Stochastic Cash Flow Management Models: A Literature Review Since the 1980s

Marcelo Botelho da Costa Moraes, Marcelo Seido Nagano and Vinicius Amorim Sobreiro

Abstract Defining cash balance is a classic problem in firms' financial management. For this reason, the aim of this study is to carry out a literature review, presenting the main cash flow management models from the Baumol and Tobin models in 1950s, to Miller–Orr model in 1960s and their development since the 1980s, focused essentially in stochastic models, with publication in economic, financial, and operation research journals. Thus, this chapter provides a review on cash balance management models in order to obtain a more consistent model on a par with investment analysis, observing the characteristics associated with cash maintenance, as well as diversification of financial applications and resources and the lack of literature in stochastic models for this problem.

Keywords Finance · Cash balance · Cash flow · Cash holding

1 Introduction

Managing the available cash balance is a constant problem in all types of firms, which happens due to the daily inflows and outflows, whether by operating activities of the firm or financial transactions. Therefore, there is a requirement to

M.B. da Costa Moraes (✉)
School of Economics, Business Administration and Accounting,
University of São Paulo, 3900 Bandeirantes Av., 14040-905 Ribeirão Preto,
SP, Brazil
e-mail: mbotelho@usp.br

M.S. Nagano
School of Engineering of São Carlos, University of São Paulo,
400 Trabalhador Sãocarlense ave, Centro, 13566-590 São Carlos, SP, Brazil
e-mail: drnagano@sc.usp.br

V.A. Sobreiro
Faculty of Economics, Management and Accounting, University of Brasília,
Campus Darcy Ribeiro - FACE North Wing, 70910-900 Brasília, DF, Brazil
e-mail: sobreiro@unb.br

P. Guarnieri (ed.), *Decision Models in Engineering and Management*,
Decision Engineering, DOI 10.1007/978-3-319-11949-6_2

control the financial resources in order to obtain better results for the firm. Considering this, cash management has the following responsibilities: to mobilize, manage, and plan the financial resources of business (Srinivasan and Kim 1986). Thus, using models to support decision-making becomes relevant as they can provide a wider perspective with better results, according to the stated objectives.

Cash balance consists of available funds at any moment in time for the firm. The cash is constantly affected by inflows and outflows from deposits and withdrawals such as income, payments, and investments in the form of expenditure of funds, all conducted by the firm. Consequently, the cash balance is the result of a cash balance at an earlier date modified by the net cash flow, which occurred on that date.

The use of models in the problem of defining the ideal level of available cash funds stems from work by Baumol (1952) and Tobin (1956). The authors proposed that the available cash balance definition is like a *commodity inventory*, in which control can be on a daily, weekly, or monthly basis, depending on the level of detail required by the firm.

The definition of an optimal cash balance follows the inventory models, which considers the financial resources available as an inventory, having certain costs related to its origin and maintenance, but that also derives benefits essential to firms. Thus, the definition of an optimal cash balance has a quantitative approach with the purpose of promoting the optimization of this fund inventory in order to minimize the costs associated with maintenance or lack of cash. Later, Miller and Orr (1966) analyzed the cash balance as having a random variable with an irregular fluctuation and proposed a stochastic model for managing the cash balance.

Despite the importance of the problem, few studies dedicated themselves to elaborate a review of the models in this problem, essentially the work of Gregory (1976), which presented a survey by the models until the mid-1970s focused on variants of the Miller–Orr model, and Srinivasan and Kim (1986), which deals only with deterministic models until the mid-1980s.

2 Literature Review

2.1 The Origin of Cash Flow Management Models

Cash management models were originally presented in Baumol (1952), whereby the author makes a parallel between cash with other firms' inventories. In the case of inventories in general, the most common approach is the economic order quantity (EOQ), which aims to find the best solution between the advantages and disadvantages of having an inventory. Nevertheless, the EOQ has restrictions when using the assumptions of fixed and predictable demand, as well as instant supply when listing an inventory is required.

According to Baumol (1952), a cash inventory is an inventory of a specific form of exchange. In Baumol model, the EOQ is adapted to optimize the cash and the best configuration based on the relationship between opportunity cost and transfer cost. So, the total transfer cost increases when the firm needs to sell bonds to accumulate cash, as the opportunity costs increase when there is a cash balance because it is a resource investment with no income associated (Baumol 1952).

The model performs the cost analysis associated with maintaining cash, for example, the opportunity cost determined by the interest rate that the firm receives or not for its investment and the cost of obtaining money by converting the investment into cash (Baumol 1952). The transfer cost represents expenditure incurred when investing in funds or withdrawals, such as interest rates and taxes. Thus, we demonstrate the model separately as follows:

$$\text{Cost} = \frac{bT}{C} \tag{1}$$

where

b fixed cost identified in investment or withdrawal transactions;
T total cash that is expected to be used in a certain period in net value as, for example, liquid cash flow; and
C initial cash balance;

The cost of holding cash is

$$\text{Cost of holding cash} = iC/2 \tag{2}$$

where

i opportunity cost of holding cash. In this case, the interest rate is defined by financial market; and
$C/2$ average cash balance, assuming that the cash decrease is a constant rate (fixed demand).

Thus, the total cost associated with the cash balance represents the sum of costs of obtaining cash, also known as trading costs, with maintenance costs that represent the opportunity cost of investment.

$$\text{Total Cost} = \frac{bT}{C} + \frac{IC}{2} \tag{3}$$

The best solution determination for cash balance C^* is the value of C in the following expression:

$$-\frac{bT}{C^2} + \frac{i}{2} = 0 \tag{4}$$

So, by the intersection of two straight lines, equalizing the formulas, we obtain the best value of C^* by

$$C^* = \sqrt{\frac{2bT}{i}} \tag{5}$$

However, as indicated by the authors cited, Baumol model has limitations when considering the constant demand for money and there is no receipt during the period, with immediate replacement of the money by withdrawal. Moreover, the model is limited with the uses of only two assets, the cash itself and any particular bond as a form of investment.

Along the same lines of development, Whalen (1966) also presents a model based on the concept of inventory, but uses a preventive approach relative to cash balance that takes into consideration three aspects:

- The cost of illiquidity;
- The opportunity cost of maintaining a precautionary cash balance;
- The average volume and variability of inflows and outflows.

Despite this, the Whalen model has no significant gains over original Baumol model. This is because both models are very similar where the cost of liquidity absence and the opportunity cost of maintaining the precautionary cash balance match, respectively, the transfer cost (obtaining) and opportunity cost (maintenance) of cash (Whalen 1966). Later, Miller and Orr (1966) presented a model that considers the assumption of random cash flows as the normal distribution. Furthermore, the Miller–Orr model considers only two assets, cash and an alternative investment is a low-risk and high-liquidity option, and the cash flow is a random variable as shown in Fig. 1, with decisions of immediate investments in bonds (moment T_1) and the decision of sale of the bonds (moment T_2).

This model attempts to define two limits for the level of cash funds: the minimum and the maximum, so the cash can reach the maximum level, represented by upper bound (h), when a financial investment is made in an amount that provides the cash balance back to best level of cash (z). Furthermore, when the minimum level is

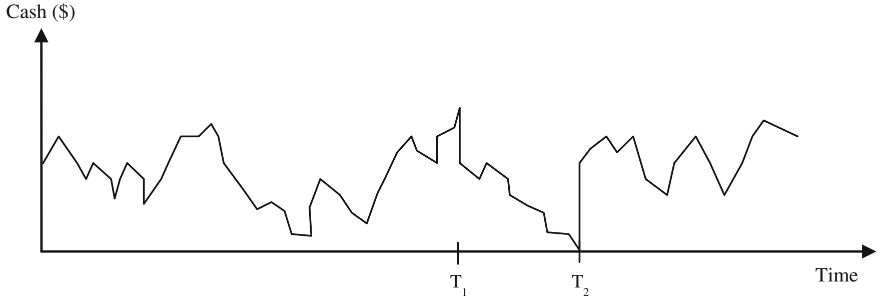

Fig. 1 Cash flow

reached with the lower bound (in the original work, this value is considered zero), a withdrawal should be made to obtain the best level of cash once again Miller and Orr (1966). Therefore, using the net cash flows (inputs minus outputs), the Miller–Orr model makes the optimization of the cash balance possible based on transfer cost (γ) and opportunity (v), obtaining the following formula (Miller and Orr 1966):

$$Z^* = \left(\frac{3\gamma}{4v}\sigma^2\right)^{1/3} \tag{6}$$

$$h^* = 3z \tag{7}$$

The symbol "*" denotes estimated optimal values, and σ^2 represents the variance of historical net cash flows. Thus, the optimal cash balance is

$$\text{Optimal Cash Balance} = 4z/3 \tag{8}$$

Despite the relative gain on the Baumol model, considering the random cash flow, the Miller–Orr model requires the definition of a lower bound as zero or some other positive values as the risk of lack of cash associated with a minimum margin of safety, an administration's choice that is not present in the model. Thus, after the Miller–Orr model, the optimization models were divided between deterministic and stochastic, and the subsequent works up to the 1970s only had modeling variations Gregory (1976).

The models showed no effective and practical solutions to the problem of three assets, the decision between investment in cash, bonds, and shares, for example. The division of investments between shares and bonds is relevant because the bonds tend to yield less and have lower transfer costs, but the development of the models did not consider the time of negotiation of the assets or the best moment to trade shares, and this last problem is an insuperable limitation Gregory (1976).

Analyzing the applicability of the models developed so far, and using simulations to define the cash balance in uncertain conditions, Daellenbach (1974) uses existing models in simulated cash flows. The author concluded that in cases where cash flows are non-stationary series, the optimization models could not make significant gains if the transfer costs are low. Therefore, the large firms perform better because of the financial volume of cash.

2.2 Cash Flow Management Models Since the 1980s

Even with the fall of development of models for this problem in the 1980s, it has not lost its relevance. The studies focused on improving methodological techniques. Since the 1980s, various authors have worked with the cash optimization problem, but because of the uncertainty related to receipts and payments from cash flow resources, what made the result a composition of random variables were implemented models with new approaches, mainly based on stochastic processes.

In this type of research, Tapiero and Zuckerman (1980) presented a stochastic model based on the premise that cash inflows and outflows have random behavior in a compound Poisson process, which consists of probability distribution of summing individual numbers distributed, according to the Poisson probability density function.

Later, Milbourne (1983) came up with a different model separating the transfer costs into two categories, β_1 the proportional cost for currency units to adjust the cash balance up and β_2 the proportional cost for currency units to adjust the cash balance down. Therefore, β_1 consists of withdrawing resources from financial assets for cash and β_2 represents the proportional cost when investing cash in other financial assets, including penalty charges for overdrawing the cash, which demands loans, or penalty charges for not performing certain payments.

From the formulation of Milbourne (1983), Smith (1986) developed a stochastic dynamic model, considering the cash flow as a diffuse process, temporally independent, as a Wiener process, known as Brownian motion. In the late 1990s, Ogden and Sundaram (1998) used the same assumptions of Baumol, considering the regular cash flow with output constant. The model incorporates the possibility of a credit line if the firm has a cash deficit, to solve the lack of cash, considering an interest rate associated with this credit line higher than the interest rate obtained on the investment used by the company. Thus, the problem of cash inflows and outflows (cash flows) according to stochastic processes is

$$f(S_n) = \begin{cases} S^- & S_n \leq S^- \\ S_n & S^- < S_n < S^+ \\ S^+ & S_n \geq S^+ \end{cases} \quad (9)$$

where the transferred amount of money at the beginning of each period $n = 1, 2, 3, \ldots$ is obtained by the transfer rule according to the model applied. This means that the amount of cash balance (S_n) at the beginning of period n is unchanged if it is within the upper (S^+) and lower (S^-) limits; otherwise, it should be adjusted to the extrapolated limit Hinderer and Waldmann (2001) in a similar way to the Miller–Orr model. More recently, some authors have been working on this problem, including Pacheco et al. (2000), who developed a genetic algorithm to determine investments in financial products available on the market based on the projected cash flow, obtaining the maximum return for specific periods.

Considering the cash balance problem as a possible use of the general and stationary Markov model in Hinderer and Waldmann (2001), the authors use a model for Markov chain processes in random environments that have a stationary process as, for example, low variation over time. Another technique used to solve this problem is linear programming. In this case, the cash flow can be developed on a specific basis of periodicity from the initial cash balance and supports input cash and a payment schedule based on estimated costs Barbosa and Pimentel (2001).

The model developed by Barbosa and Pimentel (2001) was very successful as it dealt with civil construction projects where the outflows are very predictable when planning construction work as, for example, a lower degree of randomization of cash flows. Computational approaches by Pacheco et al. (2000) and Barbosa and Pimentel (2001) tend to facilitate the practical application for firms enabling them to use software more easily.

Even with the changes in the design of the models discussed, there are some technical difficulties in defining cash balance policy or obtaining the ideal range or amount of cash. This is because the models use an approach of discrete time, even considering cash as a Markov chain, which depends on the amount existing beforehand.

This problem, with the linear condition of maintaining costs of cash, as we assume fixed proportional costs, can be avoided by reformulating continuous time, where the cash fund varies according to a *Brownian* motion with an average μ and variance σ^2. Therefore, the model is able to provide a band (range defined by upper and lower limits) of cash balance, indicating an optimal control policy in isolated moments of time Baccarin (2002).

Changing the focus of the optimization problem, Baccarin (2002) no longer searches to maximize the profitability of using financial resources or even minimizing the cost of opting to use the resource in the form of cash, but obtains the optimal level of liquidity, which presupposes that there is a negative cash balance. Their results indicate that the penalty charge is finite and the lower bound tends to be always below zero.

Another modeling variation is in Premachandra (2004), where the author develops a model of diffuse approximation for problems with two assets as proposed in the classical models. In this model, the assumptions of normal distribution of net cash flows and fixed transfer costs are relaxed in order to obtain a model closer to reality and with results significantly higher than the Miller-Orr model.

The model also uses the upper and lower bounds, and the optimal level of cash (z) is adopted by Miller and Orr, but considering the small time gap between the investment of cash and withdrawing it. In this case, with buy and sell bonds operations, we calculate the time on an hourly basis and the cost of maintaining the cash balance daily where m_1 and m_2 are the densities of probabilities of lower and upper limits, respectively, and the appropriate boundary conditions are given by $-m_2$ and m_1, as shown by Premachandra (2004). Moreover, the α in the model is a infinitesimal variation of the diffusion process, for example, the rate of oscillation of the variance of daily cash balance and β the infinitesimal mean of the diffusion process, for example, the rate of oscillation from cash balance daily average.

So taking the values b as the upper limit, a as the lower limit, and an integration constant c, the weighted daily cash balance (ABC—average daily cash balance) is

$$\begin{aligned} ABC &= \int_a^c x\frac{\lambda m_1}{\beta}\left[-1 + e^{\frac{-\beta(x-a)}{\alpha}}\right]\mathrm{d}x + \int_c^b x\frac{\lambda m_1}{\beta}\left[-1 + e^{\frac{-\beta(x-b)}{\alpha}}\right]\mathrm{d}x + m_2 b + m_1 a \\ &= -\frac{\lambda m_1}{2\beta}\left(c^2 - a^2\right) + \frac{\lambda m_2}{2\beta}\left(b^2 - c^2\right) + X(c) - Y(c) \end{aligned} \tag{10}$$

where

$$X(c) = \frac{\lambda m_1 \alpha (c y_2 - a)}{2\beta^2} - \frac{\lambda m_1 \alpha^{2(\gamma_2 - 1)}}{4\beta^3} + m_2 b + m_1 a \tag{11}$$

$$Y(c) = \frac{\lambda m_2 a (b - c\gamma_2)}{2\beta^2} - \frac{\lambda m_2 a^{2(1-\gamma_1)}}{4\beta^3} \tag{12}$$

Thus, cash flows were simulated (with inputs and outputs) under normal distribution with mean (μ) and variance (σ^2) at different magnitudes, as well as using different transfer costs for investments and withdrawals, obtaining lower costs and making it possible for the model itself to identify the upper and lower bounds Premachandra (2004). This is an advantage over the Miller–Orr model, which does not define the lower bound.

Along these lines of stochastic development, Volosov et al. (2005) present a stochastic programming model in two stages, based on scenario trees, which consider not only the problem of cash balance, but also the exposure to international currency, addressing the risk of exchange rate variation. In this model, the authors consider cash flows coming from different currencies, relating to the aspect of existing foreign exchange and the need for hedging. Thus, the authors obtain positive results in determining the optimal cash balance together with the definitions on how to use the hedge, even in a limited way.

The study conducted by Yao et al. (2006) presents a different formulation, considering the demand for money according to fuzzy logic concepts, developing a single-period model as, for example, without using past data due to historical data not being able to provide a cash demand forecast. In Yao, Chen, and Lu's model, a stochastic inventory similar to the single-period model is used where the inventory of one day cannot be used for the next day as it loses the value and is considered the opportunity cost. Therefore, the administrator must set the cash balance at a level where the value of the cumulative distribution function is equal to the cost/rate of interest.

This type of modeling considers the relationship between the costs of leftover financial resources and the cost penalty for the lack, but it is different when considering a fuzzy environment. More recently, Gormley and Meade (2007) have differentiated their work by presenting a dynamic policy for cash balance that minimizes transfer costs when cash flows are not independent or identically distributed in a general cost structure. By using this methodology, the authors used historical data to develop a time series model to forecast cash flows, promoting a conditional expectation of future cash flows and obtaining results in the reduced transfer cost. In this model, called the dynamic single policy (DSP), there is no need for the flow to be identically distributed and it is as follows:

$$E(w_{t+\tau} | W_t, W_{t-1}, \ldots) \neq E(w_t) = \text{Constant} \tag{13}$$

where
$w_t \in (-\infty, \infty)$ is an exogenous modification in the cash balance held at date (t).

There is no need to assume a distribution of cash flow, but the methodology requires a historical cash flow model for adaptation and τ corresponds to the amount of time in the future in which is desired the adjustment of cash. Thus, the DSP model calculates the costs associated with cash flow, divided into Gormley and Meade (2007):

- Holding cost: h;
- Shortage cost: u;

Transfer cost is divided into the following:

- Fixed transfer cost into account: γ_0^+;
- Fixed transfer cost from account: γ_0^-;
- Variable transfer cost into account:γ_1^+;
- Variable transfer cost from account: γ_1^-;

In this case, the authors considered that every transfer has an associated cost, which varies according to the type of operation, whether in investing cash for another asset or withdrawing from the asset for cash. Moreover, this transfer cost has a fixed amount per transaction, usually associated with bank fees or brokerage, and a variable value that typically represents a percentage of commissions or taxes related to operation. Then, the cash balance (O) on date $t + 1$ is

$$O_{t+1} = O_t + K_t + w_t \tag{14}$$

where
K represents the changes made by the management to invest and withdraw; and
w represents an exogenous change of balance from uncontrolled inflows and outflows (cash flow).

The transfer cost of money (γ) each day (t) within the period is provided as follows:

$$\Gamma(K_t) = \begin{cases} \gamma_0^- - \gamma_1^- K_t, & K_t < 0, \\ 0, & K_t = 0, \\ \gamma_0^+ - \gamma_1^+ K_t, & K_t > 0, \end{cases} \tag{15}$$

The holding cost (h) and shortage cost (u) are associated with the respective percentages of financial costs established. After formatting the DSP model, Gormley and Meade (2007) used data from a multinational company and a genetic algorithm model applied to solving continuous problems proposed by Chelouad and Siarry apud (Gormley and Meade 2007) to obtain the parameters that minimize the total cost (holding, shortage, and transfer) of cash holding, obtaining significant results, especially in a time horizon longer than 2 days.

Following this same uncertain stochastic problem model, Liu and Xin (2013) presented an adaptive algorithm with characteristics of changing the management policies at the beginning of each period to know the upper and lower demands for money, even without the knowledge of the probability distribution of demand from cash.

In another recent model, Baccarin (2009) develops the best cash balance using a standard n-dimensional Wiener process adapted to this problem using the impulse control method. The great contribution of this methodology is the multidimensional application, which eliminates the restriction on two options of assets (cash or bonds) since it is the restriction on only two financial assets, which rarely occurs in practice.

In this way, Baccarin (2009) describes the function of cost control for two bank accounts. The author assumes that those cash inflows and outflows which cannot be predicted have no systematic trend; that is, we set $b_1 = b_2 = 0$, suppose the cash stock dynamics corresponding to these cash flows are independent, with a common standard deviation. The cost function expression is as follows:

$$C(\xi_1, \xi_2) = C + c_1(|\xi_1 + \xi_2|) + c_2(|\xi_2|) \tag{16}$$

where

$C > 0$	is a fixed cost;
$c_1(\lvert\xi_1 + \xi_2\rvert)$	is a variable cost related to investments of high-liquidity purchase and sale, from accounts 1 and 2, with the corresponding random vectors ξ_i of the impulse control;
$c_2(\lvert\xi_2\rvert)$	is a variable cost paid by transferring money between different accounts, from account 1 to account 2.

Furthermore, the Baccarin model uses nonlinear functions to determine the transfer costs and maintenance/penalty in cash. Despite these advantages, the model uses stochastic process techniques, which have large computational costs when developing complex models Baccarin (2009).

The problems related to the risk of lack of cash and liquidity preference are recovered in the work of Mierzejewski (2010), where the author develops a stochastic model considering the premise of the demand for cash balance with normal distribution and applied the value-at-risk (VaR) methodology, with the limitation of market equilibrium for the cash flows and portfolio investments used, from an average rate of return.

In another model, the authors Melo and Bilich (2011) propose the use of dynamic programming to minimize the cost of the cash, considered together the cost of rupture cash. In this model, the cash flow has two parts: a deterministic part, by which the firm expects to have receipts and payments, and other stochastic. Thus, the lowest cost is

$$C_{\Delta t} = \left\{ \sum_{i=1}^{n} \left[\int_{-\infty}^{\infty} \alpha_{\Delta t} \left(B_{i,\Delta t} \right) \gamma_{i,\Delta t} P\left(B_{i,\Delta t} \right) \mathrm{d}B_{i,\Delta t} \right] \right\} \tag{17}$$

where
$C_{\Delta t} = 0$ is the minimum total cost;

Existing are N blocks with (N intervals of stochastic cash flows). So the cash flow of the previous period $B_{\Delta t-1}$ is added to the expected cash flow (deterministic), to the stochastic net cash flow and to the variable rescue decision on investment $(X_{\Delta t})$, obtaining the current cash flow $B_{\Delta t}$.

The result $(B_{\Delta t})$ is multiplied by the probability $P(B_{i,\Delta t})$, which represents the expectation of the cash balance. The interest rate that represents the cost of lack or remains of cash (depending on the positive/negative flow) $\gamma_{i,\Delta t}$ is applied. Thus, the value $X_{\Delta t}$ acts in minimizing the cost function (Melo and Bilich 2011). However, the major limitation of this model is found in the definition of the intervals corresponding to stochastic flows and determining the probability associated with them. The authors suggest the use of artificial intelligence techniques as possible applications for prediction, performing the fit of the model variables.

Considering this, from the original work to the most current, the theoretical conceptualization of optimizing the cash balance problem still has practical limitations in its application, such as restrictions in applying the model and ways of development primarily focused on stochastic models.

The major limitation of stochastic models is the need of prior knowledge about the distribution of cash flows Kachani and Langella (2005). This demonstrates the importance of searching for references on the subject and delineating models developed in this point of view, opening up perspectives for new approaches and application of computational tools for this problem. We summarize the most important literature studies concerning the cash flow management problem since the 1980s in Table 1.

3 New Perspectives

The literature on the cash balance problem shows that from the 1980s, research that deals with reasons that cause firms to maintain cash resources is presented essentially to scientific journals in the field of economics, accounting, and finance.

Table 1 Selected studies on stochastic cash flow management models

Authors	Research summary	Journal/conference	Subject of journal/ conference
Baumol (1952)	Proposes that the available cash balance is a commodity inventory	Quarterly Journal of Economics	Economics and finance
Tobin (1956)	Adjusts the Baumol model, so the number of transactions becomes a positive integer value	The Review of Economics and Statistics	Economics and finance
Miller and Orr (1966)	Analyze the cash balance as having a random variable with an irregular fluctuation and proposed a stochastic model for managing the cash balance	Quarterly Journal of Economics	Economics and finance
Whalen (1966)	Presents a model based on the concept of inventory considering the cost of illiquidity, the opportunity cost of maintaining a precautionary cash balance and the average volume and variability of inflows and outflows	Quarterly Journal of Economics	Economics and finance
Daellenbach (1974)	Concludes that in cases where cash flows are non-stationary series, the optimization models cannot make significance gains if the transfer costs are low	Journal of Financial and Quantitative Analysis	Economics and finance
Gregory (1976)	Presents a survey by the models until the mid-1970s focused on variants of the Miller and Orr (1966)	Omega	Operational research and computational optimization
Tapiero and Zuckerman (1980)	Present a stochastic model based on the premise that cash inflows and outflows have random behavior	Journal of Banking and Finance	Economics and finance
Milbourne (1983)	Presents a model separating the transfer costs into two categories, in other words, cost for currency units to adjust the cash balance up and cash balance down	International Economic Review	Economics and finance
Srinivasan and Kim (1986)	Present the principles of deterministic models until the mid-1980s	Omega	Operational research and computational optimization

(continued)

Table 1 (continued)

Authors	Research summary	Journal/conference	Subject of journal/ conference
Smith (1986)	Develops a stochastic dynamic model, considering the cash flow as a diffuse process	The Review of Economic Studies	Economics and finance
Ogden and Sundaram (1998)	Propose the utilization of a credit line if the firm gets a cash deficit considering an interest rate associated with this credit line and the assumptions of Baumol	Journal of Financial and Strategic Decisions	Economics and finance
Pacheco et al. (2000)	Develop a genetic algorithm to determine investments in financial products available on the market based on the projected cash flow	6th International Conference of Society for Computational Economics on Computing in Economics and Finance	Economics and finance
Hinderer and Waldmann (2001)	Propose the utilization of Markov chains in the problem	European Journal of Operational Research	Economics and finance
Barbosa and Pimentel (2001)	Develop and applied a model in civil construction projects very successfully	Construction Management and Economics	Economics and finance
Baccarin (2002)	Proposes a modeling variation that changes the focus of the optimization problem	Decision Economics Finance	Economics and finance
Premachandra (2004)	Shows a model considering the assumptions of normal distribution of net cash flows and that the fixed transfer costs are relaxed in order to obtain a model closer to reality	European Journal of Operational Research	Operational research and computational optimization
Volosov et al. (2005)	Develop a stochastic programming model in two states, based on scenario trees, for the problem of cash balance	Computational Optimization and Applications	Operational research and computational optimization
Yao et al. (2006)	Show a single-period model, considering the demand for money according to fuzzy logic concepts, for the problem of cash balance	European Journal of Operational Research	Operational research and computational optimization

(continued)

Table 1 (continued)

Authors	Research summary	Journal/conference	Subject of journal/ conference
Gormley and Meade (2007)	Propose the utilization of dynamic policy for cash balance that minimizes transfer costs when cash flows are not independent or identically distributed in a general cost structure	European Journal of Operational Research	Operational research and computational optimization
Liu and Xin (2008)	Propose an adaptive algorithm with characteristics of changing the management policies at the beginning of each period to know the upper and lower demands for money	Fourth International Conference on Natural Computation	Operational research and computational optimization
Baccarin (2009)	Presents a standard n-dimensional Wiener process using the impulse control method, for the problem of cash balance	European Journal of Operational Research	Operational research and computational optimization
Mierzejewski (2010)	Develops a stochastic model considering the premise of the demand for cash balance with normal distribution and applied the value at risk (VaR)	IMA Journal of Management Mathematics	Operational research and computational optimization
Melo and Bilich (2011)	Propose the use of dynamic programming to minimize the cost of cash, considering the cost de rupture cash	Journal of Economics And Finance	Economics and finance

Concerning the focus of studies on developing models for managing the cash balance, we have a revival since 2000 mostly in journals of the areas of operational research, computing, and management sciences.

Considering this, the more recent models focus on the efficiency of optimization, but do not observe all aspects of managing cash. Therefore, cash management decision must have the same importance like other investment decisions, observing the aspects of profitability, liquidity, and risk:

- Profitability: The existence of cash itself does not generate profit for the firm, but it is necessary to develop its activity. Thus, observing the cost associated with cash, the firm can attempt to improve its profitability by minimizing the associated costs;
- Liquidity: The investment in cash has total liquidity, but financial investments (typically bonds), which are the option to direct the excess cash and obtain cash when there is a need to withdraw it, may not have a ready market and this is a strong limitation for traditional models; and
- Risk: We must observe the risk by both the occurrence of cash shortage, where there is a need to take funds from third parties, and from overtrading, when the firm does not have lines of credit compared to their need for cash and insolvency may incur.

Observing these aspects, the development of models should consider investing cash resources in more than one option besides cash, with returns, minimum periods of investment/withdrawal (liquidity) and transfer costs, fixed and variable, and different for each option. Furthermore, the possibility of obtaining financial resources in case of cash deficit should be observed, once again with financial costs (interest) and transfer costs different for each funding source.

Moreover, the possibility of financial leverage, something which given the cash flow conditions and financial and investment options, as well as the opportunity and financial costs, may indicate to the firm to maintain a negative cash balance as being the best alternative. Therefore, the new developing models must meet these needs simultaneously, which exist in practice, but we not observed in recent cash management models.

The use of multiobjective functions with a combination of three factors to analyze investments (risk, profitability, and liquidity), according to the probability of the cash balance at date t being less than the maximum cash shortage than that of the firm, is able to support given the final cash flow at date $t-1$, which serves the assumption of a stochastic process and certain time limits to withdraw the investment i and the maximum cash shortage value less than zero (assumption of negative cash) or ultimately equal to zero as, for example, when the firm has no immediate lines to credit.

It is important to note that the minimization of the total risk of cash would result in zero probability of there being a cash balance below the maximum cash shortage (or a negative cash balance in case of maximum cash shortage equal to zero). This rarely occurs in practice, since it implies a high concentration of cash balance, and therefore, the definition of risk profiles to be minimized is acceptable.

Traditionally, cash management models indicate a choice between maintaining the cash resource and an investment option, usually a highly liquid asset (bonds), something that hampers the practical application of existing models, as investment options have differentiated profitability, risk, and liquidity. Moreover, the framework must foresee the use of more than one source of funds in cases of cash deficit, with different time limits and financial costs.

Applying different probability distributions simultaneously, despite the use in some studies of Poisson distribution or Brownian motion (Wiener process), the literature essentially uses the normal distribution, without comparing the models in samples with different distributions. The Poisson distribution, for example, is relevant in cases of concentrating payments or receipts over specific periods, which is what justifies its application.

On the other hand, the meta-heuristics and evolutionary computational models in this problem have not been investigated enough, using only genetic algorithms. The models presented in the literature mostly use stochastic modeling. Moreover, this option is relevant because it is easy to develop the algorithm to different patterns of cash flow distribution as there is no need to know the distribuition of the cash flow because the model can adapt based on historical or projected data.

4 Conclusions

Managing the cash balance is important in business administration, but rarely do we apply the techniques presented in this study in practice. Much of this neglect is due to the difficulty in developing models closer to reality.

Considering this, the literature review shows the importance of cash balance within firms, but the development of cash management models are still bound to formulations developed over nearly five decades, without improving the used model. Furthermore, the view of the cash balance is still limited and not regarded as an investment, which has a negative profitability (defined by total cost of the cash), immediate liquidity, and risk associated with cash deficit. Thus, it is necessary to understand the cash balance together with other financial investments as a portfolio investment and examines the investment choices in financial products according to their variable liquidity, profitability, and associated risk.

Another relevant aspect to us is the methodology in developing cash management models. The literature shows a clear preference for stochastic models, and the researchers do not use computer models.

The use of evolutionary computational algorithms, not only genetic algorithms, can reduce limitations when developing more complex models, reducing the constraints presented in this work and making computational implementation easier in accounting and financial management systems within firms.

We can also analyze the distribution of article in the journal's areas. Originally, the articles were presented in journals of economics and finance, but, with the evolution of methods and computer applications in 2000s, the major area of publication has been the operational research and computational optimization. This demonstrates a greater concern about the method, but not with the problem's formulation.

Finally, this is a classic problem in business, involving economics, accounting, and finance, and it should return to be the focus of discussions in these areas, as the existing limitations concerning the models and methods can be eliminated. We must discuss the cash balance problem not only about the method involved in optimization but also in practical application.

References

Baccarin S (2002) Optimal impulse control for cash management with quadratic holding-penalty costs. Decisions Econ Finan 25(1):19–32

Baccarin S (2009) Optimal impulse control for a multidimensional cash management system with generalized cost functions. Eur J Oper Res 196(1):198–206

Barbosa PSF, Pimentel PR (2001) A linear programming model for cash flow management in the Brazilian construction industry. Constr Manage Econ 19(5):469–479

Baumol WJ (1952) The transactions demand for cash: an inventory theoretic approach. Q J Econ, 545–556

Daellenbach HG (1974) Are cash management optimization models worth while? J Financ Quant Anal 9(4):607–626

Gormley FM, Meade N (2007) The utility of cash flow forecasts in the management of corporate cash balances. Eur J Oper Res 182(2):923–935

Gregory G (1976) Cash flow models: a review. Omega 4(6):643–656

Hinderer K, Waldmann KH (2001) Cash management in a randomly varying environment. Eur J Oper Res 130(3):468–485

Kachani S, Langella J (2005) A robust optimization approach to capital rationing and capital budgeting. Eng Econ 50(3):195–229

Liu B, Xin C (2008) An online model for managing cash: an alternative approach to the Miller-Orr model. In: 2013 international conference on computing, networking and communications (ICNC). p 314-317

Melo MAS, Bilich F (2011) Expectancy balance model for cash flow. J Econ Finan 37(2):240–252

Mierzejewski F (2010) A model of equilibrium in markets of cash balances. IMA J Manage Math 22(3):253–270

Milbourne R (1983) Optimal money holding under uncertainty. Int Econ Rev, 685–698

Srinivasan V, Kim YH (1986) Deterministic cash flow management: state of the art and research directions. Omega 14(2):145–166

Tobin J (1956) The interest-elasticity of transactions demand for cash. Rev Econ Stat, 241–247

Miller MH, Orr D (1966) A model of the demand for money by firms. Q J Econ, 413–435

Whalen EL (1966) A rationalization of the precautionary demand for cash. Q J Econ, 314–324

Tapiero CS, Zuckerman D (1980) A note on the optimal control of a cash balance problem. J Bank Finance 4(4):345–352

Smith GW (1986) A dynamic Baumol-Tobin model of money demand. Review Econ Stud 53 (3):465–469

Ogden WA, Sundaram S (1998) A model for optimal utilization of a firm's line of credit. J Finan Strateg Decis, 27–36

Pacheco MA, Noronha M, Vellasco M, Lopes C (2000) Cash flow planning and optimization through genetic algorithms. In: 6th international conference of the society for computing in economics and finance, Society for Computational Economics

Premachandra IM (2004) A diffusion approximation model for managing cash in firms: an alternative approach to the Miller-Orr model. Eur J Oper Res 157(1):218–226

Volosov K, Mitra G, Spagnolo F, Lucas C (2005) Treasury management model with foreign exchange exposure. Computational Optimization and Applications 32(1–2):179–207

Yao JS, Chen MS, Lu HF (2006) A fuzzy stochastic single-period model for cash management. Eur J Oper Res 170(1):72–90

Author Biographies

Marcelo Botelho da Costa Moraes is an assistant professor at the Departmant of Accounting at School of Economics, Business Administration and Accounting of Ribeirão Preto, University of São Paulo. He holds a PhD in production engineering. His research interests include computational models in accounting and finance.

Marcelo Seido Nagano is an adjunct professor at the School of Engineering of São Carlos in the University of São Paulo, where he currently holds the position of Head of the Operations Research Group of the Production Engineering Department. Along with his teaching duties, his research interests include decision systems and models in industry and services, including a range of decisions related to the design and optimization of processes, production and planning and scheduling, as well as knowledge management and innovation management as a supporting infrastructure. In these areas, he carried out several research projects and produced a number of refereed publications. He have some editorial duties, most notably the editor of the Journal of Engineering and Computer Innovations and member of the Editorial Board of International Journal of Industrial Engineering Computations, and Management Science Letters.

Vinicius Amorim Sobreiro is an assistant professor at the Department of Management at the University of Brasília. He holds a doctorate in production engineering. He has a bachelor's degree in economics from Antônio Eufrásio Toledo College.