# Chapter 33 Optimization of the Dimensionless Model of an Electrostatic Microswitch Based on AMGA Algorithm

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**Abstract** In this paper a micro genetic algorithm for multi-objective optimization (AMGA) is used to minimize the number of function evaluations of the dimensionless model of an electrostatic microswitch. A non-dimensional dynamic model is proposed, and three objective functions are defined: the closing dimensionless time of the first impact, the maximum dimensionless speed and the maximum dimensionless displacement of the first impact. This work has been carried out using dimensional analysis. Results demonstrate an interesting methodology based on AMGA for optimizing the closing time and displacement of the first impact in a microswitch.

**Keywords** Electrostatic microswitch · Dimensional analysis · Multi-objective optimization

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# **33.1 Introduction**

MEMS electrostatic microswitches possess high insulation of the electromechanical switches and ultra-low losses. They also have low power consumption and small size and the advantages of low cost of solid state relays manufactured with microelectronic technology. Besides, electrostatic microswitches operate in a large range of frequencies. These properties make possible the massive application of these MEMS devices to wide technology fields, in particular, to the telecommunication industry, to wireless devices such as microswitches for antennas and switches for reception-transmission, among others.

One of the main subjects in resistive microswitch design is related to the interaction between the tip and the substrate, and the damage accumulated produced by the bouncing of the tip on the substrate. It is well known that the tip of a resistance microswitch bounces several times on the substrate before reaching a permanent contact [1].

We have used dimensional analysis for measured performance of some dimensionless parameters. The use of a dimensionless model is a valuable procedure used to study engineering problems [2]. By applying Buckingham [3] theorem, dimensionless parameters are obtained.

 $\Pi$  Buckingham theorem applied to dimensionless analysis establishes that, an equation with a number of variables related between them that defines a physical problem is reduced to another similar dimensionless equation but with a lower number of variables. A dimensionless parameter consists in a group of variables joined in a way that the dimensionless expression is the unit. The number of dimensionless sets for a particular problem is equal to the difference between the total number of variables minus the number of fundamental dimensions.

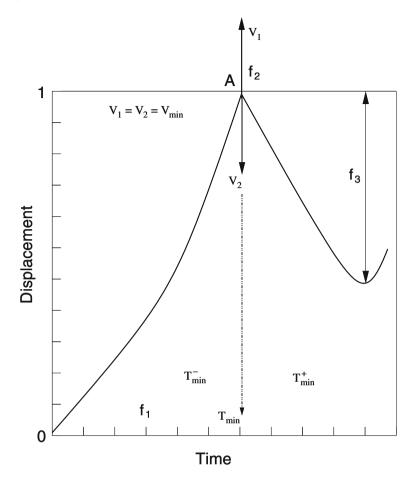
Our contribution in this work consists in designing a microswitch with the following goal: minimize the time of the first contact tip-substrate to increment the working frequency and, at the same time, minimize the maximum velocity and the oscillation of the first bouncing to decrease the number of bounces.

These goals are in contradiction because if we decrease the closing time, then, the velocity of the first impact increases and, hence, there is a greater bouncing, see Fig. 33.1. This fact indicates that we are dealing with a multiobjective optimization problem.

This paper is organized as follows. Section 33.2 presents the theoretical fundaments of the optimization multiobjective of a microswitch. Section 33.3 provides the results and discussion. Finally, in Sect. 33.4 conclusions are presented.

# 33.2 Theoretical Fundaments of the Optimization Multiobjective of a Microswitch

In general, the optimization multiobjective problem is defined as follows: find the vector  $\vec{x}^* = [x_1^*, x_2^*, \dots, x_n^*]^T$  that satisfaces the *m* restrictions of the inequalities



**Fig. 33.1** Dimensionless analysis of the velocity for  $A_1 = 0.1$ 

 $g_i(\overrightarrow{x}) \ge 0$   $i = 1, 2, \dots, m$ 

the p restrictions of the equalities

$$h_j(\vec{x}) = 0$$
  $j = 1, 2, ..., p$  (33.1)

and optimizes the vector of functions

$$\mathbf{f}(\vec{x}) = [f_1(\vec{x}), f_2(\vec{x}), \dots, f_k(\vec{x})]^T$$
(33.2)

where  $\mathbf{x} = [x_1, x_2, \dots, x_r]^T$  is the vector of variable decision.

Multiobjective optimization problems require three basic elements:

- A conflicting trade-off between two or more objective functions. They are a quantitative measure of the system to be optimized.
- The variables that affect the objective functions.
- The restrictions: a set of relations, equations and inequalities that some variables must satisfy.

For obtain an optimal design of the microswitch the dinamic behavior of the system is modelled and analyzed. A cantilever beam represents a basic physical model of microswitch. We have analyzed this beam through a lumped parameter model of mass-spring-friction and two state variables (velocity and position). In this model, we stress the interaction between the tip of the beam and the substrate that has been modeled introducing a "Lennard–Jones" force [1]. In our work, see Fig. 33.1, the three objective functions are:

- $f_1(\vec{x})$ : The closing time—to increase the working frequency of the microswitch.
- $f_2(\vec{x})$ : The first impact velocity.
- $f_3(\vec{x})$ : The first impact displacement.

We have identified six dimensionless parameters. In first place,  $A_1$ , which is a proportion between the impeller electrostatic force and the elastic force associated to the cantilever beam. Second, the quality factor Q which is inversely proportional to friction coefficient.

From the dynamical analysis, we have obtained two performance related to the time domain, the velocity and the position. From this performance we obtain important data for the design of the microswitch: the time  $T_{\min}$  to establish the first contact tip/substrate, the maximum velocity for the first contact,  $\tilde{v}_{\max}$ , and the maximum elongation after the first impact,  $\tilde{r}_{\max}$ .

The MEMS switch is modeled as a one-degree of freedom system which is the position of the tip of the cantilever *r*. It consists on a mass *m*, initially placed at a distance  $g_0$  from the substrate, a spring with elastic constant *k*, and a dashpot with damping coefficient *b*. Thus, the motion of the system is described by the classical second order linear Eq. 33.3, where  $F_{EL}$  is the electrostatic actuation force and  $F_{LJ}$  the Lennard-Jones force that provides the mechanical interaction between two facing surfaces.  $F_{EL}$  and  $F_{LJ}$  is expressed as 33.4 and 33.5, respectively. In this work, the area of interaction has been assumed as  $A = 100 \ \mu m^2$ ,  $C_1 = 10^{-20} \ Nm$  and  $C_2 = 10^{-80} \ Nm$  [1].

$$m\frac{d^{2}r}{dt^{2}} + b\frac{dr}{dt} + kr = F_{EL} + F_{LJ}$$
(33.3)

$$F_{EL} = \frac{\frac{1}{2}\varepsilon_0 A_0 V^2}{\left(g_0 + \frac{d_\varepsilon}{\varepsilon_r} - r\right)^2}$$
(33.4)

$$F_{LJ} = \frac{C_1 A}{(g_0 - r)^3} - \frac{C_2 A}{(g_0 - r)^9}$$
(33.5)

The dimensionless equations of the dynamic model are:

$$\frac{d\tilde{v}_{e}}{d\tilde{t}} = \left[\frac{A_{1}}{(1+A_{4}-\tilde{r})^{2}}\left[1+A_{5}\left(1-\tilde{r}\right)\right] - \frac{\tilde{v}_{e}}{Q} - \tilde{r} + \frac{A_{2}}{(1-\tilde{r})^{3}} - \frac{A_{3}}{(1-\tilde{r})^{9}}\right]$$
(33.6)

$$\frac{d\tilde{r}_e}{d\tilde{t}} = \tilde{v}_e \tag{33.7}$$

where  $A_1$ ,  $A_2$ ,  $A_3$ ,  $A_4$  and  $A_5$  are non dimensional parameters and Q is the quality factor.

 $A_1$  and Q are expressed as follows:

$$A_1 = \frac{\varepsilon_0 A_0 V^2}{2kg_0^3}$$
(33.8)

and

$$Q = \frac{\sqrt{km}}{b} \tag{33.9}$$

#### **33.3 Results and Discussion**

The proposed optimization problem is described as follows

$$min \quad (f_1(\vec{x}), f_2(\vec{x}), f_3(\vec{x})) \tag{33.10}$$

where

$$\overrightarrow{x} = (A_1, Q) \tag{33.11}$$

is the decision variable vector that corresponds to the dimensionless parameters  $A_1$  and Q described previously.

 $f_1(\vec{x})$  corresponds to the first dimensionless bounce time. At this instant the microswitch contacts the substrate for first time,  $f_1(\vec{x}) = T_{\min}$ , see Fig. 33.1.

 $f_2(\vec{x})$  is  $\tilde{v}_{e_{\text{max}}}$ , which is the velocity for the first bounce at  $T_{\text{min}}$  value; in this time instant, the velocity  $V_1$  for  $T_{\text{min}}^-$  is equal to  $V_2$  at  $T_{\text{min}}^+$ , see Fig. 33.1. Note that the velocity senses are opposite.

 $f_3(\vec{x})$  is  $\tilde{r}_{max}$  which is the dimensionless maximum position of the first bounce and correspond to a V = 0 velocity.

The instants  $T_{\min}$  and  $T_{\max}$  are determined in the execution time of the numerical solution. For this reason, we have analyzed two consecutive instants of the simulation, comparing the value of the variables  $\tilde{r}_e$  and  $\tilde{v}_e$ .

Table 33.1 Execution time   of the experiments	Population	Generation	Evaluations	Time (s)
	80	10	800	2,405
	80	50	4,000	20,566
	80	62	4,960	25,568
	100	62	6,200	31,996
	60	200	12,000	62,245

Differential Eqs. 33.6 and 33.7 have been solved using a RKF with fixed step programmed in C++ [4] wich uses the algorithm AMGAII-Archive-based Microgenetic Algorithm. We have chosen a real codification for each chromosome composed by the design variables  $A_1$  and Q. In general, the precision is better than the binary codification [5]. This is improved adding more bits, but it increases the simulation time. The used time step  $\Delta$  was  $10^{-6}$  s and is constant during the whole simulation.

All the simulation results were obtained in a SunFire X2200, with 2 CPU AMD Opteron 2214 Dual Core (2,2Ghz), 4 GB RAM, using the O. S. Red Hat Enterprise Linux Server 5.3.

We have solved this problem based on the concept of Pareto optimal solution applying genetic multiobjective algorithms. The numerical solution of the objective functions is highly time consuming, see Table 33.1. Hence, we have reduced the number of evaluations of the objective functions.

We propose the use of Archieve based Micro Genetic Algorithm (AMGA) [6] method because this algorithm generates a small number of new solutions in each evaluation, improving the total time for the evaluation [7].

Taking into account the first group of results of the dimensionless dynamic model, we conclude that the most important dimensionless parameters are  $A_1$  and Q.  $A_1$  represents the quotient between the minimum electrostatic force and the maximum elastic force and Q represents the quotient between the maximum energy stored in the spring and mass, and the friction looses.

Figure 33.2 illustrates the Pareto optimum with the objective functions closing time for the first impact versus the maximum velocity in the first impact. Figure 33.3 shows the Pareto optimum and represents the three objective functions mentioned in the previous section [6]. Figures 33.4 and 33.5 represent the influence of  $A_1$  in the closing time and velocity of the first impact. As  $A_1$  increases the closing time decreases and the closing speed increases.

Figure 33.6 shows the influence of the dimensionless parameter Q in the maximum oscillation of the first impact. As Q increases the closing position increases.

In this work, we have developed a methodology based on AMGA for optimizing the closing time and displacement of the first impact in a microswitch. From Figs. 33.2, 33.3, 33.4, 33.5 and 33.6, the relationship between dimensionless parameters  $A_1$  and Q and closing time and displacement is illustrated.

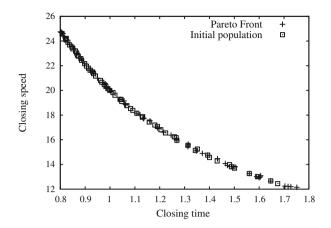


Fig. 33.2 Pareto solution time versus speed

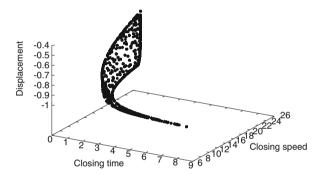


Fig. 33.3 3D Pareto AMGA method first impact

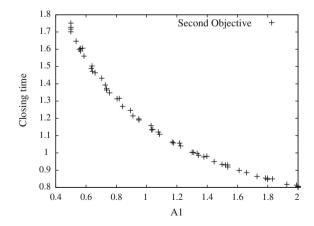


Fig. 33.4 Dimensionless time versus A1

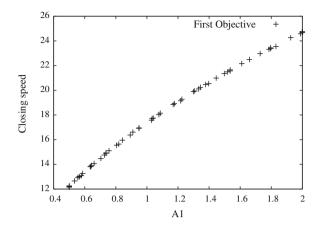


Fig. 33.5 Dimensionless speed versus A1

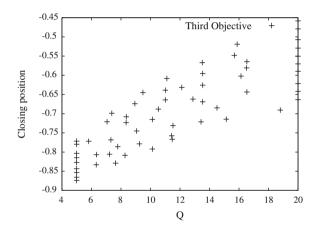


Fig. 33.6 Dimensionless speed versus Q

### **33.4 Conclusions**

Based on a dimensionless model of a microswitch for the closing time and displacement, we have analyzed the sensitivity of this Micro Electro Mechanical System— MEMS—to the variation of the dimensionless parameters of the model. In this way, we predict the dynamic behavior of a microswitch. By using multiobjective Genetic Algorithms we have optimized the bouncing of the microswitch which is one of the major designing concern. We have used the AMGA multi-objective GA for obtaining the Pareto front of the design space.

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# References

- 1. Granaldi A, Decuzzi P (2006) The dynamic response of resistive microswitches: switching time and bouncing. J Micromech Microeng 16(7):1108
- Vasca F, Verghese GC (1999) Adimensional models and participation factors for the analysis of induction motor dynamics. In: Industrial electronics, 1999. ISIE '99. Proceedings of the IEEE international symposium on, vol 2, pp 480–485
- Misic T, Najdanovic-Lukic M, Nesic L (2010) Dimensional analysis in physics and the buckingham theorem. Eur J Phys 31(4):893
- Nicolet A, Delince F (1996) Implicit runge-kutta methods for transient magnetic field computation. IEEE Trans Magn 32(3):1405–1408
- 5. Michalewicz Z (1996) Genetic algorithms + data structures = evolution programs, 3rd edn. Springer, London
- Tiwari S, Koch P, Fadel G, Deb K (2008) Amga: an archive-based micro genetic algorithm for multi-objective optimization. In: Proceedings of the 10th annual conference on genetic and evolutionary computation, GECCO '08, ACM, New York, pp 729–736
- 7. Monzon-Verona JM, Garcia-Alonso S, Sosa J, Montiel-Nelson JA (2013) Multiobjective genetic algorithms applied to low power pressure microsensor design. Eng Comput 30(8):1128–1146