Argumentative Aggregation of Individual Opinions

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Abstract. Over a new abstract model of aggregating individual issues – *abstract debates* – we introduce an entire class of *aggregating operators* by borrowing ideas from *Abstract Argumentation* to *Social Choice Theory*. The main goal was to introduce *rational* aggregation methods which do not satisfy the commonly used *independence condition* in Social Choice Theory. This type of *context dependent aggregation* is very natural, could be useful in many real world decision making scenarios, and the present paper provides the first theoretical investigation of it.

1 Introduction

Comparing and assessing different points of view in order to obtain *fair and rational* collective aggregation of them is the main research topic of *Social Choice Theory* (SCT) [4] having major philosophical, economic, and political significance. The most important methodological tool in SCT is the axiomatic method, pioneered by Arrow [3], and consisting in formulating normatively desirable properties of aggregation rules as *postulates* or *axioms*, in order to obtain precise characterizations of the aggregation rules that satisfy these properties. The AI developments, especially in the area of collective decision making in *Multiagent Systems*, have lead to the emergence of a new research area called *Computational Social Choice* (CSC), mainly concerned with the design and analysis of collective decision making mechanisms. If in classical SCT the objects of aggregation belong to *preferential knowledge* [5], recent developments apply the same methodology to other types of information: *beliefs* [14], *judgments* [16], *ontologies* [19], *graphs* [1, 13], and *argumentation frameworks* [8, 12].

Argumentation is a powerful mechanism for automating the decision making process of autonomous agents. Several recent works have studied the problem of accommodating ideas from CSC to Argumentation [17, 22, 20, 21, 7, 12]. Most of them rely on Dung's *Argumentation Frameworks* and their acceptability semantics [10].

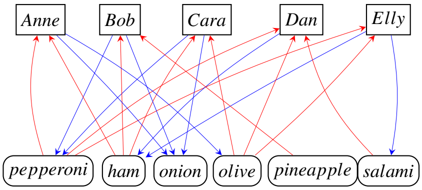
In this paper, we go beyond what we have done in [9] by borrowing ideas from Abstract Argumentation Frameworks to CSC, hence in the converse direction of the above line of research on this subject. Inspired by Dung's admissibility based semantics, we consider a new interpretation of the *collective rationality*. More precisely, we introduce a novel framework for aggregating individual *opinions* expressed as pairs of disjoint sets of positive and negative positions on a given finite set of *facts*. Each opinion having a negative position on some fact *attacks* all other opinions having positive positions on this fact. The attack digraph obtained, viewed as an argumentation framework, gives rise to rational coalition formations, whose collective opinions are viewed as aggregate opinions of the society. This represents a novel *qualitative approach* to the aggregation

E. Fermé and J. Leite (Eds.): JELIA 2014, LNAI 8761, pp. 600–608, 2014.

of individual opinions contrasting the usual quantitative voting methods.Let us consider a simple mundane choice situation. The table in Figure 1 presents the available pizza toppings, $F = \{pepperoni, salami, ham, onion, olive, pineapple\}$, and the opinions on F of a group of five friends, $S = \{Anne, Bob, Cara, Dan, Elly\}$.

	L	**DL**
Anne	$\{pepperoni, ham\}$	$\{olive, onion\}$
Bob	$\{ham, pineapple\}$	$\{pepperoni, onion\}$
Cara	$\{ham, olive\}$	$\{pepperoni, onion\}$
Dan	$\{olive, pepperoni, salami\}$	$\{ham\}$
Elly	$\{olive, pepperoni\}$	$\{ham, salami\}$
Majority	$\{ham, pepperoni, olive\}$	$\{onion\}$

(a) **Pizza Paradox**



(b) **Graph Representation**

Fig. 1. A debate and its bipartite digraph representation

As we can see, *Anne* likes (agrees) *pepperoni* and *ham* but dislikes (disagrees) *olive* and *onion*. Similarly, we can read the opinions of the other members of S. The table is entitled **Pizza Paradox** since if we consider the *majority opinion* (obtained by including each fact in one of the two sets of liked and disliked facts using the majority rule) as output, then this has the unpleasant property that **each** *individual dislikes a topping in the collective output*: $(\{ham, pepperoni, olive\}, \{onion\})$. Note that this happens despite the majority rule gives a consistent opinion, i.e. a disjoint pair of subsets of F.

The basic idea of the *argumentative aggregation of individual opinions* is to consider *collective opinions* by merging the opinions of non-conflicting coalitions of individuals. A coalition is conflict-free if the individual's opinions in the coalition does not attack each other. Such a coalition is called an *autarky* if, in addition, has the property that the collective opinion counterattacks any attack of the opinion of an individual not in coalition. This property offers a rational justification for the output opinion.
In our example, such an autarky is $\mathscr{C}_1 = \{Bob, Cara\}$ giving the output opinion $O_{\mathscr{C}_1} = (\{ham, pineapple, olive\}, \{pepperoni, onion\})$. $O_{\mathscr{C}_1}$ attacks the opinions of *Anne*, *Dan* or *Elly* (on *pepperoni*) in response of their attacks (on *olive*, or *ham*). Another autarky is $\mathscr{C}_2 = \{Elly\}$ with her vegetarian opinion $O_{\mathscr{C}_2} = (\{olive, pepperoni\}, \{ham, salami\})$.

This kind of explanatory selection of the output opinion arises in more important choice situations, where the facts could be: ethical values; drugs to be administrated to a patient; meanings of a discourse; actions, goals, propositions in political practice.

We introduce different types of autarkies, corresponding to the admissible based extensions in abstract argumentation. In fact, we show that *any argumentation framework can be viewed as a particular abstract debate.* This implies that the time complexity of the decision problems on the abstract debates is high, often beyond NP.

In the new framework, a natural way of elimination the conflicts in a coalition gives rise to *compromise autarkies* and their collective opinions enlarge the set of opinions returned by the argumentative aggregation operators. It is proved that the argumentative aggregation operators satisfy appropriate *unanimity* and *anonymity* conditions but not the analogue of Arrow's *independence condition*. This shows that the argumentative aggregation is strongly dependent on the context: *the position of the collective opinion on a fact depends not only on the positions of individual opinions on this fact, but also depends on their position on other facts.* The rest of the paper is organized as follows. The next section presents a brief description of argumentation frameworks and their semantics, as introduced by Dung [10]. It follows the main section in which we introduce opinions and their attacks, abstract debates, aggregation operators, and focus on argumentative aggregation obtained using (compromise) coalitions of individuals. The last section concludes the paper and suggests future study.

2 Dung's Theory of Argumentation

In this section we present the basic concepts used for defining classical semantics in abstract argumentation frameworks introduced by Dung in 1995, [10].

Definition 1. An *Argumentation Framework* is a digraph $AF = (A, D)$, where A is a finite and nonempty set; the vertices in A are called *arguments*, and if $(a,b) \in D$ is a directed edge, then *argument a defeats (attacks) argument b.*

Let $AF = (A, D)$ be an argumentation framework. For each $a \in A$ we denote $a^+ = \{b \in A|\ (a,b) \in D\}$ the set of all arguments *attacked* by a, and $a^- = \{b \in A|\ (b,a) \in D\}$ the set of all arguments *attacking* a. These notations can be extended to sets of arguments. The set of all arguments *attacked by* $S \subseteq A$ is $S^+ = \bigcup_{a \in S} a^+$, and the set of all arguments *attacking* S is $S^- = \bigcup_{a \in S} a^-$. We also have $\emptyset^+ = \emptyset^- = \emptyset$.

The set S of arguments *defends* an argument $a \in A$ if $a^- \subseteq S^+$ (i.e. any a's attacker is attacked by an argument in S). The set of *all arguments defended by a set S* of arguments is denoted by $F(S)$. For $\mathbb{M} \subseteq 2^A$, $\mathbf{max}(\mathbb{M})$ denotes the set of maximal (w.r.t. set-inclusion) members of $\mathbb{M}$ and $\mathbf{min}(\mathbb{M})$ denotes the set of its minimal members.

Definition 2. Let $AF = (A, D)$ be an argumentation framework.

– A *conflict-free set* in AF is a set $S \subseteq A$ with property $S \cap S^+ = \emptyset$ (i.e. there are no attacking arguments in S). We will denote $\mathbf{cf}(AF) = \{S \subseteq A | S \text{ is conflict-free set }\}$.
– An *admissible set* in AF is a set $S \in \mathbf{cf}(AF)$ with property $S^- \subseteq S^+$ (i.e. defends its elements). We will denote $\mathbf{adm}(AF) = \{S \subseteq A | S \text{ is admissible set }\}$.
– A *complete extension* in AF is a set $S \in \mathbf{cf}(AF)$ with property $S = F(S)$. We will denote $\mathbf{comp}(AF) = \{S \subseteq A | S \text{ is complete extension }\}$.
– A *preferred extension* in AF is a set $S \in \mathbf{max}(\mathbf{comp}(AF))$. $\mathbf{pref}(AF) := \mathbf{max}(\mathbf{comp}(AF))$.
– A *grounded extension* in AF is a set $S \in \mathbf{min}(\mathbf{comp}(AF))$. $\mathbf{gr}(AF) := \mathbf{min}(\mathbf{comp}(AF))$.
– A *stable extension* in AF is a set $S \in \mathbf{cf}(AF)$ with the property $S^+ = A - S$. We will denote $\mathbf{stab}(AF) = \{S \subseteq A | S \text{ is stable extension }\}$.

3 Abstract Debates

In this section we introduce our new framework of aggregating individual opinions, consider its relationship with argumentation frameworks in order to define the argumentative aggregation operators.

Let $F \neq \emptyset$ be a finite set of *facts (items)*. An *opinion on* F (shortly, F*-opinion*) is a pair $O = (L, DL)$ of disjoint sets of facts: $L, DL \subseteq F$ and $L \cap DL = \emptyset$. L is the set of *liked (agreed, accepted) facts* in O and DL is the set of *disliked (disagreed, rejected) facts* in O (the facts in $F - (L \cup DL)$ are not the subject of opinion O). $\mathscr{O}(F)$ denotes the set of all F-opinions. $O = (L, DL) \in \mathscr{O}(F)$ is a *full opinion* if $L \cup DL = F$, and a *single-minded opinion* if $|L| = 1$. An *Abstract Debate* is a tuple $AD = (F, S, \{O_s\}_{s \in S})$, where: S, the *society*, is a finite non-empty set of *individuals (agents, persons)*; $O_s \in \mathscr{O}(F)$ is the F-opinion of individual $s \in S$. We denote by $\mathscr{A}\mathscr{D}(F,S)$ the set of all abstract debates of S over F. The *graph representation* of the abstract debate $AD = (F, S, \{O_s\}_{s \in S})$ is the bipartite digraph $G_{AD} = (F, S; E)$, where $(f, s) \in E$ if and only if $f \in L_s$ and $(s, f) \in E$ if and only if $f \in DL_s$. The graph representation of the debate in the introduction is depicted in Figure 1 b). This is an intuitive and concise representation of a debate.

Note that our abstract debates correspond to *profiles* in SCT and to *agendas* in the Judgment Aggregation area. In fact, our framework is equivalent to judgment aggregation with atomic propositions only (and their negations) and with the standard requirement of *completeness* dropped. Also, note that if in the bipartite digraph G_{AD} a node $f^* \in F$ with $f^* = \arg\max_{f \in F}(|f^+| - |f^-|)$ is selected, then we obtain the well-known *dis&approval* voting procedure characterized axiomatically in [2].

Our approach is based on the following relationship between abstract debates and argumentation frameworks.

Definition 3. (Abstract Debates vs Argumentation Frameworks)

(i) Let $O_1 = (L_1, DL_1), O_2 = (L_2, DL_2) \in \mathscr{O}(F)$. O_1 *agrees with* O_2 *on* $f \in F$ if $f \notin L_1 \cap DL_2 \cup DL_1 \cap L_2$. O_1 *attacks* O_2 *on* $f \in F$ if $f \in DL_1 \cap L_2$. O_1 *agrees with* O_2 if O_1 agrees with O_2 on every $f \in F$. O_1 *attacks* O_2 if there is $f \in F$ such that O_1 attacks O_2 on f. The *argumentation framework associated to* $AD = (F, S, \{O_p\}_{p \in S}) \in \mathscr{A}\mathscr{D}(F,S)$ is $AF(AD) = (S, D)$ in which the arguments are the individuals and an individual s_1 attacks an individual s_2 if and only if O_{s_1} attacks O_{s_2}.

(ii) Let $AF = (A, D)$ be an argumentation framework such that $(a, a) \notin D, \forall a \in A$.The *abstract debate assocsiated* to AF is $AD_{AF} = (F_{AF}, S_{AF}, \{O_s\}_{s \in S_{AF}})$, where $F_{AF} = \{f_a | a \in A\}$, $S_{AF} = \{s_a | a \in A\}$, and for each $a \in A$, $O_{s_a} = (\{f_a\}, \{f_b | b \in a^+\})$.

In Figure 2 i) is illustrated the attack digraph of the argumentation framework associated to the pizza topping debate in the Introduction. Note that we labelled each attack with the set of facts on which the corresponding opinions attacks each other (h=ham, p=pepperoni, s=salami, o=olive). In Figure 2 ii) we have the attack digraph of a simple argumentation framework AF, and the bipartite digraph representation of its associated abstract debate AD_{AF}, is depicted in Figure 2 iii). Note that in AD_{AF} each fact f_a is liked by exactly one individual s_a, and all individual's opinions are single minded.

The argumentation framework $AF(AD)$ can be used to consider particular sets of compatible individuals such that their merged collective opinion defends itself against the attacks of the opinions of individuals outside these sets.

A *coalition* in $AD = (F, S, \{O_p\}_{p \in S})$ is any subset $\mathscr{C} \subseteq S$. $\mathscr{C}$ is *opinion-closed* if

$$O_{\mathscr{C}} = (L_{\mathscr{C}}, DL_{\mathscr{C}}) = \Big(\bigcup_{p \in \mathscr{C}} L_p, \bigcup_{p \in \mathscr{C}} DL_p\Big) \in \mathscr{O}(F). \qquad O_{\mathscr{C}} \text{ is the collective}$$

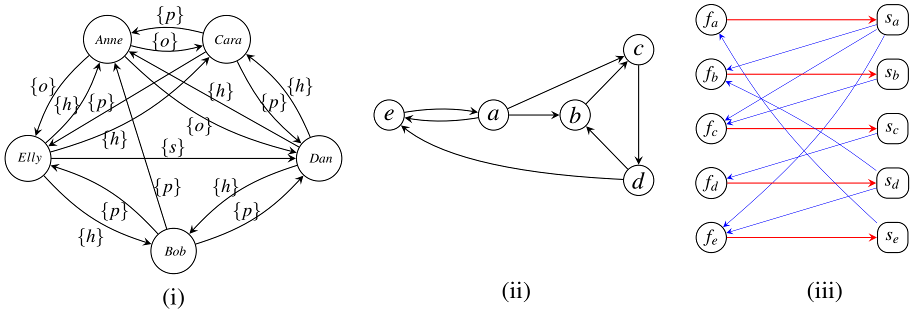


Fig. 2. Abstract Debates vs Argumentation Frameworks

opinion of the coalition $\mathscr{C}$. Note that a coalition $\mathscr{C}$ is opinion-closed if and only if $\mathscr{C}$ is a conflict-free set in $AF(AD)$. It follows that each admissible based extension in $AF(AD)$ gives rise to a collective opinion in $\mathscr{O}(F)$ and the semantics in $AF(AD)$ can be transferred to the abstract debate AD.

Definition 4. Let $AD = (F, S, \{O_p\}_{p \in S}) \in \mathscr{AD}(F, S)$ an abstract debate.

- A coalition $\mathscr{C}$ is an *autarky* in AD if $\mathscr{C}$ is an admissible set in $AF(AD)$, i.e. if it is opinion-closed and for each $p \in S - \mathscr{C}$, if O_p attacks $O_{\mathscr{C}}$ then $O_{\mathscr{C}}$ attacks O_p.
- A coalition $\mathscr{C}$ is a *strong autarky* in AD if $\mathscr{C}$ is a complete extension in $AF(AD)$, i.e. if it is an autarky, and, for each $p \notin \mathscr{C}$ such that O_p is not attacked by $O_{\mathscr{C}}$, there is $s \notin \mathscr{C}$ such that O_s attacks O_p and $O_{\mathscr{C}}$ does not attack O_s. A *minimal strong autarky* (*maximal strong autarky*) is a strong autarky such that there is no strong autarky strictly contained in it (stricly containing it).
- A coalition $\mathscr{C}$ is a *stable coalition* in AD if $\mathscr{C}$ is a stable extension in $AF(AD)$, i.e. if it is opinion-closed and $O_{\mathscr{C}}$ attacks the opinion O_p of any individual p outside $\mathscr{C}$.

Example 1. Let AD be the topping pizza debate in the introduction. The only non-empty opinion-closed coalitions are singletons and $\mathscr{C}_1 = \{Bob, Cara\}$. We can observe that $O_{\mathscr{C}_1} = \{Bob, Cara\} = (\{ham, pineapple, olive\}, \{pepperoni, onion\})$ attacks O_{Anne}, O_{Dan}, and O_{Elly}. It follows that $\mathscr{C}_1 = \{Bob, Cara\}$ is a stable coalition (hence it is an autarky, a strong autarky, and a maximal strong autarky). We can also easily see that $\{Anne\}$ and $\{Dan\}$ are not autarkies but $\mathscr{C}_2 = \{Elly\}$ is a stable coalition.
Example 2. Let AD be the abstract debate represented in the Figure 3 below.

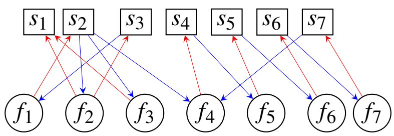


Fig. 3. Bipartite digraph representation of debate in Example 2

$\mathscr{C}_1 = \{s_5, s_7\}$ is an autarky. Indeed, $O_{\mathscr{C}_1} = (\{f_5, f_7\}, \{f_4, f_6\})$. $O_{s_4} = (\{f_4\}, \{f_5\})$ attacks $O_{\mathscr{C}_1}$ but this counterattacks O_{s_4}; $O_{s_6} = (\{f_6\}, \{f_7\})$ attacks $O_{\mathscr{C}_1}$ but this counterattacks O_{s_7}; no other O_{s_i} attacks $O_{\mathscr{C}_1}$, for $i \in \{1,2,3\}$. $\mathscr{C}_1$ is also a strong autarky, but it is not a maximal strong autarky since $\mathscr{C}_2 = \{s_1, s_3, s_5, s_7\}$ is also a strong autarky as we can easily verify. Note that $\mathscr{C}_2$ is also a stable coalition.

For the debate AD_{AF} associated to an argumentation framework AF, the above different type of coalitions translate to the corresponding admissible based extensions in AF. Hence the decision problems on argumentation frameworks can be polynomially transformed into instances on abstract debates. Two typical examples are given below.

Cred$_{\mathbf{msa}}$: Given an abstract debate $AD = (F, S, \{O_p\}_{p \in S})$, and a fact $f \in F$. Is f contained in the liked facts set $L_{\mathscr{C}}$ of some maximal strong autarky $\mathscr{C}$ in AD?
Skept$_{\mathbf{msa}}$: Given an abstract debate $AD = (F, S, \{O_p\}_{p \in S})$, and a fact $f \in F$. Is f contained in the liked facts set $L_{\mathscr{C}}$ of each maximal strong autarky $\mathscr{C}$ in AD?

Using the time complexity results on the corresponding decision problems for argumentation frameworks [11], and the above remark we obtain the following Theorem.

Theorem 5. Cred$_{\mathbf{msa}}$ *is* NP*-complete and* **Skept$_{\mathbf{msa}}$** *is* Π_2^P*-complete.*

Coalitions are very restrictive when the opinions of individuals are full-opinions (in this case, the only nonempty opinion-closed coalitions are singletons). Inspired by political practice, we consider a strategical way of coalition formation: some members of a coalition renounces at some liked facts for making the coalition opinion-closed.

Definition 6. Let $\mathscr{C}$ be a coalition in $AD = (F, S, \{O_s\}_{s \in S})$. A *$\mathscr{C}$-compromise* is a function $\alpha : \mathscr{C} \to 2^F$ such that for every $s \in \mathscr{C}$ we have $\alpha(s) \neq \emptyset$, $\alpha(s) \subseteq L_s$ and

$$O_{\mathscr{C};\alpha} = (L_{\mathscr{C};\alpha}, DL_{\mathscr{C}}) = \Big(\bigcup_{p \in \mathscr{C}} \alpha(p), \bigcup_{p \in \mathscr{C}} DL_p \Big) \in \mathscr{O}(F).$$

If α a $\mathscr{C}$-compromise, the pair $(\mathscr{C}, \alpha)$ is a *compromise* σ for $\sigma \in \{$ *autarky, strong autarky, minimal strong autarky, maximal strong autarky, stable coalition*$\}$ if $\mathscr{C}$ is a σ in the abstract debate $AD_{|\mathscr{C};\alpha} = (F, S, \{O'_s\}_{s \in S})$, where $DL'_s = DL_s$ for every $s \in S$, and $L'_s = L_s$ if $s \in S - \mathscr{C}$ and $L'_s = \alpha(s)$ if $s \in \mathscr{C}$.

Example. Let us consider again the pizza topping debate, and $\sigma =$ stable coalition. Clearly $\mathscr{C}_3 = \{Anne, Dan\}$ is not opinion-closed. But, we can obtain a $\mathscr{C}_3$-compromise by taking $\alpha_3(Anne) = \{pepperoni\}$ and $\alpha_3(Dan) = \{pepperoni, salami\}$. Then $(\mathscr{C}_3, \alpha_3)$ is a stable coalition with $O_{\mathscr{C}_3;\alpha_3} = (\{pepperoni, salami\}, \{olive, onion, ham\})$.

Note that in debates $AD = (F, S, \{O_s\}_{s \in S})$ with $L_p \neq \emptyset$, $\forall p \in S$, if $\mathscr{C}$ is σ then $\mathscr{C}$ is also a *compromise* σ (by taking $\alpha_0(s) = L_s$ for each $s \in \mathscr{C}$). Also, in the debates AD with $|L_p| = 1$, $\forall p \in S$, a coalition $\mathscr{C}$ is compromise σ if and only if $\mathscr{C}$ is σ.

We define now our argumentative aggregation operators.

Definition 7. An *argumentative aggregation operator* for abstract debates is a function $\mathbb{A}_\sigma : \mathscr{A}\mathscr{D}(F, S) \to 2^{\mathscr{O}(F)}$ such that for any $AD \in \mathscr{A}\mathscr{D}(F, S)$,

$$\mathbb{A}_\sigma(AD) = \{O_{\mathscr{C};\alpha} | (\mathscr{C}, \alpha) \text{ is a compromise } \sigma\}.$$

In the terminology of SCT, our argumentative aggregation operators, $\mathbb{A}_\sigma$, are *(irresolute) social functions*. We can reduce the set of aggregate opinions by an appropriate choosing of σ. Another possibility is to keep in $\mathbb{A}_\sigma(AD)$ only the opinions with a maximal (w.r.t. inclusion) set of liked facts. This would eliminate the trivial opinion $(\emptyset,\emptyset)$ for $\sigma =$ autarky. Other strategy of reducing the set $\mathbb{A}_\sigma(AD)$ is to retain only the opinions at minimum distance to the entire set of individuals opinion (after defining appropriate distance functions).

We now turn to conditions one may wish to impose on an aggregation operator as usually done in SCT. Let $\mathbb{A}_\sigma : \mathscr{AD}(F,S) \to 2^{\mathscr{O}(F)}$ be an argumentative aggregation operator for $\sigma \in \{$ *autarky, strong autarky, minimal strong autarky, maximal strong autarky, stable coalition*$\}$. We begin with the uncontroversial requirement that, if all individuals have the same opinion, this should be the collective one **(Unanimity)**. Clearly, if in an abstract debate $AD = (F,S,\{O_s\}_{s\in S})$ we have $O_s = O \in \mathscr{O}(F)$ for every $s \in S$, then the grand coalition S is σ and its collective opinion is $O_S = O$. Hence every $\mathbb{A}_\sigma$ satisfies the unanimity condition. Another basic democratic requirement of an aggregation operator is **Anonymity:** for every two abstract debates $AD = (F,S,\{O_s\}_{s\in S})$ and $AD' = (F,S,\{O'_s\}_{s\in S})$ such that there is a permutation $\pi : S \to S$ with $O'_s = O_{\pi(s)}$, we have $\mathbb{A}_\sigma(AD) = \mathbb{A}_\sigma(AD')$. Clearly, if $\pi : S \to S$ is a permutation, then a coalition $\mathscr{C}$ is σ if and only if $\pi(\mathscr{C}) = \{\pi(s) | s \in \mathscr{C}\}$ is σ. Hence every $\mathbb{A}_\sigma$ satisfies the anonymity condition. Similarly, we can easily argue that every $\mathbb{A}_\sigma$ satisfies **Neutrality**: the set of aggregate opinions returned by $\mathbb{A}_\sigma$ for a debate obtained by renaming the facts in a debate is obtained by renaming the facts in each aggregate opinion returned by $\mathbb{A}_\sigma$ for that debate. Also, by the definition, every $\mathbb{A}_\sigma$ satisfies **Compatibility** (each returned opinion agrees with at least one individual opinion), introduced in [18].

The main tool used in SCT to change the argument of an aggregation operator without changing the output is the *Arrow's independence of irrelevant alternatives*. $\mathbb{A}_\sigma$ satisfies **Independence** if for every $f \in F$, $AD^1 = (F,S,\{O^1_p\}_{p\in S})$, $AD^2 = (F,S,\{O^2_p\}_{p\in S})$,

> if O^1_s agrees with O^2_s on f for all $s \in S$, then for every opinion $O_1 \in \mathbb{A}(AD_1)$ there is a nontrivial opinion $O_2 \in \mathbb{A}(AD_2)$ such that O_1 agrees with O_2 on f.

Since the operators $\mathbb{A}_\sigma$ are not "fact wise" and are strongly dependent on the context of the debate on which they are applied, we have the following theorem.

Theorem 8. *Argumentative aggregation operators $\mathbb{A}_\sigma$ do not satisfy independence.*

Proof. Let $F = \{f,g,h\}$ be the set of facts and let $S = \{s_1,s_2,s_3\}$. Let $AD^1 = (F,S,\{O^1_p\}_{p\in S})$ be the abstract debate in which $O^1_{s_1} = (\{f\},\{g\})$, $O^1_{s_2} = (\{g\},\{f\})$, $O^1_{s_3} = (\{h\},\{g\})$. Since all opinions in AD^1 are single minded, we have no compromise σ in the debate AD^1. The coalition $\mathscr{C} = \{s_1,s_3\}$ is an autarky with $O_{\mathscr{C}} = (\{f,h\},\{g\})$. There is no autarky containing s_2 and other member of S since by adding s_1 or s_3 to $\{s_2\}$ the resulting coalition is not opinion-closed. The coalition $\mathscr{C}' = \{s_2\}$ is not an autarky since $O_{\mathscr{C}'} = (\{g\},\{f\})$ does not defend against the attack of $O^1_{s_3} = (\{h\},\{g\})$. It follows that $\mathbb{A}_\sigma(AD^1) = \{(\{f,h\},\{g\})\}$. Let $AD^2 = (F,S,\{O^2_p\}_{p\in S})$ be the abstract debate in which $O^2_{s_1} = (\{f\},\{h\})$, $O^2_{s_2} = (\{g\},\{f,h\})$, $O^2_{s_3} = (\{h\},\emptyset)$. All opinions in AD^2 are single minded, hence we have no compromise σ in the debate AD^2. The only autarky is

$\mathscr{C} = \{s_2\}$, therefore $\mathbb{A}_\sigma(AD^2) = \{(\{g\},\{f,h\})\}$. Hence no opinion in $\mathbb{A}_\sigma(AD^1)$ agrees with an opinion in $\mathbb{A}_\sigma(AD^2)$ on f. But, O_s^1 agrees with O_s^2 on f, $\forall s \in S$. □

4 Discussion

In spite of its proximity to the field of judgment aggregation (JA), in our approach the "facts" in F are not a priory logically related. However, it is possible to discuss problems related to "logical consistency" by considering *opinion spaces*, in which an opinion $O = (L, DL)$ is *consistent* if and only if $\overline{L} \cap DL = \emptyset$, where $\overline{L}$ is the set of facts entailed by L under a predefined entailment relation on F. The opinion attack digraph is defined now by considering that an opinion $O = (L, DL)$ attacks any opinion $O' = (L', DL')$ if and only if $DL \cap \overline{L'} \neq \emptyset$. In this way, we meet questions related to logically based AF's ([6]), since opinion-closed coalitions are not simply conflict-free sets (as we obtained for the particular case $\overline{X} = X$). However, using specific rules from judgment aggregation field (see [15] and its references), could be worthwhile, when these are applied to our set $\mathbb{A}_\sigma(AD)$ of aggregate opinions. Of course, our incipient study of the properties of argumentative operators must be developed. Most of the properties studied in SCT or JA are quantitative in nature and require introducing a structure on the set of abstract debates in order to replace the independence property as a vehicle for passing from a profile (debate) to another one to obtain impossibility or non-manipulability results.

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