On the Topological Structure of Rough Soft Sets

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Abstract. The concept of rough soft set is introduced to generalize soft sets by using rough set theory, and then the soft topologies on soft sets are introduced.

Keywords: Soft approximation space, Soft topology, Soft relation, Soft closure operator.

1 Introduction

The soft set theory introduced by Molodtsov [18], which is assumed as a mathematical tool for dealing with uncertainties, has been developed significantly with a number of applications such as it can be applied in game theory, Riemann integration, probability theory, etc. (cf. [19]). It has also been seen that the mathematical objects such as topological spaces, fuzzy sets and rough sets can be considered as a particular types of soft sets (cf., [16,18]). Recently, so many authors have tried to develop the mathematical concepts based on soft set theory, e.g., in [2,6,27,29], rough soft sets and fuzzy soft sets ; in [8], Soft rough fuzzy sets and soft fuzzy rough sets; in [10], the algebraic structure of semi-rings by applying soft set theory; in [3], fuzzy soft group; in [13], soft BCK/BCI-algebras; in [14], the applications of soft sets in ideal theory of BCK/BCI-algebras; in [5,28,1], soft set relations and functions; in [4,7,12,25], soft topology, which itself is showing the interest of researchers in this area.

Beside soft set theory, rough set theory, firstly proposed by Pawlak [20] has now been developed significantly due to its importance for the study of intelligent systems having insufficient and incomplete information. In rough set introduced by Pawlak, the key role is played by equivalence relations. In literature (cf., [15,20,21,23], several generalizations of rough set have been made by replacing the equivalence relation by an arbitrary relation. Simultaneously, the relation of rough set with topology is also studied (cf., [15,24]).

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As both the theories approaches to vagueness, it will be interesting to see the connection between both the theories. In this direction, an initiation has already been made (cf., [11,26]), in which, soft set theory is utilized to generalize the rough set model introduced by Pawlak (cf., [20]). Also, the resultant hybrid model has been applied to multicriteria group decision making (cf., [9]). It is the natural question that what will happen if rough set theory is used to generalize soft sets. This paper is toward this study. Specifically, we try to introduce the concept of rough soft set, and as topology is closely related to rough sets, we try to introduce soft topologies on soft sets with the help of rough soft sets.

2 Preliminaries

In this section, we collect some concepts associated with soft sets, which we will use in the next section. Throughout, U denotes an universal set and E, the set of all possible parameters with respect to U. The family of all subsets of U is denoted by $P(U)$.

Definition 1. [18] *A pair $F_A = (F, A)$ is called a* ***soft set*** *over U, where $A \subseteq E$ and $F : A \to P(U)$ is a map.*

In other words, a soft set F_A over U is a parameterized family $\{F(a) : a \in A\}$ of subsets of the universe U. For $\epsilon \in A$, $F(\epsilon)$ may be considered as the set of ϵ-appximate elements of the soft set F_A.

For the universe U, $S(U)$ will denote the class of all soft sets over U.

Definition 2. [22] *Let $A, B \subseteq E$ and $F_A, G_B \in S(U)$. Then F_A is* ***soft subset*** *of G_B, denoted by $F_A \subseteq G_B$, if*

(i) *$A \subseteq B$, and*
(ii) *$\forall a \in A$, $F(a) \subseteq G(a)$.*

Definition 3. [16] *F_A and G_B are said to be* ***soft equal*** *if $F_A \subseteq G_B$ and $G_B \subseteq F_A$. For a soft set $F_A \in S(U)$, $\widetilde{P}(F_A)$ denotes the set of all soft subsets of F_A.*

Definition 4. [17] *Let $A \subseteq E$ and $F_A \in S(U)$. Then F_A is called* ***soft empty****, denoted by F_ϕ, if $F(a) = \phi$, $\forall a \in A$.*

$F(a) = \phi$, $\forall a \in A$ means that there is no element in U related to the parameter $a \in A$. Therefore, there is no need to display such elements in the soft sets, as it is meaningless to consider such parameters.

Definition 5. [17] *Let $A, B \subseteq E$ and $F_A, G_B \in S(U)$. Then the* ***soft union*** *of F_A and G_B is a soft set $H_C = (H, C)$, where $C = A \cup B$ and $H : C \to P(U)$ such that $\forall a \in C$,*

$$H(a) = \begin{cases} F(a) & \text{if } a \in A - B \\ G(a) & \text{if } a \in B - A \\ F(a) \cup G(a) & \text{if } a \in A \cap B \end{cases}$$

Definition 6. [17] *Let $A, B \subseteq E$ and $F_A, G_B \in S(U)$. Then the **soft intersection** of F_A and G_B is a soft set $H_C = (H, C)$, where $C = A \cap B$ and $H : C \to P(U)$ such that $H(a) = F(a) \cap G(a)$, $\forall a \in C$.*

Definition 7. [17] *Let $E = \{e_1, e_2, e_3, ...e_n\}$ be a set of parameters. Then the **NOT** set of E is $\rceil E$ is defined by $\rceil E = \{\rceil e_1, \rceil e_2, \rceil e_3, ...\rceil e_n\}$, where $\rceil e = not\ e_i$, $\forall i = 1, 2, ..., n$.*

Definition 8. [1] *Let $A \subseteq E$ and $F_A \in S(U)$. Then the **soft complement** of F_A is $(F_A)^c$ and defined by $(F_A)^c = F_A^c$, where $F^c : A \to P(U)$ is a map such that $F^c(a) = U - F(a)$, $\forall a \in A$.*

We call F^c, the soft complement function of F. It is easy to see that $(F^c)^c = F$ and $(F_A^c)^c = F_A$. Also, $F_\phi^c = F_E$ and $F_E^c = F_\phi$.

Proposition 1. [17] *Let $F_A \in S(U)$. Then*

(i) $F_A \cup F_A = F_A$, $F_A \cap F_A = F_A$
(ii) $F_A \cup F_\phi = F_A$, $F_A \cap F_\phi = F_\phi$
(iii) $F_A \cup F_E = F_E$, $F_A \cap F_E = F_A$
(iv) $F_A \cup F_A^c = F_E$, $F_A \cap F_A^c = F_\phi$.

Definition 9. [5] *Let $A, B \subseteq E$ and $F_A, G_B \in S(U)$. Then the **cartesian product** of F_A and G_B is the soft set $H_{A\times B} = (H, A \times B)$, where $H_{A\times B} = F_A \times G_B$ and $H : A \times B \to P(U \times U)$ such that $H(a, b) = F(a) \times G(b)$, $\forall (a, b) \in A \times B$, i.e., $H(a, b) = \{(h_i, h_j) : h_i \in F(a)$ and $h_j \in G(b)\}$.*

Definition 10. [5] *Let $A, B \subseteq E$ and $F_A, G_B \in S(U)$. Then a **soft relation** from F_A to G_B is a soft subset of $F_A \times G_B$.*

In other words, a soft relation from F_A to G_B is of the form H'_C, where $C \subseteq A\times B$ and $H'(a, b) = H(a, b)$, $\forall (a, b) \in C$, and $H_{A\times B} = F_A \times G_B$ as defined in Definition 9. Any subset of $F_A \times F_A$ is called a **soft relation** on F_A.

In an equivalent way, the soft relation R on the soft set F_A in the parameterized form is as follows:

If $F_A = \{F(a_1), F(a_2), ...\}$, $a_1, a_2, ... \in A$, then ${}_{F(a_i)}R_{F(a_j)} \Leftrightarrow F(a_i)\times F(a_j) \in R$.

Definition 11. [5] *A soft relation R on a soft set $F_A \in S(U)$ is called*

(i) ***soft reflexive*** *if $H'(a, a) \in R$, $\forall a \in A$,*
(ii) ***soft symmetric*** *if $H'(a, b) \in R \Rightarrow H'(b, a) \in R$, $\forall (a, b) \in A \times A$, and*
(iii) ***soft transitive*** *if $H'(a, b) \in R$, $H'(b, c) \in R \Rightarrow H'(a, c) \in R$, $\forall a, b, c \in A$.*

Above definition can be restated as follows:

Definition 12. [28] *A soft relation R on a soft set $F_A \in S(U)$ is called*

(i) ***soft reflexive*** *if $F(a) \times F(a) \in R$, $\forall a \in A$,*

(ii) ***soft symmetric*** *if* $F(a) \times F(b) \in R \Rightarrow F(b) \times F(a) \in R$, $\forall (a, b) \in A \times A$, *and*

(iii) ***soft transitive*** *if* $F(a) \times F(b) \in R$, $F(b) \times F(c) \in R \Rightarrow F(a) \times F(c) \in R$, $\forall a, b, c \in A$.

Definition 13. [5] *Let* $A \subseteq E$ *and* $F_A \in S(U)$*. Then* $[F(a)] = \{F(a') : F(a) \times F(a') \in R, \forall a, a' \in A\}$.

Remark 1. For $A \subseteq E$ and $F_A \in S(U)$, it can be seen that $[F(a)] = (F, A_a), a \in A$ is a soft subset of F_A, where $A_a = \{a' \in A : F(a) \times F(a') \in R\}$.

Definition 14. [7] *Let* $F_A \in S(U)$ *and* $\tau \subseteq \widetilde{P}(F_A)$*. Then* τ *is called a* ***soft topology*** *on* F_A *if*

(i) F_ϕ, $F_A \in \tau$,

(ii) *for* $F_{A_i} \in \widetilde{P}(F_A)$, $i \in I$, *if* $F_{A_i} \in \tau$, *then* $\cup_{i \in I} F_{A_i} \in \tau$, *and*

(iii) *for* $F_{A_1}, F_{A_2} \in \widetilde{P}(F_A)$, *if* $F_{A_1}, F_{A_2} \in \tau$, *then* $F_{A_1} \cap F_{A_2} \in \tau$.

The pair (F_A, τ) is called **soft topological space** and soft subsets of F_A in τ are called **soft open** set. The compliment of a soft open set is called a **soft closed** set.

3 Rough Soft Set and Soft Topology

In this section, we introduce the concept of rough soft set and introduce soft topologies on soft sets. Throughout this section, F_A is a soft set over U.

Definition 15. *A pair* (F_A, R) *is called a* ***soft approximation space****, where* $F_A \in S(U)$ *and* R *is a soft relation on* F_A.

Definition 16. *Let* (F_A, R) *be a soft approximation space. Then* ***soft lower approximation*** *and* ***soft upper approximation*** *of* $G_B \subseteq F_A$*, are respectively, defined as:*

$$\underline{apr}(G_B) = \cup_{a \in A}\{F(a) \in F_A : [F(a)] \subseteq G_B\}, \text{ and}$$
$$\overline{apr}(G_B) = \cup_{a \in A}\{F(a) \in F_A : [F(a)] \cap G_B \neq F_\phi\}.$$

The pair $(\underline{apr}(G_B), \overline{apr}(G_B))$ *is called a* **rough soft set**.

Remark 2. From above definition, it is clear that $\underline{apr}(G_B)$ and $\overline{apr}(G_B)$ are soft subsets of F_A.

Example 1. Let $U = \{u_1, u_2, u_3\}, E = \{x_1, x_2, x_3\}, A = \{x_1, x_2\}$, $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\})\}$ and $G_B \subseteq F_A$, where $G_B = \{(x_2, \{u_2, u_3\})\}$. Also, consider a soft relation $R = \{F(x_1) \times F(x_1), F(x_1) \times F(x_2), F(x_2) \times F(x_2)\}$. Then $[F(x_1)] = \{F(x_1), F(x_2)\}$ and $[F(x_2)] = \{F(x_2)\}$. It can be easily seen that $\underline{apr}(G_B) = G_B$ and $\overline{apr}(G_B) = F_A$.

Proposition 2. *For a soft approximation space* (F_A, R) *and* $\forall\ G_B, H_C \subseteq F_A$,

(i) $\underline{apr}(F_\phi) = F_\phi = \overline{apr}(F_\phi)$;
(ii) $\underline{apr}(F_A) = F_A = \overline{apr}(F_A)$;
(iii) *If* $G_B \subseteq H_C$, *then* $\underline{apr}(G_B) \subseteq \underline{apr}(H_C)$ *and* $\overline{apr}(G_B) \subseteq \overline{apr}(H_C)$;
(iv) $\underline{apr}(G_B) = (\overline{apr}(G_B^c))^c$;
(v) $\overline{apr}(G_B) = (\underline{apr}(G_B^c))^c$;
(vi) $\underline{apr}(G_B \cap H_C) = \underline{apr}(G_B) \cap \underline{apr}(H_C)$;
(vii) $\underline{apr}(G_B) \cup \underline{apr}(H_C) \subseteq \underline{apr}(G_B \cup H_C)$;
(viii) $\overline{apr}(G_B \cup H_C) = \overline{apr}(G_B) \cup \overline{apr}(H_C)$;
(ix) $\overline{apr}(G_B \cap H_C) \subseteq \overline{apr}(G_B) \cap \overline{apr}(H_C)$

Proof (i) and (ii) are obvious.

(iii) Let $G_B \subseteq H_C$ and $F(a) \in \underline{apr}(G_B)$, $a \in A$. Then $[F(a)] \subseteq G_B$, and so $[F(a)] \subseteq H_C$. Thus $F(a) \in \underline{apr}(H_C)$, whereby $\underline{apr}(G_B) \subseteq \underline{apr}(H_C)$. Similarly, we can show that $\overline{apr}(G_B) \subseteq \overline{apr}(H_C)$.

(iv) $F(a) \in (\overline{apr}(G_B^c))^c \Leftrightarrow F(a) \notin (\overline{apr}(G_B^c)) \Leftrightarrow [F(a)] \cap G_B^c = F_\phi \Leftrightarrow [F(a)] \subseteq G(B) \Leftrightarrow F(a) \in G(B)$. Thus $\underline{apr}(G_B) = (\overline{apr}(G_B^c))^c$.

(v) Similar to that of (iv).

(vi) $F(a) \in \underline{apr}(G_B \cap H_C) \Leftrightarrow [F(a)] \subseteq G_B \cap H_C \Leftrightarrow [F(a)] \subseteq G_B$ and $[F(a)] \subseteq H_C \Leftrightarrow F(a) \in \underline{apr}(G_B)$ and $F(a) \in \underline{apr}(H_C) \Leftrightarrow F(a) \in \underline{apr}(G_B) \cap \underline{apr}(H_C)$. Thus $\underline{apr}(G_B \cap H_C) = \underline{apr}(G_B) \cap \underline{apr}(H_C)$.

(vii) Follows as above.

(viii) $F(a) \in \overline{apr}(G_B \cup H_C) \Leftrightarrow [F(a)] \cap (G_B \cup H_C) \neq F_\phi \Leftrightarrow [F(a)] \cap G_B \neq F_\phi$ or $[F(a)] \cap H_C \neq F_\phi \Leftrightarrow F(a) \in \overline{apr}(G_B)$ or $F(a) \in \overline{apr}(H_C) \Leftrightarrow F(a) \in \overline{apr}(G_B) \cup \overline{apr}(H_C)$. Thus $\overline{apr}(G_B \cup H_C) = \overline{apr}(G_B) \cup \overline{apr}(H_C)$.

(ix) Follows as above.

Following example support each of proposition (i) to (ix).

Example 2. Let $U = \{u_1, u_2\}, E = \{x_1, x_2, x_3\}, A = \{x_1, x_2\}$. Also, let $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$ and $F_A^i, i \in I$ denotes soft subsets of F_A. Then all soft subsets of F_A are
$F_A^1 = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\}$,
$F_A^2 = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$,
$F_A^3 = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}$,
$F_A^4 = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}$,
$F_A^5 = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}$,
$F_A^6 = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$,
$F_A^7 = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$,
$F_A^8 = \{(x_1, \{u_2\}), (x_2, \{u_1\})\}$,

$F_A^9 = \{(x_1, \{u_1, u_2\})\}$,
$F_A^{10} = \{(x_2, \{u_1, u_2\})\}$,
$F_A^{11} = \{(x_1, \{u_1\})\}$,
$F_A^{12} = \{(x_1, \{u_2\})\}$,
$F_A^{13} = \{(x_2, \{u_1\})\}$,
$F_A^{14} = \{(x_2, \{u_2\})\}$,
$F_A^{15} = F_\emptyset$,
$F_A^{16} = F_A$.
Let $R = \{F(x_1) \times F(x_1), F(x_2) \times F(x_2), F(x_1) \times F(x_2)\}$.
By definition 13 and 16 it follows that
$[F(x_1)] = \{F(x_1), F(x_2)\}, [F(x_2)] = \{F(x_2)\}$ and
$\underline{apr}(F_A^i : i = 3, 4, 10) = \{F(x_2)\}, \underline{apr}(F_A) = F_A$,
$\underline{apr}(F_A^i : i = 1, 2, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15) = F_\emptyset$, also
$\overline{apr}(F_A^i : i = 1, 2, 3, 4, 5, 6, 7, 8, 10, 13, 14, 16) = F_A$,
$\overline{apr}(F_A^i : i = 9, 11, 12) = \{F(x_1)\}$ and $\overline{apr}(F_A^{15}) = F_\emptyset$.

Proposition 3. *Let (F_A, R) be a soft approximation space and R be soft reflexive. Then $\forall G_B \subseteq F_A$,*

(i) $G_B \subseteq \overline{apr}(G_B)$, *and*
(ii) $\underline{apr}(G_B) \subseteq G_B$.

Proof Follows easily from the fact that R is reflexive.

Example 3. In Example 2, let $R = \{F(x_1) \times F(x_1), F(x_2) \times F(x_2)\}$. Then $[F(x_1)] = \{F(x_1)\}, [F(x_2)] = \{F(x_2)\}$. Thus $\underline{apr}(F_A^i : i = 1, 2, 9) = \{F(x_1)\}$, $\underline{apr}(F_A^i : i = 3, 4, 10) = \{F(x_2)\}$, $\overline{apr} = F_A$, $\underline{apr}(F_A^i : i = 5, 6, 7, 8, 11, 12, 13, 14, 15) = F_\emptyset$, $\overline{apr}(F_A^i : i = 1, 2, 3, 4, 5, 6, 7, 8, 16) = F_A$, $\overline{apr}(F_A^i : i = 9, 11, 12) = \{F(x_1)\}$, $\overline{apr}(F_A^i : i = 10, 13, 14) = \{F(x_2)\}$, $\overline{apr}(F_A^{15}) = F_\emptyset$. Clearly, $F_A^i \subseteq \overline{apr}(F_A^i) \forall i = 1, ...16$, and $\underline{apr}(F_A^i) \subseteq F_A^i, \forall i = 1, ...16$.

Proposition 4. *Let (F_A, R) be a soft approximation space and R be soft symmetric. Then $\forall\ G_B, H_C \subseteq F_A$,*

(i) $\overline{apr}(\underline{apr}(G_B)) \subseteq G_B$, *and*
(ii) $G_B \subseteq \underline{apr}(\overline{apr}(G_B))$.

Proof (i) Let $F(a) \in \overline{apr}(\underline{apr}(G_B))$, $a \in A$. Then $[F(a)] \cap \underline{apr}(G_B) \neq F_\phi$, or that, there exists $F(a') \in [F(a)], a' \in A$ such that $F(a') \in \underline{apr}(G_B)$. $F(a') \in \underline{apr}(G_B)$, implying that $[F(a')] \subseteq G_B$. Since R is symmetric and $F(a) \times F(a') \in R$, so $F(a') \times F(a) \in R$. Thus $F(a) \in [F(a')]$, and so $F(a) \in G_B$. Hence $\overline{apr}(\underline{apr}(G_B)) \subseteq G_B$.

(ii) Follows as above.

Proposition 5. *Let (F_A, R) be a soft approximation space and R be soft transitive. Then $\forall\ G_B \subseteq F_A$,*

(i) $\overline{apr}(\overline{apr}(G_B)) \subseteq \overline{apr}(G_B)$, *and*
(ii) $\underline{apr}(G_B) \subseteq \underline{apr}(\underline{apr}(G_B))$.

Proof (i) Let $F(a) \in \overline{apr}(\overline{apr}(G_B))$, $a \in A$. Then $[F(a)] \cap \overline{apr}(G_B) \neq F_\phi$, i.e., there exists $F(a') \in [F(a)], a' \in A$ such that $F(a') \in \overline{apr}(G_B)$. Now, $F(a') \in \overline{apr}(G_B) \Rightarrow [F(a')] \cap G_B \neq F_\phi$, i.e., there exists $F(a'') \in [F(a')], a'' \in A$ such that $F(a'') \in G_B$. But R being soft transitive, $F(a') \in [F(a)]$ and $F(a'') \in [F(a')]$ implying that $F(a'') \in [F(a)]$. Thus $[F(a)] \cap G_B \neq F_\phi$, whereby $F(a) \in \overline{apr}(G_B)$. Hence $\overline{apr}(\overline{apr}(G_B)) \subseteq \overline{apr}(G_B)$.

(ii) Follows as above.

Proposition 6. *If a soft relation R on F_A is soft reflexive. Then $\tau = \{G_B \subseteq F_A : \underline{apr}(G_B) = G_B\}$ is a soft topology on F_A.*

Proof In view of Proposition 2, we only need to show that if $G_{B_i} \in \tau$, then $\cup_{i \in I} G_{B_i} \in \tau$, where $G_{B_i} \in \widetilde{P}(F_A)$, $i \in I$. For which, it is sufficient to show that $\cup_{i \in I} G_{B_i} \subseteq \underline{apr}(\cup_{i \in I} G_{B_i})$. Let $F(a) \in \cup_{i \in I} G_{B_i}, a \in A$. Then their exists some $j \in J$ such that $F(a) \in G_{B_j} = \underline{apr}(G_{B_j})$, i.e., $[F(a)] \subseteq G_{B_j} \subseteq \cup_{i \in I} G_{B_i}$, or that $F(a) \in \underline{apr}(\cup_{i \in I} G_{B_i})$. Thus $\cup_{i \in I} G_{B_i} \subseteq \underline{apr}(\cup_{i \in I} G_{B_i})$, whereby $\cup_{i \in I} G_{B_i} \subseteq \underline{apr}(\cup_{i \in I} G_{B_i})$. Hence τ is a soft topology on F_A.

Proposition 7. *Let R be soft reflexive and soft symmetric. Then $\underline{apr}(G_B) = G_B$ if and only if $G_B^c = \underline{apr}(G_B^c)$.*

Proof Let $\underline{apr}(G_B) = G_B$. As, $\underline{apr}(G_B^c) \subseteq G_B^c$, we only need to show that $G_B^c \subseteq \underline{apr}(G_B^c)$. For this, let $F(a) \notin \underline{apr}(G_B^c)$, $a \in A$. Then $\exists F(b) \in F_A$ such that $F(b) \in [F(a)]$ and $F(b) \notin ((G_B)^c)$, or that, $F(b) \in G_B = \underline{apr}(G_B)$ and $F(b) \in [F(a)]$. Now, R being soft symmetric and $F(b) \in [F(a)]$ so $F(a) \in [F(b)]$. Also, $F(b) \in \underline{apr}(G_B) \Rightarrow [F(b)] \subseteq G_B$. Thus $F(a) \in G_B$, or that $F(a) \notin ((G_B)^c)$, whereby $G_B^c = \underline{apr}(G_B^c)$. The converse part can be proved similarly.

Following is an easy consequence of the above proposition.

Proposition 8. *Let R be soft reflexive and soft symmetric relation on F_A. Then (F_A, τ) is the soft topological space having the property that G_B is soft open if and only if G_B is soft closed.*

Proof As R is soft reflexive, from Proposition 6, τ is a topology on a F_A. Also, G_B is soft open if and only if $G_B \in \tau$ if and only if $\underline{apr}(G_B) = G_B$ if and only if $\underline{apr}(G_B))^c = (G_B)^c$ if and only if $(G_B)^c \in \tau$ if and only if $(G_B)^c$ is open if and only if G_B is soft closed.

Now, we introduce the following concept of soft closure and soft interior operator on a soft set.

Definition 17. *A mapping $\widetilde{c} : \widetilde{P}(F_A) \to \widetilde{P}(F_A)$ is called a* ***soft closure operator*** *if $\forall G_B, G_{B_1}, G_{B_2} \in \widetilde{P}(F_A)$,*

(i) $\widetilde{c}(F_\phi) = F_\phi$,

(ii) $G_B \subseteq \widetilde{c}(G_B)$,
(iii) $\widetilde{c}(G_{B_1} \cup G_{B_2}) = \widetilde{c}(G_{B_1}) \cup \widetilde{c}(G_{B_2})$,
(iv) $\widetilde{c}(\widetilde{c}(G_B)) = \widetilde{c}(G_B)$.

Remark 3. Let $\tau = \{G_B \subseteq F_A : \widetilde{c}(G_B^c) = G_B^c\}$. Then it can be seen that τ is a soft topology on F_A.

Definition 18. *A mapping $\widetilde{i} : \widetilde{P}(F_A) \to \widetilde{P}(F_A)$ is called a* ***soft interior operator*** *if, $\forall G_B, G_{B_1}, G_{B_2} \in \widetilde{P}(F_A)$,*

(i) $\widetilde{i}(F_A) = F_A$,
(ii) $\widetilde{i}(G_B) \subseteq G_B$,
(iii) $\widetilde{i}(G_{B_1} \cap G_{B_2}) = \widetilde{i}(G_{B_1}) \cap \widetilde{i}(G_{B_2})$,
(iv) $\widetilde{i}(\widetilde{i}(G_B)) = \widetilde{i}(G_B)$.

Remark 4. Let $\tau = \{G_B \subseteq F_A : \widetilde{i}(G_B) = G_B\}$. Then it can be seen that τ is a soft topology on F_A.

Proposition 9. *If a soft relation R on F_A is soft reflexive and soft transitive, then $\underline{apr}$ and $\overline{apr}$ are saturated[1] soft interior and saturated soft closure operators respectively.*

Proof Follows from Propositions 2, 3 and 5.

Finally, we show that each saturated soft closure operator on a soft set also induces a soft reflexive and soft transitive relation as:

Proposition 10. *Let $\widetilde{c}$ be a saturated soft closure operator on F_A. Then there exists an unique soft reflexive and soft transitive relation R on F_A such that $\widetilde{c}(G_B) = \overline{apr}(G_B), \forall G_B \subseteq F_A$.*

Proof Let $\widetilde{c}$ be a saturated soft closure operator and R be a soft relation on F_A given by $F(a) \times F(a') \in R \Leftrightarrow F(a) \in \widetilde{c}(\{F(a')\}), a, a' \in A$. As, $\{F(a)\} \subseteq \widetilde{c}(\{F(a)\})$, $F(a) \in \widetilde{c}(\{F(a)\})$, or that, $F(a) \times F(a) \in R$. Thus R is a soft reflexive relation on F_A. Also, let $F(a) \times F(a') \in R$ and $F(a') \times F(a'') \in R; a, a', a'' \in A$. Then $F(a) \in \widetilde{c}(\{F(a')\})$ and $F(a') \in \widetilde{c}(\{F(a'')\})$. Thus $F(a) \in \widetilde{c}(\{F(a')\})$ and $\widetilde{c}(\{F(a')\}) \subseteq \widetilde{c}(\widetilde{c}(\{F(a'')\})) = \widetilde{c}(\{F(a'')\})$, or that, $F(a) \in \widetilde{c}(\{F(a'')\})$, i.e., $F(a) \times F(a'') \in R$. Therefore R is a soft transitive relation on F_A. Now, let $G_B \subseteq F_A$ and $F(a) \in \overline{apr}(G_B), a \in A$. Then $[F(a)] \cap G_B \neq F_\phi$, or that, $\exists F(a') \in F_A$ such that $F(a') \in [F(a)] \cap G_B$, showing that $F(a) \in \widetilde{c}(\{F(a')\})$ and $F(a') \in G_B$. Thus $F(a) \in \widetilde{c}(G_B)$, whereby $\overline{apr}(G_B) \subseteq \widetilde{c}(G_B)$. Conversely, let $F(a) \in \widetilde{c}(G_B)$. Then $F(a) \in \widetilde{c}(\cup\{F(a') : F(a') \in G_B\}) = \cup\{\widetilde{c}(\{F(a')\}) : F(a') \in G_B\}$ (as $\widetilde{c}$ is a saturated closure operator). Now, $F(a) \in \cup\{c(\{F(a')\}) : F(a') \in G_B\} \Rightarrow F(a) \in c(\{F(a')\})$, for some $F(a') \in G_B$, or that $F(a') \in [F(a)]$, for some $F(a') \in G_B$, i.e., $[F(a)] \cap G_B \neq F_\phi$, showing that $F(a) \in \overline{apr}(G_B)$. Thus $\widetilde{c}(G_B) \subseteq \overline{apr}(G_B)$. Therefore $\widetilde{c}(G_B) = \overline{apr}(G_B)$. The uniqueness of soft relation R can be seen easily.

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[1] A soft closure operator $\widetilde{c} : \widetilde{P}(F_A) \to \widetilde{P}(F_A)$ on F_A is being called here saturated if the (usual) requirement $\widetilde{c}(G_{B_1} \cup G_{B_2}) = \widetilde{c}(G_{B_1}) \cup \widetilde{c}(G_{B_2})$ is replaced by $\widetilde{c}(\cup_{i\in I} G_{B_i}) = \cup_{i\in I} \widetilde{c}(G_{B_i})$, where $G_{B_i} \in \widetilde{P}(F_A), i \in I$.

4 Conclusion

In this paper, we tried to introduce the concept of rough soft sets by combining the theory of rough sets and that of soft sets, as well as introduce soft topologies on a soft set induced by soft lower approximation operator. As rough soft sets are generalization of soft sets with the help of rough set theory and the rough set theory has already been established much more; so this paper opens some new directions.

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