Forecasting in Fuzzy Time Series by an Extension of Simple Exponential Smoothing

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Abstract. Fuzzy Time Series was introduced to improve the forecasting made by statistical methods in vague or imprecise data and in time series with few samples available. However, the integration of these concepts is a little explored area. In this paper we introduced a new forecast model composed by a pre-processing method and a predicting method. The pre-processing method is responsible for analyzing the data and defining a suitable structure of representation. The predicting method is based on the combination of fuzzy time series concepts with the simple exponential smoothing, a traditional statistical method for prediction. The experiments performed with the TAIEX index show that, besides obtaining better accuracy rates when compared with other methods available in the literature, the predictions made over the whole time series had the same behavior and trends than the real data.

Keywords: Fuzzy time series · Simple exponential smoothing · Forecasting

1 Introduction

In the last years, Fuzzy Time Series (FTS) has been used in many forecasting problems, especially when the data in the Time Series (TS) are represented by linguistic terms, are vague, imprecise or when there are few samples available. Since it was introduced by Song and Chinson [1,2,3], several proposals have been presented to extend the main concepts with applications in different areas. Basically, the FTS follow three steps: (1) fuzzification of crisp values; (2) derivation of Fuzzy Logical Relationships (FLR); and (3) calculation of the forecasted value. In Uslu *et al.* [4], the authors use genetic algorithm to fuzzify and to derivate the FLR. The calculation of forecasted values considers the frequency of each FLR, but do not consider when the FLR occurred. A hybrid system with concepts of support vector machines, neural network and linguistic fuzzy rules is presented by Stepnicka *et al.* [5] to deal forecasting problems. Chatterjee *et al.* [6] presented a second order forecast model to multi-attribute time series. In [7] a forecasting method for FTS based on k-means is introduced.

In the literature, several researches in FTS focus on forecasting without the preprocessing of the data [8,9,10]. However, the preprocessing has an important role to improve the forecasting accuracy rate. Without the preprocessing, the forecast in FTS

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can be prejudiced by outliers and by fuzzy sets that represent inadequately the distribution of the data over the entire time series.

Some works have reported better accuracy in forecasted values by FTS than by traditional statistics methods [5], particularly in TS with few samples or irregular behavior. However, the integration of FTS concepts with the traditional statistical methods is a little explored area. An important feature frequently neglected in FTS research is the updating of obtained knowledge with the arrival of new samples. In a FTS with variation in its behavior, the update of FLR identified during the training can be essential for a good accuracy on the new forecasts. Therefore, this research arises to overcome this lack in FTS. Besides keeping the FLR base updated with the arrival of new samples, allowing the forecasting model to follow the new tendencies of the time series, the proposed model is composed of a preprocessing method and a forecasting method. The pre-processing method aims at removing the outliers and creating a suitable representation by means of linguistic terms that reflects the real structure of the data. The forecasting method is defined using a combination of FLR concepts with the extension of simple exponential smoothing, a traditional statistical method. The obtained results confirm the effectiveness of the proposal.

The remaining of this paper is organized as follows. Section 2 presents the basic concepts of fuzzy time series and simple exponential smoothing. The proposed forecasting model is introduced in Section 3. In Section 4 the results and comparisons among the proposed method and other techniques found in the literature are presented. Section 5 summarizes the main conclusions and gives indications for future work.

2 Basic Concepts

2.1 Fuzzy Time Series

Based on Zadeh's works [11,12], a fuzzy set A in the universe of discourse $U = \{u_1, u_2, ..., u_n\}$, can be defined by $A = f_A(u_1)/u_1 + f_A(u_2)/u_2 + \cdots + f_A(u_n)/u_n$ where f_A is the membership function of the fuzzy set A, $f_A: U \rightarrow [0,1]$, and $f_A(u_i)$ denotes the value of membership of u_i in the fuzzy set A, where $1 \leq i \leq n$. In the sequence, the basic definitions about fuzzy time series are presented [1,2,3]:

Definition 1: Let $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots)$, a subset of $\mathbb{R}$, be the universe in which fuzzy sets $f_i(t)$ $(i = 1, 2, \ldots)$ are defined. If $F(t)$ is a collection of $f_i(t)(i = 1, 2, \ldots)$, then, $F(t)$ is called a fuzzy time series on $Y(t)$.

Definition 2: If $F(t)$ is caused by $F(t-1), F(t-2), \ldots, F(t-n)$, then the fuzzy logical relationship between them can be represented by a high order fuzzy logical relationship as $F(t-1), F(t-2), \ldots, F(t-n) \rightarrow F(t)$ and it is called the nth-order fuzzy time series model.

According to Song and Chinson [3], the forecasting process in FTS follows five steps: (1) specify the universe of discourse U on which the fuzzy sets will be defined; (2) partition U into several equal length intervals for defining the fuzzy sets; (3) if the historical data are linguistic values, go to Step 4; otherwise fuzzify the data; (4) forecast the linguistic value; (5) defuzzify the linguistic value attained in Step 4.

2.2 Simple Exponential Smoothing

Proposed by Brown and Meyer [13], the Simple Exponential Smoothing ponders past observations by means of exponentially decreasing weights. The forecasting value is defined by Equation 1:

$$\bar{d}_{t+1} = \sum_{i=0}^{t-1} \alpha(1-\alpha)^i d_{t-i} + (1-\alpha)^t d_0 \quad (1)$$

Where t is the index of most recent sample, $\bar{d}_{t+1}$ is the forecasted value for the time $t+1$, d_{t-i} is the crisp value at time $t-i$, and α is the smoothing factor and should attend the constraint $0 \le \alpha \le 1$. If α is equal to 1, the forecasted value is equal to the most recent sample, that is, the sample at time t. If α is near 0, than the forecasted value have more influence of the oldest sample, that is, the first sample of time series.

3 Proposed Forecasting Model

In this section we present a forecasting model developed from the integration of fuzzy time series concepts with simple exponential smoothing of statistical area. The model is composed by two main processes: (1) pre-processing and; (2) forecasting method.

3.1 Pre-processing Method

The pre-processing is based on [14] and its purpose is to analyze the data to allow a better linguistic representation of TS and, in this way, improve the accuracy of forecasting values. This method consists of four main steps shown in Figure 1.

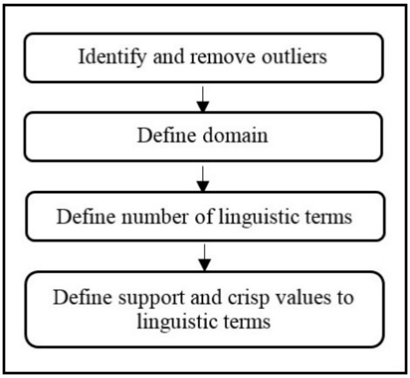

Fig. 1. Steps of the pre-processing method

The existence of one or more outliers in the sample data can exert a negative influence in all remaining steps in the process. Thus, consider the existence of n historical data in $Y(t)$, where $t = 0,1,2,\ldots,n$. The first step to detect an outlier is define the square matrix H as in Equation 2.

$$H = x(x^T x)^{-1} x^T \quad (2)$$

where

$$x = \begin{bmatrix} 1 & d_0 \\ 1 & d_1 \\ 1 & d_2 \\ \vdots & \vdots \\ 1 & d_n \end{bmatrix} \quad (3)$$

and d_i for $i = 0,1,\ldots,n$ are the observed values in the time series. In the sequence, the $Residual\ Student$ index is computed by Equation 4 for each sample in the time series [15].

$$RStudent_i = \frac{e_i}{\hat{\sigma}^{(i)}\sqrt{1-h_i}} \quad (4)$$

In Equation 4, $\hat{\sigma}^{(i)}$ is the standard deviation without the ith sample of time series, h_i is the ith diagonal element in the matrix H, and e_i is defined by Equation 5, where d_i is the ith sample of the time series. In this work, the ith observed sample is an outlier, when the $RStudent_i$ is equal or greater than 2.5. This value was defined after the execution of some preliminary experiments.

$$e_i = d_i - \frac{\sum_{t=0}^{n} d_t}{n} \quad (5)$$

Once the outliers have been identified and removed, the universe of discourse D is defined according to Equation 6, where D_{min} is the minimum value in the observed values, D_{max} is the maximum and σ is the standard deviation of the historical data.

$$D = [D_{min} - \sigma, D_{max} + \sigma] \quad (6)$$

The number of linguistic terms that will represent the data in the time series is defined by the following steps.

Step 1. Sort the n numerical data in an ascending sequence as $d_0, d_1, d_2, \ldots, d_n$, where d_0 is the smallest datum among the n numerical data and d_n is the highest datum.

Step 2. Calculate the threshold τ as the maximum distance to fuse clusters using Equation 7, where δ is the standard deviation of the difference between the sorted samples defined in Step 1.

$$\tau = \frac{\sum_{i=1}^{n-1}(d_{i+1} - d_i)}{n-1} + \delta \quad (7)$$

Step 3. Put each numerical datum into a cluster as $\{d_1\}, \{d_2\}, \ldots, \{d_i\}, \ldots, \{d_n\}$, where the symbol "{ } " denotes a cluster.

Step 4. Assume that there are p clusters. Calculate the center of cluster k by the arithmetic mean of all data that belong to it, for $1 \leq k \leq p$.

Step 5. Calculate the distance between clusters k and $k+1$ by Equation 8, where $k = 1, 2, 3, \ldots, p-1$.

$$distance_{k,k+1} = |cluster\_center_k - cluster\_center_{k+1}| \quad (8)$$

Step 6. If the shortest distance between two clusters is less than τ, then join the clusters into only one cluster and go back to Step 4. Otherwise, stop the algorithm.

In order to reflect the real structure of data, the parameter p resulting from the previous algorithm will be used as input to indicate the number of clusters in the fuzzy c-means [16]. After its execution, the centroids found will be used to represent the linguist terms of FTS. Thus, the parameters of the membership functions that represents the linguistic terms are defined as follows:

- The linguistic terms L_j, for $1 < j < p$, are represented by triangular fuzzy sets, whose center is the center of cluster j;
- The left-limit and the right-limit of the support of linguistic terms L_j, where $1 < j < p$, are defined by the cluster center $j - 1$ and $j + 1$, respectively;
- The linguistic term L_1 is represented by a trapezoidal membership function with values equal to 1 in the interval $[D_{min} - \sigma, center\_cluster_1]$, where σ is the standard deviation of the series and D_{min} is the minimum value in the observed data;
- The left-limit and the right-limit of the support of L_1 are defined, respectively, by $D_{min} - \sigma$ and $center\_cluster_2$;
- The linguistic term L_p is also represented by a trapezoidal membership function, with values equal to 1 in the interval $[center\_cluster_p\ , D_{max} + \sigma]$, where σ is the standard deviation of the series and D_{max} is the maximum value in the observed data;
- The left-limit and the right-limit of the support of L_p are defined by $center\_cluster_{p-1}$ and $D_{max} + \sigma$, respectively;

After the pre-processing of the time series, the fuzzification of data is performed and the FLR are derived to make predictions as presented in the next subsection.

3.2 Forecasting Method

In general, the forecasting methods by means of time series consider that, from the observations and analysis of available samples, it is possible to predict the values and, therefore, the future behavior of the series. However, several time series have different behavior along the entire observation period. The proposed model consider that the most recent samples should have more influence in the process of forecasting than the earliest samples and, for this, uses the statistical technique called simple exponential smoothing. Besides, other important feature of the proposed model is the update of the FLR base with the arrival of new samples, i.e., after the acquisition of knowledge from training data, the model is updated by derivation of new FLR with the arrival of new samples. This feature allows the method to enhance the accuracy rate in scenarios where the series present a different behavior when compared with the training data. The proposed forecasting method is composed by four steps: (1) fuzzification of the original data; (2) extraction of a FLR base formed by second order FLR with three consequents; (3) forecasting value using the integration of FLR with the simple exponential smoothing concepts; (4) update the FLR base.

After the definition of fuzzy sets in the pre-processing phase, the fuzzification of the time series consists of finding the linguistic term that better represents each crisp value of the series. The derivation of fuzzy logical relationships from the fuzzified FTS, as proposed in this work, uses an extension of the traditional definition of FLR presented in the Section 2.1. In this proposal, we use second order FLR with three linguist terms in the consequent where the reverse simple exponential smoothing will be applied. Thus, the knowledge is represented by a FLR base composed by $n-4$ fuzzy logical relationships of the form presented in Equation 9.

$$L_t, L_{t+1} \rightarrow L_{t+2}, L_{t+3}, L_{t+4} \tag{9}$$

where L_t is the linguistic term in time t, $t = 1, 2, \dots, n-4$, and n is the number of observed data in the FTS. With three linguistic terms in the consequent of FLR, it is possible to better identify the future trends over forecasted values. However, previous experiments have suggested that by considering more than three terms in the consequent of FLR, the accuracy rate starts to decrease.

Once the FLR base has been defined, the predictions are performed from the last two linguistic terms in the FTS, L_n and L_{n-1}. The FLR in the base with the antecedents L_n and L_{n-1} will be used in the calculation of the forecasted value. Differently than the original method [13], in which the exponential smoothing is applied over the past values of the entire time series, in the proposed method a modified version of exponential smoothing is introduced. In this work, the smoothing is performed both on the three crisp values that represent the linguistic terms in the consequents of each FLR being used in the forecasting and on the FLR themselves, so that the most recent ones have higher influence than the oldest ones. Suppose that a FLR of the form presented in Equation 9 will be used in the forecasting. This FLR then, generates a smoothing value S, defined by Equation 10.

$$S = (1-\alpha)^2 l_{t+2} + \alpha(1-\alpha) l_{t+3} + \alpha l_{t+4} \tag{10}$$

where α is the smoothing factor, l_{t+2}, l_{t+3} and l_{t+4} represents the crisp values of the corresponding linguistic terms L_{t+2}, L_{t+3} and L_{t+4}, respectively and ,as in Equation 1, the constraint $0 \leq \alpha \leq 1$ should be observed.

Consider that there are m FLR in the base with the terms L_{n-1} and L_n in the antecedents, and q is the index of the most recent value S_j, $j = 0, 1, \dots, q$, defined by Equation 10. The forecasted value d_{n+1} is calculated as presented in Equation 11.

$$d_{n+1} = \alpha \sum_{i=0}^{q-1} (1-\alpha)^i S_{q-i} + (1-\alpha)^q S_0 \tag{11}$$

where S_0 refers to the value defined by the oldest FLR with antecedents L_{n-1} and L_n in the base. Similarly to Equation 1, the α value should attend the constraint $0 \leq \alpha \leq 1$.

The continuous update of FLR base is also an important feature to attain the good accuracy in the proposed method. The arrival of new samples can mean a change in the behavior of time series. Thus, after the arrival of each new sample, a new FLR with two antecedents and three consequents is added to the FLR base. In the next section the experiments performed are described. The results show that the accuracy rate of the proposed method was better than other methods available in the literature.

4 Experiments

To validate the proposed method, four experiments with the Taiwan Stock Exchange (TAIEX) index between 2001 and 2004 were performed and compared with other methods in the literature. The training was performed with the data from January to October of each year, and the data between November and December were used in the tests. Table 1 shows the amount of samples, the domain and number of linguistic terms of each time series obtained from the pre-processing.

Table 1. Pre-processing in TAIEX index between 2001 and 2004

Time Series	Samples	Domain	Linguistic Terms	Samples Trainning	Samples Test
TAIEX 2001	244	[2732.30; 6818.20]	52	201	43
TAIEX 2002	248	[3190.59; 7121.75]	49	205	43
TAIEX 2003	249	[3580.74; 6666.89]	49	206	43
TAIEX 2004	250	[4861.44; 7489.53]	48	205	45

The next step after the pre-processing is the fuzzification of the crisp values values in the original observed data. Each value in the FTS is represented by the cluster center of the corresponding linguistic term. To illustrate the process, Table 2 shows the linguistic terms and their respective center values for TAIEX 2003 index.

Table 2. Crisp values for the representation of linguistic terms for TAIEX 2003 index

Linguistic Term	Value	Linguistic Term	Value	Linguistic Term	Value	Linguistic Term	Value
0	4439.08	13	5223.69	26	5488.22	39	5853.37
1	4537.30	14	5257.34	27	5513.87	40	5869.14
2	4590.80	15	5284.78	28	5522.88	41	5917.75
3	4648.07	16	5301.02	29	5553.36	42	5925.46
4	4691.68	17	5303.11	30	5582.89	43	5957.99
5	4827.69	18	5318.04	31	5620.80	44	6038.06
6	4893.06	19	5341.17	32	5645.91	45	6039.01
7	4939.72	20	5367.29	33	5678.97	46	6044.48
8	4970.38	21	5393.38	34	5694.98	47	6066.15
9	4997.74	22	5409.20	35	5721.49	48	6094.29
10	5075.14	23	5438.26	36	5750.26	-	-
11	5144.70	24	5448.49	37	5817.84	-	-
12	5203.78	25	5486.13	38	5818.87	-	-

The FTS is fuzzified using the linguistic terms defined on its domain. For example, the fuzzified training data set of TAIEX 2003 index using the linguistic terms presented in Table 2, is shown in Table 3.

Table 3. Training data set of fuzzy time series

$L_1, L_3, L_4, L_4, L_5, L_5, L_5, L_9, L_9, L_9, L_7, L_6, L_7, L_7, L_9, L_{10}, L_{10}, L_8, L_9, L_5, L_4, L_3, L_2, L_3, L_1, L_1,$
$L_4, L_2, L_1, L_1, L_1, L_2, L_0, L_0, L_0, L_1, L_1, L_0, L_0, L_0, L_0, L_0, L_0, L_0, L_0, L_0, L_1, L_1, L_2, L_2, L_2, L_1, L_1,$
$L_1, L_0, L_0, L_0, L_0, L_0, L_1, L_2, L_1, L_1, L_1, L_1, L_0, L_0, L_2, L_2, L_3, L_3, L_1, L_2, L_0, L_0, L_0, L_0, L_0, L_0, L_0,$
$L_0, L_0, L_0, L_0, L_0, L_0, L_0, L_0, L_0, L_0, L_0, L_0, L_0, L_0, L_0, L_0, L_0, L_1, L_1, L_4, L_4, L_4, L_4, L_5, L_5, L_5, L_6,$
$L_6, L_6, L_8, L_9, L_{10}, L_9, L_7, L_6, L_7, L_6, L_6, L_6, L_9, L_{10}, L_{10}, L_{11}, L_{18}, L_{20}, L_{20}, L_{15}, L_{13}, L_{19}, L_{19}, L_{22},$
$L_{16}, L_{15}, L_{13}, L_{14}, L_{15}, L_{22}, L_{21}, L_{24}, L_{19}, L_{17}, L_{18}, L_{21}, L_{20}, L_{14}, L_{13}, L_{14}, L_{13}, L_{13}, L_{14}, L_{23}, L_{23},$
$L_{26}, L_{27}, L_{27}, L_{29}, L_{31}, L_{32}, L_{33}, L_{29}, L_{29}, L_{28}, L_{32}, L_{34}, L_{35}, L_{33}, L_{31}, L_{32}, L_{35}, L_{33}, L_{31}, L_{32},$
$L_{31}, L_{34}, L_{36}, L_{36}, L_{36}, L_{33}, L_{33}, L_{35}, L_{34}, L_{32}, L_{32}, L_{31}, L_{30}, L_{34}, L_{36}, L_{39}, L_{39}, L_{38},$
$L_{40}, L_{43}, L_{42}, L_{42}, L_{44}, L_{46}, L_{47}, L_{47}, L_{46}, L_{43}, L_{41}, L_{43}, L_{47}, L_{48}, L_{48}.$

The next step is to derivate the fuzzy logical relationships from the FTS for, in the sequence, calculate the forecasted values. For instance, according to Table 3, the first second-order FLR with three consequents to be inserted in the FLR as proposed in this model is $L_1, L_3 \rightarrow L_4, L_4, L_5$, the second is $L_3, L_4 \rightarrow L_4, L_5, L_5$ and so on. After the FLR base has been defined, the forecasted values were calculated. The actual and forecasted TAIEX indexes for November and December 2003 are shown in Table 4.

Table 4. Actual and forecasted index for the months of November and December of 2003

Date	Actual	Forecasted	Date	Actual	Forecasted
2003/11/03	6087.45	6044.48	2003/12/03	5884.97	5917.75
2003/11/04	6108.99	6094.29	2003/12/04	5920.46	5869.14
2003/11/05	6142.32	6053.94	2003/12/05	5900.05	5917.75
2003/11/06	6013.40	6053.94	2003/12/08	5847.15	5917.75
2003/11/07	6056.83	6038.06	2003/12/09	5859.56	5853.37
2003/11/10	6059.03	6066.15	2003/12/10	5803.42	5799.73
2003/11/11	6022.08	6024.02	2003/12/11	5867.05	5817.84
2003/11/12	5982.75	6038.06	2003/12/12	5858.32	5869.14
2003/11/13	6035.44	5957.99	2003/12/15	5924.24	5853.37
2003/11/14	6044.77	6038.06	2003/12/16	5887.23	5925.46
2003/11/17	5952.32	6063.99	2003/12/17	5752.01	5818.87
2003/11/18	5939.47	5936.21	2003/12/18	5768.76	5750.26
2003/11/19	5865.51	5947.49	2003/12/19	5759.23	5791.74
2003/11/20	5834.24	5869.14	2003/12/22	5835.11	5791.74
2003/11/21	5830.06	5818.87	2003/12/23	5845.51	5818.87
2003/11/24	5821.58	5818.87	2003/12/24	5857.87	5833.78
2003/11/25	5861.18	5818.87	2003/12/25	5853.70	5812.87
2003/11/26	5860.61	5853.37	2003/12/26	5857.21	5812.87
2003/11/27	5740.57	5837.30	2003/12/29	5804.89	5812.87
2003/11/28	5771.77	5750.26	2003/12/30	5866.75	5873.35
2003/12/01	5870.17	5709.97	2003/12/31	5890.69	5861.44
2003/12/02	5911.45	5869.14	-	-	-

To illustrate the advantage of updating the FLR base and considering all the FLR with matching antecedent in the forecasting process, consider the forecasting for December 29, 2003. The two preceding actual values L_n and L_{n-1} in the time series are represented, respectively, by the linguistic terms L_{39} and L_{39}. In the FTS showed in Table 3, we can identify the FLR $L_{39}, L_{39} \rightarrow L_{38}, L_{40}, L_{43}$ to calculate the forecasted value S_0. If we consider, as usually is done in traditional forecasting, only this FLR and $\alpha = 0.1$ in Equation 10, the weight of each linguistic term in the FLR consequent would be 0.81, 0.09 and 0.1. Considering the linguistic terms in Table 2, and by the employment of Equation 11, the forecasted index would be 5837.30. However, before December 29, the FLR base was updated by the arrival of new samples after the initial training. With the samples of November 25 to 30 and December 1st, the FLR $L_{39}, L_{39} \rightarrow L_{36}, L_{36}, L_{40}$, was included in the FLR base. Similarly, the samples of December 08 to 12 generate the FLR $L_{39}, L_{39} \rightarrow L_{37}, L_{40}, L_{39}$.

Our method considers all the FLR whose antecedent matches the values used in the forecast. Thus, from the matching of the three FLR, we have $S_0 = 5837.30$, $S_1 = 5762.14$ and $S_2 = 5826.01$. Considering $\alpha = 0.5$ in Equation 11, the forecasted value is 5812.87, which is closer to the real value than the previous value. The graphics of actual index and forecasted index for TAIEX in the years 2001, 2002 and 2004, are shown in Figures 2, 3 and 4, respectively.

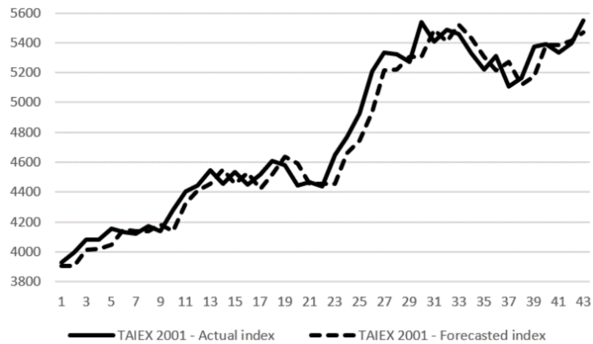

Fig. 2. Actual and forecasted indexes for November and December 2001

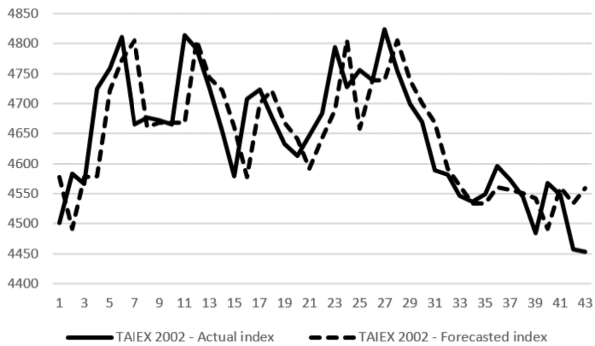

Fig. 3. Actual and forecasted indexes for November and December 2002

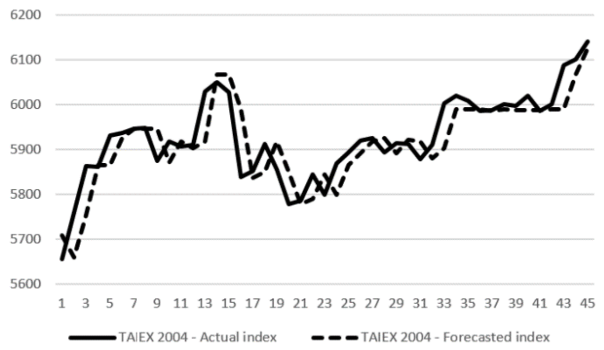

Fig. 4. Actual and forecasted indexes for November and December 2004

The obtained results were compared with six different methods by the Root Mean Square Error (RMSE) calculated from Equation 12.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(d_i - \overline{d_i})^2}{n}} \tag{12}$$

where n is the total number of forecasting, d_i is the actual index and $\bar{d}_i$ is the forecasted index. Table 5 shows the results.

Table 5. RMSE of TAIEX index

Method	2001	2002	2003	2004	Average
Huarng, et. al. [17]	124.02	93.48	65.51	72.35	88.84
Yu and Huarng [18]	120	69	52	60	75.25
Chen and Chen [19]	115.33	71.01	58.06	57.33	75.43
Chen and Chang [20]	113.33	66.82	53.51	60.48	73.53
Chen, et. al.[21]	114.47	67.17	52.49	52.84	71.74
Brown and Meyer [13]	123.19	66.07	52.68	56.97	74.73
Proposed method	106.26	66.73	51.12	52.90	69.25

The methods proposed in [17,18,19,20,21] are based on the FTS concepts and the method presented in [13] is the simple exponential smoothing from the statistical area. Besides obtaining better accuracy when compared with others methods available in the literature, the series predicted by the proposed method maintained the same behavior and trend of the actual index, as can be observed in the preceding graphics.

5 Conclusions

The behavior presented by the time series can change during the observation period. For instance, time series with growth trend can present decline periods, or yet, to change the intensity in which the variation occurs. In this work we proposed a forecasting method that aims at identifying and considering the chances in the series behavior in the forecasting process. The proposed method combines the concepts of the simple exponential smoothing with the fuzzy logical relationships. While in the traditional method the smoothing is applied in the past samples, in our work the smoothing is applied to the consequents of the FLR. Another important aspect of the method proposed is the constant update of FLR base toghether with the consideretion of all the FLR whose antecedent match the last two values in the series in the porecasting. Furthermore, the pre-processing has an essential role, given that the suitable linguistic representation of the data structure is fundamental for the proposed model. In this way, with the analysis of the obtained results is possible to assert that the proposed model is able to deal with different trends in the time series and to attain good results.

For future work will be considered the development of a method to analyze and classify the time series by means of clustering, where each time series will be considered an element in the cluster. The main purpose is to predict not only the time series values, but also changes in their behavior.

References

1. Song, Q., Chissom, B.S.: Fuzzy Time Series and its Models. Fuzzy Sets and Systems **54**, 269–277 (1993)
2. Song, Q., Chissom, B.S.: Forecasting Enrollments With Fuzzy Time Series – Part I. Fuzzy Sets and Systems **54**, 1–9 (1993)
3. Song, Q., Chissom, B.S.: Forecasting Enrollments With Fuzzy Time Series – Part II. Fuzzy Sets and Systems **62**, 1–8 (1994)
4. Uslu, V.R., Bas, E., Yolcu, U., Egrioglu, E.: A Fuzzy Time Series Approach Based on Weights Determined by the Number of Recurrences of Fuzzy Relations. Swarm and Evolutionary Computation **15**, 19–26 (2014)
5. Stepnicka, M., Cortez, P., Donate, J.P., Stepnicková, L.: Forecasting Seasonal Time Series With Computational Intelligence: On Recent Methods and the Potential of their Combinations. Expert Systems with Applications **40**, 1981–1992 (2013)
6. Chatterjee, S., Nigam, S., Singh, J.B., Upadhyaya, L.N.: Application of Fuzzy Time Series in Prediction of Time Between Failures & Faults in Software Reliability Assessment. Fuzzy Information and Engineering **3**, 293–309 (2011)
7. Kai, C., Fang-Ping, F., Wen-Gang,C.: A Novel Forecasting Model of Fuzzy Time Series Based on K-Means Clustering. In: Proc. Second International Workshop on Education Technology and Computer Science, pp. 223–225 (2010)
8. Joshi, B.P., Kumar, S.: A Computational Method for Fuzzy Time Series Forecasting Based on Difference Parameters. International Journal of Modeling, Simulation, and Scientific Computing **4**(1), 1250023-1–1250023-12 (2013)
9. Chu, H., Chen, T., Cheng, C., Huang, C.: Fuzzy Dual-Factor Time-Series for Stock Index Forecasting. Expert Systems With Applications **36**, 165–171 (2009)
10. Qiu, W., Liu, X., Li, H.: A Generalized Method for Forecasting Based on Fuzzy Time Series. Expert Systems With Applications **38**, 10446–10453 (2011)
11. Zadeh, L.A.: Fuzzy Set. Information and Control **8**, 338–353 (1965)
12. Zadeh, L.A.: The Concept of a Linguistic Variable and its Application to Approximate Reasoning - Part 1. Information Sciences **8**, 199–249 (1975)

13. Brown, R.G., Meyer, R.F.: The fundamental theory of exponential smoothing. Operations Research **9**, 673–685 (1961)
14. Santos, F.J.J., Camargo, H.A.: Preprocessing in Fuzzy Time Series to Improve the Forecasting Accuracy. In: 12th International Conference on Machine Learning and Applications, pp. 170–173 (2013)
15. Barnett, V., Lewis, T.: Outliers in Statistical Data, 3rd ed. John Wiley & Sons, NY (1994)
16. Bezdek, J.C., Tsao, E.C., Pal, N.R.: Fuzzy Kohonen Clustering Networks. In: IEEE International Conference on Fuzzy Systems, pp. 1035–1043 (1992)
17. Huarng, K., Yu, H.K., Hsu, Y.W.: A Multivariate Heuristic Model for Fuzzy Time-Series Forecasting. IEEE Trans. Syst. Man, Cybern. B, Cybern. **37**(4), 836–846 (2007)
18. Yu, T.H.K., Huarng, K.H.: A Bivariate Fuzzy Time Series Model to Forecast the TAIEX. Expert Systems With Applications **34**, 2945–2952 (2008)
19. Chen, S.M., Chen, C.D.: TAIEX Forecasting Based on Fuzzy Time Series and Fuzzy Variation Groups. IEEE Trans. Fuzzy Syst. **19**, 1–12 (2011)
20. Chen, S.M., Chang, Y.C.: Multi-Variable Fuzzy Forecasting Based on Fuzzy Clustering and Fuzzy Rule Interpolation Techniques. Information Sciences: an International Journal. **180**(24), 4772–4783 (2010)
21. Chen, S.M., Chu, H.P., Sheu, T.W.: TAIEX Forecasting Using Fuzzy Time Series and Automatically Generated Weights of Multiple Factors. IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans **42**, 1485–1495 (2012)