Cellular Automaton Model with Non-hypothetical Congested Steady State Reproducing the Three-Phase Traffic Flow Theory

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Abstract. A new assumption is assumed to explain the mechanisms of traffic flow that in the noiseless limit, vehicles' space gap will oscillate around the desired space gap, rather than keep the desired space gap, in the homogeneous congested traffic flow. It means there are no steady states of congested traffic and contradicts with the fundamental diagram approach and three-phase traffic flow theory both of which admit the existence of steady states of congested traffic. In order to verify this assumption, a cellular automaton model with non-hypothetical congested steady state is proposed, which is based on the Nagel-Schreckenberg model with additional slow-to-start and the effective desired space gap. Simulations show that this new model can produce the synchronized flow, the transitions from free flow to synchronized flow to wide moving jams, and multiple congested patterns observed by the three-phase theory.

Keywords: Cellular automaton, three-phase traffic flow, fundamental diagram.

1 Introduction

In order to understand the mechanism of traffic congestion, many models and analysis have been carried out to explain the empirical findings [1–5]. Generally speaking, these models can be classified into the fundamental diagram approach or the three-phase theory. The fundamental diagram is the idealized form of the flow-density curve in traffic flow, which goes through the origin with at least one maximum. It describes the theoretical relationship between density and flow in the stationary homogeneous traffic, i.e., the steady state of identical driver-vehicle units[5]. In the last century, almost all traffic flow models belong to the fundamental diagram approach. In microscopic models, the fundamental

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diagram is linked to the steady states of car-following (CF) or cellular automaton (CA) models. For example, in the Optimal Velocity Model (OV model) [6], the fundamental diagram corresponds to the optimal velocity function itself. In the Nagel-Schreckenberg cellular automaton model (NaSch model), it could be derived in terms of the steady state in the deterministic limit [7]. In the macroscopic or mesoscopic models, it has been directly applied (e.g. the LWR theory [8, 9]) or incorporated into the momentum equation (e.g. the PW theory [10]).

The majority of models in fundamental diagram approach belongs to the two-phase models [6–13], which refers to the free flow phase (F) and the jammed phase (J). The phase transitions involved are the transition from free flow to jams (F→J transition) and the transition from jam to free flow (J→F transition). The fundamental diagram approach explains the jam formation mainly by excess demand, i.e., the traffic inflow exceeds the static capacity defined by the maximum of the fundamental diagram. Additionally, instabilities of traffic flow, which are caused by finite speed adaption time (due to finite accelerations) or reaction time, can lead to jam formation even before static capacity is reached. For the detailed discussion of stability, one can refer to [5, 14].

Based on the long-term empirical analysis, Kerner [3, 4] argues that two-phase models could not reproduce the empirical features of traffic breakdown as well as the further development of the related congested region properly. Then the three-phase theory is introduced, in which there are (1) free traffic flow (2) synchronized flow and (3) wide moving jams. The fundamental hypothesis of the three-phase theory is that the hypothetical steady states of the synchronized flow cover a two-dimensional region in the flow-density plane[1] , in other words there is no fundamental diagram of traffic flow. Over the time, many models within the framework of three-phase theory are proposed [15–25].

In three-phase traffic theory, traffic breakdown is a phase transition from free flow to synchronized flow (F→S transition). Wide moving jams can occur spontaneously in synchronized flow only (S→J transition), i.e. due to a sequence of F→S→J transitions. Empirical observations show that General Patterns (GPs) and Synchronized Patterns (SPs) are two main types of congested patterns at an isolated bottleneck. After the synchronized flow occurs upstream of the bottleneck, the wide moving jams continuously emerge in that synchronized flow and propagate upstream, and then this congested pattern is often called as the General Patterns (GP). However, if the wide moving jams discontinuously emerge on the road, there will just have one or few wide moving jams appearing in that synchronized flow, then this congested pattern is often called as the dissolving General Patterns (DGP). If there is only synchronized flow upstream of the bottleneck, no wide moving jams emerge in the synchronized flow, then this congested pattern is often called as the Synchronized Patterns. And as a result of the F→S transition, various synchronized flow patterns can occur at the bot-

[1] Two-dimensional steady states refer to a two-dimensional manifold of steady states parameterized by the associated flow and density. In the flow-density diagram, this is represented by a two-dimensional area of possible states, hence the name.

tleneck, such as the widening synchronized pattern (WSP), local synchronized pattern (LSP), and moving synchronized pattern (MSP).

In this paper, another assumption is conceived to explain the mechanisms of traffic flow that in the noiseless limit, vehicles' space gap will oscillate around the desired space gap, rather than keep the desired space gap, in the homogeneous congested traffic flow, which means there are no steady states of congested traffic. In order to validate this assumption, a new cellular automation model is established in section 2. Empirical findings of three-phase theory are simulated and discussed in section 3. Finally, the conclusion is given in section 4.

2 The New Model

In the fundamental diagram approach, the unique relationship between the equilibrium space gap and speed is assumed in the stationary homogeneous traffic: if the equilibrium space gap is smaller than the actual gap, vehicles tend to accelerate; if the equilibrium space gap is greater than the actual gap, vehicles tend to decelerate; otherwise vehicles tend to keep the uniform speed. However in the three-phase traffic theory, there is no unique relationship between the equilibrium space gap and speed in the stationary homogeneous traffic: if the actual gap is greater than the synchronized gap, vehicles tend to accelerate; if the actual gap is smaller than the safe gap, vehicles tend to decelerate; otherwise vehicles tend to adjust their speed according to the speed of their formers. Although three-phase traffic theory denies the fundamental diagram, both admit the existence of stationary homogeneous traffic in the congested traffic.

Comparing with taking the unique relationship between the equilibrium space gap and speed into the models within the fundamental diagram approach, models in the framework of the three-phase traffic theory are often complicated due to the two-dimensional region, which makes them less practical. We wonder whether there is another assumption that considers more reality than the fundamental diagram approach but less complicated than the three-phase traffic theory. Since there is seldom any evidence confirming the existence of stationary homogeneous traffic in the congested traffic that tends to keep oscillating in reality, the following assumption is proposed: **in the noiseless limit, vehicles' space gap will oscillate around the desired space gap, rather than keep the desired space gap, in the homogeneous congested traffic flow, which means there are no steady states of congested traffic.**

In order to validate this assumption, the following cellular automaton model is proposed whose main mechanisms incorporating this assumption are embodied in the randomization process of vehicles. The parallel update rules are as follows.

1. Determination of the randomization parameter $p_n(t+1)$ and deceleration extent Δv:

$$p_n(t+1) = \begin{cases} p_a: & \text{if } d_n^{\text{eff}}(t) < d_n^*(t) \\ p_b: & \text{if } v_n(t) = 0 \text{ and } t_n^{\text{st}}(t) \geq t_c \\ p_c: & \text{in all other cases} \end{cases} \tag{1}$$

$$\Delta v(t+1) = \begin{cases} b_{\text{defens}}: & \text{if } d_n^{\text{eff}}(t) < d_n^*(t) \\ 1: & \text{in all other cases} \end{cases} \quad (2)$$

2. Acceleration:
$v_n(t+1) = \min(v_n(t)+1, v_{\max})$
3. Deceleration:
$v_n(t+1) = \min(d_n^{\text{eff}}(t), v_n(t+1))$
4. Randomization with probability:
if$(rand() < p_n(t+1))$ then $v_n(t+1) = \max(v_n(t+1) - \Delta v(t+1), 0)$
5. The determination of $t_n^{\text{st}}(t+1)$:
if$(v_n(t+1) = 0)$ then $t_n^{\text{st}}(t+1) = t_n^{\text{st}}(t) + 1$
if$(v_n(t+1) > 0)$ then $t_n^{\text{st}}(t+1) = 0$
6. Car motion:
$x_n(t+1) = x_n(t) + v_n(t+1)$

where $d_n(t)$ is the space gap between vehicle and its preceding vehicle $n+1$, $d_n(t) = x_{n+1}(t) - x_n(t) - L_{\text{veh}}$, $x_n(t)$ is the position of vehicle n (here vehicle $n+1$ precedes vehicle n) and L_{veh} is the length of the vehicle. $v_n(t)$ is the speed of the vehicle n, and $v_{\max}$ is the maximum speed. $d_n^*(t) = Tv_n(t)$ is the effective desired space gap between vehicle n and $n+1$, and T is the effective safe time gap between vehicle n and $n+1$ at the steady state. $d_n^{\text{eff}}(t)$ is the effective gap, $d_n^{\text{eff}}(t) = d_n(t) + max(v_{\text{anti}}(t) - g_{\text{safety}}, 0)$, where $v_{\text{anti}} = \min(d_{n+1}(t), v_{n+1}(t)+1, v_{\max})$ is the expected speed of the preceding vehicle in the next time step. g_{safety} is the parameter to control the effectiveness of the anticipation. Accidents are avoided only if the constraint $g_{\text{safety}} \geq b_{\text{defens}}$ is satisfied. The speed anticipation effect is considered in order to reproduce the real time headway distribution, which has a cut off at the small time headway less than one second [22]. $t_n^{\text{st}}(t)$ denotes the time since the last stop for standing vehicles, while $t_n^{\text{st}}(t) = 0$ for moving vehicles.

The basis of the new model is the rules of the NaSch model with randomization parameter p_c to which a slow-to-start rule and the effective desired space gap $d_n^*(t)$ has been added. The slow-to-start effect is characterized by an increase of the randomization parameter from p_c to p_b $(> p_c)$, which is the element to realize the transition from synchronized flow to wide moving jams. The new model assumes the driver tends to keep the effective gap no smaller than $d_n^*(t)$, otherwise the driver will become defensive. The actual behavioral change is characterized by increasing the spontaneous braking probability from p_c to p_a. Moreover, the associated deceleration will change from 1 to b_{defens} (≥ 1). This effect is the factor to reproduce the transition from free flow to synchronized flow in the new model.

In the following, the steady states of the new model are analyzed in the unperturbed, noiseless limit. For microscopic traffic flow models, the steady state requires that the model parameters are the same for all drivers and vehicles. In that case, the steady state is characterized by the following two conditions [5]:

1) Homogeneous traffic: All vehicles move at the same speed and keep the same gap behind their respective leaders.

2) No accelerations: all vehicles keep a constant speed.

Since the mechanisms associated with the hypothetical congested steady state analysis are all embodied in the randomization process, the noiseless limit should be taken as $p_a = 1, p_b = 0, p_b = 1$ or $p_a = 1, p_b = 1, p_c = 1$. However, all vehicles will keep a constant speed no matter how long distance between vehicles is in the latter case, which is obviously unrealistic. Thus, we consider the former. According to the model rules, if $d^{\text{eff}}/T \geq v_{\max}$, all vehicles will move with $v_{\max}$; if $d^{\text{eff}}/T < v_{\max}$, all vehicles' speed will take turns to change simultaneously over time between $\max(v - b_{\text{defens}}, 0)$ and v, where $v \in [d^{\text{eff}}/T, \min(v_{\max}, d^{\text{eff}})]$ and $\max(v-1,0) < d^{\text{eff}}/T$. It means there are no steady states of congested traffic in the new model. Vehicles space gaps oscillate around the desired gap, i.e., deviations from the steady state are caused by local instabilities (representing the inability of the drivers to keep the desired gap), not by the driver heterogeneity [2], which is consistent with the empirical findings by [27]. Therefore, this model is named the cellular automaton model with non-hypothetical congested steady state (NH model).

3 Simulation Investigation

In this section, simulations are carried out on a road with the length $L_{\text{road}} = 1000L_{\text{cell}}$. Both the cell length and vehicle length are set as $7.5m$, i.e. $L_{\text{cell}} = 7.5m$ and $L_{\text{veh}} = 1L_{\text{cell}} = 7.5m$. One time step corresponds to $1s$. During the simulations, the first 50000 time steps are discarded to let the transients die out. The parameters are shown in Tab.1.

Table 1. Model parameters of NH model

Parameters	L_{cell}	L_{veh}	$v_{\max}$	T	b_{defens}	p_a	p_b	p_c	g_{safety}	t_c
Units	m	L_{cell}	L_{cell}/s	s	L_{cell}/s^2	-	-	-	L_{cell}	s
Value	7.5	1	5	1.8	1	0.95	0.55	0.1	2	8

Traffic patterns that emerge near an on-ramp are studied under open boundary condition. The vehicles drive from left to right. The left-most cell corresponds to $x = 1$. The position of the left-most vehicle is x_{last} and that of the right-most vehicle is x_{lead}. At each time step, if $x_{\text{last}} > v_{\max}$, a new vehicle with speed $v_{\max}$ will be injected to the position $\min(x_{\text{last}} - v_{\max}, v_{\max})$ with probability $q_{\text{in}}/3600$ and q_{in} is the traffic flow entering the main road. At the right boundary, the leading vehicle moves without any hindrance. If $x_{\text{lead}} > L_{\text{road}}$, the leading vehicle will be removed and the following vehicle becomes the leader.

[2] Driver heterogeneity, or, more specifically, inter-driver heterogeneity, refers to different parameterizations of every vehicle representing variations of the individual driving style and vehicle performances. If there is no inter-driver heterogeneity (identical drivers and vehicles), all deviations from a steady state are either due to intrinsic randomness (a random term in the accelerations as in the NaSch Model with the randomization probability p $\neq$ 1 or zero), or dynamic instabilities. In the deterministic case of our model, we have only the latter, i.e. "no" internal/intrinsic randomness.

We adopt a simple method to model the on-ramp, which is similar to that of [26]. Assuming the position of the on-ramp is x_{on}, a region $[x_{\text{on}}, x_{\text{on}} + L_{\text{ramp}}]$ is selected as the inserting area of the vehicle from on-ramp. At each time step, we find out the longest gap in this region. If the gap is large enough for a vehicle, then a new vehicle will be inserted at the cell in the middle of the gap with probability $q_{\text{on}}/3600$ and q_{on} is the traffic flow from the on-ramp. The speed of the inserted vehicle is set as the speed of its preceding vehicle, and the stop time is set to zero. In this paper, the parameters are set as $x_{\text{on}} = 0.8L_{\text{road}}$ and $L_{\text{ramp}} = 10L_{\text{cell}}$.

In Fig.1(a), the reproduced spatial-temporal features of the congested pattern named moving synchronized flow (MSP) are shown (see the empirical figure 7.6 in [4]). In this pattern, synchronized traffic flow spontaneously emerges in the free flow. Fig.1(b) exhibits the widening synchronized flow (WSP, see the empirical figure 7.4 in [4]). For this pattern, wide moving jams do not emerge in synchronized flow. The downstream front of WSP is fixed at the on-ramp and the upstream front of WSP propagates upstream continuously over time. In Fig.1(c), both the downstream and the upstream front of synchronized flow are fixed at the on-ramp, thus, it belongs to the local synchronized pattern (LSP). Moreover, the width of LSP in the longitudinal direction changes over time, which is in accordance with empirical observations (see the empirical figure 7.2 in [4]). Fig.1(d) shows the dissolving General Patterns (DGP) in which just one wide moving jam emerges in the synchronized flow. Fig.1(e) shows the spatial-temporal features of General Pattern (GP).

In order to emphasize the significance of the two-dimensional steady states of synchronized flow, Kerner and Klenov (2006) proposed the Speed Adaption Models (SAMs) in the framework of fundamental diagram approach. The basic hypothesis of SAMs is the double Z-characteristic for the sequence of phase transitions from free flow to synchronized flow to wide moving jams (F→S→J transitions). Based on this hypothesis, SAMs can reproduce both the traffic breakdown and the emergence of wide moving jams in synchronized flow as found in empirical observations. However, SAMs are not able to reproduce the local synchronized patterns (LSPs) consistent with empirical results as well as some of empirical features of synchronized flow between wide moving jams within general patterns (GPs). Kerner et al. attribute these drawbacks of SAMs to the lacking of the two-dimensional steady states of synchronized flow.

Although only free outflow exists in the downstream of wide moving jams in GP of the NH model, it can be easily improved if we decrease the slow-to-start probability p_b or adjust the values of T and b_{defens}, see Fig.1(f),(g). Thus, all the above simulation results are well consistent with the well-known results of the three-phase traffic theory. Therefore it is reasonable to conclude that the two-dimensional steady states of synchronized flow are not an essential requirement for the spatiotemporal dynamics.

Moreover, in Fig.1(a) and (d), one could obtain the propagation velocity of the downstream MSP front is nearly −26.8km/h and the propagation velocity of the downstream jam front is nearly −13km/h which is about half that of

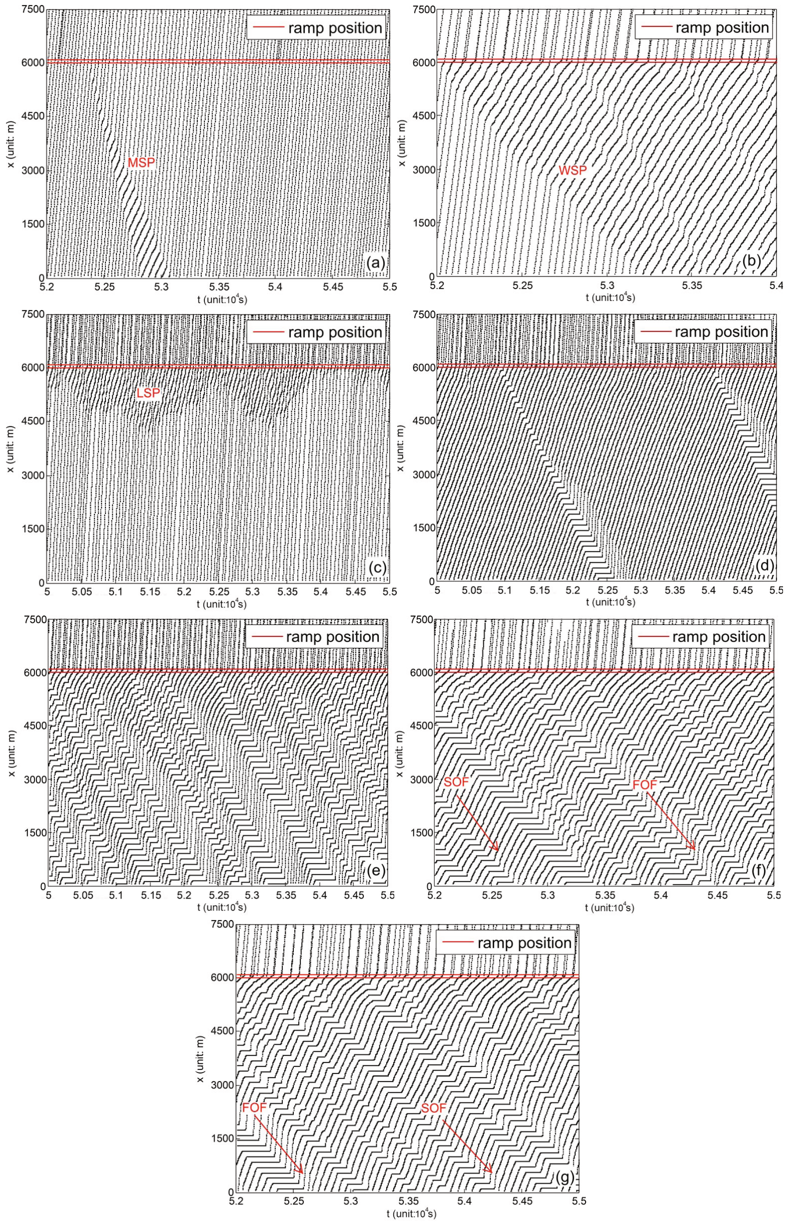

Fig. 1. Trajectories of every 20^{th} vehicle of the NH model. (a) $q_{\text{in}} = 2339, q_{\text{on}} = 19$ (MSP), (b) $q_{\text{in}} = 1728, q_{\text{on}} = 968$ (WSP), (c) $q_{\text{in}} = 1440, q_{\text{on}} = 823$ (LSP), (d) $q_{\text{in}} = 1134, q_{\text{on}} = 1123$ (DGP), (e) $q_{\text{in}} = 920, q_{\text{on}} = 1304$ (GP), (f) $q_{\text{in}} = 931, q_{\text{on}} = 1304$ (GP), (g) $q_{\text{in}} = 933, q_{\text{on}} = 1011$ (GP) (unit: veh/h). The horizontal direction (from left to right) is time and the vertical direction (from down to up) is space. (f) $P_b = 0.5$, (g) $P_b = 0.55, T = 1.6, g_{\text{safety}} = b_{\text{defens}} = 2$. 'SOF' and 'FOF' represent the synchronized outflow and free outflow of wide moving jams, respectively.

the downstream MSP front. This is better than the results in most three-phase models which often have propagation velocities as negative as −40km/h or even more negative.

4 Conclusion

The fundamental diagram approach and three-phase traffic flow theory were established to explore the mechanisms of traffic flow. The fundamental diagram approach assumes the existence of the unique space-gap-speed relationship, while the three-phase theory presumes drivers can make arbitrary choice of the space gap within some gap range. One of the most important similarities between both theories is that they both admit the existence of stationary homogeneous traffic in the congested traffic.

In this paper, another assumption is assumed to explain the mechanisms of road traffic flow that in the noiseless limit, vehicles' space gap will oscillate around the desired space gap, rather than keep the desired space gap, in the homogeneous congested traffic flow. It means there are no steady states of congested traffic. In order to verify this assumption, a new model named as the cellular automaton model with non-hypothetical congested steady state (NH model) is proposed. Simulations obtained from an open road with an on-ramp show that NH model can produce the synchronized flow, two kinds of phase transitions i.e. F→S transition and S→J transition, and multiple congested patterns observed by the three-phase theory.

In summary, the NH model produces the same spatiotemporal dynamics as many of the more complex three-phase models. Besides many aspects that are consistent with traffic data, it also includes a feature that is at variance with observations: the propagation velocity of MSP is twice than that of the downstream jam front, while observations indicate that both velocities are of the same order (with values between −20 and −15km/h). It illustrates that the two-dimensional steady states of synchronized flow are not an essential requirement for the spatiotemporal dynamics.

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