)\\&=\theta_{r_k}\Big(\eta_1\ e^T(t)e(t)+\eta_2\ e^T(t-\tau(t))e(t-\tau(t))\Big). \quad (8)\end{aligned}$$

It follows from (5)-(8) that for $t\in[t_k,t_{k+1})$,

$$\begin{aligned}&\mathcal{L}V(t)\le e^T(t)\Big\{-P_{r_k}C^N-(P_{r_k}C^N)^T+\frac{1}{\varepsilon_{1,r_k}}P_{r_k}B^N(P_{r_k}B^N)^T\\&+\varepsilon_1(L_1^T L_1)^N+\frac{1}{\varepsilon_{2,r_k}}P_{r_k}D^N(P_{r_k}D^N)^T+\frac{1}{\varepsilon_{3,r_k}}P_{r_k}\mathbf{A}_{r_k}\mathbf{A}_{r_k}^T P_{r_k}+\theta_{r_k}\eta_1 I_{nN}\\&+\alpha_{r_k}P_{r_k}\Big\}e(t)-\alpha_{r_k}e^T(t)P_{r_k}e(t)+\beta_{r_k}e^T(t-\tau(t))P_{r_k}e(t-\tau(t))\\&+e^T(t-\tau(t))\Big[\varepsilon_{2,r_k}(L_1^T L_1)^N+\theta_{r_k}\eta_2 I_{nN}-\beta_{r_k}P_{r_k}\Big]e(t-\tau(t))\\&+e^T(t-\rho(t))\Big[\varepsilon_{3,r_k}(L_2^T L_2)^N-\gamma_{r_k}P_{r_k}\Big]e(t-\rho(t))\\&+\gamma_{r_k}e^T(t-\rho(t))P_{r_k}e(t-\rho(t))\\&\le -\alpha_{r_k}V(t)+\beta_{r_k}V(t-\tau(t))+\gamma_{r_k}V(t-\rho(t)). \quad (9)\end{aligned}$$

Integrating on both sides of (9) from t to $t+\triangle t$ for any $\triangle t>0$ and taking mathematical expectation. Let $m(t)=EV(t)$, associating with the properties of the Itô's integral and Dini derivation, we can derive from (9) that

$$D^+m(t)\le -\alpha_{r_k}m(t)+\beta_{r_k}m(t-\tau(t))+\gamma_{r_k}m(t-\rho(t)), \quad t\in[t_k,t_{k+1}).$$

When $t=t_{k,l_k}$, we can easily derive that

$$m(t_{k,l_k})=(1+\mu_{l_k})^2E[e^T(t_{k,l_k}^-)P_{r_k}e(t_{k,l_k}^-)]\le \mu m(t_{k,l_k}^-).$$

For any $\varepsilon>0$, let $y(t)$ be a unique solution of the following delay system:

$$\begin{cases}\dot{y}(t)=-\alpha y(t)+\beta y(t-\tau(t))+\gamma y(t-\rho(t))+\varepsilon,\ t\neq t_{k,l_k}\\ y(t_{k,l_k})=\mu y(t_{k,l_k}^-), \quad t=t_{k,l_k}\\ y(t)=m(t),\ t_k-\aleph\le t\le t_k.\end{cases} \quad (10)$$

By the formula for the variation of parameters, it follows from (10) that for $t \in [t_k, t_{k+1})$,

$$y(t) = P(t,t_k)y(t_k) + \int_{t_k}^{t} P(t,s)\big[\beta y(s-\tau(s)) + \gamma y(s-\rho(s)) + \varepsilon\big] ds, \tag{11}$$

where $P(t,s),\ t,s > t_k$ is the Cauchy matrix of the linear system

$$\begin{cases} \dot{y}(t) = -\alpha y(t),\ t \neq t_{k,l_k} \\ y(t_{k,l_k}) = \mu y(t^-_{k,l_k}), \quad t = t_{k,l_k}. \end{cases} \tag{12}$$

According to the representation of Cauchy matrix, one can get the following estimation:

$$P(t,s) = e^{-\alpha(t-s)}\mu^{N_\delta(s,t)} \leq \mu^{-\delta}e^{-\alpha^*(t-s)},$$

where $\alpha^* = \alpha - \frac{ln\mu}{T_a}$. Define $s(\varsigma) = \varsigma - \alpha^* + \mu^{-\delta}(\beta e^{\varsigma\tau} + \gamma e^{\varsigma\rho})$. From $(\mathbf{H}_2)$ we know $s(0) = -\alpha^* + \mu^{-\delta}(\beta+\gamma) < 0$. Since $\dot{s}(\varsigma) > 0$ and $\lim\limits_{\varsigma\to+\infty} s(\varsigma) = +\infty$, there exists a unique $\lambda > 0$ such that $s(\lambda) = 0$, i. e., $\lambda - \alpha^* + \mu^{-\delta}(\beta e^{\lambda\tau} + \gamma e^{\lambda\rho}) = 0$. Let $\xi = \mu^{-\delta}\|y(t_k)\|_\aleph = \mu^{-\delta} \sup\limits_{t_k-\aleph\leq t\leq t_k} \|y(t)\|$. In the following, we shall prove the following inequality is satisfied:

$$y(t) < \xi e^{-\lambda(t-t_k)} + \frac{\varepsilon}{\alpha^*\mu^\delta - \beta - \gamma}, \quad t_k - \aleph \leq t \leq t_{k+1}. \tag{13}$$

It is obvious that $y(t) \leq \mu^\delta\xi < \xi < \xi e^{-\lambda(t-t_k)} + \frac{\varepsilon}{\alpha^*\mu^\delta-\beta-\gamma}$ for $t_k - \aleph \leq t \leq t_k$. When $t_k < t < t_{k+1}$, we will prove the inequality (13) is still satisfied by the way of contradiction. If there exists a $t^* \in (t_k, t_{k+1})$ such that

$$y(t^*) \geq \xi e^{-\lambda(t^*-t_k)} + \frac{\varepsilon}{\alpha^*\mu^\delta - \beta - \gamma}, \tag{14}$$

and

$$y(t) < \xi e^{-\lambda(t-t_k)} + \frac{\varepsilon}{\alpha^*\mu^\delta - \beta - \gamma},\ t \in (t_k - \aleph, t^*) \tag{15}$$

Note that $\tau(t) \leq \tau, \rho(t) \leq \rho$ and $e^{\lambda\tau}\beta + e^{\lambda\rho}\gamma = \mu^\delta(\alpha^* - \lambda)$, then by some simple computation, we can derive from (11) and (15) that

$$\begin{aligned} & y(t^*) \\ & < \xi e^{-\alpha^*(t^*-t_k)} + \int_{t_k}^{t^*} \mu^{-\delta}e^{-\alpha^*(t^*-s)}\big[\xi(e^{\lambda\tau}\beta + e^{\lambda\rho}\gamma)e^{-\lambda(s-t_k)} + \frac{\alpha^*\mu^\delta\varepsilon}{\alpha^*\mu^\delta - \beta - \gamma}\big] ds \\ & = \xi e^{-\lambda(t^*-t_k)} + \frac{\varepsilon}{\alpha^*\mu^\delta - \beta - \gamma} - \frac{\varepsilon}{\alpha^*\mu^\delta - \beta - \gamma}e^{-\alpha^*(t^*-t_k)} \\ & < \xi e^{-\lambda(t^*-t_k)} + \frac{\varepsilon}{\alpha^*\mu^\delta - \beta - \gamma}, \end{aligned}$$

which contradicts with (14). Thus, (13) is always satisfied for $t_k - \aleph \leq t < t_{k+1}$. Let $\varepsilon \to 0$, one can obtain $y(t) \leq \xi e^{-\lambda(t-t_k)}$. Then it follows from Lemma 1 that

$m(t) \le y(t) \le \xi e^{-\lambda(t-t_k)} = \mu^{-\delta}\|m(t_k)\|_{\aleph} e^{-\lambda(t-t_k)}$ for $t_k \le t < t_{k+1}$. We will show by induction that

$$m(t) \le \mu^{-\delta}\Upsilon^k \|m(t_0)\|_{\aleph} e^{-\lambda(t-t_0)}, \qquad t_k \le t < t_{k+1}, \tag{16}$$

where $\Upsilon = \mu^{-\delta}\max\{\frac{\overline{p}}{\underline{p}}, e^{\lambda\aleph}\}$ and $\overline{p} = \max_{r_k\in\mathfrak{M}}\{\lambda_{max}(P_{r_k})\}, \underline{p} = \min_{r_k\in\mathfrak{M}}\{\lambda_{min}(P_{r_k})\}$. When $t \in [t_0, t_1), m(t) \le \mu^{-\delta}\|m(t_0)\|_{\aleph} e^{-\lambda(t-t_0)}$. Assume (16) holds for $1 \le k \le j, j \in \mathbb{N}^+$, then we will show that (16) holds for $k = j+1$. Since

$$m(t) \le \mu^{-\delta}\Upsilon^j \|m(t_0)\|_{\aleph} e^{-\lambda(t_{j+1}-\aleph-t_0)} = \mu^{-\delta}\Upsilon^j e^{\lambda\aleph}\|m(t_0)\|_{\aleph} e^{-\lambda(t_{j+1}-t_0)}$$

for $t_{j+1} - \aleph \le t < t_{j+1}$, and note that $t_{j+1} < t_{j+1,1}$, which follows that

$$\begin{aligned} m(t_{j+1}) &= E(e^T(t_{j+1})P_{r_{j+1}}e(t_{j+1})) = E(e^T(t_{j+1}^-)P_{r_{j+1}}e(t_{j+1}^-)) \\ &\le \frac{\overline{p}}{\underline{p}} m(t_{j+1}^-) \le \frac{\overline{p}}{\underline{p}}\mu^{-\delta}\Upsilon^j\|m(t_0)\|_{\aleph} e^{-\lambda(t_{j+1}-t_0)}, \end{aligned}$$

then it follows that

$$\|m(t_{j+1})\|_{\aleph} \le \Upsilon\Upsilon^j\|m(t_0)\|_{\aleph} e^{-\lambda(t_{j+1}-t_0)} = \Upsilon^{j+1}\|m(t_0)\|_{\aleph} e^{-\lambda(t_{j+1}-t_0)}.$$

Thus we can get

$$\begin{aligned} m(t) &\le \mu^{-\delta}\|m(t_{j+1})\|_{\aleph} e^{-\lambda(t-t_{j+1})} \le \mu^{-\delta}\Upsilon^{j+1}\|m(t_0)\|_{\aleph} e^{-\lambda(t_{j+1}-t_0)} e^{-\lambda(t-t_{j+1})} \\ &= \mu^{-\delta}\Upsilon^{j+1}\|m(t_0)\|_{\aleph} e^{-\lambda(t-t_0)}, \quad t_{j+1} \le t < t_{j+2} \end{aligned}$$

Thus, (16) can be derived for $\forall t \in [t_k, t_{k+1})$ and $\forall k \in \mathbb{N}^+$ by the induction principle. For an arbitrarily given $t > t_0, \exists k \in \mathbb{N}^+$, such that $t \in [t_k, t_{k+1})$. Note that $k \le N_\delta(t, t_0)$, then it follows from (16) that

$$\begin{aligned} m(t) &\le \mu^{-\delta}\Upsilon^k\|m(t_0)\|_{\aleph} e^{-\lambda(t-t_0)} \le \mu^{-\delta}\Upsilon^{N_\delta(t,t_0)}\|m(t_0)\|_{\aleph} e^{-\lambda(t-t_0)} \\ &\le \mu^{-\delta}\Upsilon^{\delta+\frac{t-t_0}{T_a}}\|m(t_0)\|_{\aleph} e^{-\lambda(t-t_0)} = \Big(\frac{\Upsilon}{\mu}\Big)^\delta \|m(t_0)\|_{\aleph} e^{-\lambda^*(t-t_0)}, \end{aligned}$$

where $\lambda^* = \lambda - \frac{ln\Upsilon}{T_a} > 0$. Then we have

$$\underline{p}E(\|e(t)\|^2) \le m(t) \le \Big(\frac{\Upsilon}{\mu}\Big)^\delta \|m(t_0)\|_{\aleph} e^{-\lambda^*(t-t_0)} \le \overline{p}\Big(\frac{\Upsilon}{\mu}\Big)^\delta E(\|e(t_0)\|_{\aleph}^2) e^{-\lambda^*(t-t_0)},$$

which follows that

$$E\Big(\sum_{i=1}^N \|x_i(t) - s(t)\|^2\Big) \le M \sup_{t_0-\aleph\le\iota\le t_0} E\Big(\sum_{i=1}^N \|x_i(\iota) - s(\iota)\|^2\Big) e^{-\lambda^*(t-t_0)},$$

where $M = \frac{\overline{p}}{\underline{p}}(\frac{\Upsilon}{\mu})^\delta > 1$. This shows that the dynamical network (1) is globally exponentially synchronized to $s(t)$ in mean square. This completes the proof.

4 Numerical Simulation

Example 1. In system (1), we select $\tilde{f}(t, x_i(t)) = (\tilde{f}_1(t, x_i(t)), \tilde{f}_2(t, x_i(t)))^T$ and $\tilde{f}_1(t, x_i(t)) = \frac{\sqrt{2}}{8}x_{i1}(t) + \frac{\sqrt{3}}{8}(|x_{i2}(t)+1| - |x_{i2}(t)-1|)$, $\tilde{f}_2(t, x_i(t)) = \frac{\sqrt{6}}{8}(|x_{i2}(t)+1| - |x_{i2}(t) - 1|)$, which follows that $L_1 = diag\{0.25, 0.75\}, L_2 = diag\{0.5, 0.25\}$. Let $\mu_{l_k} = -0.1$ for $\forall l_k \in \mathbb{N}^+$, and $C_1 = 4.5I_2, C_2 = 3.8I_2$,

$$\tilde{g}(t, x_i(t), x_i(t-\tau(t))) = 0.1 = diag\{x_i(t), x_i(t-\tau(t))\}, \tau(t) = 0.3sint,$$

$$\rho(t) = 0.2cost, B_1 = \begin{pmatrix} 0.8 & 0.9 \\ -0.6 & 0.8 \end{pmatrix}, B_2 = \begin{pmatrix} 1 & 0.5 \\ -0.9 & 1 \end{pmatrix}, D_1 = \begin{pmatrix} 0.5 & 0.4 \\ 0.8 & 0.5 \end{pmatrix},$$

$$D_2 = \begin{pmatrix} 0.9 & 0.5 \\ 0.6 & 0.8 \end{pmatrix}, A_1 = \begin{pmatrix} -0.9 & 0.5 & 0.4 \\ 0.8 & -1 & 0.2 \\ 0.5 & 0.5 & -1 \end{pmatrix}, A_2 = \begin{pmatrix} -1 & 0.4 & 0.6 \\ 0.6 & -1.1 & 0.5 \\ 0.4 & 0.6 & -1 \end{pmatrix}.$$

Assuming that the coupled neural networks switches in a random order between two networks, i. e., $\mathfrak{M} = \{1, 2\}$. The switching scheme is shown in Fig. 1. Select $\alpha_1 = 4.5, \beta_1 = \gamma_1 = 0.42, \alpha_2 = 4.25, \beta_2 = \gamma_2 = 0.78$, then by using Matlab LMI tool we can obtain $\varepsilon_{1,1} = 1.3931, \varepsilon_{1,2} = 1.2767, \varepsilon_{2,1} = 0.5423, \varepsilon_{2,2} = 0.9266, \varepsilon_{3,1} = 1.4451, \varepsilon_{3,2} = 3.1661, \theta_1 = 2.1717, \theta_2 = 1.6614, \underline{p} = 0.6353, \overline{p} = 1.3182$. The impulsive sequence is constructed by taking $T_a = 4.6$ and $\delta = 4$, then by solving the nonlinear equation $-\alpha + \frac{ln\mu}{T_a} + \lambda + \mu^{-\delta}(\beta e^{\lambda\tau} + \gamma e^{\lambda\rho}) = 0$, we can get $\lambda = 0.3431$. So by virtue of the Theorem in this paper, it can be concluded that the considered network can be exponentially synchronized onto the objective trajectory. The following Figure shows that the errors between the networks' states and converge to zero under the given conditions.

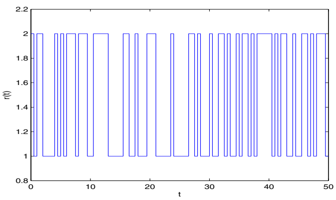

Fig.1. The switching scheme

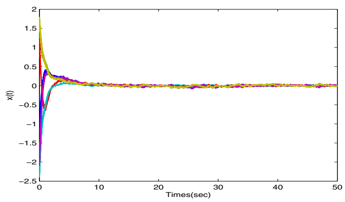

Fig.2. The state variables$x_{ir}(t)$

5 Conclusion

The exponential synchronization of switched coupled neural networks with mixed time-varying delays and stochastic disturbances is investigated in this paper. The main contribution of this paper contains three aspects. Firstly, as discussed in the section of Introduction, the network model considered in this paper is more practical in real world. Secondly, different from the average dwell time approach used in many existing literatures, there is no upper bound for switching interval, which is only assumed to be greater than the maximum of delays. As for the impulsive scheme, the named average impulsive interval is utilized to get less conservative synchronization criterion. Thirdly, by using multiple Lyapunov function, we have shown that the exponential synchronization can be achieved by solve some LMIs and nonlinear equations, which are easy to check.

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