Adaptive Pinning Synchronization of Coupled Inertial Delayed Neural Networks

Jianqiang Hu[1](✉), Jinde Cao[2], and Quanxin Cheng[2]

[1] School of Automation, Southeast University, Nanjing 210096, China
jqhuseu@seu.edu.cn

[2] Department of Mathematics, Research Center for Complex Systems and Network Sciences, Southeast University, Nanjing 210096, China
jdcao@seu.edu.cn

Abstract. In this paper, adaptive pinning synchronization (i.e., leader-following synchronization) is considered for an array of linearly coupled inertial delayed neural network. By applying feedback control on a small fraction of network nodes with the dynamical feedback gains turning adaptively and combining the Lyapunov function method, an easy-to-verify sufficient condition is derived for globally asymptotically synchronization for the coupled network. Meanwhile, the coupling configuration matrix is not necessary to be symmetric or irreducible. Finally, an illustrative example is given to show the effectiveness of the obtained theoretical results.

Keywords: Inertial delayed neural networks · Asymptotical synchronization · Adaptive pinning control

1 Introduction

Synchronization of complex networks has received notable attentions in the past decade due to its potential applications in various fields, see [1–3]. As a special class of complex networks, neural networks have also been intensively investigated [4,5], where the network nodes are neurons and the network coupling is the connection weight matrix. Synchronization of coupled neural networks means multiple neural networks can achieve a common trajectory, such as a common equilibrium, limit cycle or chaotic trajectory. Based on Lyapunov functional methods, global synchronization was investigated in [6,7] for linearly and diffusively coupled identical delayed neural networks.

Inertial electronic neural networks with one or two neurons were considered in [8], where it was found that when the neuron couplings were of an inertial nature, the dynamics could be more complex compared with the simpler behavior displayed in the standard resistor-capacitor variety. The dynamical behaviors of a single delayed neuron model with inertial terms were investigated in [9]; bifurcation problems were investigated in [10,11] for low-order neural networks. While most of the published investigations in the literature concerning inertial

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neural networks are always focusing on small-scale neural networks with only one or two neurons, the general network coupled by multiple neural networks are rarely seen in the literature.

Recently, the authors in [12] considered the stability and existence of periodic solutions for the general inertial BAM neural networks with time delays. Furthermore, stability analysis was carried out in [13] for the general inertial Cohen–Grossberg-type neural networks with time delays. While in [14], the stability of an inertial delayed neural network was investigated by matrix measure strategies and drive-response synchronization was considered as an application at the end of the paper. On the other hand, pinning synchronization of coupled neural networks has been investigated, such as the synchronization of a general weighted neural network with coupling delay was investigated in [15,16] by adaptive pinning control. More studies concerning pinning synchronization of neural networks can be found in [17,18] and references cited therein. Inspired by the above discussions, this paper investigates the pinning synchronization of coupled inertial delayed neural networks.

2 Model Description and Preliminaries

Consider an array of linearly coupled inertial delayed neural networks consisted of N identical nodes with dynamics of the ith node described by the following equation:

$$\begin{aligned}\frac{\mathrm{d}^2 x_i(t)}{\mathrm{d}t^2} =& - D\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} - Cx_i(t) + Af(x_i(t)) + Bf(x_i(t-\tau(t))) + I(t)\\ &+ c\sum_{j=1}^{N} G_{ij}\Gamma\Big(\frac{\mathrm{d}x_j(t)}{\mathrm{d}t} + x_j(t)\Big) + u_i(t), \quad i = 1,\dots,N,\end{aligned} \tag{1}$$

where $x_i(t) = (x_{i1}(t),\dots,x_{in}(t))^T \in \mathbb{R}^n$ is the state vector of the ith neural network, and $u_i(t)$ is the control input imposed on the ith node; $D = \mathrm{diag}\{d_1,\dots,d_n\}$, $C = \mathrm{diag}\{c_1,\dots,c_n\}$ are constant positive definite matrices. $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ denote the connection weight matrix and the delayed connection weight matrix, respectively. The nonlinear function $f(x_i) = (f_1(x_{i1}),\dots,f_n(x_{in}))^T$ is the activation function for the inertial neural network; and $I(t) = (I_1(t),\dots,I_n(t))^T$ is the external input vector. The second derivative of $x_i(t)$ is called an inertial term of system (1). The positive constant c is the network coupling strength and Γ is the inner coupling matrix. $G = (G_{ij})_{N\times N}$ is the constant coupling configuration matrix defined to be diffusive: $G_{ij} \geq 0 (i \neq j)$ and $G_{ii} = -\sum_{j=1,j\neq i}^{N} G_{ij}$. The coupling matrix G is not required to be symmetric or irreducible.

The initial conditions associated with system (1) are given as $x_i(\omega) = \phi_i(\omega) \in \mathcal{C}^{(1)}([-\tau,0],\mathbb{R}^n)$, $i = 1,\dots,N$, where $\mathcal{C}^{(1)}([-\tau,0],\mathbb{R}^n)$ denotes the set of all n-dimensional continuous differentiable functions defined on the interval $[-\tau,0]$ with $\tau = \sup_{t\geq 0}\{\tau(t)\}$.

The isolated node of network (1) is given by the following inertial delayed neural network:

$$\frac{\mathrm{d}^2 s(t)}{\mathrm{d}t^2} = -D\frac{\mathrm{d}s(t)}{\mathrm{d}t} - Cs(t) + Af(s(t)) + Bf(s(t-\tau(t))) + I(t), \qquad (2)$$

where $s(t) = (s_1(t), \ldots, s_n(t)) \in \mathbb{R}^n$. The initial condition for system (2) is given as $s(\omega) = \varphi(\omega) \in \mathcal{C}^{(1)}([-\tau, 0], \mathbb{R}^n)$.

To proceed, the following assumptions and definition are given.

Assumption 1. *The activation functions $f_i(\cdot) : \mathbb{R} \to \mathbb{R}, 1 \le i \le n$ are bounded and satisfy Lipschitz condition, i.e., there exist constants F_i and M_i such that $|f_i(x) - f_i(y)| \le F_i|x - y|$ and $|f_i(x)| \le M_i$ for all $x, y \in \mathbb{R}$.*

Assumption 2. *The time delay $\tau(t) \ge 0$ in systems (1) and (2) is a bounded and differentiable function of time t satisfying $\dot{\tau}(t) \le \rho < 1$ for all $t \ge 0$, where $\rho > 0$.*

Definition 1. *The coupled inertial neural network (1) is said to be globally asymptotically synchronizable to the goal trajectory $s(t)$ if the discriminant relations $\lim_{t\to\infty} \|x_i(t) - s(t)\| = 0$, $i = 1, 2 \ldots, N$ hold for all initial functions.*

3 Main Results

In this section, we will investigate the global synchronization of the coupled inertial neural network by adaptive pinning control. The feedback injections are only placed on a small fraction of the total network nodes and the feedback gains are turned adaptively.

By letting the synchronization error $e_i(t) = x_i(t) - s(t)$, one can derive the following error system:

$$\begin{aligned}\frac{\mathrm{d}^2 e_i(t)}{\mathrm{d}t^2} = &- D\frac{\mathrm{d}e_i(t)}{\mathrm{d}t} - Ce_i(t) + Ag(e_i(t)) + Bg(e_i(t-\tau(t))) \\ &+ c\sum_{j=1}^{N} G_{ij}\Gamma\Big(\frac{\mathrm{d}e_j(t)}{\mathrm{d}t} + e_j(t)\Big) + u_i(t) \quad i = 1, \ldots, N,\end{aligned} \qquad (3)$$

where $g(e_i) = (f_1(e_{i1} + s_1) - f_1(s_1), \ldots, f_n(e_{in} + s_n) - f_n(s_n))^T$.

Next, by introducing the following variable transformation:

$$r_i(t) = \frac{\mathrm{d}e_i(t)}{\mathrm{d}t} + e_i(t), \qquad i = 1, \ldots, n,$$

the error system (3) can be written as

$$\begin{cases}\dfrac{\mathrm{d}e_i(t)}{\mathrm{d}t} = -e_i(t) + r_i(t), \\ \dfrac{\mathrm{d}r_i(t)}{\mathrm{d}t} = -\boldsymbol{C}e_i(t) - \boldsymbol{D}r_i(t) + Ag(e_i(t)) + Bg(e_i(t-\tau(t))) \\ \qquad\qquad + c\displaystyle\sum_{j=1}^{N} G_{ij}\Gamma r_j(t) + u_i(t),\end{cases} \qquad (4)$$

for $i = 1, \dots, N$, where $\boldsymbol{C} \triangleq C + I_n - D$ and $\boldsymbol{D} \triangleq D - I_n$.

The pinning controller is designed as follows:

$$u_i(t) = -\sigma_i(t)\Gamma r_i(t) \quad i = 1, \dots, N, \tag{5}$$

where $\sigma_i(t)$ is the time-varying feedback control gain designed as

$$\dot{\sigma}_i(t) = \begin{cases} \sigma_i r_i^T(t)\Gamma r_i(t), \ \sigma_i(0) > 0, \text{ for } i \in \mathcal{V}_{pin}, \\ 0, \qquad\qquad\qquad \sigma_i(0) = 0, \text{ for } i \notin \mathcal{V}_{pin}, \end{cases}$$

where $\sigma_i > 0$ is a constant and $\mathcal{V}_{pin}$ is the set of the pinning nodes.

Thus, under the control input (5), the error system (4) turns out to be the following one

$$\begin{cases} \dfrac{\mathrm{d}e_i(t)}{\mathrm{d}t} = - e_i(t) + r_i(t), \\ \dfrac{\mathrm{d}r_i(t)}{\mathrm{d}t} = - \boldsymbol{C}e_i(t) - \boldsymbol{D}r_i(t) + Ag(e_i(t)) + Bg(e_i(t - \tau(t))) \\ \qquad\qquad + c\displaystyle\sum_{j=1}^{N} G_{ij}\Gamma r_j(t) - \sigma_i(t)\Gamma r_i(t). \end{cases} \tag{6}$$

The coupled inertial neural network (1) can be synchronized if the above error system (6) is globally asymptotically stable. The following theorem gives the synchronization criterion.

Theorem 1. *Under Assumptions 1 and 2, the coupled inertial neural network (1) is globally asymptotically synchronized if there exists a positive definite matrix P such that*

$$\Phi = \begin{bmatrix} I_N \otimes [-P + (\frac{1}{2}F^2 + \eta)I_n] & \frac{1}{2}I_N \otimes (P - \boldsymbol{C}) \\ * & Q \end{bmatrix} < 0, \tag{7}$$

where $F = \max_{1\le i\le N}\{F_i\}$, $\eta > \max_{1\le i\le N}\left\{\frac{F_i^2}{2(1-\rho)}\right\}$ is a positive constant, $\boldsymbol{C} \triangleq C + I_n - D$, $\boldsymbol{D} \triangleq D - I_n$ and $Q = I_N \otimes (-\boldsymbol{D} + \frac{AA^T + BB^T}{2} + c\frac{G+G^T}{2} \otimes \Gamma - M \otimes \Gamma)$ with $M = \mathrm{diag}\{\sigma_1^*, \dots, \sigma_N^*\} \ge 0$, in which $\sigma_i^* = 0$ for $i \in \mathcal{V}_{pin}$ and $\sigma_i^* > 0$ when $i \notin \mathcal{V}_{pin}$.

Proof. To prove the result, one just need to show that the error system (6) is globally asymptotically stable. Consider the following Lyapunov-Krasovskii functional candidate:

$$\begin{aligned} V(t) =& \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)Pe_i(t) + \eta\sum_{i=1}^{N}\int_{t-\tau(t)}^{t} e_i^T(s)e_i(s)\mathrm{d}s \\ &+ \frac{1}{2}\sum_{i=1}^{N} r_i^T(t)r_i(t) + \sum_{i=1}^{N}\frac{(\sigma_i(t) - \sigma_i^*)^2}{2\sigma_i}. \end{aligned} \tag{8}$$

Calculating the time derivative of $V(t)$ along the trajectories of system (6), one can obtain

$$\begin{aligned}
\dot{V}(t) \leq & \sum_{i=1}^{N} e_i^T(t)P\big(-e_i(t)+r_i(t)\big)+\eta\sum_{i=1}^{N} e_i^T(t)e_i(t) \\
& -\eta(1-\rho)\sum_{i=1}^{N} e_i^T(t-\tau(t))e_i(t-\tau(t)) \\
& -\sum_{i=1}^{N} r_i^T(t)\boldsymbol{C}e_i(t)-\sum_{i=1}^{N} r_i^T(t)\boldsymbol{D}r_i(t)+\sum_{i=1}^{N} r_i^T(t)Ag(e_i(t)) \\
& +\sum_{i=1}^{N} r_i^T(t)Bg(e_i(t-\tau(t)))+c\sum_{i=1}^{N}\sum_{j=1}^{N} r_i^T(t)G_{ij}\Gamma r_j(t) \\
& -\sum_{i=1}^{N}\sigma_i(t)r_i^T(t)\Gamma r_i(t)+\sum_{i\in\mathcal{V}_{pin}}\sigma_i(t)r_i^T(t)\Gamma r_i(t) \\
& -\sum_{i\notin\mathcal{V}_{pin}}\sigma_i^* r_i^T(t)\Gamma r_i(t).
\end{aligned}$$

It follows from Assumption 1 that

$$\sum_{i=1}^{N} r_i^T(t)Ag_i(e_i(t)) \leq \sum_{i=1}^{N}\Big(\frac{1}{2}r_i^T(t)AA^Tr_i(t)+\frac{1}{2}F_i^2e_i^T(t)e_i(t)\Big) \tag{9}$$

and

$$\begin{aligned}
\sum_{i=1}^{N} r_i^T(t)Bg_i(e_i(t-\tau(t))) \leq & \sum_{i=1}^{N}\Big(\frac{1}{2}r_i^T(t)BB^Tr_i(t) \\
& +\frac{1}{2}F_i^2e_i^T(t-\tau(t))e_i(t-\tau(t))\Big).
\end{aligned} \tag{10}$$

Combining inequalities (9) and (10), we have

$$\begin{aligned}
\dot{V}(t) \leq & \sum_{i=1}^{N} e_i^T(t)\big(-P+(\eta+\frac{1}{2}F_i^2)I_n\big)e_i(t)+\sum_{i=1}^{N} e_i^T(t)(P-\boldsymbol{C})r_i(t) \\
& +\sum_{i=1}^{N} r_i^T(t)\big(-\boldsymbol{D}+\frac{1}{2}(AA^T+BB^T)\big)r_i(t) \\
& +c\sum_{i=1}^{N}\sum_{j=1}^{N} r_i^T(t)G_{ij}\Gamma r_j(t)-\sum_{i=1}^{N}\sigma_i^* r_i^T(t)\Gamma r_i(t) \\
= & \ \psi^T(t)\Phi\psi(t),
\end{aligned}$$

where $\psi(t)=[e^T(t),r^T(t)]^T$. Thus, by LMI (7) we have $\dot{V}(t)<0$ for $\psi(t)\neq 0$, which indicates that $\lim_{t\to\infty} e(t)=\mathbf{0}$ and $\lim_{t\to\infty} r(t)=\mathbf{0}$. Therefore, the pinning controlled network (6) can be globally asymptotically synchronized to the objective trajectory.

4 Illustrative Example

In this section, one illustrative example is presented to demonstrate the effectiveness of the obtained theoretical results.

Example 1. Consider the following coupling inertial delayed neural networks with 12 nodes:

$$\begin{aligned}\frac{\mathrm{d}^2 x_i(t)}{\mathrm{d}t^2} =& -D\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} - Cx_i(t) + Af(x_i(t)) + Bf(x_i(t-\tau(t))) + I_i(t)\\ &+ c\sum_{j=1}^{12} G_{ij}\Gamma\Big(\frac{\mathrm{d}x_j(t)}{\mathrm{d}t} + x_j(t)\Big) + u_i(t), \quad i = 1, \ldots, 12,\end{aligned} \tag{11}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t))^T$, $f(x_i(t)) = \big(\tanh(x_{i1}(t)), \tanh(x_{i2}(t))\big)^T$, $I(t) = (2,4)^T, 1 \leq i \leq 12$ and the time delay $\tau(t) = 0.15e^t/(1+e^t)$. So, it is easy to get $F_i = 1$, $\tau = 0.15$ and $\rho = 0.0375$. The coefficient matrices and inner coupling matrix of (11) are given as

$$D = \begin{bmatrix} 2.6 & 0 \\ 0 & 2.4 \end{bmatrix},\ C = \begin{bmatrix} 4.6 & 0 \\ 0 & 3.8 \end{bmatrix},\ A = \begin{bmatrix} 0.2 & -0.2 \\ -0.4 & 0.3 \end{bmatrix},\ B = \begin{bmatrix} -4 & -5 \\ -2 & -5 \end{bmatrix},\ \Gamma = \begin{bmatrix} 6 & 1 \\ 1 & 4 \end{bmatrix}.$$

The coupling matrix G is determined by the directed topology given in Fig. 1 with $G_{ij} = 0, 1 (i \neq j)$.

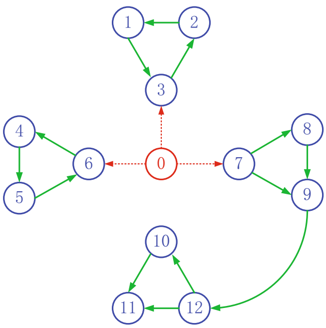

Fig. 1. Communication topology $\mathcal{G}$ and node 0 is the isolated objective node

Let the initial state of the objective system be $\tilde{\phi} = [3, -3]^T$ on the interval $[-0.15, 0]$ and initial functions for system (11) are chosen randomly. We use the quantity $E(t) = \sqrt{(1/12)\sum_{i=1}^{12} e_i^T(t)e_i(t)}$ to measure the quality of the synchronization process. Setting the pinning node set $\mathcal{V}_{pin} = \{3, 6, 7\}$ (see Fig. 1),

$\eta = 0.5205$ and the coupling strength $c = 30$, it is easy to check that the LMI (7) has a positive definite solution. Theorem 1 ensures that the whole coupled neural network system (11) can be synchronized to the given goal trajectory asymptotically.

The objective trajectory of the pinning controlled system (1) is shown in Fig. 2; and the state trajectories of (11) are given in Fig. 3. The synchronization error and the pinning feedback gains $\sigma_3(t)$, $\sigma_6(t)$ and $\sigma_7(t)$ are illustrated, respectively, in Fig. 4 and Fig. 5.

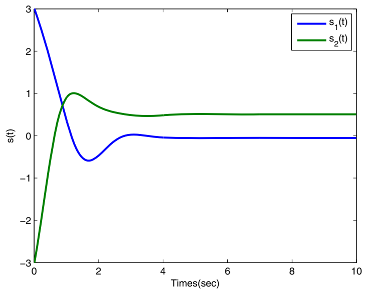

Fig. 2. State trajectory $s(t)$ in system (2)

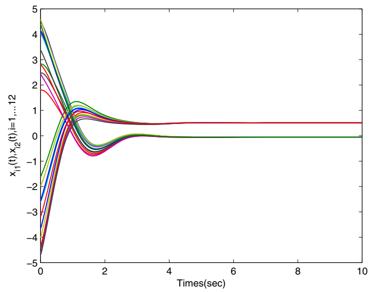

Fig. 3. State trajectories $x_i(t)$ in system (11)

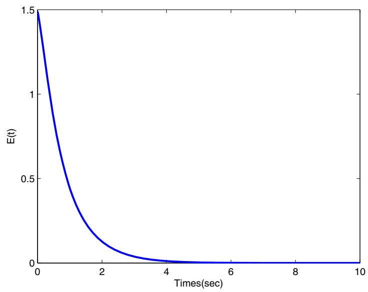

Fig. 4. Time evolution of synchronization error $E(t)$

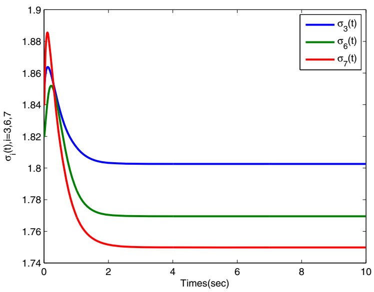

Fig. 5. Variations of pinning feedback gains

5 Conclusions

In this paper, the synchronization control problem of coupled inertial neural network systems is formulated based on adaptive pinning control strategy. By Lyapunov stability theory and LMI technique, some sufficient criteria have been established for the global asymptotically synchronization of the coupled system. A numerical example has been given to illustrate the usefulness of the obtained results.

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