Theon of Smyrna and Ptolemy on Celestial Modelling in Two and Three Dimensions

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Ptolemy, as is well known, devoted the great part of his *Almagest* (properly, *Mathematical Composition*) to modelling the motions of the Sun, Moon, and planets by means of combinations of circular motions that are represented in his geometrical analyses by two-dimensional assemblages of circles, straight lines, and points. It is a little less well known that his later work, the *Planetary Hypotheses*, proposed systems of three-dimensional bodies, composed of ether (αἰθήρ, the Aristotelian fifth, celestial element) and having spherical and planar surfaces, that were supposed to be the physical realities performing the various revolutions that the *Almagest*'s circles describe geometrically.

The three-dimensional, physical cosmology of the *Planetary Hypotheses* was not an abrupt innovation that Ptolemy imposed after the fact on the geometrical theories he worked out the *Almagest*. Its foundations include assumptions expressed, albeit sometimes briefly and in passing, in the *Almagest* and in works that Ptolemy composed before the *Almagest*, as well as ideas that were already current in Platonist and Peripatetic philosophy before Ptolemy's time, for which we have a valuable witness in the one surviving work by the Platonist philosopher Theon of Smyrna. This paper attempts to delineate some of this background and to show what elements were indeed new in the *Planetary Hypotheses*.

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1 Theon of Smyrna

Theon of Smyrna "the Platonist," the author of an extant (though incomplete) book *On the Mathematics Useful for Reading Plato* in addition to several other works of Platonic scholarship that have not come down to us, is almost certainly identical to a "Theon the Platonist Philosopher" whose portrait bust, purchased in Smyrna in the 17th century, is now in the Capitoline Museum.[1] The inscription on the bust's base identifies not only the subject but also the person who commissioned it, Theon "the Priest," who was the philosopher's son. Technical as well as stylistic grounds securely date the portrait to the later part of Hadrian's reign, in the 120s or 130s of our era, making Theon an older contemporary of Ptolemy, though we have no reason to believe that Ptolemy was acquainted with Theon or his writings.[2]

Theon's book consists of several sections on various branches of mathematics (broadly conceived), of which the one on astronomy is the longest.[3] The title accurately indicates Theon's purpose and intended readership: philosophical students who had a limited mathematical education—chiefly the earlier parts of the *Elements* (H16)—and who could benefit from some background on topics such as properties of whole numbers, harmonics, and astronomy that are alluded to in the dialogues of Plato that were widely read at this period, among which the *Timaeus* and *Republic* were prominent. Theon acknowledges a heavy dependence on earlier works, mostly by philosophical writers in the Platonist or Peripatetic traditions rather than mathematicians or scientists.[4] Most of the astronomical section, according to his own repeated statements, came from a certain Adrastos, whom we know from other authors who refer to him as a Peripatetic from Aphrodisias. In his commentary on Ptolemy's *Harmonics* (ed. Düring 1932, 96) Porphyrios (3rd century AD) quotes as from Adrastos's *Commentary on the Timaeus* a passage that we find *verbatim* in Theon (H50–51), and this was presumably the source work for the rest of Theon's Adrastian material.[5] There are also extensive and very close parallels in Calcidius's Latin *Timaeus* commentary (4th century AD) to passages in the part of Theon dependent on Adrastos. Calcidius does not acknowledge his

[1] Musei Capitolini inv. 529. Richter (1965, 3.285).

[2] For more detailed discussion of the evidence for Theon's biography see Jones (forthcoming). The "Theon the mathematician" from whom Ptolemy obtained reports of observations of Mercury and Venus at greatest elongation (*Almagest* 9.9 and 10.1–2) can hardly have been Theon of Smyrna, as has often been suggested. No astronomer of the second century AD could have been unaware that the planets exhibit two anomalies or could have believed that their stationary points occur when they are at greatest elongation from their mean longitudes.

[3] References to passages in Theon's book will be by the pages of Hiller's edition, Hiller (1878).

[4] Hence the absence of references to Ptolemy or of knowledge of the *Almagest*'s contents can hardly stand an argument that Theon's book was written before the *Almagest* (completed soon after AD 146/147, the date of Ptolemy's *Canobic Inscription*). The *only* good evidence for Theon's date is the bust, which portrays him as a man in his prime, so if he was still living when it was made, he might still have been living and writing in the 150s.

[5] Earlier Porphyrios (ed. Düring 1932, 7) quoted another passage from Adrastos, without specifying the work's title, that is matched *verbatim* in Theon (H50).

source; but if he was using Theon, it is hard to explain why he avoided drawing on anything that Theon identifies as his own contribution or as coming from source texts other than Adrastos, so it seems most probable that Calcidius had access either directly to Adrastos's book or to a digest independent of Theon's. If this is correct, Theon hewed closely to Adrastos in wording as well as content.

Adrastos is usually assigned to the period immediately preceding or even contemporary with Theon, but while this is certainly possible, nothing in his reported writing is incompatible with a date anywhere within the first century AD or even the first century BC.[6] The latest earlier author to whose work he is known to have referred is Hipparchos (fl. 160s–120s BC). The hypotheses for the Sun's motion that he discussed are versions of the one that Ptolemy attributes to Hipparchos (though Adrastos, at least as we read him through the filters of Theon and Calcidius, does not expressly associate them with Hipparchos), and he apparently believed that the planets, like the Sun, exhibited only the single periodic anomaly manifested by their synodic cycles. Ptolemy (*Almagest* 9.2) tells us that Hipparchos had written a book showing that hypotheses for the planets that assumed only the synodic anomaly were inconsistent with observations. It does not follow, of course, that the publication of such a book would have led all astronomers, let alone philosophers, to instantly abandon single-anomaly theories; nevertheless, the closer one gets to Ptolemy's time, the less likely it would be that an author with up-to-date knowledge of the astronomical literature would have been unaware of the planets' zodiacal anomaly and the consequent need for hypotheses more complicated than Adrastos's and Theon's simple epicycles or eccenters.[7] We know that such hypotheses were circulating at least as early as the mid first century AD, because Pliny (d. AD 79), in his confused discussion of the planets (2.13/63–64), reveals an awareness, if not an understanding, of hypotheses that involved an eccentricity to account for zodiacal anomaly. If Adrastos lived as late as is generally supposed, therefore, he was quite out of touch with contemporary astronomy, something that can be said about Theon without reservation.

A point of interest concerning Adrastos is that a philosopher who identified himself as a Peripatetic in this period (i.e. anywhere between the 1st century BC and the early 2nd century AD, given the uncertainty about his date), and whose other known writings were devoted to Aristotelian topics,[8] would have composed a didactic commentary on the *Timaeus*, and moreover one on which a subsequent

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[6] See for example Moraux (1984, pp. 294–295), and (Sharples 2010, p. 22), neither of whom give any reason for their confidence that Adrastos wrote in the first half of the 2nd century AD except for the *terminus ante quem* provided by Theon.

[7] The resistance of astronomers and astrologers, even after Ptolemy's time, to accept Hipparchos's discovery of precession is a striking illustration of how scientific arguments that may appear cogent to us (reading them as they are presented in Ptolemy's *Almagest*, since the relevant works by Hipparchos have not survived) could be disregarded in the contemporary scientific community; see Jones (2009). The inadequacy of single-anomaly planetary models would have been harder to ignore, however, especially after the Babylonian predictive models for planetary phenomena had become prevalent in the Greek world.

[8] Moraux (1984, pp. 314–330).

Platonist philosopher would rely for so much of his material. This was symptomatic of the closeness that existed between Middle Platonism and contemporary Peripateticism; inasmuch as Theon is our most abundant informant for the approach to astronomical cosmology that supplied the building blocks for Ptolemy's, we cannot label this approach exclusively as belonging to one of these schools or the other.

Because Theon's *The Mathematics Useful for Reading Plato* is a comparatively neglected text, I will summarize his treatment of celestial modelling in some detail, in addition to providing in an appendix an English translation of the passage that is most central to this paper's concerns.[9]

The section on astronomy in Theon's book (H120–205) begins with material, drawn from Adrastos, that is similar in scope to *Almagest* 1.2–8 (H120–133): the spherical shape and revolution of the heavens and the spherical shape, central location, and minuscule relative size of the Earth, supported by mostly empirical arguments; definitions of the celestial equator, tropic circles, and arctic circles; and explanation of the inclined zodiacal belt as the apparent pathway of the Sun, Moon, and planets. After a general discussion of the planets and their motion and phenomena (H134–147), Theon takes up the topic of their apparent anomalies.

The first portion of this passage (H147–152) is ascribed through repeated citations to Adrastos. It begins with the phenomena that, while the Sun and Moon are always seen to move eastward relative to the fixed stars, the planets exhibit apparent reversals of direction. The reality, however, is that the cosmos consists effectively of three tiers of entities in motion: the outermost, which moves with uniform revolution concentric with the cosmos; below this, the circular but *apparently* nonuniform motion of the Sun, Moon, and planets, and innermost, the truly irregular motion of bodies that experience generation and corruption. The entities of the outer tiers are described as divine and eternal. Physical necessity dictates that the other heavenly bodies must, like the fixed stars, move in a simple, uniform, and orderly manner, where Adrastos defines uniform motion as travelling equal intervals in equal times and orderly motion as motion without stoppings and reversals of direction. Nevertheless all the heavenly bodies (other than the fixed stars) exhibit apparent nonuniformity while some (the five planets) also exhibit disorderly motion, i.e. changes of apparent direction. Two causes are given for the appearance of nonuniformity, firstly that the bodies travel on circles that are distinct from (i.e. not concentric with) the zodiac that they are seen as traversing, and secondly that they travel on combinations of more than one circular path.

To illustrate this general explanation (H152–166), Theon cites the phenomenon that the intervals between the equinoxes and solstices are observed as being unequal in duration, which means that the Sun appears to traverse the four equal quadrants of the zodiacal circle in unequal times. Two hypotheses are shown to be able to account for this: the eccentric hypothesis, according to which the Sun travels uniformly on a circle that is not concentric with the cosmos, and the epicyclic

[9] Theon's book has twice been translated into French (Dupuis 1892 and Delattre Biencourt 2010). I am not aware of any complete translations into other modern languages except for the English version of Lawlor and Lawlor (1979), which is derived from Dupuis's French, not Theon's Greek.

hypothesis, according to which the Sun travels uniformly on an epicyclic circle that itself travels uniformly on a circle concentric with the cosmos.[10] Theon castigates the "mathematicians" (μαθηματικοί) for disagreeing among themselves whether the motions of the heavenly bodies are produced only by epicycles on deferents or only by eccenters, since, he says, it will be shown subsequently that the heavenly bodies trace (γράφοντες) all three kinds of circle as nonessential consequences (κατὰ συμβεβηκός). (It will only later become clear that Theon means this in two ways, firstly that each hypothesis, taken as the valid one, generates the circular paths of the other hypothesis incidentally, and secondly that there exists a third hypothesis, the valid one, that incidentally generates the paths of both the epicyclic and eccentric hypotheses.) Using diagrams similar in kind to those of the *Almagest*, Theon demonstrates how each of these hypotheses "save the phenomena," i.e. lead, under appropriate conditions, to the observed inequality of the seasons.

Theon now (H166–172) raises the question (which he tells us Hipparchos considered worthy of mathematical understanding) of how it is that two seemingly very different hypotheses can lead to the same phenomena.[11] In response, he tells the reader that Adrastos demonstrated that the eccentric hypothesis arises as a nonessential consequence of the epicyclic, and vice versa.[12] In fact Theon gives two demonstrations of the former claim, one applying just to the situations where the center of the epicycle is situated on the apsidal line or on the line through the center of the cosmos and perpendicular to the apsidal line, while the second demonstration is for the general situation. The demonstration of the converse is also general.

These demonstrations, Theon continues (H172–174), can be extended to the Moon and planets, with the qualification that whereas the Sun has *almost* identical periods of revolution in longitude, latitude, and anomaly so that a model with a fixed eccenter or one with the same periods of revolution for the epicycle eastward around the center of the cosmos and of the Sun around its epicycle (in the opposite sense of revolution) suffice, whereas these periods are significantly different from each other for each of the remaining bodies. This diversity of periods, which in the case of the five planets is manifested in their retrogradations, has to be reflected in

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[10] Theon cites the specific intervals 94 1/2 days from vernal equinox to summer solstice, 92 1/2 days from summer solstice to autumnal equinox, 88 1/8 days from autumnal equinox to winter solstice, and 90 1/8 days from winter solstice to vernal equinox; the first two of these are the ostensibly observed intervals from which Ptolemy derives his eccenter model, which he attributes to Hipparchos (*Almagest* 3.4). Theon's eccenter model also has identical parameters to Ptolemy's, with eccentricity 1/24 of the eccenter's radius in the direction of Gemini 5 1/2°, and his epicyclic model's parameters are equivalent. Subsequently (H188), Theon will claim that Hipparchos expressed a preference for the epicyclic model, which is not something that one can infer from anything Ptolemy says.

[11] Tannery (1893, pp. 60–61), followed later by van der Waerden (1988, p. 180), tendentiously interpreted this passage as stating that Hipparchos admitted to not understanding the kinematic equivalence of the epicyclic and eccentric hypotheses, and on this basis both maintained, perversely, that Hipparchos could not have been a competent mathematician.

[12] The construction of Theon's sentence implies that Adrastos gave both the derivation of the eccentric hypothesis from the epicyclic and that of the epicyclic from the eccentric; the parenthetic phrase "as I say" (ὥς δὲ ἐγώ φημι) is probably not an indication that Theon has added the second demonstration in his own right but just a reminder that the subject was mentioned earlier.

the model; thus in an epicyclic model the period of the planet's revolution around the epicycle is in some cases faster and in other cases slower than that of the epicycle around the center of the cosmos. Using simple *Almagest*-style diagrams, Theon stipulates (H175–177) that for an epicyclic model to "save the phenomena," the direction of revolution of the body around the epicycle must be opposite to that of the epicycle's own eastward revolution, but for the planets both revolutions are in the same (eastward) direction; and again, for an eccentric model, the center of the eccenter must revolve in the eastward direction on a circle that is concentric with the cosmos and equal in size to the epicycle of the alternative model (presuming that the eccenter is the same size as the deferent of the other model), while the eccenter revolves in an unspecified direction around its center, carrying the body with it.

In bringing this passage to a close, Theon tells us that he has been following Adrastos's account of how the hypotheses of the "mathematicians" can be reconciled (προσοικειῶσαι) with each other. This repeated mention of the "mathematicians" marks the transition to the crucial passage (H177–189) translated in the appendix of this paper. There are no further nods to Adrastos in the passage. However, parts of Theon's discussion of Aristotle's cosmology (H178–179) are paralleled in Calcidius (ed. Waszink 1962, 135–136) so that this much, at least, probably comes from Adrastos, and since this includes a compact outline of the principles of Theon's "physical" hypotheses for the heavenly bodies later in the passage, it appears that Theon was at most working out a specific implementation of an approach that was already known in Peripatetic (and presumably Platonist) cosmology. It may in fact, as Evans has suggested, be the kind of planetary modelling that Geminos (mid 1st century BC) refers to as σφαιροποιία, "sphere-assemblage," though Geminos provides no details.[13]

Theon likens the "mathematicians" to the Babylonian, Chaldean, and Egyptian astronomers of old, who, he alleges, fitted mathematical methods to the phenomena with a view only to prediction. An allusion to the *Epinomis* (which Theon regards as an authentic work of Plato's) highlights the pejorative implication of this association of the "mathematicians" with non-Greeks, since it is meant to call to mind "Plato's" remark (987d) that "whatever Greeks take over from barbarians is in the end turned by them into something finer." In this instance what the Greeks—but not the "mathematicians" among them—have contributed is reasoning according to nature (φυσιολογία), that is, explanation in terms of physical causes.[14]

[13] Evans (2003). Evans further speculates that the three-dimensional physical approach to modelling was associated with the epicyclic and eccentric hypotheses already at the time of their introduction (which he attributes to Apollonios of Perge); but if that was the case, it is hard to understand why both Theon and Geminos (see next note) assert that mathematical astronomers took no account of physical causation.

[14] Similarly, Geminos, in a passage from his lost digest of Posidonios's *Meteorology* quoted by Simplikios (*Commentary on Aristotle's Physics* 291–292) by way of Alexander of Aphrodisias, contrasts the methods used to address celestial phenomena in astronomy (ἀστρολογία), which are mathematical, and those used in φυσιολογία, with a certain bias in favor of the latter.

Aristotle is made to supply these physical causes by way of his adaptation of the systems of "homocentric" spheres of Eudoxos and Kallippos as outlined in *Metaphysics* Λ and *De Caelo*. These spheres, being three-dimensional bodies composed of the Aristotelian "fifth body" or ether, are an appropriate cause for the motions of the heavenly bodies, as the circles and more complicated "spiral" curves of the "mathematicians" are not. According to Theon (in one of the presumed Adrastian sentences that also appear in Calcidius), not all of Aristotle's etherial spheres were shells concentric with the cosmos, but some were solid spheres embedded in the interval between the interior and exterior surfaces of spherical shells (τὰς ἐν τ βάθει τούτων [scil. τῶν κοίλων σφαιρῶν]… στερεάς), which is, to put it mildly, a bold reading of Aristotle's cosmology. It turns out presently that this is how Theon, or Adrastos, interprets Aristotle's "unwinding spheres": they are lodged like ball bearings or (in Theon's remarkable simile) gears in the intervals between revolving shells to effect the transfers and reversals of their spinning motions.

Having thus procured Aristotle's blessing for the principle that the heavens can be composed of an assemblage of spherical shells concentric with the cosmos and of solid spheres embedded in the thickness of the shells, Theon shows how a hypothesis can be formed out of such bodies that results in the path for a heavenly body identical to the path generated by an epicyclic or eccentric hypothesis. It is essentially a three-dimensional filling out of an epicyclic model (Fig. 1, showing the epicycle in four positions), though the diagram illustrating it in Theon's text (Fig. 5) is necessarily just a cross-section of the hypothesis in the plane of the orbit. The planar circle of the epicyclic model becomes a great circle of a solid sphere that revolves on an axis perpendicular to the plane of the epicycle. The visible heavenly body is fixed on the surface of this solid sphere. The solid sphere in turn is assumed

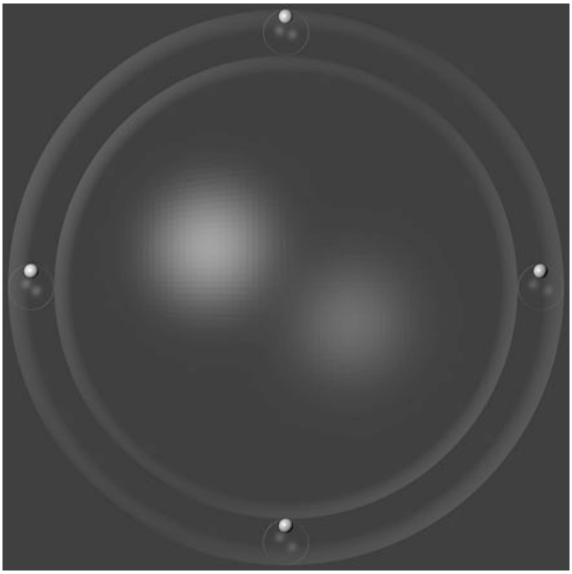

Fig. 1 Theon's three-dimensional epicyclic hypothesis for the Sun

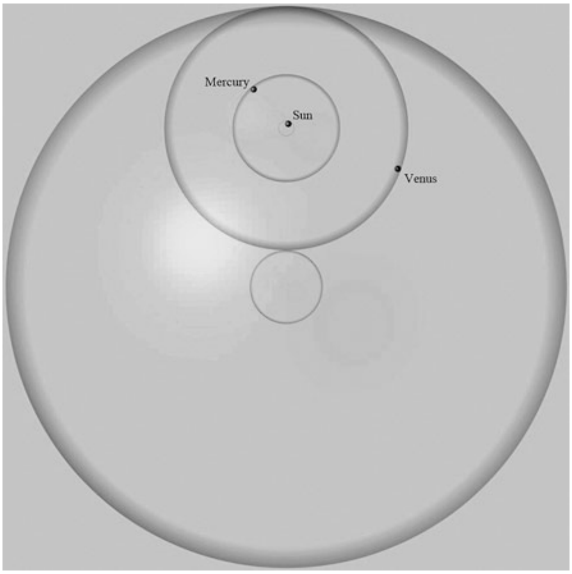

Fig. 2 Theon's nested epicyclic hypothesis for the Sun, Mercury, and Venus

to be embedded in a spherical shell concentric with the cosmos, such that the exterior and interior surfaces of the shell are tangent to the solid sphere. The shell revolves on an axis perpendicular to the plane of the deferent, so that the center of the solid sphere traces out the deferent, while the heavenly body traces out the epicycle; or, taking into consideration the combined motions of the solid sphere and the shell, the heavenly body traces out the eccenter.

Theon further points out that a common hypothesis of this kind could serve for the Sun, Mercury, and Venus; in this case, the single solid sphere tangent to the exterior and interior surfaces of the revolving shell is replaced by a spherical shell bearing Venus, enclosing a second shell bearing Mercury, which in turn encloses a solid sphere bearing the Sun (Fig. 2). It is noteworthy that Theon, while giving this model only as a possible alternative to having separate hypotheses for each body, expresses a "suspicion" that the combined hypothesis is correct, on the grounds that it would provide a suitable seat for the cosmic soul; notwithstanding the apparently mechanical conception of how revolving motions are imparted to the spheres that was ascribed to Aristotle, Theon believes that the cosmos is a single living being.

Theon's cosmological system is, in comparison to the one Ptolemy was to devise, a rather modest affair. The Sun, Moon, and planets are each assumed to have just a single periodic anomaly, and the building blocks of Theon's hypotheses, spherical "deferent" shells concentric with the cosmos in combination with solid "epicyclic" spheres or concentric shells lodged in the thickness of the shells, only suffice to model a body with a single anomaly. Although his reading of such embedded spheres into Aristotle seems anachronistic and perverse, the approach does make sense as a way of preserving a significant part of the Eudoxian cosmology while making it possible to explain anomaly as a function of motion in "depth," i.e. by means of a kind of epicyclic motion. Theon has a fairly precise conception of how to build a viable quantified model for the Sun, but his accounts of how to extend such models to the Moon and planets suffer from vagueness with respect to the dimensions and rates of revolution of the components as well as how latitudinal motion is to be effected. A technically oriented "mathematician" astronomer of his time would likely have dismissed the whole enterprise as simplistic.

2 Ptolemy's Planetary Hypotheses

The *Planetary Hypotheses* consists of several sections.[15] First (1A.1–2/BM81b–82a), Ptolemy gives a brief introduction explaining the work's purpose and relationship to the *Almagest*. The next section (1A.3–14/BM82a–88a), accounting for a little over half of Book 1, is a technical description of Ptolemy's "hypotheses" or models for the Sun, Moon, and planets as consisting of combinations of circles. This is followed by a brief section (1B.1/BM88a–88b) reviewing in a qualitative manner the various kinds of north-south motion exhibited by each of the heavenly bodies according to their models. The fourth, longer, section (1B.2–5/BM88a–92a) takes up the topic of the sizes and distances of the heavenly bodies. A discussion of the visibility conditions of the stars and planets and visual misjudgment of their apparent sizes (1B.6–7/BM92a–92b) brings Book 1 to a close. The first section of Book 2 (2.1–10/BM93a–96b) is devoted to general considerations with respect to the nature, shape, and motion of the etherial bodies that make up the systems for the heavenly bodies, followed by a detailed description (2.11–16/BM96b–101b) of the configuration of each system.[16] Ptolemy gives alternative versions for the systems of the Sun, Moon, and planets, in one of which the bodies are all bounded by complete spherical surfaces, while in the other some of the bodies are truncated by planes perpendicular to the bodies' axes of revolution, so that they take the form of

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[15] Only a part of Book 1, from the beginning to close to the end of the description of the models (ending at ed. Heiberg 104 line 23 "ἰσοταχῶς"), survives in Greek; the completion of this section that appears in some manuscripts is a later restoration. Heiberg (1907, pp. 70–107) presents the Greek text facing L. Nix's German translation of the corresponding part of the Arabic version. We will use "1A" to designate this first part of Book 1. Nix's translation of the Arabic version of Book 2 follows on pp. 111–145. Morelon (1993) has edited and translated into French the Arabic version of Book 1, and Goldstein (1967, pp. 13–55) provides a facsimile of the manuscript BM Arab. 426 with an apparatus including collation of Leid. Arab. 1155; for his English translation of the part of Book 1 not extant in Greek ("1B") see pp. 5–9. Unfortunately the patchwork state of publication of the *Planetary Hypotheses* makes a consistent and convenient system of reference difficult. We will refer to passages in two ways: first by the numbers of the sections into which the text has been divided by Heiberg and Goldstein (unfortunately in his translation only, and numbered starting with 1 instead of continuing Heiberg's sequence), and secondly by the folios of BM Arab. 426, which are indicated in Goldstein's facsimile and the margins of Morelon's text. Thus "1A.2/BM81b" means Sect. 2 of the first part of Book 1 (appearing on pp. 72–73 in Heiberg 1907) and f. 81^b of BM Arab. 426 (appearing on p. 13 of Goldstein 1974 and pp. 16–17 of Morelon 1993).

[16] In the absence of a critical edition of Book 2, the analysis of this part by Murschel (1995) is indispensable.

disks or rings. Then a short section (2.17/BM101b–102a) concerns the number of distinct etherial bodies in Ptolemy's entire cosmic system, and in conclusion (2.18/BM102a–102b) Ptolemy explains how a "table-top" model of his system could be made to show the agreement of the theories with observation, with the components set in appropriate positions according to a set of mean motion tables appended to the end of the *Planetary Hypotheses* (the tables have not survived).

The somewhat disjointed structure of the *Planetary Hypotheses* is at least in part a reflection of the fact that Ptolemy did not compose the work with a single category of reader in mind. In his preface (1A.1/BM81b) he identifies his intended readership as "ourselves and those who choose to set these things [*scil.* the celestial models] in an instrumental construction" (ὑπό τε ἡμῶν αὐτῶν καὶ τῶν εἰς ὀργανοποιίαν ἐκτάσσειν αὐτὰ προαιρουμένων). Whom does he mean by "ourselves"? Ptolemy's addressee is the same Syros, otherwise unknown to us, to whom he dedicated most of his astronomical writings, including the *Almagest*, as well as the astrological *Tetrabiblos* (but not the *Criterion*, *Harmonics*, or *Geography*). Syros must therefore have been adept in mathematics and mathematical astronomy; but it is not primarily in this capacity that Ptolemy seems to be speaking of "ourselves" in the *Planetary Hypotheses* but rather as philosophers interested in the cosmological issues that flowed chiefly from Plato and—especially—Aristotle.[17]

Thus the section in Book 1 in which Ptolemy singles out for special attention the north-south aspect of the apparent motions of the heavenly bodies, and differentiates between the kinds of north-south motion that are due respectively to the obliquity of the ecliptic, the inclination of the eccenter, and the inclination of the epicycle, seems pointless from either an astronomical or a mechanical point of view. Its ultimate motivation is in Aristotle, *On Generation and Corruption* 2.10, where the annual north-south motion of the Sun caused by the obliquity of the ecliptic is characterized as alternate approach and recession, and assigned the key role of instigating the cycles of generation and corruption in the sublunary part of the cosmos. Theon (H148–149) shows how this had come to be generalized to apply to the other heavenly bodies: "For the sake of the numbering of time and the transformation of the things near the Earth and far from the Earth, the travel of the wandering (stars) came to be; for the things here (below the heavens) also transform in all ways together with their (scil. the heavenly bodies') turnings (τροπαί) as they approach and recede." Extension of the term τροπαί, which normally refers to solstices, to the reversals of north-south motion of the Moon and planets meant that their latitudinal cycles were also regarded as significant in maintaining the

[17] Ptolemy's philosophical positions, though influenced in some respects by the other Hellenistic sects, are closest to Middle Platonism and contemporary Peripateticism (Feke and Jones 2010). In some outstanding recent discussions of the *Planetary Hypotheses* (Murschel 1995, p. 36; Swerdlow 2005, p. 66) the significance of the "ourselves" in the preface to the *Planetary Hypotheses* is passed over, resulting (in my opinion) in an overemphasis on the construction of instruments to simulate and represent the celestial bodies as if this was the primary or sole purpose of the work.

processes of change in the sublunary world; hence Ptolemy's concern with analysing the more complex patterns of north-south motion into their constituents.

In Book 2, Ptolemy's presentation of the three-dimensional physical systems for the heavenly bodies is intermixed with extended criticisms of the cosmology of Aristotle's *Metaphysics* Λ.8. Again from the point of view of the mathematical astronomer this appears to be a strange preoccupation with a long abandoned theory, one to which Ptolemy had not made any allusion in the *Almagest*; but we have seen from the example of Theon that for a Platonist or Peripatetic of Ptolemy's time the introduction of eccentric and epicyclic motion into the apparatus of astronomical modelling had not made Aristotle's, or indeed Plato's, cosmological pronouncements irrelevant. Theon diminishes the conceptual interval between the fourth century BC cosmologies and the not-quite-as-out-of-date simple eccentric and epicyclic models that he describes by attributing opinions to Plato and Aristotle that we would consider anachronistic: the interpretation of Aristotle's "unwinding spheres" as epicyclic spheres rolling between Eudoxos's homocentric spheres, and the suggestion that the Myth of Er reveals Plato as an advocate of epicyclic modelling. For Ptolemy, how corporeal spheres transmit or do not transmit their motions to the spheres that they enclose is a problem that remains crucial for contemporary physical cosmology, and that Aristotle mishandled.

Aristotle's purpose in *Metaphysics* Λ.8 was not to describe for its own sake a physical system of spherical shells conforming to the astronomical hypotheses of Eudoxos and Kallippos, but to obtain a tally of the number of divine unmoved movers that must exist on the hypothesis that one mover is required for each sphere; he concludes that it is "plausible" (εὔλογον) that there forty-seven movers for forty-seven spheres. It is without doubt in response to Aristotle that Ptolemy takes the trouble of counting the number of moving parts in his own cosmic system, finding the total to be as few as thirty-four or indeed as few as twenty-two, depending on whether one requires all the spheres and spherical shells to be complete or allows some to be truncated.[18] Ptolemy does not ascribe the cause of their motion to unmoved movers, but rather to planetary souls; each of the visible heavenly bodies is the seat of a single soul that causes and governs the motions of all the etherial bodies, visible and invisible, that constitute that body's system. Hence each heavenly body's system is a kind of divine animal whose mobile components can be likened to the limbs of a terrestrial animal. One passage (2.3/BM93a) appears to speak also of the entirety of the cosmos as a single animal, though such a unitary conception does not arise in any obvious way in his detailed account of the systems and their operation.[19] Like the question of how many moving bodies exist in the heavens, the question of the causes of their motions was extraneous to the

[18] In the "complete spheres" systems some of the bodies both are enclosed within and enclose other bodies that have identical revolutionary motions. Truncation allows these neighboring bodies to become a single body wrapping around the top and bottom.

[19] Murschel (1995, p. 38). The expression in question (*al-ḥayawān al-kullī*) may, however, render a Greek expression such as (μόρια) τοῦ παντὸς ζ ου, meaning just "the animal taken as a whole," as Nix interprets it.

mathematical astronomy of the *Almagest*, but in writing the *Planetary Hypotheses* Ptolemy evidently anticipated readers for whom these were more important than the technical details of planetary motion.

Those technical details, as laid out in terms of *Almagest*-style circles in Book 1 ("for the time being fitting the motions to the circles themselves, as if they were liberated from the spheres that contain them", 1A.2/BM82a) and in terms of solids in Book 2, would obviously be directed at the craftsmen whom Ptolemy exhorts in his preface to construct tangible models of the heavens that make visible the realities rather than the mere phenomena, that is, the eccenters and epicycles rather than the apparent changing speeds and the alternation of direct and retrograde motion. Beyond this insistence that the models should show what we *cannot* directly see in the sky, not just mimic what we *can* see,[20] Ptolemy is not very specific about the form the models should take, and he allows for both models composed of unconnected pieces that can be slid manually into appropriate orientations and models in which the motions are coordinated in appropriate ratios by "mechanical methods" (διὰ τῶν μηχανικῶν ἐφόδων), probably meaning gear trains. Book 1's descriptions of the celestial models are expressed in the language of geometrical construction, which is conceptual (e.g. employing such passive imperatives as ὑποκείσθω, "let there be hypothesized," and νοείσθω, "let there be imagined"), not the active language of mechanical construction; a glance at the chapters of the *Almagest* (e.g. 1.12 and 5.1) where Ptolemy describes how to construct instruments of observations shows the difference. In the *Planetary Hypotheses* Ptolemy is not giving instructions for making instruments but rather laying out the theory that someone else must design instruments to reproduce as best they can.

One kind of hand-set instrument that Ptolemy probably has in mind is an equatorium, in which the deferents and epicycles are simulated by revolvable graduated disks that can each be put in its appropriate position for a given date according to the tables of mean motions that originally stood at the end of Book 2.[21] This could serve as an analogue computing device, but Ptolemy apparently sees its value not so much in practical application (say for calculating planetary data for astrology) as in allowing one to confirm graphically that the theories are in precise agreement with observed phenomena (2.18/BM102a). It is worth keeping in mind that astronomical demonstration-models were valued in antiquity as instruments of philosophical instruction: Posidonios possessed one (Cicero, *De Natura Deorum* 2.88), Cicero repeatedly invoked their properties in his philosophical dialogues (*De Re Publica* 1.21–22, *Tusc. Disp.* 1.36, in addition to the passage just cited), and Theon of Smyrna, whose likening of the transfer of motion in celestial spheres to

[20] Ptolemy invokes the same rationale in *Geography* 1.1 for maps of the world, where the map is said to allow us to grasp the reality of the spherical Earth and our place on it, which we cannot see directly because we are too close to it and it is too big.

[21] Swerdlow (2005, p. 66) asserts that the subject of the first part of Book 1 *is* the construction of equatoria, though he concedes that the text does not explain how one would physically implement the three-dimensionality of the models with their variously inclined circles.

astronomical gearwork has already been mentioned, had a model of Plato's spindle and whorls (H146).[22] So the dichotomy among Ptolemy's intended readers is not as profound as it might initially appear.

When we say that Ptolemy in the *Planetary Hypotheses* is viewing the modelling of the heavens more from a philosopher's point of view than from a mathematical astronomer's, we must not forget that Ptolemy himself rejected such a dichotomy. His position, as set out in *Almagest* 1.1, was that mathematical astronomy is a part of mathematics proper (and not merely a science *dependent* on mathematics), that mathematics is one of the divisions of theoretical philosophy together with theology and physics, and that for human beings mathematics is the most valuable of the three because its subject matter, encompassing spatial properties and motions of unchanging bodies, is stable and knowable. Mathematical knowledge, he asserts, even confers some limited degree of illumination on theology (whose subject, the ultimate causes of motion and change, is inaccessible to the senses) and physics (whose subject, material properties of bodies, is unstable and irregular). In some sense the *Planetary Hypotheses* could be seen as an effort in this direction, using the "firm" results of mathematical astronomy to obtain an understanding of the physics and theology of the divine heavens. That he still regarded this understanding as less secure than the mathematical models is apparent in his frequent appeals in the *Planetary Hypotheses* to plausibility and likelihood, his use of analogies, and his readiness to offer more than one answer to a problem while making it clear which answer he favors.

Theon of Smyrna's physical modelling, as we saw, was based on three varieties of etherial body: spherical shells concentric with the cosmos to serve the function of the deferents in epicyclic models, solid spheres embedded within the "deferent" shells to serve the function of the epicycles, and (for the special case of the Sun-Mercury-Venus system) spherical shells again embedded within the "deferent" shells to serve the function of epicycles that are concentric with other epicycles. Ptolemy's theories of planetary motion were significantly more complex than Theon's, yet surprisingly in his first approach to giving them a physical interpretation he added only one new basic body type to Theon's repertoire, namely eccentric spherical shells embedded within "deferent" shells to serve the function of eccenters. In the case of the Sun, the visible body is directly embedded in such an eccenter shell (Fig. 3, showing the planet in four positions), while for the Moon and planets the solid or solids acting as the epicycle are embedded in the shell (Fig. 4). The idea of eccentric shells could well have been older than Ptolemy; the fact that Theon does not speak of it may indeed merely mean that he considered it to be physically less satisfactory than his epicyclic spheres.

We saw that Theon took over from Adrastos a definition of uniform motion as "traversing equal intervals in equal times" (τὸ τὰ ἴσα διαστήματα ἐν ἴσοις χρόνοις διανύειν, H151); this is expressed rather naïvely since the meaning of "intervals" when the motion is not rectilinear is not explained, but in the demonstrations Theon

[22] I have argued in Jones (2012) that the gearwork Antikythera Mechanism was likewise intended as a didactic instrument, not as a computer.

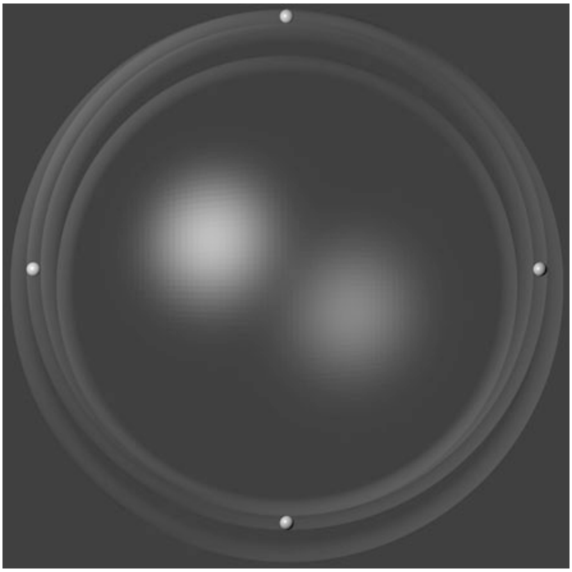

Fig. 3 Ptolemy's three-dimensional eccentric hypothesis for the Sun

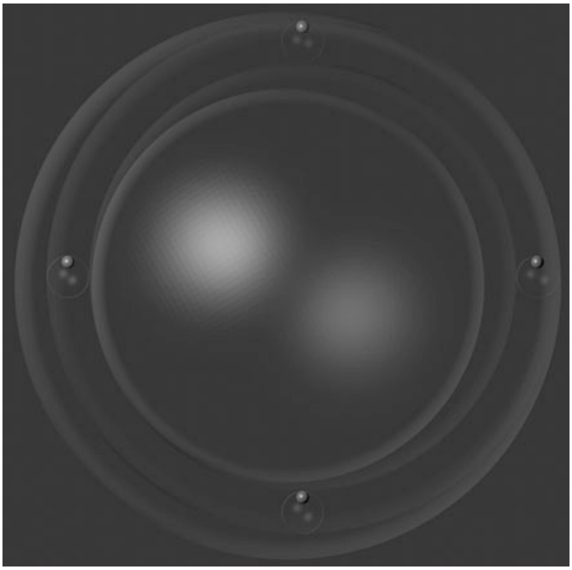

Fig. 4 Ptolemy's three-dimensional epicycle-and-eccenter hypothesis

clearly assumes that a uniformly moving body must be traversing equal arcs on its actual circular path. Again rather naïvely, he sees no need to define the frames of reference for the motions; in his practice, the frame of reference turns out to be zodiacally fixed for motion of a body on an eccenter or of an epicyle on a concentric deferent, but geocentrically fixed for motion of a body on an epicycle.

By contrast, Ptolemy takes it as a given, though in neither the *Almagest* nor the *Planetary Hypotheses* does he offer an explicit definition of the principle, that a motion along a circular path is uniform if there is *some* fixed point inside the circular path of the moving body or point such that the radius from the fixed point to the body or point sweeps out equal angles in equal times. In his eccentric solar model this point is indeed the center of the eccenter, but for the Moon and planets the center of uniform revolution of the epicycle's center is always a defined point (i.e. an "equant") different from the eccentric deferent's center.[23] Moreover, Ptolemy *does* state as a general principle that the frame of reference for the revolution of a body around an epicycle is the radius from the equant to the epicycle's center.[24] It may seem surprising that the descriptions of the physical models in *Planetary Hypotheses* 2 contain no allusion to the equants. A spherical "deferent" shell with an embedded epicycle revolving according to an equant model would have to exhibit a periodic oscillation in its rate of rotation, which seems intuitively harder to accept as "uniform motion" than the motion of the epicycle thought of in isolation from its corporeal context. Ptolemy probably believed that a revolution sweeping out equal angles with respect to a noncentral point was sufficiently explained by ascribing the cause of motion to a rational soul, which could through sheer volition maintain the required oscillation of speed just as it maintains the exact rates of revolution of each of its components through eternity.

It is in its treatment of planetary motion in latitude that the *Planetary Hypotheses* truly embraces the three-dimensionality of Ptolemy's cosmology. Theon has vague references to latitudinal motion, but his geometrical discussions of the epicyclic and eccentric hypotheses ignore it, restricting themselves to the relation between motion in "depth" (or anomaly) and apparent longitudinal motion. His shells and spheres are really little more than a two-dimensional diagram of an epicyclic model that has sprouted a third dimension after the fact. In the *Almagest* Ptolemy always starts his empirical working out of a model as if all the circles were in a single plane, treating anomalistic motion as something that can be analysed independently of motion in latitude; inclinations of the circles with respect to the plane of the ecliptic are added on to the models as a last stage, with Ptolemy relying on the comparatively small angles of inclination to ensure that the validity of the previous planar analyses is not undermined by the neglect of them. Ptolemy's planetary tables in the *Almagest* and *Handy Tables* also treat the calculation of planetary latitude as a kind of add-on.

[23] The equants in the *Almagest* consist of both an equant point and an "equant circle" centered on this point, such that radii from the equant point through the epicycle's center sweep out equal *arcs* on the equant circle. In the *Planetary Hypotheses* the equants are introduced simply as points "around which the center of the epicycle is moved with constant speed (ἰσοταχῶς)".

[24] It is noteworthy that Ptolemy states this principle in *Almagest* 5.5 when he is on the point of making an exception to the rule for his lunar model, and also noteworthy that the description of the lunar model in the *Planetary Hypotheses*, though otherwise unaltered from the *Almagest*, eliminates this exception without drawing any attention to the fact (Murschel 1995, p. 50).

Released in the *Planetary Hypotheses* from the obligation to show how his models and their parameters can be deduced from observations, Ptolemy no longer treats the models like paper dolls that lie flat until one punches their limbs out from the page. In Book 1, as each circle of a model is introduced, any inclination that it has with respect to the ecliptic or to the circle that bears it is specified immediately, so that the model grows in three dimensions all at once. The circles of Ptolemy's solar model, of course, lie entirely in the plane of the ecliptic, while the Moon's model lies entirely in a single inclined plane. Things become more interesting when we turn to the models for the five planets.

In the Book 1 "bare circles" descriptions of the models, Ptolemy states that the plane of each planet's eccentric deferent is tilted at a constant inclination relative to the ecliptic, though the inclinations for Mercury and Venus are very slight, just 1/6°. Revolving around this deferent circle with equant-uniform motion is a *sphere* that Ptolemy calls the "epicyclic sphere" (ἡ ἐπίκυκλος σφαῖρα), and on this sphere we are to imagine two "little circles" (κυκλίσκοι), both being great circles of the epicyclic sphere in the inclined plane of the deferent and thus apparently identical in location though not in motion:[25] the first circle is fixed relative to the radius from the equant point to the epicycle's center, while the second revolves relative to the first "with an equal course to the aforesaid (course) of the epicycle's center" but in the opposite sense. To this second circle is attached a third circle at a fixed inclination to the second circle, and the planet revolves around this third circle with uniform angular speed (ἰσοταχῶς) with respect to its center.

Ptolemy does not explain the kinematic functions of these "little circles." The first is really there just to establish a frame of reference for motions around the epicycle; in the systems of etherial solids of Book 2 it has no counterpart. The second circle, since it is revolving uniformly around the equant together with the epicyclic sphere on which it lies while simultaneously it is revolving *on* the epicyclic sphere with an equal but opposite motion in the same plane, is actually maintaining a stationary orientation according to the frame of reference of the whole model, as defined by its apsidal line. In Book 2 the solid corresponding to this is a spherical epicyclic shell, a device that we have seen applied by Theon to a different and more elementary purpose. The third circle (in Book 2, a solid epicyclic sphere enclosed by the shell), being fixed at an inclination to the second, also maintains a stationary orientation in the model's frame of reference while always being tilted relative to the plane of the deferent. Ptolemy tells us the angle of this epicyclic tilt but, oddly, omits to specify its direction relative to the tilt of the deferent. For Mars, Jupiter, and Saturn, however, the angle of epicyclic tilt is exactly equal to the tilt of the deferent relative to the ecliptic, and there can scarcely be doubt that he intends the two tilts to cancel each other out for these planets, so that the third circle always lies in a plane parallel to the ecliptic. From a modern perspective these *Planetary Hypotheses* models come close to an "ideal" approximation of the geocentric latitudinal motions of the planets by means of an epicycle-

[25] The radii of these circles are actually arbitrary since their function is entirely to do with their inclinations and revolutions, but the natural interpretation of the phrase ἐν τ ἐπικύκλῳ σφαίρᾳ, which applies to both, is that they are on the surface of the sphere.

and-eccenter model, with the circle in the model corresponding to the Earth's orbit around the Sun being either in the plane of the ecliptic or parallel to it, and the circle corresponding to the planet's orbit around the Sun maintaining a fixed inclination to the plane of the ecliptic; the small inclinations Ptolemy gives to the deferents of Venus and Mercury are the only defect. Yet it is striking that Ptolemy does not point out that it is a consequence of his models that the epicycle's orientation is constant and, in the case of the superior planets, parallel to the ecliptic, which one might have expected to be a fact about the planetary systems deserving some attention.

The planetary latitude models of the *Planetary Hypotheses* present further enigmas when we try to relate them to his earlier efforts to model planetary latitude; we can only touch on the complex issues here.[26] The very complicated latitude models of *Almagest* 13, as well as the modified ones underlying the planetary latitude tables in the *Handy Tables* which he published after the *Almagest* and before the *Planetary Hypotheses*, are fundamentally geocentric (or perhaps we should say "equant-centric"), in that they define the tilts of the epicycles with reference to the radius from the equant to the epicycle's center, and the tilts have oscillations. Unfortunately we know nothing directly about planetary latitude theories previous to Ptolemy, but an indirect argument would suggest that models similar to those of the *Planetary Hypotheses*, with the epicycle's inclination fixed in the frame of reference of the deferent (and in the case of the superior planets, parallel to the ecliptic), may have been known in his time.[27] If this is correct, then Ptolemy must have had reasons, either empirical or coming out of an a priori belief that epicycles *ought* to operate according to geocentrically-based principles of motion, or perhaps a combination of the two, for rejecting the fixed-orientation epicycles. Though the models of the *Planetary Hypotheses* are undoubtedly superior to Ptolemy's earlier models, one cannot be certain why he came to adopt them. He does say in very general terms at the beginning of the work (1A.2/BM81b) that, as a result of continued observation, he has made modifications of the models in the *Almagest* that include not only periodicities and dimensions but also structural changes in the models themselves; and aside from the abandonment of the nonstandard definition of the epicycle's apogee in the lunar model, the *only* structural changes are in the theories of planetary latitude. However, he also writes a little further on in his preface (1A2/BM82a) as follows:

> In the case of the placements and arrangement of the circles that produce the anomalies we will employ the simpler ones among the approaches for the sake of facilitating the construction of instruments, even if some little difference (παραλλαγή) results …

[26] See Swerdlow (2005) for an excellent treatment of the developments in Ptolemy's latitude theory, arguing that they were driven by empirical considerations.

[27] The basis of the argument, which I owe to Dennis Duke (private communication), is that algorithms for computing planetary latitudes existed in Indian astronomical texts that approximated the behavior of models like those in the *Planetary Hypotheses* (Kennedy and Ukashah 1969). Greek astronomical texts not dependent on Ptolemy are a plausible source for many features of the Indian models (Duke 2005).

which seems to mean that, in contrast to the *Almagest*, some choices in the model structures have been made not for the sake of theoretical correctness but to make mechanical modelling easier, even though these changes would result in small discrepancies in the predicted appearances. Again it is difficult to see anything else that this could refer to besides the planetary anomalies. Maybe both his accounts tell part of a story that involved both continued empirical refinement of the parameters of planetary latitude and a genuine uncertainty about whether the geocentric framework of the *Almagest* models might after all be incorrect, so that he allowed considerations of mechanical simplicity to tip the scale.[28] In any case the *Planetary Hypotheses* latitude models evince a remarkable degree of three-dimensional imagination on Ptolemy's part, even if he might not have entirely believed in them.

3 Ptolemy Before the Planetary Hypotheses

The short philosophical essay *On the Criterion and the Governing Faculty* is likely to be Ptolemy's earliest surviving work.[29] Although it is not about astronomy or cosmology, its second part, which is devoted to an attempt to localize the "governing faculty" (ἡγεμονικόν) of the human body and soul, contains some glancing remarks that show points of continuity in certain presuppositions between Theon (a representative of what was probably typical Middle Platonist cosmology), the young Ptolemy, and the Ptolemy of the *Planetary Hypotheses*. In particular, *On the Criterion* (ed. Lammert in Lammert and Boer 1952, 19) presents an Aristotelian five-element theory, with earth and water as the "more material" and passive elements, air and fire as "more motive" (κινητικώτερα) and both passive and active, and ether as "always in the same condition" (ἀεὶ ὡσαύτως ἔχοντα) and solely active. Ptolemy's concern in this work is not with the heavens but with human beings, but it is obvious that if he believed ether to be present in the cosmos at all, he would have believed it to be the sole or at least primary constituent of the heavens.[30] But ether is also the most characteristic and governing element in our souls, so there is not an absolute divide between the sublunary cosmos as composed only of the four transitory elements and the heavens as composed of ether. Through the presence of ether in the human soul, we possess an intellective capacity that Ptolemy speaks of as the "more divine" (θειότερον) aspect of the soul in *both* humans and the cosmos (ed. Lammert in Lammert and Boer 1952, 22). Hence the

[28] It is surely relevant that in *Almagest* 13.2 Ptolemy conceded that his latitude models would be difficult to reproduce in a mechanical model, but there he insisted that empirical evidence has to override considerations of simplicity, especially when the criterion of simplicity is reproducibility in non-etherial materials.

[29] Its authenticity, concerning which doubts have been raised, is assured by the presence of several words and word forms that are idiosyncratic favorites of Ptolemy but rare or unattested in other authors.

[30] Aristotle, *De Generatione Animalium* 737a comes close to positing an etherial component in the soul when he speaks of the generative warmth (θερμόν) in the seed as being not fire but a *pneuma* whose nature is "analogous to the element of the stars".

Ptolemy of the *Criterion* already conceived of the cosmos as an animated being, though apparently a single "cosmic animal" with its governing faculty high up in the heavens, as that of the human being is in the head.

In the *Almagest*, although the mathematical modelling is carried out in terms of two-dimensional circles (albeit with inclinations that bring them into a three-dimensional space), Ptolemy makes it clear from the outset that both the visible and the invisible bodies in the heavens are spherical and composed of ether (*Almagest* 1.3), and there are intermittent mentions of the spheres elsewhere in the work. What is not so clear is whether at this stage Ptolemy was committed to the kind of cosmology that the *Planetary Hypotheses* sets out, with discrete, non-overlapping, but tightly packed systems of bodies.

A crucial document in addressing this question is the *Canobic Inscription*, a summary of the parameters of his astronomical models that Ptolemy erected in AD 146/147 shortly before completing the *Almagest* as we have it, so that while most of the data in the inscription agree with the *Almagest*, a few were subjected to rethinking at a late stage in the composition of the treatise.[31] Among the data that Ptolemy altered after the inscription were the mean distances from the Earth of the Sun and of the Moon at syzygies, which in the inscription are respectively 64 and 729 Earth-radii, whereas according to the *Almagest* they are 59 and 1,210 Earth-radii. The inscription does not say anything directly about the absolute or relative distances of the other planets, but it concludes with a scheme matching numbers representing musical pitches with each of the heavenly bodies and the four elements that clearly reflects at least their relative distances from the center of the cosmos:

Sphere of the fixed stars	36
Saturn	32
Jupiter	24
Mars	21 1/3
Sun	18
Venus and Mercury	16
Moon	12
Fire and air	9
Water and Earth	8

Thus Venus and Mercury appear to be situated above the Moon but below the Sun, as they are in the *Planetary Hypotheses*, but they are assigned a single pitch-number, as if they could not be ranked in terms of relative distance from the center.

In *Almagest* 9.1 Ptolemy presents it as uncontroversial that the "order of the spheres" in the cosmos from outermost inwards is that of the fixed stars, Saturn, Jupiter, and Mars, and that the Moon's is innermost, but he presents it as an open question whether Venus and Mercury are further from the center or closer to the

[31] Hamilton et al. (1987), Jones (2005).

center than the Sun, though he favors the latter arrangement, giving as his reason that it would be plausible for the Sun to separate the planets that can reach all elongations from the Sun from those that cannot. Here he says nothing about the distances of Venus and Mercury relative to each other, though whenever in the *Almagest* he presents material relating to all five planets, the order is Mercury, Venus, Mars, Jupiter, Saturn.

Now as *Planetary Hypotheses* 1B shows, it would be possible for an implementation in terms of etherial solids of the *Almagest* models for Venus and Mercury —in either order—to fit snugly in the space between the outer boundary of the solid version of the *Almagest*'s lunar model and the inner boundary of the *Almagest*'s solar model. But obviously this was not possible at the *Canobic Inscription* stage of Ptolemy's thought, since the gap was much smaller then. One can only wonder whether Ptolemy had a resolution of this difficulty that might be hinted at by the single harmonic number shared by the two planets in the inscription. Perhaps he thought it might be possible for Mercury's epicycle to be within that of Venus, though the mechanics of such an arrangement would not be as trivial as Theon's onion-like triple epicycle since Ptolemy's models have different eccentric deferents. A system in which Mercury's model is subsumed within Venus's would not have entirely filled the interval between the inscription's lunar and solar models, so close packing could not have been envisioned.

Since the rediscovery of *Planetary Hypotheses* 1B in the 1960s, it has usually been assumed that it was only after completing the *Almagest* that Ptolemy noticed the coincidence that the Venus and Mercury models would fit neatly in the Moon-Sun gap, with its implications for the possibility of a close-packed cosmology. Christián Carman has recently shown that a plausible argument can be made that Ptolemy deliberately chose his revised solar distance in the *Almagest* with such a nesting of the models in mind, and contrived through numerical fudging so that this distance would appear to have come from analysis of observations of the Sun and Moon.[32] If this reconstruction is correct, however, it then becomes a puzzle why Ptolemy did not allude to the nesting until he came to write the *Planetary Hypotheses*.

There remains one further passage in the *Almagest* that attempts to describe something about the physics of the etherial bodies. It forms part of Ptolemy's defense of his models for planetary latitude in 13.2, the very models that were radically supplanted in the *Planetary Hypotheses* by the ones described above. The characteristic of these models that is at issue is that the planetary epicycles, and in the case of Venus and Mercury also the eccentric deferents, are supposed to have periodically oscillating inclinations. Ptolemy models these oscillations by hypothesizing that the points of the wobbling circles that mark their northern or southern limits of latitude ride on small wheel-like revolving circles in planes perpendicular to the ecliptic. Significantly, these perpendicular circles would not lend themselves easily to a three-dimensional interpretation as spherical bodies. Ptolemy's concern in 13.2 is not this, however, but that a critic might object that the models are too

32 Carman (2008).

complicated to be credible, and moreover that it would be a practical impossibility to make a working "table-top" model of these devices, so how could they function in the heavens?

Ptolemy's rejoinder has two parts: firstly, that the materials that we use to make mechanical imitations of celestial models have different properties from the etherial material of the heavens, so that it is not legitimate to infer from a failed mechanical simulation that the proposed model is unviable, and secondly that, presuming a model is viable, we have no right to object to it on the grounds that it violates criteria of simplicity that we base on our mundane experience. This second part of the defense is a dangerous (and perhaps desperate) move on Ptolemy's part since one might wonder whether it leaves any place for simplicity arguments in astronomy. But the part that concerns us here is Ptolemy's characterization of the properties of etherial bodies:

> There is no hindering nature in them, but one that is commensurate [συμμέτρου] with respect to yielding and going along with each one's natural motions, even if they prove to be opposed, such that all can pass through and be seen through absolutely all the fluids [χυμάτων], and this kind of thing can flow freely [εὐοδεῖν] not only around the particular circles but also around the spheres themselves and the axes of revolutions.

The word I have translated as "fluids," χύμα, means something that is liquid or molten and thus flows (χέω) or that has solidified from a molten state (e.g. an ingot). Ptolemy does not apply it elsewhere to the invisible etherial stuff filling the heavens, but it occurs in this sense in much later authors, especially John Philoponos, and it must have had an existence that we cannot now trace in earlier cosmological texts of the Platonist-Peripatetic tradition since this obscure passage of Ptolemy is surely not where the later writers discovered it. In any event, ether was always in principle the lightest, most evanescent of the elements, but the modelling of Theon and the *Planetary Hypotheses* nevertheless treats the bodies composed of it as having rigid forms, so in effect "solid" in consistency as well as in geometry. In the *Almagest* passage, on the other hand, ether is not merely fluid in principle but actually flowing around other bodies of etherial matter. We obtain here a fleeting glimpse of a different approach to physical cosmology that Ptolemy seems to have entertained but not fully articulated in his *Almagest* days, and that he abandoned in favor of the Theonic approach.

4 Appendix: Translation of Theon of Smyrna on the Solid Models (H177–189)[33]

And (Adrastos) expounds these things at greater length with a view to accommodating to each other the hypotheses and approaches of the mathematicians, who, while giving regard only to the appearances and the motions of the wandering

[33] This translation (based on the text and diagrams of Hiller's edition) attempts, by avoiding conventional "technical" renderings of the Greek expressions, to approximate the way

(stars) that occur by happenstance, having observed them for long time-intervals because of the natural suitability of their country—(I mean) the Babylonians and Chaldeans and Egyptians—eagerly sought certain first principles and hypotheses, to which the appearances fit; by means of this (they would be able) to make judgment with respect to the things found before and to forecast with respect to things going to happen, some of them adducing certain numerical methods, like the Chaldeans, and others (adducing) graphical (methods), like the Egyptians, but all (of them) making their methods incomplete without reasoning according to nature, whereas it is needful at the same time to make examination concerning these things in a nature-related manner; and this is the very thing that those among the Greeks who engaged in astronomy tried to do, taking the first principles and the observations of the appearances from these (people), just as Plato discloses in the *Epinomion*, as will be clear a little later when his statements have been laid out.

And Aristotle, having demonstrated at length concerning the stars collectively in *On the Heavens* that they neither are borne through the etherial body which stands still, nor rush along with it as it is borne, just as if they are liberated and by themselves, nor indeed (are they) whirled about or rolled about, but rather the nonwandering (stars) are borne by that (etherial body), being multiple, by one common (sphere) that is outside, while each one of the wandering (stars) (is borne) by multiple spheres, again in *Metaphysics* Λ says that Eudoxos and Kallippos move the wanderers by means of certain spheres. For the (principle) according to nature is that neither do the stars move, with respect to themselves, along certain circular or spiral-shaped lines and oppositely to the whole nor do certain circles themselves whirl around their own centers while bearing the stars fixed upon them, some of them in the same direction as the whole and others oppositely. For how is it even possible for such great bodies to be bound on bodiless circles?

He says that it is fitting that there are certain spheres of the fifth body situated and borne in the depth of the whole, some of them higher up, others placed below them, and some of them greater, some of them smaller, and moreover some of them hollow, and again some of them solid, (being situated) in the depth of these (hollow spheres), on which the wanderers, being fixed on in the manner of nonwandering stars, appear as being moved, by a travel that is the simple one of those (nonwandering stars), but at unequal speed because of the places (where they are situated), by happenstance in a complicated manner, and (they appear as) describing certain eccentric circles or again (describing circles that are) situated on certain other circles or (as describing) certain spirals, on which the mathematicians think that they are moved, being deceived by the turning about. So since they appear as being

(Footnote 33 continued)
that Theon's intended readers—philosophical students without training in astronomy—would have encountered his book. For details see Jones (forthcoming), which includes a translation on the same principles of the immediately preceding passage, overlapping the present excerpt in its final paragraph. The most crucial way that conventional modern translation practices distort Theon's meaning is in the representation of "forward" and "backward" directions of celestial motion.

borne about together by the whole with the daily shift from risings to settings, but (also as) being borne in the opposite direction with the shift towards the trailing (stars) along the zodiacal (circle) which is inclined, and (as) also being moved a bit in breadth,[34] being seen more to the north and more to the south, and besides these things (in) height and depth, sometimes being beheld as further from the Earth and at other times as nearer to the Earth, Aristotle says that each (wanderer) is borne by means of multiple spheres that were previously hypothesized. Eudoxos says that Sun and Moon are fixed by three spheres, one (being) the (sphere) of the nonwandering (stars) that is whirled about the poles of the whole and that drags along by strength all the other (spheres) collectively from risings to settings, and a second (sphere) being borne around an axis at right angles to the (circle) through the middle of the zodiacal signs, by means of which again, each (of the Sun and Moon) appears as making the shift in length towards the trailing zodiacal signs collectively, and a third (sphere being borne) about an axis at right angles to the circle that is inclined with respect to the (circle) through the middle (of the zodiacal signs) in the breadth of the zodiacal signs, by means of which each is borne in its distinct motion in breadth, one in a greater and the other in a smaller interval, coming to be further north and further south of the (circle) through the middles of the zodiacal signs; and each of the other wandering (stars) by means of four (spheres), with another (sphere) added in the case of each, by means of which each one's[35] depth will be made. He says that Kallippos, with Kronos (Saturn) and Zeus (Jupiter) excepted, added certain other spheres to the others, by two for Sun and Moon, and by one for the rest. Next he additionally reasons that if (the spheres) when put together are going to save the phenomena, in the case of each of the wandering (stars) there are still other spheres, fewer by one than the bearing (spheres), (namely) the unwinding (spheres), either uttering his own opinion or that of those (*scil.* Eudoxos and Kallippos).

For since they thought that it was according to nature that all things be borne in the same direction, seeing the wandering (stars) also shifting in the opposite direction, they assumed that there had to be certain other spheres, obviously solid ones, between bearing (spheres), which unwind by their own motion the bearing (spheres) in the opposite direction, being in contact with them, just like the so-called "disks" in mechanical sphere-constructions, being moved about the center with a certain distinct motion, by the engagement of the teeth move and unwind in the opposite direction the underlying and juxtaposed (disks). The (principle) according to nature in fact is that all the spheres are borne in the same direction, being drawn about by the outermost (sphere), but according to their distinct motion, because of the arrangement of the placement and the places and the sizes, some of them travel faster, some of them slower in the opposite direction about distinct axes that are also inclined with respect to the sphere of the nonwandering (stars); so that the stars on them, being borne by the simple and uniform motion of these (spheres),

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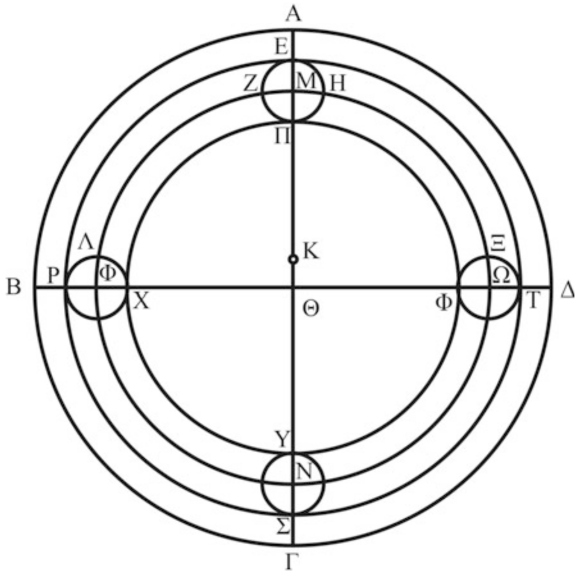
[34] inserting ⟨κατὰ⟩ πλάτος.

[35] reading ἑκάστου for the manuscript's ἕκαστον.

Fig. 5 Theon's diagram



seem by happenstance to make certain compound and nonuniform and complicated travels. And they describe different circles, some of them concentric, some eccentric, and some epicycles. For the sake of comprehension of the things that have been said (above), it is fitting to set out briefly also concerning these things, in accordance with the diagram that seems to us to be necessary for the sphere-constructions (Fig. 5).

Let there be a hollow sphere of the nonwandering (stars) ΑΒΓΔ about center Θ of the whole in depth ΑΕ; (and let there be) diameters of it ΑΓ, ΒΔ; and let circle ΑΒΓΔ be imagined, a greatest (circle) and through the middles of the zodiacal signs; and (let there be imagined) a certain other hollow sphere of a wanderer below it around the same center, ΕΡΣΤ and ΠΧΥΨ, in depth ΕΠ; and in this depth (let there be imagined) a solid sphere ΕΖΠΗ, bearing the wandering (star) fixed upon itself at Ε. And let them all be borne in the same direction uniformly in simple motions from risings to settings, but let the (sphere) that delimits the breadth of the wanderer alone be borne in the opposite direction, or (let it be borne) in the same direction but let it be trailing because of its slowness; for either way the phenomena will be saved. But *let*[36] the (sphere) of the nonwandering (stars be borne) about an axis at right angles to *the plane of the equinoctial (circle); and (let) the hollow (sphere) of the wanderer (be borne) about an axis at right angles to the* same plane in which also is the circle delimiting the breadth, which is inclined with respect to the (circle) through the middles of the zodiacal signs. And let the sphere of the unwandering (stars) be borne fastest; and the hollow (sphere) of the wanderer more slowly than this one in the opposite direction, so that in a certain defined time-period it goes

[36] Italics represent text restored in a lacuna by Martin.

about the whole (sphere) of the unwandering (stars), or, as some think, it trails; which is the truer opinion will be stated elsewhere; and let it bear the solid sphere that has the wandering (star); the solid sphere, being borne about its own axis uniformly, will be restituted to the same (situation), being borne in the same direction as the unwandering (sphere); either it will be restituted to the same (situation) in an equal time-period to that in which also the hollow (sphere) of the wandering (star), being borne in the opposite direction, goes about the (sphere) of the unwandering (stars) or it trails, either faster or slower.

Let it first be restituted in the same (time-period); and let M be center of the sphere; and with center Θ and radius ΘM let circle ΜΛΝΞ be described; and with straight (line) EY having been divided in two at K, with center K and radius KE let circle ΕΛΥΞ be described, being eccentric with respect to the whole. It is obvious that in the time-period in which the hollow sphere of the wandering (star) trails by the (sphere) of the unwandering (stars) while bearing the solid (sphere), center K of the solid sphere will go through concentric circle ΜΛΝΞ, while seeming to be borne in the opposite direction while also bringing along the solid sphere, and the wandering (star) at E will describe the circle ΕΗΠΖ on the solid sphere, which (circle) becomes an epicycle on concentric ΜΛΝΞ, while itself being borne in the same direction as the whole; but by happenstance it will also describe the eccentric (circle) ΕΛΥΞ which is equal to the concentric, circumscribing it in the opposite direction to the whole; it will also seem to those who see from Θ to traverse the zodiacal (circle) ΑΒΓΔ, advancing in the direction of the trailing (zodiacal signs) oppositely to the travel of the whole; it will also appear as being moved over a breadth in proportion of the inclination of the plane with respect to the (circle) through the middles of the zodiacal signs, to which plane the axes of its spheres are at right angles; it will always make its greatest distance at the same place and it will seem to move the least, say at point A of the zodiacal (circle), whenever the center of the solid sphere is on straight (line) AΘ at M, and the wandering (star) itself at E; contrarily the least distance will always be restituted and it will seem to move the most, say at point Γ of the zodiacal (circle), whenever, with the hollow sphere having changed its position in the opposite direction, the center of the solid (sphere) comes to be at N on straight (line) ΘΓ, and the wandering (star) itself at Γ, that is, at Y. However, it will make its mean distances and mean motions in two places, when it comes to be at the bisections of the epicycle ΕΖΠΗ and of the concentric (circle) ΜΛΝΞ, say the (bisections) Z, H, which, because of the change of position of the spheres in the opposite direction or the trailing, come to be the same as the bisections Λ, Ξ of the eccentric circle ΕΛΥΞ and of the concentric (circle) ΜΛΝΞ, appearing at the points in between A, Γ on either side, (namely) B, Δ on the zodiacal (circle), say Φ, Ω; all these things appear concerning the Sun, because of the fact that its time-periods of restitution all are equal so far as sense perception is concerned or are found very close to one another—I mean the (time-period) of the length and of the breadth and of the depth—and the corresponding points of both spheres always are seen to coincide at their corresponding motions at the same places and in the same zodiacal signs.

Since as a consequence of a travel of the spheres of such a kind and according to nature, uniform and simple and ordered, but inclined and only because of slowness trailing the unwandering (stars) or only the (sphere) bearing the solid (sphere), that is the epicycle, being borne in the opposite direction, by happenstance there arises a complicated and compound and nonuniform travel of the wandering (star), because of *the travel of the hollow (sphere)*[37] towards the trailing zodiacal signs which arises[38] either in reality or by trailing, and because of the inclination being beheld in a certain breadth of the zodiacal signs, and because of the whirling of the solid (sphere) about its own axis, sometimes seeming to be in height and because of this slow, sometimes in depth and because of this faster, and simply nonuniform, and because of this also seeming to be on the epicycle and on the eccentric (circle), it is also obvious that reasonably the mathematicians' hypotheses of their travel, (namely) the (hypothesis) according to epicycle and (that) according to eccentric (circle), follow each other and are in concord with each other, since both are consequences of the (hypothesis) according to nature, but by happenstance, at which Hipparchos marvels, especially in the case of the Sun because of the equality of time-periods of the travels of its spheres which is accurately completed, while in the case of the others (it is not completed) so accurately because of the fact that the solid sphere of the wanderer is not restituted in the same time-period in which the hollow (sphere) of the unwandering (stars) either trails or goes about in the opposite direction, but in the case of some of them faster, in the case of others slower, so that their corresponding motions do not coincide at the same points of the spheres at the same places, but always diverge, and also the inclinations of the spheres is in multiple breadths, and because of these things their time-periods of restitution of length and breadth and depth are unequal and different, and they make their greatest and least and mean distances and motions at different places at different times and in all the zodiacal signs, and moreover, because of the fact that, as we say, the corresponding motions diverge also (?) at the corresponding points of the spheres, the wandering (stars) do not seem to describe circles by their by-happenstance motions, but certain spirals. So in the case of each of the wandering (stars) one must think of the hollow sphere as distinct and bearing in its depth the solid (sphere), and (one must think of) the solid (sphere) as distinct, bearing on its distinct surface again the wandering (star).

In the case of the Sun and Light-bearer (Venus) and Gleamer (Mercury) it is possible for both (spheres) to be distinct for each, but (for) the hollow (spheres) of the three (spheres), being of equal course, to go about the sphere of the unwandering (stars) in the opposite direction in an equal time-period, while the solid (spheres) have their centers on one straight line, that of the Sun being smaller, that of Gleamer being larger, and that of Light-bearer being still greater than this one. But it is also possible for the hollow sphere of the three to be one, and the solid (spheres) of the three to be in the depth of this (hollow sphere) about the same center as each other, the (sphere) of the Sun being smallest and really solid, and the

[37] Hiller's restoration of a lacuna.

[38] reading γινομένης for the manuscript's γινομένη.

(sphere) of Gleamer about this one, and the (sphere) of Light-bearer thereupon enclosing both and filling the whole depth of the hollow and common (sphere); through this, these three make their trailing or travel in the opposite direction in length through the zodiacal signs of equal course, but the others not likewise, they are always seen catching up with each other and being caught up and standing in front of each other, with Hermes (Mercury) standing away at most about twenty degrees on either side of the Sun to the setting or to the rising, and the (star) of Aphrodite (Venus) (standing away) at most fifty degrees. One might suspect that the truer placement and arrangement is this one, so that this would be the place of the ensoulment of the cosmos, as cosmos and living thing, the Sun being as it were heart of the whole, greatly warm because of its motion and size and the companionship in journey of the things about it. For in ensouled beings the middle of the creature, that is of the living thing in the way that it is a living thing, is one thing, and (the middle) of the size is another thing; for example, as we said, the middle of the ensoulment of ourselves, as human beings and living things, is one thing, (namely) the (place) around the heart, which is always moving and greatly warm and because of these things is beginning of every power of the soul, for example spiritual (power) and (power) of impulse with respect to place and (power) of appetite and (power) of imagination and (power) of intellect, while there is another middle of our size, such as the (place) about the navel. Similarly too the middle of the whole cosmos, to liken the greatest and most worthy of honor and divine on the basis of slight and random and mortal things, is the (place) about the earth, which is chilled and immobile; but as cosmos and in the way that it is cosmos and living thing, the middle of its ensoulment is the (place) about the Sun, as it were being heart of the whole, starting from which they relate that its soul makes its way through the whole body, being stretched from the extremities.

It is obvious that whereas because of the aforesaid causes both hypotheses follow each other, the (hypothesis) according to the epicycle seems more collective and more general and very close to the (hypothesis) according to nature; for the greatest circle of the solid sphere, which the wandering (star) describes on it by its travel on it, is the epicycle; but the eccentric (circle) is absolutely separated from the (hypothesis) according to nature and is described more by happenstance. Comprehending this, Hipparchos too praises the hypothesis according to epicycle as being his own, saying it is more credible that all the heavenly things are situated in equilibrium and joined together likewise with respect to the middle of the cosmos; nevertheless he himself, because he did not proceed from reasoning according to nature, did not comprehend accurately what is the travel of the planets that is according to nature and accordingly true is, and what is the (travel that is) by happenstance and apparent; even he hypothesizes that each one's epicycle is moved on the concentric circle, and the wandering (star) on the epicycle.

Plato too seems to believe that the (hypothesis) according to epicycle is more powerful, but that the things that bear the wandering (stars) are not spheres but circles, just as at the end of the *Republic* he riddles by the whorls fitted in one another; but he uses the words in a more common manner, and often calls spheres "circles" and "poles," and axes as "poles."

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