A Note on Objective-Based Rough Clustering with Fuzzy-Set Representation

Ken Onishi[1], Naohiko Kinoshita[1], and Yasunori Endo[2]

[1] Graduate School of Sys. and Info. Eng., University of Tsukuba
Tennodai 1-1-1, Tsukuba, Ibaraki, 305-8573, Japan
{s1320619,s1220594}@u.tsukuba.ac.jp
[2] Faculty of Eng., Info. and Sys., University of Tsukuba
Tennodai 1-1-1, Tsukuba, Ibaraki, 305-8573, Japan
endo@risk.tsukuba.ac.jp

Abstract. Clustering is a method of data analysis. Rough k-means (RKM) by Lingras et al. is one of rough clustering algorithms[3]. The method does not have a clear indicator to determine the most appropriate result because it is not based on objective function. Therefore we proposed a rough clustering algorithm based on optimization of an objective function [7]. This paper will propose a new rough clustering algorithm based on optimization of an objective function with fuzzy-set representation to obtain better lower approximation, and estimate the effectiveness through some numerical examples.

Keywords: clustering, rough clustering, optimization, fuzzy set.

1 Introduction

Data have become large-scale and complex in recent years. We cannot get useful information without computers. The importance of data analysis techniques has been increasing accordingly and various data analysis methods have been proposed. Clustering is one of the major techniques in pattern recognition. Clustering is a technique automatically classifying data into some clusters. Many researchers have been interested in clustering as a significant data analysis method.

Types of clustering are divided broadly into hierarchical and non-hierarchical clustering. The standard techniques of non-hierarchical clustering are called objective-based clustering. The objective-based clustering is constructed to minimize a given objective function. Therefore, the objective function plays many important role in objective-based clustering.

From the viewpoint of the membership of an object to each cluster, called membership grade, types of clustering are divided into crisp and fuzzy. The value of membership grade is 0 or 1 in crisp clustering. The value is included into the unit interval [0,1] in fuzzy clustering. Fuzzy clustering allows an object to belong more than one cluster at the same time. That is why fuzzy clustering can be regarded as more flexible than crisp clustering. On the other hand, it is pointed out that the fuzzy degree of membership may be too descriptive for

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interpreting clustering results. In such cases, rough set representation is a more useful and powerful tool[1][2].

Recently, clustering based on rough set theory has attracted some attention[3]. Rough clustering represents a cluster by using two layers, upper and lower approximations. We can regard rough clustering as three-value clustering, that is, into the cluster, out of the cluster and unknown. The lower approximation means that an object surely belongs to the set and the upper one means that an object possibly belongs to the set. Clustering based on rough-set representation could provide a solution that is less restrictive than conventional clustering and less descriptive than fuzzy clustering, and therefore clustering based on rough set representation has attracted increasing interest of researchers[4][5][6].

However, traditional rough clustering does not have an objective function. For that reason the problem is pointed out that we cannot evaluate the result quantitatively. In order to solve this problem, a rough clustering algorithm based on optimization of an objective function was proposed[7]. But the algorithm has a problem that an object cannot belong to more than two upper approximations.

This paper proposes new rough clustering algorithms based on optimization of an objective function with fuzzy-set representation and estimate the effectiveness through some numerical examples.

2 Conventional Rough Clusterings

2.1 Rough Sets

Let U be the universe and $R \subseteq U \times U$ be an equivalence relation on U. R is also called equivalence relation. The pair $X = (U, R)$ is called approximation space. If $x, y \in U$ and $(x, y) \in R$, we say that x and y are indistinguishable in X.

Equivalence class of the relation R is called elementary set in X. The family of all elementary sets is denoted by U/R. The empty set is also elementary in every X.

Since it is impossible to distinguish each element in an equivalence class, we may not be able to get a precise representation for an arbitrary subset $A \subseteq U$. Instead, any A can be represented by its lower and upper bounds. The upper bound $\overline{A}$ is the least composed set in X containing A, called the best upper approximation or, in short, upper approximation. The lower bound $\underline{A}$ is the greatest composed set in X containing A, called the best lower approximation or, briefly, lower approximation. The set $\mathrm{Bnd}(A) = \overline{A} - \underline{A}$ is called the boundary of A in X.

The pair $(\underline{A}, \overline{A})$ is the representation of an ordinary set A in the approximation space X, or simply a rough set of A. The elements in the lower approximation of A definitely belong to A, while elements in the upper bound of A may or may not belong to A.

2.2 Rough *k*-Means

In this section, we explain rough k-means (RKM) by Lingras. From the above section of rough sets, we can define the following conditions for clustering.

(C1) An objet x can be part of at most one lower approximation.
(C2) If $x \in \underline{A} \Longrightarrow x \in \overline{A}$
(C3) An object x is not part of any lower approximation if and only if x belongs to two or more boundaries.

Cluster centers are updated by

$$v_i = \begin{cases} \underline{\omega} \times \dfrac{\sum_{x_k \in \underline{A}_i} x_k}{|\underline{A}_i|} + \overline{\omega} \times \dfrac{\sum_{x_k \in \mathrm{Bnd}(A_i)} x_k}{|\,\mathrm{Bnd}(A_i)|}, & (\underline{A}_i \neq \emptyset \wedge \mathrm{Bnd}(A_i) \neq \emptyset) \\ \dfrac{\sum_{x_k \in \overline{A}_i} x_k}{|\overline{A}_i|}. & \text{(otherwise)} \end{cases}$$

The coefficients $\underline{\omega}$ and $\overline{\omega}$ are weights of lower approximations and boundaries, respectively. $\underline{\omega}$ and $\overline{\omega}$ satisfy as follows:

$$\underline{\omega} > 0, \quad \overline{\omega} > 0, \quad \underline{\omega} + \overline{\omega} = 1, \quad 1 \leq k \leq n, \quad 1 \leq i \leq c.$$

Lower approximations and boundaries are calculated as follows:

$$d_{ki} = \|x_k - v_i\|^2, \quad d_{km} = \min_{1 \leq i \leq c} d_{ki}$$
$$T = \{i \mid d_{ki} - d_{km} \leq \text{threshold}\} \qquad (i \neq m)$$
$$T \neq \emptyset \Rightarrow x_k \in \overline{A}_m \text{ and } x_k \in \overline{A}_i \qquad (\forall i \in T)$$
$$T = \emptyset \Rightarrow x_k \in \underline{A}_m \ .$$

Algorithm 1. RKM

RKM0 Give initial cluster centers.
RKM1 Calculate lower approximations and boundaries.
RKM2 Calculate cluster centers.
RKM3 If the stop criterion satisfies, finish. Otherwise back to **RKM1**.

2.3 Rough *c*-Means

RKM has the following problems.

- Since RKM does not have objective function, there is no guidance to estimate the validity of the obtained results.
- There is no guidance to determine the threshold.

Rough c-means (RCM) which is based on optimization of an objective function was proposed to solve the above problems by Endo et al[7].

The objective function of RCM is defined as follows:

$$J_{\mathrm{RCM}} = \sum_{i=1}^{c} \sum_{k=1}^{n} \sum_{l=1}^{n} (\nu_{ki} u_{li} (\underline{\omega} d_{ki} + \overline{\omega} d_{li}) + (\nu_{ki} \nu_{li} + u_{ki} u_{li}) D_{kl}) \ .$$

ν_{ki} represents a membership grade of x_k to a lower approximation of cluster of i. u_{li} represents a membership grade of x_l to a boundary of cluster of i. Here, $d_{si} = \|x_s - v_i\|^2$ and $D_{kl} = \|x_k - x_l\|^2$.

The constraints are as follows:

$$\underline{\omega} + \overline{\omega} = 1, \quad \nu_{ki} \in \{0,1\}, \quad u_{ki} \in \{0,1\}, \quad \sum_{i=1}^{c} \nu_{ki} \in \{0,1\}, \quad \sum_{i=1}^{c} u_{ki} \neq 1,$$

$$\sum_{i=1}^{c} \nu_{ki} = 1 \Longleftrightarrow \sum_{i=1}^{c} u_{ki} = 0.$$

Those constraints obviously satisfy the above conditions **C1**, **C2** and **C3**. Actually, those constraints are rewritten as:

$$\sum_{i=1}^{c} \nu_{ki} = 0 \Longleftrightarrow \sum_{i=1}^{c} u_{ki} = 2.$$

The cluster center v_i is calculated as follows:

$$v_i = \underline{\omega} \times \frac{\sum_{x_k \in \underline{A}_i} x_k}{|\underline{A}_i|} + \overline{\omega} \times \frac{\sum_{x_k \in \mathrm{Bnd}(A_i)} x_k}{|\,\mathrm{Bnd}(A_i)|}.$$

The optimal solutions to N and U are updated as follows:

$$\nu_{ki} = \begin{cases} 1, & (J_k^{\nu} < J_k^{u} \wedge i = p_k) \\ 0, & (\text{otherwise}) \end{cases}$$

$$u_{ki} = \begin{cases} 1, & (J_k^{\nu} > J_k^{u} \wedge (i = p_k \vee i = q_k)) \\ 0. & (\text{otherwise}) \end{cases}$$

Here p_k, q_k, J_k^{ν} and J_k^{u} are calculated as follows:

$$p_k = \arg\min_i d_{ki}, \quad q_k = \arg\min_{i \neq p_k} d_{ki},$$

$$J_k^{\nu} = \sum_{l=1, l\neq k}^{n} \nu_{kp_k} (u_{lp_k} (\underline{\omega} d_{kp_k} + \overline{\omega} d_{lp_k}) + 2\nu_{lp_k} D_{kl}),$$

$$J_k^{u} = \sum_{i=p_k, q_k} \sum_{l=1, l\neq k}^{n} u_{ki} (\nu_{li} (\underline{\omega} d_{li} + \overline{\omega} d_{ki}) + 2u_{li} D_{kl}).$$

Algorithm 2. RCM

RCM0 Give initial cluster centers.
RCM1 Calculate lower approximations and boundaries.
RCM2 Update cluster centers.
RCM3 Calculate $\min_V J_{\mathrm{RCM}}$ and update V.
RCM4 If the stop criterion satisfies, finish. Otherwise back to **RCM1**.

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3 Proposed Method 1 — RCM-FU

We propose RCM-FU (RCM with fuzzy upper approximation) which is constructed by introducing fuzzy-set representation into membership of boundary.

3.1 Objective Function

The objective function of RCM-FU is defined as follows:

$$J_{\text{RCM-FU}} = \sum_{k=1}^{n}\sum_{l=1}^{n}\sum_{i=1}^{c}(u_{ki}^{m}\nu_{li}(\underline{\omega}d_{li} + \overline{\omega}d_{ki}) + (\nu_{ki}\nu_{li} + u_{ki}^{m}u_{li}^{m})D_{kl}).$$

The constraints are as follows:

$$\underline{\omega} + \overline{\omega} = 1, \quad \nu_{ki} \in \{0,1\}, \quad u_{li} \in [0,1], \quad \sum_{i=1}^{c}\nu_{ki} \in \{0,1\},$$

$$\sum_{i=1}^{c}\nu_{ki} = 1 \Longleftrightarrow \sum_{i=1}^{c}u_{ki} = 0,$$

$$\sum_{i=1}^{c}\nu_{ki} = 0 \Longleftrightarrow \sum_{i=0}^{c}u_{ki} = 1.$$

3.2 Derivation of the Optimal Solution and Algorithm

A cluster center v_i is calculated as follows:

$$v_i = \begin{cases} \dfrac{\sum_{x_k \in \underline{A}_i} x_k}{|\underline{A}_i|}, & (\mathrm{Bnd}(A_i) = \emptyset) \\ \dfrac{\sum_{k=1}^{n} u_{ki}^{m} x_k}{\sum_{k=1}^{n} u_{ki}^{m}}, & (\underline{A}_i = \emptyset) \\ \underline{\omega} \times \dfrac{\sum_{x_k \in \underline{A}_i} x_k}{|\underline{A}_i|} + \overline{\omega} \times \dfrac{\sum_{k=1}^{n} u_{ik}^{m} x_k}{\sum_{k=1}^{n} u_{ik}^{m}}. & (\text{otherwise}) \end{cases}$$

The optimal solutions to N and U are updated as follows:

In case that x_k belongs to the lower approximation of a cluster, the cluster is C_{p_k} $(p_k = \arg\min_i d_{ki})$ and

$$u_{ki} = 0, \quad (\forall i)$$

$$\nu_{ki} = \begin{cases} 1, & (i = p_k) \\ 0. & (\text{otherwise}) \end{cases}$$

Therefore, we calculate u_{ki} that minimizes J_k^{ν} as follows:

$$J_k^{\nu} = \sum_{l=1}^{n}(u_{lp_k}^{m}(\underline{\omega}d_{kp_k} + \overline{\omega}d_{lp_k}) + 2\nu_{lp_k}D_{kl}).$$

In case that x_k belongs to boundaries of some clusters, $\nu_{ki} = 0$. Thus, the objective function is represented by

$$J_k^u = \sum_{l=1}^{n} \sum_{i=1}^{c} (u_{ki}^m \nu_{li}(\underline{\omega} d_{li} + \overline{\omega} d_{ki}) + 2u_{ki}^m u_{li}^m D_{kl}).$$

We calculate the optimal solutions by Lagrange multiplier as follows:

$$u_{ki} = \frac{\left(\frac{1}{\sum_{l=1}^{n}(\nu_{li}(\underline{\omega} d_{li}+\overline{\omega} d_{ki})+4u_{li}^m D_{kl})}\right)^{\frac{1}{m-1}}}{\sum_{j=1}^{c}\left(\frac{1}{\sum_{l=1}^{n}(\nu_{lj}(\underline{\omega} d_{lj}+\overline{\omega} d_{kj})+4u_{lj}^m D_{kl})}\right)^{\frac{1}{m-1}}}.$$

In comparison with the above two cases, we obtain the optimal solutions on ν_{ki} and u_{ki} as follows:

$$\nu_{ki} = \begin{cases} 1, & (J_k^\nu < J_k^u \land i = p_k) \\ 0, & (\text{otherwise}) \end{cases}$$

$$u_{ki} = \begin{cases} 0, & (J_k^\nu < J_k^u \land i = p_k) \\ \dfrac{\left(\frac{1}{\sum_{l=1}^{n}(\nu_{li}(\underline{\omega} d_{li}+\overline{\omega} d_{ki})+4u_{li}^m D_{kl})}\right)^{\frac{1}{m-1}}}{\sum_{j=1}^{c}\left(\frac{1}{\sum_{l=1}^{n}(\nu_{lj}(\underline{\omega} d_{lj}+\overline{\omega} d_{kj})+4u_{lj}^m D_{kl})}\right)^{\frac{1}{m-1}}}. & (\text{otherwise}) \end{cases}$$

Algorithm 3. RCM-FU

RCM-FU0 Give initial cluster centers.
RCM-FU1 Calculate lower approximations and boundaries.
RCM-FU2 Calculate cluster centers.
ERCM-FU3 Calculate $\min_V J_{\text{RCM-FU}}$ and update V.
ERCM-FU4 If the stop criterion satisfies, finish. Otherwise back to **RCM-FU1**.

4 Proposed Method 2 — Entropy RCM-FU

We propose Entropy RCM-FU(ERCM-FU) by introducing an entropy regularizer into RCM.

4.1 Objective Function

The objective function of ERCM-FU is defined as follows:

$$J_{\text{ERCM-FU}} = \sum_{k=1}^{n} \sum_{l=1}^{n} \sum_{i=1}^{c} (u_{ki}\nu_{li}(\underline{\omega} d_{li} + \overline{\omega} d_{ki}) + (\nu_{ki}\nu_{li} + u_{ki}u_{li})D_{kl})$$
$$+ \lambda \sum_{k=1}^{n} \sum_{i=1}^{c} u_{ki} \log u_{ki}.$$

The constraints are the same as ones of RCM-FU.

4.2 Derivation of the Optimal Solution and Algorithm

The cluster center v_i is calculated as follows:

$$v_i = \begin{cases} \dfrac{\sum_{x_k \in \underline{A}_i} x_k}{|\underline{A}_i|}, & (\mathrm{Bnd}(A_i) = \emptyset) \\ \dfrac{\sum_{k=1}^{n} u_{ki}^m x_k}{\sum_{k=1}^{n} u_{ki}^m}, & (\underline{A}_i = \emptyset) \\ \underline{\omega} \times \dfrac{\sum_{x_k \in \underline{A}_i} x_k}{|\underline{A}_i|} + \overline{\omega} \times \dfrac{\sum_{k=1}^{n} u_{ik}^m x_k}{\sum_{k=1}^{n} u_{ik}^m}. & (\text{otherwise}) \end{cases}$$

The optimal solutions to N and U are updated as follows:

In case that x_k belongs to the lower approximation of a cluster, the cluster is C_{p_k} ($p_k = \arg\min_i d_{ki}$) and

$$u_{ki} = 0, \quad (\forall i)$$
$$\nu_{ki} = \begin{cases} 1, & (i = p_k) \\ 0. & (\text{otherwise}) \end{cases}$$

Therefore, we calculate u_{ki} that minimizes J_k^ν as follows:

$$J_k^\nu = \sum_{l=1}^{n} (u_{li^*}(\underline{\omega} d_{ki^*} + \overline{\omega} d_{li^*}) + 2\nu_{li^*} D_{kl} + \lambda u_{li^*} \log u_{li^*}).$$

In case that x_k belongs to boundaries of some clusters, $\nu_{ki} = 0$. Thus, the objective function is represented by

$$J_k^u = \sum_{l=1}^{n} \sum_{i=1}^{c} (u_{ki}\nu_{li}(\underline{\omega} d_{li} + \overline{\omega} d_{ki}) + 2u_{ki}u_{li}D_{kl} + \lambda u_{ki} \log u_{ki}).$$

We calculate the optimal solutions by Lagrange multiplier as follows:

$$u_{ki} = \exp(\lambda^{-1}(\sum_{l=1}^{n}(-\nu_{li}(\underline{\omega} d_{li} + \overline{\omega} d_{ki}) - 4u_{li}D_{kl}) - \lambda$$
$$- \lambda \log \sum_{j=1}^{c}(\exp(\lambda^{-1}(-\sum_{l=1}^{n}(\nu_{lj}(\underline{\omega} d_{lj} + \overline{\omega} d_{kj}) - 4u_{lj}D_{kl}) - \lambda))))$$

In comparison with the above two cases, we obtain the optimal solutions to ν_{ki} and u_{ki} as follows:

$$\nu_{ki} = \begin{cases} 1, & (J_k^\nu < J_k^u \wedge i = p_k) \\ 0, & \text{(otherwise)} \end{cases}$$

$$u_{ki} = \begin{cases} 0, (J_k^\nu < J_k^u \wedge i = p_k) \\ \exp(\lambda^{-1}(\sum_{l=1}^n(-\nu_{li}(\underline{\omega} d_{li} + \overline{\omega} d_{ki}) - 4u_{li}D_{kl}) - \lambda \\ -\lambda \log \sum_{j=1}^c(\exp(\lambda^{-1}(-\sum_{l=1}^n(\nu_{lj}(\underline{\omega} d_{lj} + \overline{\omega} d_{kj}) - 4u_{lj}D_{kl}) - \lambda))))). \\ \quad \text{(otherwise)} \end{cases}$$

The Algorithm of ERCM-FU is as same as the one of RCM-FU.

5 Numerical Examples

In this section, we use two artificial datasets (Fig. 1 and Fig. 4) and one real dataset to compare the proposed methods with the conventional ones. We examine the effectiveness of proposed methods (RCM-FU and ERCM-FU).RKM has no evaluation criteria so that we cannot evaluate the outputs of RKM. Therefore, we consider an objective function based on the objective function of HCM as the evaluation criterion as follows:

$$J = \underline{\omega} \times \sum_{i=1}^{c} \sum_{x_k \in \underline{A}_i} d_{ki} + \overline{\omega} \times \sum_{i=1}^{c} \sum_{x_k \in \mathrm{Bnd}(A_i)} d_{ki}.$$

Table 1. Algorithms used for comparison

Algorithm	Parameters
RCM-FU	proposed method: $\underline{\omega} = 0.55,\ m = 2.0$
ERCM-FU	proposed method: $\underline{\omega} = 0.35,\ \lambda = 0.7$
RCM	$\underline{\omega} = 0.55$
RKM	$\underline{\omega} = 0.55$, threshold=0.01
HCM	–
FCM[8]	fuzzy parameter : 2.0

5.1 Artificial Dataset

We show result of artificial dataset in Fig. 1. The membership of each objects to boundaries in Table 2. The membership of boundary is fuzzy.

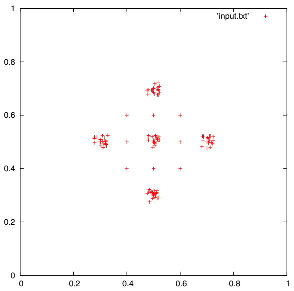

Fig. 1. Original data

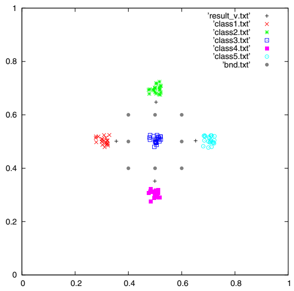

Fig. 2. Artificial data(RCM-FU, $\underline{\omega} = 0.55$)

Table 2. The membership of each objects to upper approximations (RCM-FU)

(x,y)	class1	class2	class3	class4	class5
(0.4,0.6)	0.258989	0.327004	0.237458	0.078667	0.097882
(0.4,0.5)	0.310618	0.150450	0.304497	0.136976	0.097460
(0.4,0.4)	0.254435	0.090088	0.241740	0.315333	0.098405
(0.5,0.6)	0.155314	0.405250	0.241697	0.064693	0.133046
(0.5,0.4)	0.156579	0.075916	0.248490	0.386302	0.132713
(0.6,0.6)	0.123476	0.296644	0.213485	0.073845	0.292549
(0.6,0.5)	0.121365	0.135507	0.255305	0.120102	0.367722
(0.6,0.4)	0.127512	0.087803	0.222790	0.277909	0.283987

We show the result by ERCM-FU in Fig. 3.

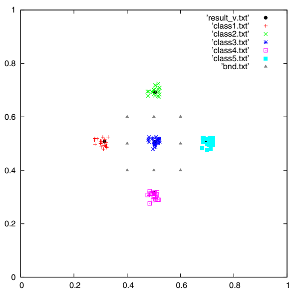

Fig. 3. Artificial data (ERCM-FU, $\underline{\omega} = 0.35$, $\lambda = 0.7$)

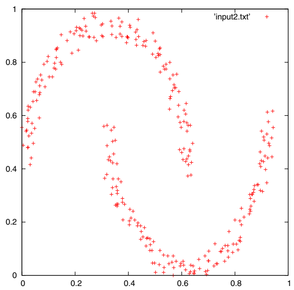

Fig. 4. Original data

The membership of each objects to boundaries in Table 3. The membership of boundary is fuzzy.

Table 3. The membership of each objects to boundaries(ERCM-FU)

(x,y)	class1	class2	class3	class4	class5
(0.4,0.6)	0.169420	0.625175	0.202058	0.000057	0.000067
(0.4,0.5)	0.631929	0.129328	0.216806	0.019321	0.000191
(0.4,0.4)	0.174968	0.006356	0.064091	0.754525	0.000059
(0.5,0.6)	0.004080	0.657372	0.329219	0.000075	0.006546
(0.5,0.4)	0.003527	0.005084	0.080845	0.905825	0.004677
(0.6,0.6)	0.000020	0.427396	0.384281	0.000026	0.188276
(0.6,0.5)	0.000072	0.082251	0.392387	0.010462	0.514828
(0.6,0.4)	0.000029	0.005078	0.146973	0.620485	0.223996

We show the results of crescents data by proposed methods and RCM in Fig 5, 6 and 7.

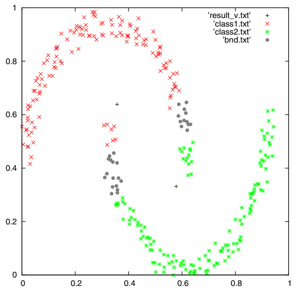

Fig. 5. Crescents data(RCM, $\underline{\omega} = 0.55$)

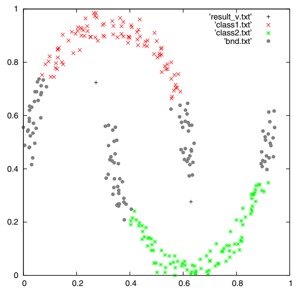

Fig. 6. Crescents data(RCM-FU, $\underline{\omega} = 0.55$)

From the above results, we can find as follows:

- Some objects are classified into incorrect lower approximation by RCM and ERCM-FU.
- There are more objects which are classified into correct lower approximation by RCM-FU than RCM.
- There are more objects which we classified into boundary by RCM-FU than RCM.

5.2 Comparison of Proposed Methods with Conventional Ones

We compare the proposed methods to conventional ones through Iris dataset (150 objects, 4 dimensions, 3 clusters). We define the ratio of correct answers as (the number of correct answers)/(the number of objects). We assign the objects which were classified into boundaries to the cluster to which the membership is maximum. Table 4 shows as follows:

Table 4. The ratio of correct answers

Algorithm	lower approximation			boundary			total	
	numbers	correct	ratio	numbers	correct	ratio	numbers of correct	ratio
RCM-FU (m=1.5)	116	115	0.991	34	20	0.588	135	0.9
RCM-FU (m=2.0)	68	67	0.985	82	62	0.756	129	0.86
ERCM-FU ($\lambda = 0.5$)	145	137	0.944	5	5	1.0	142	0.947
ERCM-FU ($\lambda = 2.0$)	109	109	1.0	41	32	0.780	141	0.94
RCM ($\underline{\omega} = 0.55$)	139	128	0.921	11	5	0.455	133	0.887
RCM ($\underline{\omega} = 0.75$)	135	126	0.933	15	11	0.733	137	0.913
RKM (threshold=0.01)	150	134	0.893	0	0	—	134	0.893
RKM (threshold=3.0)	76	75	0.987	74	40	0.541	115	0.767
FCM	150	134	0.893	0	0	—	134	0.893
HCM	150	134	0.893	0	0	—	134	0.893

- There are more objects which are classified into correct lower approximations by RCM-FU and ERCM-FU than RCM.
- More objects are classified into boundaries as the parameter m increases by RCM-FU.
- All objects are classified into correct lower approximations when the parameter λ is suitable by ERCM-FU.
- Less objects are classified into lower approximation as the parameter λ increases by ERCM-FU.
- No objects are classified into boundaries as the threshold=0.01 by RKM.
- We get the same results by HCM and FCM.

The optimal parameter for lower approximation is different from the optimal one for the whole in both RCM-FU and ERCM-FU.

5.3 Consideration of Parameters

We consider the relation between parameters and the number of objects which are classified into boundaries.

Fig. 8 shows the relation between $\underline{w}$ and the number of objects which are classified into boundaries by RCM. Horizontal- and vertical-axes mean $\underline{w}$ and the number of objects which are classified into boundaries, respectively. Fig. 8 shows that $\underline{w}$ is ineffective at the number of objects which are classified into boundaries. This means that it is difficult to adjust the ratio of the number of objects into boundaries to the number of all objects by the parameter $\underline{w}$.

Fig. 9 shows the relation between m and the number of objects which are classified into boundary by RCM-FU. Horizontal- and vertical-axes mean m and the number of objects which are classified into boundaries, respectively. Fig. 9 shows that m is effective at the number of objects which are classified into boundaries. This means that it is easy to adjust the ratio of the number of objects into boundaries to the number of all objects by the parameter m.

Fig. 10 shows the relation between λ and the number of objects which are classified into boundaries by RCM-FU. Horizontal- and vertical-axes mean λ

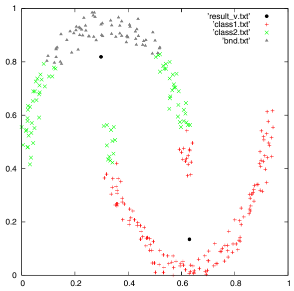

Fig. 7. Crescents data(ERCM-FU, $\underline{\omega} = 0.35,\ \lambda = 1.0$)

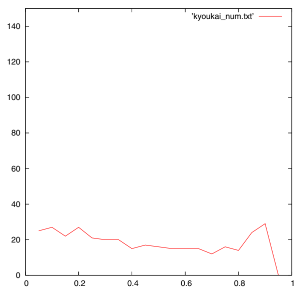

Fig. 8. The relation between $\underline{w}$ and the number of objects which are classified into boundaries by RCM

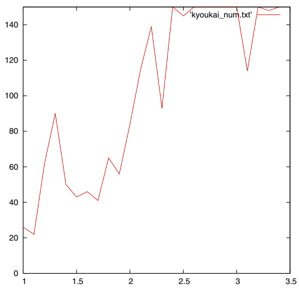

Fig. 9. The relation between m and the number of objects which are classified into boundaries by RCM-FU

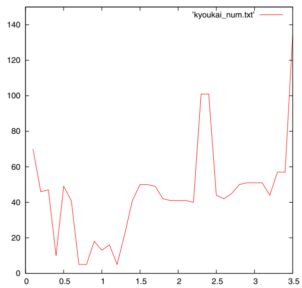

Fig. 10. The relation between λ and the number of objects which are classified into boundaries by ERCM-FU

and the number of objects which are classified into boundaries, respectively. We fixed $\underline{w} = 0.35$.

6 Conclusion

This paper proposed new rough clustering algorithms based on optimization of objective functions. The proposed methods based on optimization of objective functions with fuzzy-set representation can classify more objects into correct lower approximations than conventional ones proposed by Endo et al [7], and we can adjust the ratio of the number of objects into boundaries to the number of all objects by parameters. The conventional rough clustering algorithm [7] has a problem that an object cannot belong to more than two upper approximations. We introduced fuzzy-set representation into membership of boundary to solve such a problem. Thus, each object into boundaries has a membership grade in [0,1], and we can classify the object according to the value of the grade. Consequently, we can classify all objects into clusters like FCM [8].

References

1. Pawlak, Z.: Rough Sets. International Journal of Computer and Information Sciences 11(5), 341–356 (1982)
2. Lingras, P., Yao, Y.: Data Mining Using Extensions of Rough Set Model. Journal of the American Society for Information Science, 415–422 (1998)
3. Lingras, P., Peters, G.: Rough clustering. WIREs Data Mining and Knowledge Discovery 1(1), 64–72 (2011)
4. Hirano, S., Tsumoto, S.: An Indiscernibility-Based Clustering Method with Iterative Refinement of Equivalence Relations. Journal of Advanced Computational Intelligence and Intelligent Informatics 7(2), 169–177 (2003)
5. Lingras, P., West, C.: Interval Set Clustering of Web Users with Rough K-Means. Journal of Information Systems, 5–16 (2004)
6. Peters, G.: Some refinements of rough k-means clustering. Pattern Recognition 39, 1481–1491 (2006)
7. Endo, Y., Kinoshita, N.: On Objective-Based Rough c-Means Clustering. In: Proc. of the 2012 IEEE International Conference on Granular Computing (2012)
8. Bezdek, J.C.: Pattern Recognition with Fuzzy Objective Function Algorithms. Plenum Press, New York (1981)