A Novel Neural Network Based Adaptive Control for a Class of Uncertain Strict-Feedback Nonlinear Systems

Baobin Miao and Tieshan Li(✉)

Navigational College, Dalian Maritime University, Dalian, 116026, China
tieshanli@126.com

Abstract. In this paper, a novel robust adaptive tracking control approach is presented for a class of strict-feedback single input single output nonlinear systems. In the controller design process, all unknown functions at intermediate steps are passed down, and only one neural network is used to approximate the lumped unknown function of the system at the last step. Although some similar design themes have been proposed, the approach presented in this paper is more reasonable and simpler. The most contribution in this paper is that a new concept named "filter technique" is proposed for how to avoid generating new unknown functions when derivation of virtual control law in the backstepping based control methods. So the neural network is just used to approximate the finite or less unknown functions and the good capabilities in function approximation of neural network are guaranteed. Stability analysis shows that the uniform ultimate boundedness of all the signals in the closed-loop system can be guaranteed, and the steady state tracking error can be made arbitrarily small by appropriately choosing control parameters. Simulation results demonstrate the effectiveness of the proposed scheme.

Keywords: Filter technique · Neural network · Adaptive control · Nonlinear systems

1 Introduction

In the past years, backstepping based nonlinear adaptive control has been paid considerable attentions and a great deal of progress had been achieved for the adaptive control of strict-feedback nonlinear systems with linearly parameterized uncertainty [1]. Although significant progresses have been made by combining backstepping methodology with neural network technologies, there are still some problems that need to be solved for practical implementations. The main drawback of the aforementioned control design methods is the problem of complexity [2]. That is, the complexity of the designed controller grows drastically as the system order increase, this phenomenon is caused by four reasons. The first reason is that the repeated differentiations of the virtual control laws in the traditional backstepping approach. The second reason is that neural network is used to approximate major terms of unknown functions.

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The third reason is that with an increase of neural network nodes, the number of parameters to be estimated will increase significantly. The last reason is that the use of multiple approximators. All of above reasons make the complexity growing problem harder for implementation.

For the first reason behind the complexity growing problem, in [3], the authors addressed a modification that obviated the repeated differentiations of the command derivatives by introducing command filter technique in the backstepping design. A dynamic surface control technique was proposed to solve the complexity growing problem in [4]. In addition, the virtual control laws were modeled as portions of unknown functions that were approximated during operation in [5]. The point is that there is not only one question in the typical adaptive backstepping control, a new concept named "filter technique" is proposed for how to avoid generating new unknown functions when derivation of virtual control law. That is, the virtual control law a_i include the elements of $[x_1, x_2, \dots, x_i]$, while in the backstepping based control design process, it needs to calculate the derivative of the virtual control law, so it will generate new terms of unknown functions, i.e., $\frac{\partial a_i}{\partial x_j}\dot{x}_j$, where x_j for $j \le i$ is an element of the state vector. In order to solve this problem, the most contribution in this paper, a first-order filtering of the synthetic input is introduced at each step of the traditional backstepping approach. So the neural network is just used to approximate the finite or less unknown functions and the good capabilities in function approximation of neural network are guaranteed. Consequently, the second reason behind the complexity growing problem is removed.

In many practical applications on the control of uncertain nonlinear systems, neural network based control methods are shown to be more efficient compared with other modern control techniques and many remarkable results have been obtained. The most useful property of neural network is their ability to approximate arbitrary linear or nonlinear mapping through learning. Although there are significant advantages by employing neural network to control uncertain nonlinear systems, these neural network based schemes suffered from some limitations. For example, many approximators are still used to construct virtual control laws and actual control law in these methods. That is, for solving the uncertainty of nonlinear systems, every virtual control laws and actual control law are constructed by at least one neural network to approximate the unknown functions. So, if the uncertain nonlinear systems order is more than three, the computational burden grows due to the adaptive computation of these approximators. Although some novel themes had been proposed for solving this problem in [4], I think there exist some problems in their literatures: (1) the virtual control laws are composed by parts of unknown function, it is unreasonable. (2) the control signal u include the derivative of a_n, which requires the second derivative of a_{n-1}, which requires the third derivative of a_{n-2}, and so on, i.e., the repeated differentiations of virtual control laws and generating new unknown functions. Another central issue within approximation based adaptive control schemes is that the number of adaptation laws depends on the number of the neural network nodes. With an increase of neural network nodes to improve approximation accuracy, the number of parameters to be estimated will increase significantly. As a result, the on-line learning time will become prohibitively large. To solve this problem, the norm of the ideal weighting vector in neural network is considered as the estimation parameter instead of the elements of weighting vector. Thus, the number of adaptation laws is reduced considerably. So, both the third and the last reasons behind the complexity growing problem are removed.

In this paper, a neural network approximation based adaptive control approach is presented for a class of uncertain strict-feedback nonlinear systems. The most contributions are that a new concept named “filter technique” is proposed for how to avoid generating new unknown functions and a reasonable controller design procedure about using one approximator is proposed. In addition, by using a first-order filter, both problems of the repeated differentiations of the virtual control laws and generating new unknown functions are solved. These features guarantee that the computational burden of the algorithm can drastically be reduced and that the algorithm is convenient to implement in applications. Stability analysis shows that all the closed-loop system signals are uniformly ultimately bounded, and the steady state tracking error can be made arbitrarily small by appropriately choosing control parameters. Theoretical result is illustrated by simulation results.

2 Problem Formulation and Preliminaries

Consider a class of uncertain nonlinear dynamical systems in the following form:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(\bar{x}_i) & 1 \le i \le n-1 \\ \dot{x}_n = u + f_n(\bar{x}_n) \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, \ldots, x_i]^T \in R^i$, $i = 1, \ldots, n$, $u \in R$ and $y \in R$ are system state variables, system input, and output, respectively; $f_i(\bar{x}_i)$, $i = 1, \ldots, n$, are unknown smooth nonlinear functions.

The control objective is to design an adaptive controller for the system (1), such that all the close-loop system signals remain uniformly ultimately bounded, and the system output y follows the reference signal $y_r(t)$.

Notation 1. $\|\cdot\|$ stands for Frobenius norm of matrices and Euclidean norm of vectors, i.e., given a matrix B and a vector Q, the Frobenius norm and Euclidean norm are given by $\|B\|^2 = tr(B^T B) = \sum_{i,j} b_{ij}^2$ and $\|Q\|^2 = \sum_i q_i^2$.

3 Controller Design and Stability Analysis

In the following part, for the purpose of simplicity, the time variable t and the state vector $\overline{x}_i$ will be omitted from the corresponding functions.

Step 1: Let $z_1 = x_1 - y_r$, the derivative of z_1 is

$$\dot{z}_1 = x_2 + f_1 - \dot{y}_r \tag{2}$$

The virtual control law a_2^0 is chosen as follows:

$$a_2^0 = -k_1 z_1 + \dot{y}_r \tag{3}$$

where k_1 is a positive real constant which will be specified later.

Introduce a new state variable a_2 and let a_2^0 pass through a first-order filter with time constant e_2 to obtain a_2

$$e_2 \dot{a}_2 + a_2 = a_2^0 \tag{4}$$

Define the filter error as follows

$$p_2 = a_2 - a_2^0 \tag{5}$$

Let $z_2 = x_2 - a_2$ and consider the (3)-(5), we can get

$$\dot{z}_1 = z_2 + p_2 + f_1 - k_1 z_1 \tag{6}$$

Consider the following Lyapunov function candidate

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} p_2^2 \tag{7}$$

Differentiating V_1 yields

$$\dot{V}_1 = -k_1 z_1^2 + z_1 z_2 + z_1 p_2 + z_1 f_1 + p_2 \dot{p}_2 \tag{8}$$

It is worth noting that

$$\dot{p}_2 = -\frac{p_2}{e_2} + B_2\left(z_1, z_2, p_2, y_r, \dot{y}_r, \ddot{y}_r\right) \tag{9}$$

where $B_2(\cdot)$ is a continuous function and has a maximum value M_2 [3].

Using the facts that

$$z_1 p_2 \le z_1^2 + \frac{p_2^2}{4} \tag{10}$$

$$p_2 B_2 \le \frac{p_2^2 B_2^2}{2} + \frac{1}{2} \tag{11}$$

Apparently, if we choose $k_1 - 1 \ge a_0$, $\frac{1}{e_2} \ge \frac{1}{4} + \frac{M_2^2}{2} + a_0$ and consider the (9)-(11), the (8) can be rewritten as

$$\dot{V}_1 \le -a_0 z_1^2 - a_0 p_2^2 + z_1 z_2 + z_1 f_1 + \frac{1}{2} \tag{12}$$

Step i $(2 \le i \le n-1)$: Let $z_i = x_i - a_i$, the derivative of z_i is

$$\dot{z}_i = x_{i+1} + f_i - \dot{a}_i \tag{13}$$

The virtual control law a_{i+1}^0 is chosen as follows:

$$a_{i+1}^0 = -k_i z_i + \dot{a}_i - z_{i-1} \tag{14}$$

where k_i is a positive real constant which will be specified later.

Introduce a new state variable a_{i+1} and let a_{i+1}^0 pass through a first-order filter with time constant e_{i+1} to obtain a_{i+1}

$$e_{i+1}\dot{a}_{i+1} + a_{i+1} = a_{i+1}^0 \tag{15}$$

Define the filter error as follows

$$p_{i+1} = a_{i+1} - a_{i+1}^0 \tag{16}$$

Let $z_{i+1} = x_{i+1} - a_{i+1}$ and consider the (14)-(16), we can get

$$\dot{z}_i = z_{i+1} + p_{i+1} + f_i - k_i z_i - z_{i-1} \tag{17}$$

Consider the following Lyapunov function candidate

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{2} p_{i+1}^2 \tag{18}$$

Differentiating V_i yields

$$\dot{V}_i = \dot{V}_{i-1} + z_i z_{i+1} - z_i z_{i-1} + z_i p_{i+1} + z_i f_i - k_i z_i^2 + p_{i+1}\dot{p}_{i+1} \tag{19}$$

It is worth noting that

$$\dot{p}_{i+1} = -\frac{p_{i+1}}{e_{i+1}} + B_{i+1}\left(\overline{z}_{i+1}, p_2, \ldots, p_{i+1}, y_r, \dot{y}_r, \ddot{y}_r\right) \tag{20}$$

where $B_{i+1}(\cdot)$ is a continuous function and has a maximum value M_{i+1}.

Using the facts that

$$z_i p_{i+1} \le z_i^2 + \frac{p_{i+1}^2}{4} \tag{21}$$

$$p_{i+1} B_{i+1} \le \frac{p_{i+1}^2 B_{i+1}^2}{2} + \frac{1}{2} \tag{22}$$

Apparently, if we choose $k_i - 1 \ge a_0$, $\frac{1}{e_{i+1}} \ge \frac{1}{4} + \frac{M_{i+1}^2}{2} + a_0$, the (19) can be rewritten as

$$\dot{V}_i \le -a_0 \sum_{l=1}^{i} z_l^2 - a_0 \sum_{l=1}^{i} p_{l+1}^2 + \sum_{l=1}^{i} z_l f_l + z_i z_{i+1} + \frac{i}{2} \tag{23}$$

Step n: Let $z_n = x_n - a_n$, the derivative of z_n is

$$\dot{z}_n = u + f_n - \dot{a}_n \tag{24}$$

Consider the following Lyapunov function candidate

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2} \tilde{\theta}^2 \tag{25}$$

where $\tilde{\theta} = \theta - \hat{\theta}$. Differentiating V_n yields

$$\dot{V}_n = \dot{V}_{n-1} + z_n \left(u + f_n - \dot{a}_n \right) - \tilde{\theta} \dot{\hat{\theta}} \tag{26}$$

Choose the actual control law as

$$u = -k_n z_n - z_{n-1} + \dot{a}_n + \left(-\hat{\theta} \right)^{\frac{1}{2}} \tag{27}$$

one has

$$\dot{V}_n \le -a_0 \sum_{l=1}^{n-1} z_l^2 - a_0 \sum_{l=1}^{n-1} p_{l+1}^2 + \sum_{l=1}^{n} z_l f_l + \frac{n-1}{2} - k_n z_n^2 + z_n \left(-\hat{\theta} \right)^{\frac{1}{2}} - \tilde{\theta} \dot{\hat{\theta}} \tag{28}$$

Given a compact set $\Omega \subset R^n$, and let W^* and ε be such that for any $Z \in \Omega$,

$$\sum_{l=1}^{n} z_l f_l = W^{*T} S\left(Z \right) + \varepsilon \tag{29}$$

where $\left| \varepsilon \right| \le \varepsilon^*$.
the (28) can be rewritten as

$$\dot{V}_n \le -a_0 \sum_{l=1}^{n} z_l^2 - a_0 \sum_{l=1}^{n-1} p_{l+1}^2 + \varepsilon^* + \frac{n-1}{2} + \frac{\left\| S\left(Z \right) \right\|^2}{4} + \tilde{\theta} - \tilde{\theta} \dot{\hat{\theta}} \tag{30}$$

The adaptive law is chosen as

$$\dot{\hat{\theta}} = -k_0\hat{\theta} + 1 \tag{31}$$

where k_0 is a positive constant.

It is worth noting that

$$k_0\tilde{\theta}\hat{\theta} \le -\frac{k_0}{2}\tilde{\theta}^2 + \frac{k_0}{2}\theta^2 \tag{32}$$

Choose $\frac{k_0}{2} \ge a_0$, one has

$$\dot{V}_n \le -2a_0V_n + d \tag{33}$$

where $d = \frac{\left\|S(Z)\right\|^2}{4} + \varepsilon^* + \frac{n-1}{2} + \frac{k_0}{2}\theta^2$.

From (33), one has

$$V_n(t) \le \frac{d}{2a_0} + \left(V_n(t_0) - \frac{d}{2a_0}\right)e^{-2a_0(t-t_0)} \tag{34}$$

It follows that, for any $\mu_1 > (b_0/a_0)^{1/2}$, there exists a constant $T > 0$ such that $z_1(t) \le \mu_1$ for all $t \ge t_0 + T$, and the tracking error can be made small, since μ_1 can arbitrarily be made small if the design parameters are appropriately chosen.

4 Simulation Examples

In this section, an example will be used to test the effectiveness of the proposed controller. Consider the following nonlinear system:

$$\begin{cases} \dot{x}_1 = x_2 + x_1^2 \\ \dot{x}_2 = x_3 + x_1^2 + x_2^2 \\ \dot{x}_3 = u + x_1x_3 \end{cases} \tag{35}$$

The reference signal is given as $y_r = 0.5\left(\sin(t) + \sin(1.5t)\right)$. The design paremeters of the above controller are $k_1 = 10$, $k_2 = 5$, $k_3 = 5$, $e_2 = 0.01$, $e_3 = 0.01$, $k_0 = 10$. The simulations are run with the initial conditions $x(0) = [2,0,0]^T$ and $\hat{\theta}(0) = -1$. The simulation results are shown in Figs. 1-2.

As it can be seen from the simulation results, good tracking accuracy and the stability of the closed-loop system are guaranteed under the proposed controller.

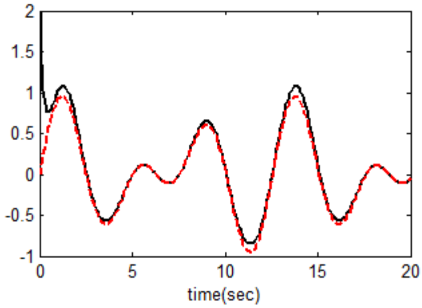

Fig. 1. Output tracking performance (y—solid line and yr—dashed line)

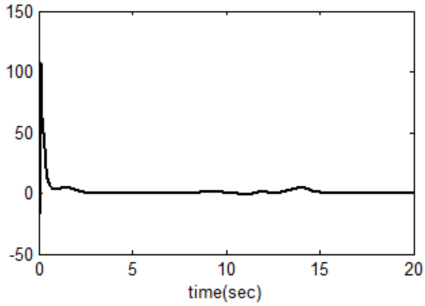

Fig. 2. The control law u

5 Conclusion

In this paper, the tracking control problem has been considered for a class of uncertain nonlinear systems with a strict-feedback structure. In the controller design process, only one neural network approximator is used to address the lumped unknown function of the system. In addition, by using the "filter technique", both problems of the repeated differentiations of the virtual control laws and generating new unknown functions are solved. By this approach, the structure of the controller can be simplified observably, and the computational burden can be reduced drastically. The main feature of the control scheme proposed in this paper is simplicity. In particular, no matter how many neural network nodes are used, there is only one parameter to be updated online. The proposed controller is derived in the sense of Lyapunov function, thus the system can be guaranteed to be asymptotically stable. Simulation results demonstrate the effectiveness of the proposed scheme.

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