Chapter 13
Purchasing Transportation Services from Ocean Carriers

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Abstract Reducing transportation costs is priority number one for global shippers who need to move their cargo containers all over the world. To achieve such cost reduction, a shipper can use what is called a reverse auction mechanism to purchase transportation services, by inviting carriers, i.e. liner shipping companies, to bid competitively to sell their services. As part of the process, carriers often seek commitments from the shipper, and internal business units of the shipper often express their own preferences when it comes choosing the carriers, which naturally complicates the shipper's decisions. In this chapter, we first review existing studies on the transportation service procurement problem. Based on a new general optimization model, we then discuss extensions to the existing known results, as well as present several results new to the literature.

13.1 Introduction

In the container shipping market, the key players are shippers (as buyers) and carriers (as sellers), where shippers, such as manufacturers and retailers, are companies who need to move their cargo containers, and carriers, such as shipping liners, are companies who provide transportation services to ship the containers. With the huge expansion in global supply chains, shippers today have a huge demand for transportation services from carriers to transport their cargo, which may include raw materials or finished products. As a result, transportation services are often listed among the top categories of spending by global shippers, providing large opportunities for cost savings (Xu 2007). In fact, it is common for a global shipper to spend more than a \$ 100million annually on transportation services (Lim et al. 2012).

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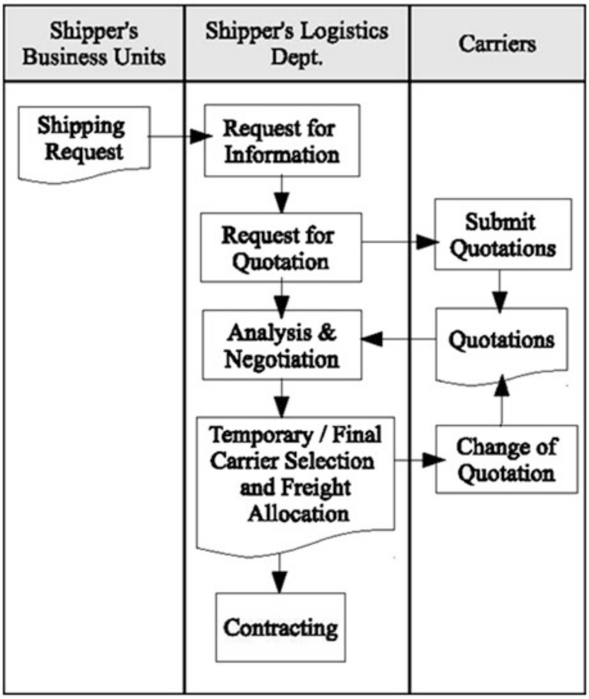

Fig. 13.1 Reverse auction mechanism for transportation service procurement

For shippers, the transportation services are often purchased by their logistics departments, and typically follow a reverse auction mechanism that consists of the following four stages (Xu 2007), as shown in Fig. 13.1:

- Stage 1—Request for Information (RFI): The logistics department collects shipping request information from different business units (or departments) of the company, and based on the information collected, it then forecasts cargo volumes for the coming period.
- Stage 2—Request for Quotation (RFQ): The logistics department invites a number of carriers to make quotations of shipping prices for lanes between different origin-destination pairs, for different service levels in terms of shipping times, and for different weights of cargo, etc.
- Stage 3—Analysis and Negotiation: The logistics department analyzes the quotations from the carriers, estimates the total transportation cost under different scenarios, and negotiates with the carriers by bargaining over the shipping prices and conditions.
- Stage 4—Signing Contract: The logistics department makes its decisions on the selection of carriers and the allocation of shipping volumes to the selected carriers, so as to finalize the prices and conditions with the carriers and then sign contracts.

Before finalizing and signing contracts, the shipper may go through multiple rounds of analysis and negotiations, with a view to minimizing the total transportation cost.

During the above process, particularly at the various stages of analysis and negotiation, as well as at the stage of signing contracts, a shipper often needs to solve optimization problems with regard to selecting carriers and allocating cargo to the selected carriers in order to minimize its total transportation cost. Such problems are challenging, since they are often triggered by various constraints that reflect different business considerations, some imposed by the external carriers who provide the shipping services (Lim et al. 2008a, 2006), and others imposed by the internal business units who require the shipping services (Lim et al. 2012). Moreover, unlike the trading of physical goods, shipping services are of a combinatorial nature, as the price of shipping services is often imposed not only on a single lane but also on a group of lanes of different origin-and-destination pairs (Lim et al. 2008a). It is also well-known that the shipping market is very volatile, as both the demands and the sport-market shipping prices vary significantly all the time. Due to this, the key players, including both shippers and carriers, sometimes may not strictly follow the contracts in actual operation, at times maybe breaking them to suit their own interests.

The procurement of transportation services is challenging, and has raised several interesting research questions that fall into the following three categories:

1. On Models: How should the problems be defined? What are the useful properties of the optimal solutions to these problems? To answer these questions, it is necessary to formulate the corresponding optimization or decision problems as mathematical programming models, as well as to analyze the properties of the models.
2. On Tractability: Do efficient algorithms exist that can solve the problems to optimality? Answering this question, it requires an understanding of the computational complexities of the corresponding optimization or decision problems, as well as being able to identify special cases that have practical applications and that can also be efficiently solved to optimality.
3. On Algorithms: How can exact or near optimal solutions to the problems be found in affordable running times? To answer this question, it is necessary to develop exact algorithms or heuristic algorithms, and to show by either theoretical analysis or numerical experiments that these algorithms can guarantee good performance.

In this chapter, we review recent studies that have addressed some of these research questions related to transportation service procurement problems. Since most of the existing studies are focused only on problems with specific constraints, it is also of interest to know how their results, models, and algorithms can be extended to solving more general problems having broader applications. For this purpose, we also introduce in this chapter a generalized optimization model for the transportation service procurement problem, which is defined and formulated in Sect. 2, and we then discuss how existing results from the literature can be applied to this generalized model. These results include computational complexities, relaxations of mathematical models, exact algorithms, and heuristic algorithms, which are all discussed in Sects. 3–6, respectively. The chapter is concluded in Sect. 7 with discussions on future research directions.

13.2 Problem Formulations: Generic Model, Side Constraints, and Generalization

13.2.1 Generic Model

Consider a shipper who has to make decisions on purchasing transportation services to move containers of its cargo for a time horizon of T periods. The shipper has a set of business units, denoted by B, who use the services potentially from m carriers, which are denoted by $I = \{i{:}1 \le i \le m\}$. The cargo for each period in the time horizon is for n lanes in total, these being defined as pairs of the cargo's origins and destinations, and are denoted by $J = \{j{:}1 \le j \le n\}$. At the beginning of the time horizon, the shipper collects information from its business units so as to forecast demands for transportation services for each period $t \in \{1, 2,\ldots, T\}$, which are given by $d_{bjt} \in R_+$ for each business unit $b \in B$ and each lane j, where R_+ indicates the set of non-negative real numbers. The forecast demand d_{bjt} is then released to the carriers, and each carrier i responds by quoting a quote of a shipping rate $p_{ijt} \in R_+$ according to its bidding strategy. In addition to such a reverse auction mechanism, the shipper can also purchase shipping services from the spot market, so as to minimize its total shipping cost. The forecast spot market shipping rates are represented by $s_{jt} \in R_+$. To reflect the fact that carriers often quote prices on groups of lanes, for each carrier i we assume that the lanes that it operates are partitioned into a collection $J_i = \{J_{i1}, J_{i2},\ldots, J_{i,|J|}\}$ of lane groups, where $J_{ik} \subseteq J$ for $1 \le k \le |J_i|$. We assume that lane groups are disjoint. Let $c_{ik} \in Z_+$ indicate the capacity that carrier i can ship for all the lanes in J_{ik} for all the T periods of the planning time horizon.

The shipper needs to make the following decisions, so as to minimize its total transportation cost:

- Decisions on selecting carriers: This can be represented by binary variables $y_{ibk} \in \{0, 1\}$ and binary variables y_{ik}, where $y_{ibk} = 1$ if and only if carrier i is selected to serve lanes in J_{ik} for the cargo of business unit b, and $y_{ik} = 1$ if and only if carrier i is selected to serve lanes in J_{ik};
- Decisions on allocating cargo to carriers: This can be represented by variables $x_{ibjt} \in \mathrm{Z}_+$, where Z_+ is the set of non-negative integers, and each x_{ibjt} indicates the number of containers allocated to carrier i for shipping the cargo of business unit b on lane j in period t.

Thus, the total transportation cost for the shipper is $\sum_{b\in B}\sum_{j\in J}\sum_{t=1}^{T}\left[\sum_{i\in I} p_{ijt}x_{ibjt} + s_{jt}(d_{bjt} - \sum_{i\in I} x_{ibjt})\right]$. The generic integer programming model (Generic IP) for the transportation service procurement problem, or in short the *TSPP*, can then be formulated as follows:

$$(Generic\ IP) = \min \sum_{b\in B}\sum_{j\in J}\sum_{t=1}^{T}\left[\sum_{i\in I} p_{ijt}x_{ibjt} + s_{jt}\left(d_{bjt} - \sum_{i\in I} x_{ibjt}\right)\right] \quad (13.1)$$

$$\text{s.t.} \sum_{i\in I} x_{ibjt} \le d_{bjt}, \forall b \in B, j \in J, 1 \le t \le T, \quad (13.2)$$

$$\sum_{b \in B} \sum_{j \in J} \sum_{t=1}^{T} x_{ibjt} \leq c_{ik} y_{ik}, \forall i \in I, 1 \leq k \leq |J_i|, \tag{13.3}$$

$$\sum_{b \in B} y_{ibk} \leq |B| y_{ik}, \forall i \in I, 1 \leq k \leq |J_i|, \tag{13.4}$$

$$x_{ibjt} \in Z_+, \forall i \in I, b \in B, j \in J, 1 \leq t \leq T, \tag{13.5}$$

$$y_{ik} \in \{0,1\}, \forall i \in I, 1 \leq k \leq |J_i|, \tag{13.6}$$

$$y_{ibk} \in \{0,1\}, \forall i \in I, b \in B, 1 \leq k \leq |J_i|. \tag{13.7}$$

In the (Generic IP), the objective (1) is to minimize the total transportation cost for the shipper. Constraint (2) ensures that for each lane j, period t and business unit b, the total volume allocated to all carriers does not exceed the total demand d_{bjt} of b. Constraint (3) ensures that the total volume of cargo allocated to carriers can never exceed their capacities for each of their lane groups. Constraint (4) ensures that y_{ibk} equals zero as long as y_{ik} equals zero. Constraint (5) restricts shipment allocations to be integers, because shippers are often required to buy spaces of full size containers in TEUs (twenty-foot equivalent units), which can be expensive. It is not difficult to see that decision variables y_{ibk} and y_{ik} are redundant in the (Generic IP). We leave y_{ibk} and y_{ik} to model various side constraints that reflect different business considerations in our formulations in the latter part of this chapter. It can be seen that the model above has taken into account carriers' lane groups, and has formulates basic demand and capacity constraints

13.2.2 Side Constraints

There are two main sources of side constraints, those from external carriers and those from internal business units (Xu 2007).

As for external carriers, in addition to the basic capacity constraint, they often also request for a volume guarantee from the shipper. When carriers submit quotations to the shipper, they make assumptions about demand, and their quotations can be either too low or too high (Caplice and Sheffi 2005). As a result, the carrier that wins the quotation can be the one that underestimates its service cost, and will thus suffer from the "winner's curse" (Caplice 2003; Sawhney 2003). A volume guarantee from the shipper can thus be helpful in resolving this problem, since it can reduce risk and uncertainty for the carriers, which enables the carriers to bid in a more realistic manner.

There are two types of volume guarantee that are commonly requested by carriers and have been studied in the literature. One is called a minimum quantity commitment (MQC) constraint, which is motivated by the stipulation of the United States Federal Maritime Commission that restricts a fixed minimum quantity for the total volume of cargo shipped by each carrier to cities in the US (Lim et al. 2006). The commitment to a minimum quantity has commonly been applied in transportation service procurement for various shippers, where carriers negotiate and quote for a minimum volume for each lane group. Let $q_{ik} \in R_+$ indicate the minimum quantity for shipments for lanes in J_{ik}, which limits the volume for carrier i to carry in all the periods to be either none or above q_{ik}. Accordingly, the MQC constraint can be formulated as follows:

$$q_{ik} y_{ik} \leq \sum_{b \in B} \sum_{j \in J_{ik}} \sum_{t=1}^{T} x_{ibjt}, \forall i \in I, 1 \leq k \leq |J_i|, \tag{13.8}$$

where the right hand side of (13.8) is the total volume of cargo allocated to carrier i for lanes in J_{ik}.

The other type of volume guarantee constraint is called the maximum-to-average ratio commitment (MARC), which is motivated by common practice in the shipping industry (Lim et al. 2008a). This commitment requires that the volume of each shipment that the shipper can ship through a carrier cannot exceed a fixed proportion of the average volume shipped through the carrier during the term of the contract. This proportion is usually referred to as the *maximum-to-average ratio*, which is quoted by the carrier and can be negotiated with the shipper. Under this condition, the shipper has to buy sufficient volume from the carrier that can be spread over the duration. This is because the shipper's shipments at any time can be made only if commensurate volume is shipped throughout the duration. For example, the shipper must ship sufficient volume in off-peak seasons if shipments are planned for peak seasons. This stipulation translates into a volume guarantee to the carrier which, in effect, smoothes shipments throughout the duration of the contract.

To formulate a constraint for the (MARC), let us take $r_{ikt} \in R_+$ to be the maximum-to-average ratio for shipments in J_{ik}, which limits the volume that carrier i is willing to carry in period t in excess of the average volume that it will carry for all T periods of the contract (Lim et al. 2008a). We take $\xi \in Z_+$ to represent a small excess that is allowed in the maximum-to-average ratio commitment in practice. Letting $\alpha_{ikt} = r_{ikt}/T$, the constraint related to the maximum-to-average ratio commitment can be formulated as follows:

$$\sum_{b \in B} \sum_{j \in J_{ik}} x_{ibjt} \leq \alpha_{ikt} \left(\sum_{b \in B} \sum_{j \in J_{ik}} \sum_{t=1}^{T} x_{ibjt} \right) + \xi, \forall i \in I, 1 \leq k \leq |J_i|, 1 \leq t \leq T, \tag{13.9}$$

where the left hand side represents the total volume of cargo shipped by carrier i during period t for lanes in J_{ik}, and the right hand side is the average volume shipped by carrier j and for lanes in J_{ik} of all T periods.

Besides constraints from carriers, the shipper's decision maker often also needs to satisfy constraints from its own internal business units, the ones who are the actual users of transportation service for shipping and receiving cargo. In the literature, three types of such constraints have been introduced and studied. One is the carrier number constraint, which imposes a lower bound and/or an upper bound on the number of carriers selected for a group of business units and a group of lanes (Lim et al. 2012). This is motivated by practical considerations, whereby a smaller number of carriers may reduce the management cost, but it also restricts the flexibility of business units in choosing shipping dates. To formulate this constraint, let BG denote a collection of groups of business units, and LG denote a collection of lane groups. For each group of business unit $B_h \in BG$, and lane group $L_g \in LG$, let $\underline{n}_{hg}$ and $\bar{n}_{hg}$ indicate the minimum and maximum numbers of carriers that can be selected to ship cargo for business units in B_h and for lanes in L_g. The carrier number constraint can be formulated as follows:

$$\underline{n}_{hg} \leq \sum_{i \in I} z_{ihg} \leq \bar{n}_{hg}, \forall B_h \in BG, L_g \in LG, \tag{13.10}$$

$$\sum_{b \in B_h} \sum_{k: J_{ik} \cap L_g \neq \varnothing} y_{ibk} \leq M z_{ihg}, \forall i \in I, B_h \in BG, L_g \in LG, \tag{13.11}$$

$$z_{ihg} \in \{0,1\}, \forall i \in I, B_h \in BG, L_g \in LG, \tag{13.12}$$

where z_{ihg} indicates whether or not carrier i is selected to ship cargo for business units in B_h and for lanes in L_g, and M is a sufficiently large constant.

The second type of shipper's constraint is called the preference constraint (Xu 2007), under which cargo of a certain business unit cannot be assigned to particular carriers, or must be assigned to particular other carriers. This is motivated by current practice, where business units have their own preferences as to the choice of carriers based on their previous experience. To impose such constraints, the logistics department needs to collect preference information from business units. For each group of business units $B_h \in BG$, and lane group $L_g \in LG$, let I_{hg}^{+} indicate a set of carriers that cannot be assigned to any business unit in B_h and any lane in L_g, and let I_{hg}^{-} indicate a set of carriers that cannot be assigned to any business unit in B_h and any lane in L_g. Thus, the preference constraint can be formulated as:

$$\sum_{b \in B_h} \sum_{k: J_{ik} \cap L_g \neq \varnothing} y_{ibk} \geq 1, \forall i \in I_{hg}^{+}, B_h \in BG, L_g \in \mathrm{LG}, \tag{13.13}$$

$$\sum_{b \in B_h} \sum_{k: J_{ik} \cap L_g \neq \varnothing} y_{ibk} = 0, \forall i \in I_{hg}^{-}, B_h \in BG, L_g \in LG. \tag{13.14}$$

Another type of shipper's constraint is called the fairness constraint, which restricts the assignment of carriers so as to be fair to different business units. This is motivated by current practice, where different business units may all request to be assigned to carriers that quote the lowest shipping price, but such carriers may only provide limited capacities. As a result, the available capacities of these low-price carriers have to be allocated fairly to the various business units (Lim et al. 2012). One way to reflect such a fairness concern is to impose a constraint such that, for each lane, the gap between the actual total shipping cost to a business unit and its minimum possible shipping cost shall not exceed a given percentage, denoted by $\eta\%$. As a result, we can formulate the fairness constraint as follows:

$$\sum_{i\in I}\sum_{t=1}^{T}[(p_{ijt}-s_{jt})x_{ibjt}+s_{jt}d_{bjt}] \le (1+\eta\%)\sum_{t=1}^{T}[(p_{i'jt}-s_{jt})x_{i'bjt}+s_{jt}d_{bjt}]$$
$$+M(1-y_{i'k}), \forall 1\le k\le |J_{i'}|, j\in J_{i'k}, b\in B, i'\in I, \tag{13.15}$$

where the left hand side indicates the actual shipping cost for business unit b and its cargo on lane j, the right hand side indicates its shipping cost for cargo on lane j if carrier i' is assigned, and M is a sufficiently large constant.

13.2.3 Generalization

By generalizing the various side constraints presented earlier, we can obtain a general mathematical programming model (General MP) for the TSPP as follows:

$$\text{(General MP)} = \min \sum_{b\in B}\sum_{j\in J}\sum_{t=1}^{T}\left[\sum_{i\in I} p_{ijt}x_{ibjt} + s_{jt}\left(d_{bjt}-\sum_{i\in I}x_{ibjt}\right)\right]$$

$$\text{s.t.}(2)-(7).\ (\mathbf{x},\mathbf{y})\in EDom\bigcap IDom,$$

where $EDom$ indicates the domain restricted by certain side constraints raised by the carriers, and $IDom$ indicates the domain restricted by certain side constraints raised by the shipper's internal business units.

Moreover, the (General MP) can be reformulated into a bi-level optimization model, which separates determinations of decision variables y_{ibk} and x_{ibjt}. In other words, letting $\mathbf{y}=\{y_{ibk}: i\in I, b\in B, 1\le k\le |J_i|\}$, $EDom(\mathbf{y})=\{\mathbf{x}:(\mathbf{x},\mathbf{y})\in EDom\}$, and $IDom(\mathbf{y})=\{\mathbf{x}:(\mathbf{x},\mathbf{y})\in IDom\}$, we have

$$(\textbf{General MP}) = \min\ \text{(General MP)}(y),\ \text{s.t.}7$$

$$\text{(GeneralMP)}(y) = \min(1), \text{s.t.}(2)-(15)\mathbf{x}\in EDom(\mathbf{y})\bigcap IDom(\mathbf{y}).$$

We make the point that the general model given here can include considerations other than those mentioned in Sect. 2.2, such as carriers' capacities in each period, and other commitment constraints on lanes.

13.3 Problem Complexities and Tractability

In this section, we discuss the computational tractability of different variations of the TSPP. Since the (Generic IP) model is equivalent to the classical transportation problem, with carriers as supply points and with triples (b, j, t) for $b\in B$, $j\in J$,

$1 \leq t \leq T$ as demand points, it can be solved by a min-cost network flow algorithm in polynomial time.

For the problem with the MQC constraint, it is easy to see that when q_{ik} equals one, the MQC constraint can be relaxed, implying that the problem is equivalent to the (Generic IP) model and can thus be solved in polynomial time. Moreover, it is known that the problem is strongly NP-hard whenever the minimum quantity q_{ik} is greater than or equal to three (Lim et al. 2006). The proof is based on a reduction from the set cover problem, which is well known to be strongly NP-complete. However, it still remains an open question as to whether or not the problem is NP-hard when q_{ik} equals two. We now consider a new special case when only one lane and one business unit is taken into account, i.e., $|J| = |B| = 1$, and establish a new tractability result for the TSPP as follows:

Theorem 1 *Solving the (Generic IP) model with the MQC constraint (13.8) and with $|J| = |B| = 1$ is NP-hard in the strong sense. It is NP-hard but has a pseudo-polynomial time algorithm for any fixed T.*

Proof. Given $|J| = |B| = 1$, we can reformulate the problem as follows:

$$(\textbf{MQC1 IP}) = \min \sum_{t=1}^{T} \left[\sum_{i \in I} p_{it} x_{it} + s_t \left(d_t - \sum_{i \in I} x_{it} \right) \right] \tag{13.16}$$

$$\text{s.t.} \sum_{i \in I} x_{it} \leq d_t, \forall 1 \leq t \leq T, \tag{13.17}$$

$$q_i y_i \leq \sum_{i \in I} x_{it} \leq c_i y_i, \forall i \in I, \tag{13.18}$$

$$x_{it} \in Z_+, \forall i \in I, 1 \leq t \leq T, \tag{13.19}$$

$$y_i \in \{0,1\}, \forall i \in I, \tag{13.20}$$

where $d_t := d_{11t}$ indicates the demand for period t, x_{it} indicates the volume of cargo assigned to carrier i for period t, and y_i indicates whether or not carrier i is selected.

The strong NP-hardness of model (MQC1 IP) can be shown by a reduction from the following unary NP-complete problem:

Cover By 3-Sets (X3C) (Garey and Johnson 1983): Given a set $X = \{1, \ldots, 3k\}$ and a collection $C = \{C_1, \ldots, C_m\}$ with each member $C_i \subseteq X$ and $|C_i| = 3$ for $i = 1, \ldots, m$, does C contain an exact cover for X, i.e. a sub-collection $C' \subset C$ such that every element of X occurs in exactly one member of C'?

For any arbitrary instance of *X3C*, consider the instance of model (MQC1 IP) where $I = \{i{:}1 \leq i \leq m\}$, $T = |X|$, $d_t = 1$ for $1 \leq t \leq T$, $q_i = 3$ for $i \in I$, $c_i \geq 3$ for $i \in I$, $p_{it} = 0$ for $i \in I$ and for $t \in C_i$, $p_{it} = \infty$ for $i \in I$ and for $t \notin C_i$, and $s_t = \infty$ for $1 \leq t \leq T$. We can show as follows that the (MQC1 IP) has a minimum total cost of zero if and only if the *X3C* has an exact cover.

On one hand, if the *X3C* has an exact cover, then we can set $y_i = 1$ if $C_i \in C'$ and $y_i = 0$ otherwise, and set $x_{it} = 1$ if $t \in C_i$ and $C_i \in C'$, and $x_{it} = 0$ otherwise. It can be seen that this leads to a feasible solution to (MQC1 IP) of a total cost equal to zero.

On the other hand, if (MQC1 IP) has a feasible solution of zero cost, let C' include C_i if and only if $y_i = 1$. Since $d_t = 1$, $q_i = 3$, and the total cost equals zero, it can be seen that each element $t \in X$ is covered by C' exactly once, which implies that C' is an exact cover. This completes the proof of the strong NP-hardness of (MQC1 IP).

Next, consider the case when T is fixed, and its NP-hardness can be shown by a reduction from the following NP-complete problem:

Partition (Garey and Johnson 1983): Given a set X, size $s(a)$ for $a \in X$, positive integer S, does X have a subset X' such that $\sum_{a \in X'} s(a) = S$?

For any arbitrary instance of *Partition*, consider the instance of model (MQC1 IP) where $I = X$, $q_a = c_a = s(a)$ for $a \in X$, $d_1 = S$, $p_{a1} = 0$, $s_1 = \infty$, and $d_t = p_{at} = s_t = 0$ for $2 \le t \le T$. If this instance has a feasible solution of zero cost, then letting X' include a with $y_a = 1$ leads to $\sum_{a \in X'} s(a) = d_1 = S$. On the other hand, if there exists $X' \subseteq X$ with $\sum_{a \in X'} s(a) = S = d_1$, then setting $y_a = 1$ only for $a \in X'$ and setting $x_{a1} = s(a)$ and $x_{at} = 0$ for $2 \le t \le T$ lead to a feasible solution of zero cost. Thus, the case with $T = 1$ is NP-hard.

Now, for any fixed T, we can use the following dynamic programming algorithm to solve (MQC1 IP). Define $f(i, Q_1, \ldots, Q_T)$ as the minimum total cost to assign the cargo of Q_t for time $t = 1, 2, \ldots, T$ to carriers $1, 2, \ldots, i$, such that the capacity and MQC constraints for carriers $1, 2, \ldots, i$ are satisfied. For this, we can establish recurrence equations as follows:

$$f(i, Q_1, \ldots, Q_T) = \min \{f(i-1, Q_1, \ldots, Q_T), g(i, Q_1, \ldots, Q_T)\} \tag{13.21}$$

$$g(i, Q_1, \ldots, Q_T) = \min_{\mathbf{x}} f(i-1, Q_1 - x_{i1}, \ldots, Q_T - x_{iT}) + \sum_{t=1}^{T} p_{it} x_{it}$$

$$\text{s.t. } q_i \le \sum_{t=1}^{T} x_{it} \le c_i \tag{13.22}$$

where $f(0, 0, \ldots, 0) = 0$. Hence, the optimal objective value equals

$$\min_{Q_1, \ldots, Q_T} f(n, Q_1, \ldots, Q_T) + \sum_{t=1}^{T} s_t (d_t - Q_T) \tag{13.23}$$

It can be seen that the total time complexity of the above dynamic programming algorithm is $O(n(\max\{c_i : i \in I\})^T + (\max\{d_t : 1 \le t \le T\})^T)$, which is pseudo-polynomial when T is fixed. □

For the problem with the MARC constraint, it is known that the problem is strongly NP-hard for any fixed $\xi \ge 0$ even when only a single lane is considered (Lim et al. 2008a). The proof is also based on a reduction from the set cover problem. When $T = 1$, it can be seen that the problem with the MARC constraint is equivalent to the (Generic IP) model, and can thus be solved in polynomial time. Moreover, when only a single carrier is considered, i.e., $|I| = 1$, the problem can be reformulated as a min-cost network flow problem, which can also be solved in polynomial time. This special case also has a simple greedy algorithm (Lim et al. 2008a), which can guarantee a polynomial running time if the total demand is polynomially bounded.

For the problem with the constraints on the number of selected carriers, it can be transformed to the classical k-median problem, and thus it is strongly NP-hard (Arya et al. 2001; Bozkaya et al. 2002; de Farias 2001; Hochbaum 1982; Lorena and Senne 2004; Rolland et al. 1997; Senne et al. 2005), where carriers correspond to candidate locations of facilities, and triples (b, j, t) indicate the locations of demand points, even when $|J| = 1$ or $T = 1$. When the maximum number of selected carriers is fixed, the problem can be solved in polynomial time, as one can enumerate all the possible combinations of carriers, and solve model (Generic IP) on only selected carriers to obtain optimal cargo allocations. In this case, even if the MQC constraint is included, the problem can still be solved in polynomial time. Now, consider a new special case where both $|J|$ and T equal to one, for which we can derive a new tractability result as follows.

Theorem 2 *It is strongly NP-hard to solve the (Generic IP) model with constraints (13.10)—(13.12) on the number of carriers and with $|J| = T = 1$, but it has a polynomial time algorithm when $|B|$ is fixed.*

Proof. For the (Generic IP) model with constraints (13.10)—(13.12) on the number of carriers and with $|J| = T = 1$, we can reformulate it as follows.

$$(\textbf{NUM1 IP}) = \min \sum_{b \in B} \left[\sum_{i \in I} p_i x_{ib} + s \left(d_b - \sum_{i \in I} x_{ib} \right) \right] \tag{13.24}$$

$$\text{s.t.} \sum_{i \in I} x_{ib} \le d_b, \forall b \in B, \tag{13.25}$$

$$\sum_{b \in B} x_{ib} \le c_i y_i, \forall i \in I, \tag{13.26}$$

$$\sum_{b \in B} y_{ib} \le |B| y_i, \forall i \in I, \tag{13.27}$$

$$\underline{n}_b \le \sum_{i \in I} y_{ib} \le \bar{n}_b, \forall b \in B, \tag{13.28}$$

$$x_{ib} \in Z_+, \forall i \in I, b \in B, \tag{13.29}$$

$$y_i \in \{0,1\}, \forall i \in I, \tag{13.30}$$

$$y_{ib} \in \{0,1\}, \forall i \in I, b \in B, \tag{13.31}$$

where x_{ib} indicates the total volume of cargo of business unit b assigned to carrier i, and y_{ib} indicates whether or not carrier i is selected to serve business unit b. We now show its NP-hardness as follows by a reduction from the following NP-complete problem:

3-Partition (Garey and Johnson 1983): Given a set X of $3k$ elements, a bound S, and a size $s(a)$ for $a \in X$ such that $S/4 < s(a) < S/2$ and $\sum_{a \in X} s(a) = kS$, can X be partitioned into $A_1, \ldots, A_k$ such that for $1 \leq r \leq k$, $\sum_{a \in A_r} s(a) = S$?

For any arbitrary instance of *3-Partition*, consider the instance of model (NUM1 IP) where $|J| = T = 1$, $I = X$, $|B| = k$, $c_a = s(a)$ for $a \in X$, $d_b = S$ for $b \in B$, $\underline{n}_b = \bar{n}_b = 3$ for $b \in B$, $p_i = 0$ for $i \in I$, and $s = \infty$. If this instance has a feasible solution of zero cost, then letting $A_b := \{i \in I : y_{ib} = 1\}$ include those carriers i with $y_{ib} = 1$ for $b \in B$. Since $\underline{n}_b = \bar{n}_b = 3$ for $b \in B$, $|B| = k$, and $\sum_{a \in X} s(a) = kS$, it can be seen that no carrier can serve more than one business unit, which implies that $A_1, \ldots, A_k$ is feasible to the *3-Partition* problem. On the other hand, if the *3-Partition* problem has a feasible partition $A_1, \ldots, A_k$, then by setting $y_{ib} = 1$ and $x_{ib} = s(i)$ for $i \in A_b$ and $b \in B$, we obtain a feasible solution to (NUM1 IP) of zero cost. Thus, the case is strongly NP-hard.

Next, consider the case when $|B|$ is fixed. Consider any y_{ib} for $i \in I$ and $b \in B$ that satisfy the constraints on the number of selected carriers. The total number of such possible combinations is polynomially bounded, since $|B|$ is fixed. We can now extend the (Generic IP) model to obtain a min-cost network flow model as follows, so as to determine optimal allocations for the cargo:

$$\min \sum_{b \in B} \left[\sum_{i \in I} p_i x_{ib} + s \left(d_b - \sum_{i \in I} x_{ib} \right) \right] \tag{13.32}$$

$$\text{s.t.} \sum_{i \in I} x_{ib} \leq d_b, \forall b \in B, \tag{13.33}$$

$$\sum_{b \in B} x_{ib} \leq c_i y_i, \forall i \in I, \tag{13.34}$$

$$x_{ib} \in Z_+, \forall i \in I, b \in B. \tag{13.35}$$

Hence, this special case can be solved in polynomial time.

For the problem with the preference constraints, this can still be transformed to a min-cost network flow problem, and thus can be solved in polynomial time. However, in the literature, it is often imposed simultaneously with the constraints on the number of selected carriers (Lim et al. 2012), which in general is strongly NP-hard.

For the problem with the fairness constraints, its tractability is unknown in the literature. Lim et al. (2012) studied a generalized problem that has taken into account fairness constraints, and carrier number constraints, as well as preference constraints, and they showed that it is strongly NP even to find a feasible solution to this problem, and also that it has a polynomial time algorithm that can obtain a feasible solution when all shipping prices are identical.

From the tractability results above, we know that the generalized problem (General MP) is strongly NP-hard. However, for those special cases of (General MP), where only two types of side constraints are taken into account, few results showing their tractability are known in the literature.

13.4 Problem Relaxations

In this section, we discuss various relaxations of the models proposed in Sect. 2 for the TSPP. Although optimal solutions to these relaxations may not be feasible for the original models, they can be used to estimate the optimal objective values of the original models, as well as to construct feasible solutions for the original models.

Consider the problem with the MQC constraints, where $|B|$ is assumed to be one for ease of presentation. We can reformulate this problem as the following integer programming model:

$$(\textbf{MQC IP}) = \min \sum_{j\in J} \sum_{t=1}^{T} \left[\sum_{i\in I} p_{ijt} x_{ijt} + s_{jt} \left(d_{jt} - \sum_{i\in I} x_{ijt} \right) \right] \quad (13.36)$$

$$\text{s.t.} \sum_{i\in I} x_{ijt} \le d_{jt}, \forall j \in J, 1 \le t \le T, \quad (13.37)$$

$$\sum_{j\in J_{ik}} \sum_{t=1}^{T} x_{ijt} \le c_{ik} y_{ik}, \forall i \in I, 1 \le k \le |J_i|, \quad (13.38)$$

$$q_{ik} y_{ik} \le \sum_{j\in J_{ik}} \sum_{t=1}^{T} x_{ijt}, \forall i \in I, 1 \le k \le |J_i|, \quad (13.39)$$

$$x_{ijt} \in Z_+, \forall i \in I, j \in J, 1 \le t \le T, \quad (13.40)$$

$$y_{ik} \in \{0,1\}, \forall i \in I, 1 \le k \le |J_i|, \quad (13.41)$$

where x_{ijt} indicates the total number of containers assigned to carrier i for cargo of lane j and period t. For this problem, it has a linear programming relaxation where **x** and **y** are relaxed to take fractional values:

$$z_{MQC}^{LP} = \min \sum_{j\in J} \sum_{t=1}^{T} \left[\sum_{i\in I} p_{ijt} x_{ijt} + s_{jt} \left(d_{jt} - \sum_{i\in I} x_{ijt} \right) \right]$$

$$\text{s.t.}(37) - (39), \quad (13.42)$$

$$x_{ijt} \in R_+, \forall i \in I, j \in J, 1 \le t \le T, \quad (13.43)$$

$$0 \le y_{ik} \le 1, \forall i \in I, 1 \le k \le |J_i|. \quad (13.44)$$

The above linear programming relaxation model can be strengthened by the following valid constraints of (MQC IP), which are extended from Lim et al. (2006)

and Lim et al. (2008b). First, by (13.37) and (13.38), we have

$$x_{ijt} \leq d_{jt} y_{ik}, \forall i \in I, j \in J_{ik}, 1 \leq k \leq |J_i|, 1 \leq t \leq T.$$

Next, suppose that pairs (*i*, *k*) are sorted on a non-decreasing order of q_{ik}, and let $K_{\max}$ indicate the smallest position of the order, such that the sum of q_{ik} for the first $K_{\max}$ pairs of (*i*, *k*) exceeds *D*, where $D := \sum_{j \in J} \sum_{t=1}^{T} d_{jt}$. By (13.37) and (13.39), we have

$$\sum_{i \in I} \sum_{k=1}^{|J_i|} y_{ik} \leq K_{\max}.$$

Moreover, by extending the arguments from Lim et al. (2006, 2008b), we can show that the above two valid constraints both define facets of the convex hull of the integer programming model (MQC IP) under some mild conditions.

In addition to the linear programming relaxation, we can obtain a Lagrangian relaxation of model (MQC IP) by dualizing the demand constraint (13.37). Let $\mu_{jt} \geq 0$ indicate the Lagrangian multiplier, associated with constraint (13.37). Let $z_{MQC}^{LR1}(\mu)$ denote the optimal objective value for the following problem:

$$z_{MQC}^{LR1}(\mu) = \min \sum_{j \in J} \sum_{t=1}^{T} \sum_{i \in I} (p_{ijt} - s_{jt} + \mu_{jt}) x_{ijt} + \sum_{j \in J} \sum_{t=1}^{T} (s_{jt} - \mu_{jt}) d_{jt}$$

$$\text{s.t.}(38) - (41).$$

The above $z_{MQC}^{LR1}(\mu)$ can be decomposed by carriers $i \in I$ and lane groups in J_i into $\sum_{i \in I} |J_i|$ sub-problems, with each corresponding to a continuous knapsack problem, and thus it can be solved in polynomial time. Thus, one can apply a subgradient algorithm to maximize $z_{MQC}^{LR1}(\mu)$ over multipliers μ, so as to obtain a Lagrangian relaxation lower bound, denoted by z_{MQC}^{LR1}, for model (MQC IP).

Furthermore, by dualizing the capacity constraint (13.38) and the MQC constraints (13.39), we can derive a new Lagrangian relaxation of model (MQC IP) as follows, where π_{ik} and γ_{ik} indicate the associated Lagrangian multipliers.

$$z_{MQC}^{LR2}(\pi, \gamma) = \min \sum_{j \in J} \sum_{t=1}^{T} \sum_{i \in I} (p_{ijt} - s_{jt} + \pi_{ik} - \gamma_{ik}) x_{ijt} + \sum_{i \in I} (q_{ik}\gamma_{ik} - c_{ik}\pi_{ik}) y_{ik}$$

$$+ \sum_{j \in J} \sum_{t=1}^{T} s_{jt} d_{jt} \quad \text{s.t. } (37), (40), (41).$$

Based on the signs of $(q_{ik}\gamma_{ik} - c_{ik}\pi_{ik})$, we can determine values of $\mathbf{y}$, and then the remaining problems on $\mathbf{x}$ can be decomposed by (*j*, *t*) for $j \in J$ and $1 \leq t \leq T$ into $|J|T$ sub-problems, with each sub-problem equivalent to a continuous knapsack problem, and thus it can be solved in polynomial time. By applying a subgradient algorithm, we can thus maximize $z_{MQC}^{LR2}(\pi, \gamma)$ to obtain a lower bound on the optimal objective value of (MQC IP), denoted by z_{MQC}^{LR2}.

We can also derive an LP relaxation of the Dantzig-Wolfe reformulation of model (MQC IP) as follows. Let $\mathbf{x}^{ih}$for $h = 1,2,\ldots, H$ indicate all the feasible cargo allocations for carrier i which satisfy the capacity constraint (13.38) and the MQC constraint (13.39), and we denote the cost of each $\mathbf{x}^{ih}$ by $c^{ih} := \sum_{j\in J}\sum_{t=1}^{T}(p_{ijt} - s_{jt})x_{jt}^{ih}$. Let λ_{ih} indicate a binary variable that equals 1 if and only if cargo allocation $\mathbf{x}^{ih}$is assigned to carrier i. Thus, model (MQC IP) can be reformulated as follows:

$$\min \sum_{i\in I}\sum_{h=1}^{H} c^{ih}\lambda_{ih} \tag{13.45}$$

$$\text{s.t.} \sum_{h=1}^{H}\lambda_{ih} = 1, \forall i \in I, \tag{13.46}$$

$$\sum_{i\in I}\sum_{h=1}^{H}\lambda_{ih}x_{jt}^{ih} \le d_{jt}, \forall j \in J, 1 \le t \le T, \tag{13.47}$$

$$\lambda_{ih} \in \{0,1\}, \forall i \in I, 1 \le h \le H. \tag{13.48}$$

The linear programming relaxation of the above model, denoted by z_{MQC}^{LPDW}, can be solved by column generation, for which the pricing problem is equivalent to a continuous knapsack problem, and thus can be solved in polynomial time.

Proposition 1 below reveals that the four relaxations above are equally tight:

Proposition 1. $z_{MQC}^{LP} = z_{MQC}^{LR1} = z_{MQC}^{LPDW} = z_{MQC}^{LR2}$

Proof. The convex hull of feasible solutions to $z_{MQC}^{LR2}(\pi,\gamma)$ is the same as the convex hull of its linear programming relaxation, which implies that $z_{MQC}^{LP} = z_{MQC}^{LR2}$. The convex hull of feasible cargo allocations for each carrier i is the same as the convex hull of its linear programming relaxation, which implies that $z_{MQC}^{LP} = z_{MQC}^{LPDW}$. Finally, the convex hull of feasible solutions to $z_{MQC}^{LR1}(\mu)$ is the same as the convex hull of its linear programming relaxation, which implies that $z_{MQC}^{LP} = z_{MQC}^{LR1}$. Hence, the proposition is proved. □

The proposition above implies that the four relaxations mentioned above are equally tight. To derive tighter relaxations, one needs to introduce more valid constraints. For example, by (13.37) and (13.39) we can obtain the following valid constraint

$$\sum_{i\in I}\sum_{k=1}^{|J_i|} q_{ik}y_{ik} \le \sum_{j\in J}\sum_{t=1}^{T} d_{jt}. \tag{13.49}$$

By extending the argument in Lim et al. (2008b), it can be shown that by including (13.49) in $z_{MQC}^{LR1}(\mu)$, one can obtain a stronger Lagrangian relaxation, which can be transformed to a multiple-dimensional knapsack problem and solved by a dynamic programming algorithm. Thus, the resulting lower bound on the optimal objective value to (MQC IP) is tighter than z_{MQC}^{LP}.

Next, consider the problem with the MARC constraint, which can be formulated as follows:

$$(\textbf{MARC IP}) = \max \sum_{j\in J}\sum_{t=1}^{T}\sum_{i\in I}(s_{jt} - p_{ijt})x_{ijt} \tag{13.50}$$

$$\text{s.t.} \sum_{i\in I} x_{itj} \le d_{jt}, \forall j \in J, 1 \le t \le T, \tag{13.51}$$

$$\sum_{j\in J_{ik}}\sum_{t=1}^{T} x_{ijt} \le c_{ik}, \forall i \in I, 1 \le k \le |J_i|, \tag{13.52}$$

$$\sum_{j\in J_{ik}} x_{ijt} \le \alpha_{ikj}\left(\sum_{j\in J_{ik}}\sum_{t=1}^{T} x_{ijt}\right) + \xi, \forall i \in I, 1 \le k \le |J_i|, 1 \le t \le T, \tag{13.53}$$

$$x_{ijt} \in Z_+, \forall i \in I, j \in J, 1 \le t \le T. \tag{13.54}$$

It also has a linear programming relaxation by relaxing $\mathbf{x}$ to take fractional values:

$$(\textbf{MARC IP}) = \max \sum_{j\in J}\sum_{t=1}^{T}\sum_{i\in I}(s_{jt} - p_{ijt})x_{ijt} \tag{13.55}$$

$$\text{s.t. } (51) - (53)$$

$$x_{ijt} \ge 0, \forall i \in I, j \in J, 1 \le t \le T. \tag{13.56}$$

The above linear programming relaxation model can also be strengthened by introducing some valid constraints of the model (MARC IP). For example, from (13.52) and (13.53), we have $\sum_{j\in J_{ik}} x_{ijt} \le \alpha_{ikj}c_{ik} + \xi, \forall i \in I, 1 \le k \le |J_i|, 1 \le t \le T$. Thus,

$$\sum\nolimits_{j\in J_{ik}} x_{ijt} \le \lfloor \alpha_{ikj}c_{ik} \rfloor + \xi, \forall i \in I, 1 \le k \le |J_i|, 1 \le t \le T. \tag{13.57}$$

We can establish the following theorem to show that under some mild conditions, the valid constraint (13.57) defines a facet of model (MARC IP), and thus is necessary.

Theorem 3. *If* $\lfloor \alpha_{ikj}c_{ik} \rfloor + \xi < \sum_{j\in J_{ik}} d_{jt}$ *and* $\sum_{t=1}^{T}\left(\lfloor \alpha_{ikj}c_{ik} \rfloor + \xi\right) < c_{ik}$, *then (*13.57) defines a facet of model (MARC IP).

Proof. To prove this theorem, we only need to show that if all feasible solutions to model (MARC IP) that satisfy (13.57) for some $i \in I$ and t with $1 \le t \le T$ at equality also satisfy

$$\sum_{i\in I}\sum_{j\in J}\sum_{t=1}^{T} a_{ijt}x_{ijt} \le \theta, \tag{13.58}$$

at equal, then (13.58) is equivalent to (13.57).

First, since $\lfloor \alpha_{ikj} c_{ik} \rfloor + \xi < \sum_{j \in J_{ik}} d_{jt}$ and $\sum_{t=1}^{T} \left(\lfloor \alpha_{ikj} c_{ik} \rfloor + \xi \right) < c_{ik}$, there exists a feasible $\mathbf{x^1}$such that (13.57) is satisfied at equality for i, k and t, and such that constraints (13.51), (13.52) and (13.53) are all satisfied but not at equality. For any with i', j', t' with $i' \neq i$, or $j' \in J_{ik}$, or $t' \neq t$, consider $\mathbf{x^2}$, which is equal to $\mathbf{x^1}$except that

$$x^2_{i'j't'} = x^1_{i'j't'} + \varepsilon. \tag{13.59}$$

Thus, it can be seen that there exists $\varepsilon > 0$, such that $\mathbf{x^2}$ is feasible to model (MARC IP) and satisfies (13.57) for i, k, t at equal. Substituting $\mathbf{x^1}$ and $\mathbf{x^2}$ into (13.58) and subtracting one from the other results in $a_{i'j't'} = 0$.

Next, for any j and j' in J_{ik}, consider $\mathbf{x^3}$, which is equal to $\mathbf{x^1}$ except that

$$x^3_{ijt} = x^1_{ijt} + \varepsilon. \tag{13.60}$$

$$x^3_{ij't} = x^1_{ij't} - \varepsilon. \tag{13.61}$$

It can be seen that there exists $\varepsilon > 0$, such that $\mathbf{x^3}$ is feasible to model (MARC IP) and satisfies (13.57) for i, k, t at equal. Substituting $\mathbf{x^3}$and $\mathbf{x^1}$into (13.58) and subtracting one from the other results in $a_{ijt} = a_{ij't}$.

Thus, we can assume $a_{ijt} = a$ for $j \in J_{ik}$ and (13.58) can be represented as:

$$\sum_{j \in J_{ik}} a x_{ijt} \leq \theta. \tag{13.62}$$

Thus, since $\sum_{j \in J_{ik}} x^1_{ijt} = \lfloor \alpha_{ikj} c_{ik} \rfloor + \varepsilon$, we obtain that (13.58) is equivalent to (13.57).

Next, by dualizing the demand constraint (13.51), we can obtain a Lagrangian relaxation of model (MARC IP). Let $\mu_{jt} \geq 0$ indicate the Lagrangian multiplier, associated with constraint (13.51). Define $z^{LR}_{MARC}(\mu)$ as follows:

$$z^{LR}_{MARC}(\mu) = \max \sum_{j \in J} \sum_{t=1}^{T} \sum_{i \in I} (s_{jt} - p_{ijt} - \mu_{jt}) x_{ijt}$$

$$\text{s.t. } (52) - (54). \tag{13.63}$$

It can be seen that $z^{LR}_{MARC}(\mu)$ can be decomposed by carriers and lane groups into $\sum_{i \in I} |J_i|$ sub-problems, with each being equivalent to a single-carrier problem, and thus it can be solved in polynomial time (Lim et al. 2008a).

Moreover, we can also derive an LP relaxation of the Dantzig-Wolfe reformulation of model (MARC IP) as follows. Let $\mathbf{x}^{ih}$for $h = 1,2,\ldots, H$ indicate all the feasible cargo allocations for carrier i, which satisfy the capacity constraint (13.52) and the MARC constraint (13.53), and we denote each saving by $c^{ih} := \sum_{j \in J} \sum_{t=1}^{T} (s_{jt} - p_{ijt}) x^{ih}_{jt}$. Let λ_{ih} indicate a binary variable that equals 1 if and only if cargo allocation $\mathbf{x}^{ih}$ is assigned to carrier i. Thus, model (MQC IP) can be reformulated as follows:

$$\max \sum_{i \in I} \sum_{h=1}^{H} c^{ih} \lambda_{ih} \tag{13.64}$$

$$\text{s.t.} \sum_{h=1}^{H} \lambda_{ih} = 1, \forall i \in I, \tag{13.65}$$

$$\sum_{i \in I} \sum_{h=1}^{H} \lambda_{ih} x_{jt}^{ih} \leq d_{jt}, \forall j \in J, 1 \leq t \leq T, \tag{13.66}$$

$$\lambda_{ih} \in \{0,1\}, \forall i \in I, 1 \leq h \leq H. \tag{13.67}$$

The linear programming relaxation of the above model, denoted by z_{MARC}^{LPDW}, can be solved by column generation, for which the pricing problem is equivalent to a single carrier problem, and thus can be solved in polynomial time.

The following proposition reveals the tightness of the three relaxations above:

Proposition 2. $z_{MARC}^{LP} \leq z_{MARC}^{LR} = z_{MARC}^{LPDW}$.

Proof. Noticing that both the Lagrangian dual and the pricing problem are equivalent to a single-carrier problem, we can obtain that $z_{MARC}^{LR} = z_{MARC}^{LPDW}$. Moreover, the convex hull of feasible solutions to the single-carrier problem is a superset of the convex hull of its linear programming relaxation, which implies that $z_{MARC}^{LP} \leq z_{MARC}^{LR} = z_{MARC}^{LPDW}$, completing the proof. □

Similarly, by dualizing the demand constraint, we can further derive Lagrangian relaxations, as well as the LP relaxation of the Dantzig-Wolfe reformulation, for problems with constraints on the number and preference of selected carriers, as well as on the fairness. Such relaxation techniques can also be applied to the (General MP) having *EDom* and *IDom* containing all the various constraints. It is of interest to investigate the tightness of these new relaxations as well as how to strengthen them. For example, for the problem with constraints on the number of selected carriers, we can derive a valid constraint directly from (13.11) as follows:

$$\sum_{b \in B_h} \sum_{k: J_{ik} \bigcap L_g \neq \varnothing} y_{ibk} \leq |B_h| \left| \left\{ k : J_{ik} \bigcap L_g \neq \varnothing \right\} \right| z_{ihg}, \forall i \in I, B_h \in BG, L_g \in LG. \tag{13.68}$$

For the problem with the fairness constraint, we can derive a valid constraint from (13.15) by replacing M with $\sum_{i \in I} \sum_{t=1}^{T} s_{jt} d_{bjt}$, since the total cost for shipping the cargo of each lane should not exceed the cost of shipping them all using the spot-market price.

13.5 Exact Algorithms

Since the TSPP, as well as most of its special cases, are often computationally intractable, it is of great interests to develop algorithms that can produce optimal solutions to relatively small sized instances of the TSPP in affordable running time.

To solve the model (General MP) for cases where (General MP)($\mathbf{y}$) can be solved to optimum efficiently, one can follow a branch-and-bound algorithm to search an optimal value of $\mathbf{y}$ that minimizes (General MP)($\mathbf{y}$). As it goes down the search tree, the branch-and-bound algorithm determines values of y_{ibk} one by one, and keeps the current best feasible solution denoted by $(\mathbf{x}^*, \mathbf{y}^*)$. At each node of the search tree, let Π_0 indicate the set of triples (i, b, k) with $y_{ibk} = 0$ and let Π_1 indicate the set of triples (i, b, k) with $y_{ibk} = 1$. Let $\Pi := \Pi_0 \cup \Pi_1$ indicate the set of triples (i, b, k) for determined y_{ibk}, and $\bar{\Pi}$ indicate the set of triples (i, b, k) for un-determined y_{ibk}. Hence (Π_0, Π_1) can be used to represent a partial solution.

Before assigning values to those un-determined y_{ibk}, the algorithm computes a lower bound on the best possible objective value that can be obtained by completing the current partial solution (Π_0, Π_1). This can be achieved by solving relaxations of the models on the remaining problems, such as the linear programming relaxation and the Lagrangian relaxation described in Sect. 4. If the obtained lower bound is not less than the objective value of the current best solution $(\mathbf{x}^*, \mathbf{y}^*)$, then the node can be pruned. Otherwise, the algorithm will select an un-determined y_{ibk} for $(i, b, k) \in \bar{\Pi}$, and assign y_{ibk} either 0 or 1, so as to generate two new nodes of the search tree. Given the partial solution of each new node, we can construct feasible solutions by various heuristics, which will be introduced later in Sect. 6. If the obtained solution has a better objective value than the current best solution $(\mathbf{x}^*, \mathbf{y}^*)$, the algorithm will update $(\mathbf{x}^*, \mathbf{y}^*)$. This branch-and-bound algorithm can be summarized in Algorithm 1.

Algorithm 1 The Branch-and-Bound Algorithm

Let *NList* represent the list of nodes of the search tree to be expanded, and set the initial value of *NList* to be the set that contains only the root node. Let *UB* indicate the objective value of the current best feasible solution $(\mathbf{x}^*, \mathbf{y}^*)$, and set the initial value of *UB* to be ∞.
while *NList* is not empty **do**
    Choose a node *p* in *NList*, exclude *p* from *NList*, and consider its associated partial solution (Π_0, Π_1).
    Compute a lower bound *LB* on the best possible objective value that can be obtained by completing (Π_0, Π_1).
    if *LB* < *UB* **then**
        Let $\Pi := \Pi_0 \cup \Pi_1$
        for each $(i, b, k) \in \bar{\Pi}$ **do**
            for $v \in \{0, 1\}$ **do**
                Construct a new feasible solution $(\hat{\Pi}_0, \hat{\Pi}_1)$, where $\hat{\Pi}_v := \Pi_v \cup \{(i,b,k)\}$ and $\hat{\Pi}_{1-v} := \Pi_{1-v}$.
                Construct a feasible solution from the new partial solution $(\hat{\Pi}_0, \hat{\Pi}_1)$. If the feasible solution has a smaller objective value than *UB*, then update *UB* and $(\mathbf{x}^*, \mathbf{y}^*)$.
                Add to *NList* the new node *p* that is associated with $(\hat{\Pi}_0, \hat{\Pi}_1)$.
            end for
        end for
    end if
end while
Return the current best solution $(\mathbf{x}^*, \mathbf{y}^*)$.

To enhance the branch and bound algorithm, we can strengthen the relaxations of the problem so as to obtain better lower bounds. For example, from the linear programming relaxation, we can obtain a fractional solution, which may not be feasible to the original model (General MP). In this case, an intuitive way to strengthen the relaxation is to introduce new valid constraints that can exclude the fractional solutions. This approach is usually referred to as a branch-and-cut algorithm (Lim et al. 2006).

The above branch-and-bound algorithm has been applied in the literature to solving the problem (MQC IP) (Lim et al. 2006). Given **y**, the model (MQC IP) can be reformulated as follows:

$$(\textbf{MQC IP})(y) = \min \sum_{j \in J} \sum_{t=1}^{T} \left[\sum_{i \in I} p_{ijt} x_{ijt} + s_{jt} \left(d_{jt} - \sum_{i \in I} x_{ijt} \right) \right] \tag{13.69}$$

$$\text{s.t.} \sum_{i \in I} x_{ijt} \le d_{jt}, \forall j \in J, 1 \le t \le T, \tag{13.70}$$

$$q_{ik} y_{ik} \le \sum_{j \in J_{ik}} \sum_{t=1}^{T} x_{ijt} \le c_{ik} y_{ik}, \forall i \in I, 1 \le k \le |J_i|, \tag{13.71}$$

$$x_{ijt} \in Z_+, \forall i \in I, j \in J, 1 \le t \le T, \tag{13.72}$$

which is equivalent to a min-cost network flow model, and thus can be solved efficiently. Moreover, when given a partial solution (Π_0, Π_1), the remaining problem can be formulated as follows:

$$(\textbf{MQC IP})(\Pi_0, \Pi_1) = \min \sum_{t=1}^{T} \sum_{j \in J} s_{jt} d_{jt} + \sum_{t=1}^{T} \sum_{(i,k) \in \overline{\Pi_0}} \sum_{j \in J_{ik}} (p_{ijt} - s_{jt}) x_{ijt} \tag{13.73}$$

$$\text{s.t.} \sum_{(i,k) \in \overline{\Pi_0}, j \in J_{ik}} x_{ijt} \le d_{jt}, \forall j \in J, 1 \le t \le T, \tag{13.74}$$

$$q_{ik} \le \sum_{j \in J_{ik}} \sum_{t=1}^{T} x_{ijt} \le c_{ik}, \forall (i,k) \in \Pi_1, \tag{13.75}$$

$$q_{ik} y_{ik} \le \sum_{j \in J_{ik}} \sum_{t=1}^{T} x_{ijt} \le c_{ik} y_{ik}, \forall (i,k) \in \overline{\Pi} \tag{13.76}$$

$$x_{ijt} \in Z_+, \forall (i,k) \in \overline{\Pi_0}, j \in J_{ik}, 1 \le t \le T, \tag{13.77}$$

$$y_{ik} \in \{0,1\}, \forall (i,k) \in \overline{\Pi}. \tag{13.78}$$

Thus, relaxations of (MQC IP)(Π_0, Π_1) can provide valid lower bounds for the branch and bound algorithm. This model for the remaining problem can be further strengthened by including the valid constraints presented in Sect. 4.

Furthermore, the framework in Algorithm 1 can also be applied to problems with constraints on the number as well as on the shipper's preference for selected carriers. This is because these constraints are only associated with **y**, and thus, given **y**, the problems with these constraints are equivalent to the classical transportation problem, and can be solved efficiently in polynomial time.

For (General MP)(**y**), searching for an exact optimal solution is more complicated, since it needs to explore possible values for both **x** and **y**. However, in some cases, we may still be able to reduce search space by introducing some auxiliary variables. Consider the problem with the maximum-to-average-ratio commitment constraint (MARC IP). Define $v_{ik} := \sum_{j\in J_{ik}} \sum_{t=1}^{T} x_{ijt}$ as new auxiliary variables that represent the total volume of cargo assigned to carrier i for lanes in J_{ik} and for all the T periods. Given **v**, the model (MARC IP) can be reformulated as follows:

$$(\textbf{MARC IP})(v) = \max \sum_{i\in I} \sum_{j\in J} \sum_{t=1}^{T} (s_{jt} - p_{ijt}) x_{ijt} \tag{13.79}$$

$$\text{s.t.} \sum_{i\in I} x_{itj} \le d_{jt}, \forall j \in J, 1 \le t \le T, \tag{13.80}$$

$$x_{ijt} \in Z_+, \forall i \in I, j \in J, 1 \le t \le T, \tag{13.81}$$

$$\sum_{j\in J_{ik}} x_{ijt} \le \alpha_{ikj} v_{ik} + \xi, \forall i \in I, 1 \le k \le |J_i|, 1 \le t \le T, \tag{13.82}$$

which can be transformed to a min-cost network flow problem, and thus can be solved efficiently. Therefore, to solve (MARC IP), we can develop a branch and bound algorithm to find optimal values of **v** so as to maximize (MARC IP)(**v**).

However, since v_{ik} are not binary variables, the branch and bound algorithm for (MARC IP) needs to narrow the value ranges of v_{ik} iteratively. Thus, a partial solution needs to be represented here by a vector of pairs $(\underline{v}_{ik}, \bar{v}_{ik})$, where $\underline{v}_{ik}$ and $\bar{v}_{ik}$ indicate the lower and the upper bound of each v_{ik}, respectively. At each decision node associated with the partial solution $\underline{v}_{ik}, \bar{v}_{ik}) : i \in I, 1 \le k \le |J_i|\}$, the algorithm first computes the lower bound LB on the best possible objective value that can be obtained by completing $\{(\underline{v}_{ik}, \bar{v}_{ik}) : i \in I, 1 \le k \le |J_i|\}$. This can be achieved by solving a relaxation of the integer programming model for given $\{(\underline{v}_{ik}, \bar{v}_{ik}) : i \in I, 1 \le k \le |J_i|\}$. Next, the algorithm selects any (i, k) with $\underline{v}_{ik} < \bar{v}_{ik}$, and computes the midpoint p of $[\underline{v}_{ik}, \bar{v}_{ik}]$, so that two new nodes can be generated with the range of v_{ik} being $[\underline{v}_{ik}, p]$ and $[p+1, \bar{v}_{ik}]$, respectively. For each new node, we can construct feasible solutions to update the objective value UB of the existing best feasible solution.

13.6 Heuristic Algorithms

To tackle the TSPP and its special cases that are computationally intractable, one solution approach is to develop heuristic algorithms that can produce feasible solutions close to the optimum in affordable running time. Recall that for a minimization (or maximization) problem, a heuristic algorithm is said to be a ρ-approximation algorithm if, for every instance of the problem, the algorithm has a polynomial running time and returns a feasible solution that has an objective value at most ρ times the minimum objective value (or at least $1/\rho$ times the maximum objective value). The value of ρ is referred to as the approximation ratio of the algorithm.

The constant-ratio approximation algorithms known in the literature on transportation service procurement are mainly for the special case where only MQC constraints are taken into account. One such algorithm follows a greedy approach (Lim et al. 2006). In this greedy algorithm, two operators, *selection*(i, k) and *assignment*(i, k), are defined for the construction of feasible solutions. For each unassigned pair of carrier $i \in I$ and $1 \leq k \leq |J_i|$, the operator *selection*(i, k) selects carrier i and lanes in J_{ik}, and assigns carrier i the cheapest q_{ik} units of unassigned cargo of lanes in J_{ik} so as to satisfy the minimum quantity commitment constraint. For each assigned carrier $i \in I$ and $1 \leq k \leq |J_i|$, the operator *assignment*(i, k) assigns carrier i the cheapest unassigned cargo of lanes in J_{ik} for delivering. Based on these two operators, the algorithm constructs a feasible solution to the problem iteratively, and during each iteration, it applies the operator with the minimum average cost, until all the cargo has been assigned. Here, the average cost of *selection*(i, k) is measured by $\sum_{(j,t)\in A} (p_{ijt} - s_{jt})/q_{ik}$, where A is the set of q_{ik} cargo of lanes in J_{ik} newly assigned to carrier i, and the average cost of *assignment*(i, k) is measured by $(p_{ijt} - s_{jt})$.

We summarize the above greedy algorithm in Algorithm 2, which extends the one in Lim et al. (2006), as constraints presented in this chapter are more general, with time periods being taken into account. It can be seen that after each iteration of Algorithm 2, at least one of the three following events must happen: (i) a carrier i and lanes in J_{ik} are newly selected; (ii) the capacity of a selected carrier i is fully assigned for lanes in J_{ik}; (iii) the demand of lane j and period t is fully satisfied. This implies that the total number of iterations is $O(\sum_{i\in I} |J_i| + |J|T)$. Since each iteration has a polynomial running time, we obtain that Algorithm 2 runs in polynomial time. Moreover, by following a similar argument as in Lim et al. (2006), it can be shown that Algorithm 2 has an approximation ratio of b if all the carriers have unlimited capacity, if their minimum quantities q_i all equal a constant b, if they have only one lane group, and if the shipping price $(p_{ijt} {-}_{jt} s)$ forms a metric.

Algorithm 2 Greedy Algorithm

Set the selected set, Π_1, equal to empty, and set assigned quantity $\hat{d}_{jt}$ equal to zero.
while NOT all cargo has been assigned, i.e. there exists (j, t) such that $\hat{d}_{jt} < d_{jt}$ **do**
    Choose an operator σ with minimum cost among all *selection*(i, k) for $(i, k) \notin \Pi_1$ and *assignment*(i, k) for $(i, k) \in \Pi_1$, breaking ties by the quantity of newly assigned cargo.
    if σ is *selection*(i, k) **then**
        Select carrier i and lanes in J_{ik} by setting $y_{ik} \leftarrow 1$ and $\Pi_1 \leftarrow \Pi_1 \cup \{(i, k)\}$.
        Let A denote the multiset of q_{ik} unassigned cargos (j, t) that are of the q_{ik} cheapest $(p_{ijt} - s_{jt})$ among all $j \in J_{ik}$ and $1 \le t \le T$ with $\hat{d}_{jt} < d_{jt}$.
        For each cargo $(j, t) \in A$, assign it to carrier i for delivering, so that both x_{ijt} and $\hat{d}_{jt}$ are increased by the number of copies of (j, t) in A.
    else if σ is *assignment*(i, k) **then**
        Let (j, t) denote the undelivered cargo (j, t) that minimizes the transportation cost $(p_{ijt} - s_{jt})$ among all $j \in J_{ik}$ and $1 \le t \le T$ with $\hat{d}_{jt} < d_{jt}$.
        Assign the cargo (j, t) to carrier i for delivering, so that both x_{ijt} and $\hat{d}_{jt}$ are increased by $\min\{c_{ik} - \sum_{j \in J_{ik}} \sum_{t=1}^{T} x_{ijt}, (d_{jt} - \hat{d}_{jt})\}$
    end if
end while
Return $(\mathbf{x}, \mathbf{y})$ as an approximation solution.

When $T = |B| = 1$, $|J_i| = 1$ for $i \in I$, $c_{i1} = \infty$ for $i \in I$, and $q_{i1} = b$ for $i \in I$, the above problem with the MQC constraint is equivalent to a facility location problem, where the number of customers assigned to an open facility cannot be less than b. In addition to the above b-approximation algorithm, two bi-criteria algorithms are known in the literature (Guha et al. 2000; Karget and Minkoff 2000) which achieve constant approximation ratios with regards to the optimal total cost, but which violate the lower bound constraint by a constant factor. Svitkina (2010) has recently developed a constant approximation algorithm for this problem, by transforming it to a *capacity facility location problem*, where the number of customers assigned to an open facility cannot exceed a given capacity, and for which several constant-factor approximation algorithms are known. Its approximation ratio has further been improved by Ahmadian and Swamy (2013).

For the problem with the MARC constraint, a linear programming relaxation heuristic (LP heuristic) is known to have a good worst-case performance (Lim et al. 2008a). The basic idea of the algorithm is to use a fractional solution, obtained from the linear programming relaxation of the problem, so as to decompose the problem into a number of sub-problems such that each sub-problem consists of only a single carrier. Consider a special case of the problem that consists only of a single carrier i, and for each $1 \le k \le |J_i|$, we still use $v_{ik} := \sum_{j \in J_{ik}} \sum_{t=1}^{T} x_{ijt}$ to represent the total volume of cargo assigned to carrier i for lanes in J_{ik} and for all the T periods. The problem for carrier i and lanes in J_{ik}, denoted by $\mathbf{IP}_{ik}$ $(\mathbf{d}, \xi)$, can be formulated as follows:

$$\mathrm{IP}_{ik}(d,\xi) = \max_{0 \le v_{ik} \le c_{ik}} \mathbf{IP_{ik}}(\mathbf{d},\xi,v_{ik}) \tag{13.83}$$

where

$$\mathbf{IP}_{ik}(\mathbf{d},\xi,v_{ik}) = \max \sum_{j \in J_{ik}} \sum_{t=1}^{T} (s_{jt} - p_{ijt})x_{ijt} \tag{13.84}$$

$$\text{s.t.}\, x_{itj} \le d_{jt}, \forall j \in J_{ik}, 1 \le t \le T, \tag{13.85}$$

$$x_{ijt} \in Z_+, \forall j \in J_{ik}, 1 \le t \le T, \tag{13.86}$$

$$\sum_{j \in J_{ik}} x_{ijt} \le \alpha_{ikt} v_{ik} + \xi, \forall 1 \le t \le T. \tag{13.87}$$

As shown earlier in Sect. 3, the model $\mathbf{IP}_{ik}$ $(\mathbf{d}, \xi, v_{ik})$ can be solved in polynomial time by a standard network flow algorithm or a simplified greedy algorithm.

Consider the fractional optimal solution $\hat{\mathbf{x}}$ to the linear programming relaxation of model (MARC IP), denoted by $\mathbf{LP}(\mathbf{d}, \xi)$. From $\hat{\mathbf{x}}$ we can construct an instance of the single carrier problem for each carrier $i \in I$ and lanes in J_{ik}, denoted by $\mathbf{IP}_{ik}$ $(\mathbf{d}^{(\mathbf{i})}, \xi)$, where $d_{jt}^{(i)} := \hat{x}_{ijt}$ for $j \in J_{ik}$ and $1 \le t \le T$. Let $\mathbf{x}$ denote the union of the obtained solutions to sub-problems $\mathbf{IP}_{ik}$ $(\mathbf{d}^{(\mathbf{i})}, \xi)$ for $i \in I$ and $1 \le k \le |J_i|$. Since $\sum_{i \in I} \hat{x}_{ijt} \le d_{jt}$, it can be seen that $\mathbf{x}$ is feasible to (MARC IP). Moreover, Lim et al. (2008a) shows that such an LP based heuristic guarantees a worst case approximation ratio of $(\frac{\xi-1}{\xi})(\frac{\xi-1}{\xi-1+\sigma})(\frac{\tau-1}{\tau})$, where parameters σ and τ are defined as follows, and which depend on the instance of the problem:

$$\sigma = 1 + \max\left\{|J_{ik}|\,\alpha_{ikj} T : i \in I, 1 \le k \le |J_i|, 1 \le t \le T\right\}, \tag{13.88}$$

$$\tau = \min \begin{cases} \dfrac{c_i}{|J_{ik}|T}, & \text{for } i \in I, 1 \le k \le |J_i|, \\ \dfrac{\xi}{|J_{ik}|}, & \text{for } i \in I, 1 \le k \le |J_i|, \\ \dfrac{d_{jt}}{|I|}, & \text{for } j \in J_{ik}, 1 \le t \le T, \text{and existing } i \text{ with } s_{jt} - p_{ijt} > 0. \end{cases} \tag{13.89}$$

It can be seen that such an approximation ratio is close to one when σ is small, and ξ or τ is large, which are often true in practice.

As for the problem with constraints on the number of selected carriers, this contains the k-median problem as a special case, for which a number of constant approximation algorithms are known (Arya et al. 2001; Li and Svensson 2013; Vazirani 2001). However, it still remains unknown as to whether or not these approximation algorithms can guarantee constant approximation algorithms for more general cases with BG containing multiple business units.

To develop heuristic algorithms for more general cases of the TSPP, we next introduce two approaches as follows, one based on rounding of the fractional solutions, and the other based on neighborhood search.

For the problems that can be formulated as integer programming models, we can first solve its linear programming relaxation and obtain a fractional solution denoted by $(\hat{\mathbf{x}}, \mathbf{y})$. If $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ contains only integer values, then a feasible solution is obtained. Otherwise, we can select one or more variables that have fractional values, and round them to integers. Fixing the values of these variables, we can obtain an integer programming model with a smaller scale, and its linear programming relaxation can be solved for the next iteration of rounding. As shown in Algorithm 3, this process is iterated until we obtain a feasible solution.

Algorithm 3 Linear Programming Rounding Heuristic

1: Let X indicate the list of x_{ibjt} whose values have been decided. Let Y indicate the list of y_{ibk} whose values have been decided. Set X and Y to initially be an empty set.
2: Solve a linear programming relaxation of the problem for the given values of variables in X and Y. Denote the obtained fractional solution by $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$.
3: **if** no variables in $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ are fractional **then**
4:     Return $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$.
5: **else**
6:     Select x_{ibjt} or y_{ibk} having a fractional value, round it to an integer, and add it to X or Y.
7:     Go to Step 2.
8: **end if**

For different specific problems or heuristics, we can use different ways to select and round variables that have fractional values in Algorithm 3. For example, the linear programming rounding heuristic has been applied to solving the model (MQC IP) (Lim et al. 2006). Since this model is equivalent to a min-cost network flow problem when $\mathbf{y}$ is given, the heuristic always selects y_i of the largest fractional value and rounds it to one. The numerical experiments have shown that such a linear programming rounding method outperforms some other heuristic methods.

For a problem that can be efficiently solved when values of $\mathbf{y}$ are given, we can explore near-optimal heuristic solutions by a neighborhood search approach.

For every feasible selection $\mathbf{y}$ of carriers, let $N(\mathbf{y})$ denote a subset of selections other than $\mathbf{y}$, defined as the neighborhood of $\mathbf{y}$. The neighborhood search approach iteratively moves the current feasible selection $\mathbf{y}$ to another feasible selection from its neighborhood $N(\mathbf{y})$, and returns the best feasible solution obtained after several iterations. We can summarize this approach as follows:

Algorithm 4 Neighborhood Search Heuristic

1: Let $(\mathbf{x}^*, \mathbf{y}^*)$ indicate the best feasible solution obtained.
2: **while** Stop condition is not satisfied **do**
3: Choose a feasible selection $\mathbf{y}'$ from $N(\mathbf{y})$.
4: Compute $\mathbf{x}'$ by solving (**General MP)(** $\mathbf{y}'$).
5: **if** $(\mathbf{x}', \mathbf{y}')$ is better than $(\mathbf{x}^*, \mathbf{y}^*)$ **then**
6: Set $(\mathbf{x}^*, \mathbf{y}^*)$ to be $(\mathbf{x}', \mathbf{y}')$.
7: **end if**
8: If certain conditions are satisfied, then update $\mathbf{y}$ by $\mathbf{y}'$.
9: **end while**

For different specific problems or heuristics, we can adopt different neighbor-hoods, different ways of choosing new feasible selections, and different conditions to update the current feasible selection. For the problem with fairness constraints, the neighborhood search heuristic has been applied (Lim et al. 2012), where the neighborhood $N(\mathbf{y})$ is defined by two operators on $\mathbf{y}$, including one that removes a selected carrier and another that inserts a new carrier. The heuristic algorithm chooses a new feasible selection from $N(\mathbf{y})$ by random picking, and updates the current selection $\mathbf{y}$ only when the new selection produces a better feasible solution. In order to avoid trapping in local optimal solutions, Lim et al. (2012) has further extended this neighborhood search heuristic to a Tabu search algorithm, so that certain selections that have been made in the neighborhood will be forbidden for several iterations. Numerical results have shown that this randomized Tabu search algorithm significantly outperforms commercial optimization solvers.

13.7 Future Research Directions

In this chapter, we have introduced a general optimization model for the transportation service procurement problem (TSPP), and have reviewed various existing solution methods for different variants and extensions of it. We have discussed existing results and developed some new results for the problem, including its tractability, relaxations, exact algorithms, approximation algorithms, and heuristic algorithms. From this, there are several themes that can be identified for future research.

When choosing carriers during the procurement process, shippers are concerned not only about the shipping cost but also about the shipping time, since a slow or unreliable shipping time may increase the shipper's production cost, or increase its risk of losing sales (Lu et al. 2014). Therefore, it would be of interest to also take into account transit times, and to factor this into the optimization model for the selection of carriers. However, as this may require making joint decisions with regard to transportation service procurement, production, and sales, the problem could be challenging.

In the existing literature on transportation service procurement, models and solution methods are mainly based on deterministic settings. However, due to the high volatility of the shipping market, there are significant uncertainties involved with both spot-market shipping rates and the actual shipping demands, and these can be taken into account in the future research. This will no doubt present new challenges when one develop optimization models and solution methods for this problem.

The volatility of the shipping market presents challenges not only for optimizing decision making over transportation service procurement, but also in coordinating the carriers and shippers. In practice, carriers are sometimes reluctant to purchase transportation services from carriers in advance, since they are concerned that the spot market price may suddenly drop. Thus, it would be interesting to design and study various forms of contracts that can facilitate the business between carriers and shippers, such as contracts that enable the sharing of risks and costs (An et al. 2014; Lee et al. 2014).

Furthermore, it would also be interesting to study the design of mechanisms for transportation service procurement. Since most problems faced in practice concern computational intractability, only heuristic algorithms are available for them. Therefore, Vickrey-based payment rules (Clarke 1971; Groves 1973; Vickrey 1961) are often not able to guarantee positive revenue for the shipper (Conitzer and Sandholm 2006; Parkes et al. 2001). Thus, in future work we can investigate effectiveness of different payment rules for situations that use heuristic algorithms (instead of exact algorithms) for the shipper to determine cargo allocations (Huang and Xu 2013; Xu and Huang 2013; Xu and Huang 2014).

Other interesting research directions may include the development of efficient solution methods for the general optimization model for the TSPP, as well as for those problems with more complicated cost structures, such as discounts that carriers may offer for multiple tiers of shipping volume (Qin et al. 2012).

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