

The Caterpillar Gallery: Quadratic Surface Theorems, Parametric Design and Digital Fabrication

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Abstract The use of certain quadratic surface theorems has mainly been associated in architecture with the design of classical vaults, domes and piping. The work presented by the authors is intended to explore the potential of these theorems to be used in the generation law for more complex shapes in contemporary architecture. The paper shows the case study of a built full-scale prototype, The Caterpillar gallery, a project stemming from the combination of geometric research and teaching innovation.

Formal and structural experimentation take place in this project where, by starting from geometrical considerations, an efficient way of generating longitudinal spaces is proposed. One of the mentioned theorems applied to rotational cones provides the starting point for the generation of a set of concatenated surfaces that, once assembled, constitute a very stable self-supporting structure with a variety of possible applications.

1 Origin and Purpose

The use of parametric design and digital fabrication techniques is becoming widespread in architectural practice and provides the focus for many studies. The capacity of today's and tomorrow's software shifted conventional process into the realm of digital (Bentscheff and Gengnagel 2010), hence this should have direct repercussions within the academic sphere. Indeed, a number of courses on these topics are offered, both in undergraduate and graduate studies. However, these technologies tend to be implemented in the last years of the curriculum and are commonly unfastened from the beginnings of the training period (Fig. 1).

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Fig. 1 Photographs of detail and general view of the installation



Fig. 2 Virtual model of the project. *Left*: East-front view. *Right*: South-side view

The Caterpillar is a project to materialise research carried out over several years to blend advances in geometry and technology with those of teaching, and to make them available to students at the earliest stage of their undergraduate training. The idea is that students learn the fundamentals of architectural geometry within a context where technological advances are commonplace Pottmann et al. (2007) and Weinstock (2010). This would allow students to acquire competences in the use of these technologies so that they can be developed and rendered during the rest of their training period in a more motivating and efficient way. On the other hand, attention must be paid in order not to miss essential competences and generate gaps of knowledge. Although these academic matters related to the development of the project were also revealing, they lie outside the scope of this paper, which focuses on the case study of the architectural process itself and the potential of its findings for application or use in other pieces of work or research (Fig. 2).

Within this context, the challenge was to investigate the creation of a motivating and attractive project to develop, fabricate and assemble a full-scale prototype. The main components were: simple generation law, covering of the fundamentals of architectural geometry throughout the whole process (with an emphasis on the use of developable surfaces), and the possibility of rendering the results through further research and development. Computational systems, basic parametric design, and digital fabrication provided the tools for the project to come to fruition.

The generation of a ten-meter-long gallery was proposed which symbolically connects the two schools (Architecture and Building Engineering of the University



Fig. 3 Photographs of the built full-scale prototype. *Left:* top view. *Right:* interior view

of Seville – Spain) in their adjoining gardens. The longitudinal path had predominance over the other dimensions and the generative law was articulated around that path to finally result in a set of conical surfaces. Parameterisation was also useful to respond to the location and environmental requirements during the design process (Fig. 3).

2 Quadratic Surface Theorems and Generation of the Form

Although simple uses of these theorems can be found in architecture in ancient times, no written evidence has been found about their systematisation until the modern day. The geometers who contributed towards the statement of theorems of the intersection of quadrics come from the French tradition in the 18th century; this statement was later developed in the nineteenth and twentieth centuries, and has even been continued by certain authors in recent times (Gentil-Baldrich 1997).

As a result of several studies on classic vaults, where theorems of the intersection of quadrics were involved, it was understood that their implicit geometric relationships were also applicable as a resource for the generation of compositions of a more complex nature through the use of parametric design. For the generative law proposed in this paper, the following statement by Gaspard Monge, has been chosen (Taibo 1983): “*If two quadratic surfaces $C1$ and $C2$ are circumscribed about a third $C3$ along their contact curves $c1$ and $c2$, their intersection curve decomposes into two planar curves $i1$ and $i2$, which pass through the points A and B , which are common to the contact curves $c1$ and $c2$.*”

By *circumscribed* is meant that $C1$ and $C2$ osculate or touch $C3$ along curves on which the surfaces share the local tangent plane. In this work, quadrics or quadratic surfaces $C1$ and $C2$ are rotational cones and quadric $C3$ is a sphere. This ensures that the intersection curves of the cones are planar curves, particularly conic sections and, in this case, ellipses.

The interest of forcing the curves to be planar lies mainly in the analysis and construction simplicity. Cones are surfaces which offer an easy-to-model geometry

and, since they are developable, are also easy to construct. Apart from this, from the design point of view, cones provide the possibility of defining various degrees of spaciousness since their geometry converges at a point.

2.1 Geometry

Due to the lineal nature of the required space, the first element of the generation law consists of a 3D polygonal line that travels through a determined path. This polygonal line is composed of vertices whose positions are alternately near the level of the floor or near the average height of a human. Each of the segments of this polygonal line constitutes the axis of the rotational cones that form the gallery. These cones should be built by circumscribing them about spheres so that their intersections result in planar curves (ellipses). From these curves, only those that connect the inner space of each pair of cones should be preserved. Finally, only the fragment of the conical surface between the planar curves that connect the cones and the plane defined by the floor is maintained (Figs. 4, 5, and 6).

At every point of the 3D polygonal line, there is the centre of a sphere which defines the spaciousness required by every point position along the path, thereby providing the possibility of generating a different space at the floor level from the space at the ceiling level. In the end, the parameter to control the volume of the interior space is the radius of the relevant sphere. For this purpose, six spheres at the lower level and five spheres at the upper level were created.

Figure 5 shows the polygonal, the spheres and the boundary resulting from the intersection between the set of cones and the plane defined by the floor. This boundary not only defines the limits of the gallery on a ground level, but it also

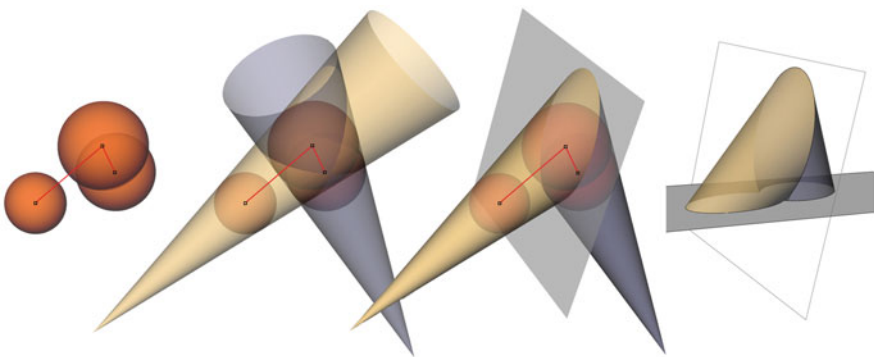


Fig. 4 Perspective view of the generation law for the first pair of conical fragments. From *left to right*: polygonal line and spheres at every vertex of the line, cones circumscribed about the spheres, cutting plane containing the common ellipse of connection for the two inner spaces, cutting plane defined by the floor

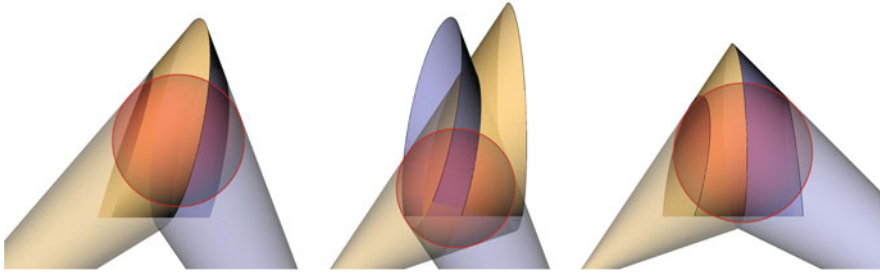


Fig. 5 Front view of the growth law for the first four conical fragments. *Left:* cones 1 and 2 circumscribed about the upper sphere 2. *Middle:* cones 2 and 3 circumscribed about the lower sphere 3. *Right:* cones 3 and 4 circumscribed about the upper sphere 4

plays a decisive role as an auxiliary element of support and acquisition of the specific curvature of each wooden surface.

2.2 Parametric Definition

The generation law was defined parametrically in Grasshopper, under Rhinoceros, to facilitate the creation and control of the form within the environment provided for the installation. The geometric principles defined in previous sections and the implicit relationship between the three-dimensional shapes and certain two-dimensional constructions were key concepts to define the parameterisation. The plane defined by the two axes of every pair of adjacent cones was the construction plane to carry out the relevant two-dimensional geometric constructions.

The definition was thought so that different solutions were easily created by modifying the two groups of input data: (i) the set of points defining the path of the 3D polygonal line and (ii) the spaciousness desired at every point (radius of the relevant sphere), where odd points refer to the lower level and even points refer to the upper level (Fig. 7).

These two groups of parameters control the position and the space created by the definition and are opened to be modified by the final user without modifying the geometric principles which govern the definition.

3 Structural Analysis and Experimentation

The use of developable surfaces considerably facilitated the experimentation with paper and cardboard as well as the relevant tests to transfer the scale model behaviours to the full-scale model. These are also surfaces which are simple to understand, generate and handle.

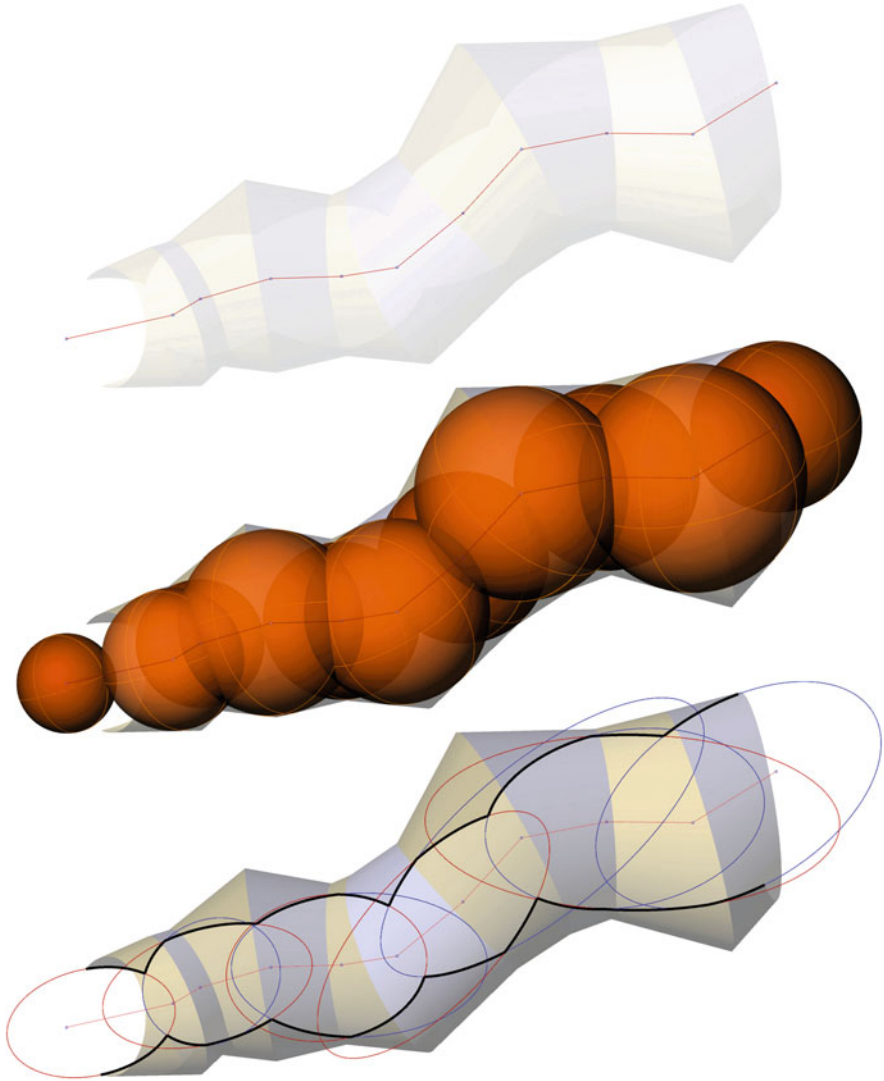


Fig. 6 Top view of the generation and growth processes. *Top*: axes defining the generation path. *Middle*: lower spheres (*dark*) and upper spheres (*light*) defining the spaciousness at the two levels. *Bottom*: intersection ellipses between the conical surfaces and floor defining the boundary at the floor level

The scale models turned out to be very useful to physically experiment with structural and constructive behaviour and with the assembly process. Furthermore, finite-element analysis was also implemented before the final step was taken. For this purpose, and before full-scale experimentation, two scale models were built; the first at 1:10 scale, and the second at 1:4 scale (Fig. 8).

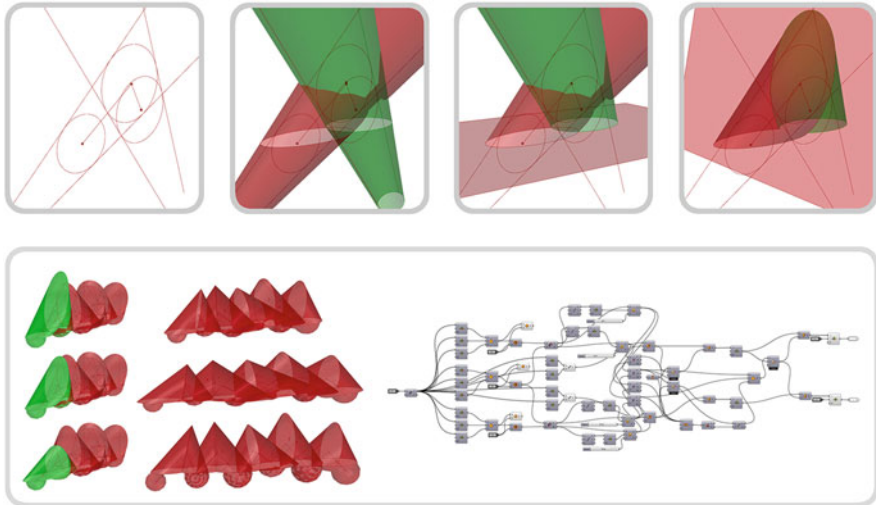


Fig. 7 Parametric definition of the model in Grasshopper for a pair of adjacent cones. *Top:* 2D construction to start the definition for a pair of cones. *Bottom left:* different variants of the resulting model after changing input data. *Bottom right:* Grasshopper screen definition



Fig. 8 Pictures of the students working on the initial model with an auxiliary structure. *Left:* interior view. *Right:* external view during the assembly process

3.1 Initial Model in Paper at 1:10 Scale

This model was mainly used as an introduction to the laser cutter machine within the academic tasks. It was useful to check certain spatial relationships as well as some perceptive matters. However, technical experimentation was very limited due to the small size and the difference in behaviour between the paper and the wood for the final prototype. An auxiliary structure made up of elliptic arches (common planar curves) was laid out as a guide to bend the paper and make it acquire the 3D form of the conical surfaces. It was later demonstrated that this structure was unnecessary. Nonetheless, this model was successful in generating sponsorship from companies.

3.2 Approaching the Real Behaviour; 1:4 Scale Model

After the first model, a new model was needed on a bigger scale (1:4) for the following purposes:

- To approach the real size, and consequently, the real structural behaviour of the set of surfaces. This allowed us to check that surfaces, once linked together, constituted a stable set with considerable rigidity.
- To define details and test their behaviour. Two major developments were achieved in this regard: (i) The use of joining strips for panel assembly, which had to satisfy the main target on this matter; to ensure curvature continuity between adjacent panels after bending the planar developments. For this purpose, the necessary width for the strips was established at 150 mm. Moreover, the joint had to be executed with four lines of screws; two along the longest boundaries of the strips and another two on both sides of the panel junction line. (ii) The solution of the junctions between the surfaces and the floor; thirty-millimetre-thick wooden wedges were used to hold the surface and cover the angle between the surface and the floor.
- To find a procedure to turn the installation into an easy-to-assemble construction. In this regard, and considering the ephemeral nature of the gallery, the use of nylon cable ties proved highly convenient. The key geometrical principle was the fact that joining points from the planar developments of different adjacent surfaces, whose positions in the space coincide, would make the surfaces acquire their 3D shape automatically. Thus the procedure consisted of using the nylon ties by way of threads to make the relevant points meet at their common position in the space. In order to ensure the correct operation of this principle, it was essential to first lay out the surface-floor edge along the rigid ellipses defined by the wedges. Otherwise, there would have been an infinite number of undesired 3D forms different from the original model.

3.3 Experimenting at 1:1 Scale

Finally, experimentation at 1:1 scale was also carried out, especially to test the behaviour differences as well as to check the feasibility of the assembly and of the set-up procedure. The main points and decisions were:

- Approve the material. After considering metal sheets and wooden panels, the final election was the use of only five-millimetre-thick black poplar wood laminate panels to materialise the project. The test carried out with an isolated conical surface revealed considerable deformations in all three dimensions as well as instability, with symptoms of near collapse. Nevertheless, the behaviour of the joining strips was very successful and curvature continuity after bending was no problem at all as can be appreciated in Fig. 9. Moreover, as expected,

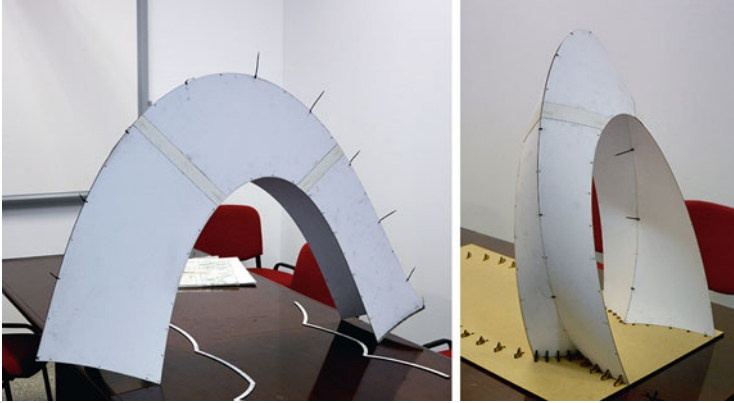


Fig. 9 *Left:* undesired form acquired by the linked surfaces without having been previously set to the rigid shape defined by the wedges on the floor. *Right:* in contrast, this picture shows the simple way to acquire the form by linking the surfaces first to the rigid shape of the floor. Joining strips between pieces to make up the planar unfolding of the surfaces can be appreciated, as well as the continuity of the surface after bending

the curvature radius was perfectly supported by the panels without cracking; the minimum existing radius in the project was 793 mm while the radius supported by the panels in the tests was under 600 mm before starting to crack.

- Test the structural behaviour of the linked surfaces. Although the test of an isolated surface was not successful in terms of structural behaviour, the gallery was expected to behave successfully after linking all the surfaces. The emergence of this new structural behaviour was proved with a finite-element structural analysis. Whereas the deformed shape of the isolated surface showed displacement values over 700 mm in the z axis, the analysis of the whole set of linked surfaces showed displacement values under 20 mm at the same point (Fig. 10).
- Check the feasibility of the assembly procedure. The trials carried out with the first two surfaces were enough to demonstrate that the procedure stated for the 1:4 scale model was perfectly valid at 1:1 scale. The details are described in the following section.

4 Fabrication and Assembly

The digital fabrication of the installation was carried out in Fab Lab Sevilla, with the collaboration of the Higher Technical School of Architecture and the Higher Technical School of Building Engineering of the University of Seville. A laser cutter machine was used for the fabrication of the scale models and a three-axis CNC milling machine for the full-scale prototype. The entire panel fabrication was done with 2D cutting profiles. That is not to say that fabricator was not working in 3D,

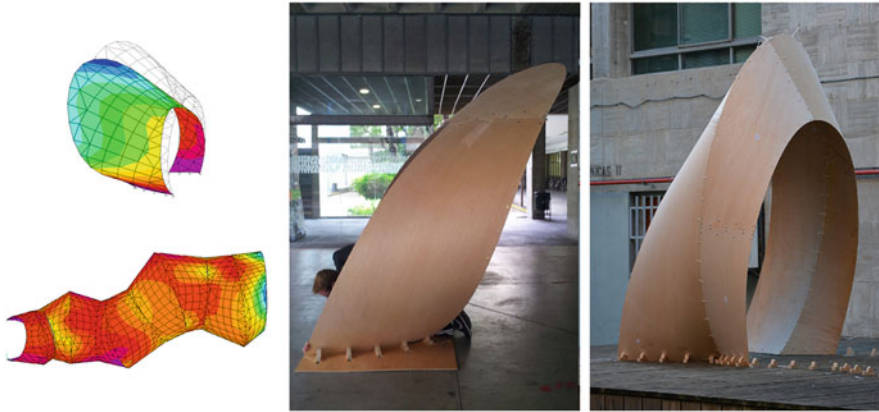


Fig. 10 *Left*: finite-element analysis. Maximum deformations are represented in *blue*: 700 mm for the isolated surface (*top*) and 40 mm for the set of surfaces (*bottom*). Photographs of the full-scale experiments. *Middle*: unacceptable deformation of the isolated conical surface. *Right*: successful behaviour of the joining strips to ensure curvature continuity after bending

but the understanding of 3D had more to do with assembly and fabrication of actual materials in space instead of digital models (Meredith and Kotronis 2012).

As stated in previous sections, the first step required for the correct wood bending and assembly of the different surfaces is the installation of the wedges. These were fabricated from 30-millimetre-thick black poplar plywood panels and would have to be laid out accurately in order to help wood acquire the correct shape. For this purpose, a laser total station combined with full-scale wooden templates were used to set them out on the definite location. Due to the different angle (measured on a vertical plane) that the surfaces form with the floor at every point, 100 hundred different wedges were fabricated and numbered (10 pieces per surface). The right position of these wedges facilitated considerably the later easy installation of the wooden conical fragments (Figs. 11 and 12).

Each conical surface needed a minimum of three standard ($1,250 \times 2,500$ mm) plywood panels and a maximum of seven. For the development of the digital fabrication, the details were generated with the conventional use of CAD drawings and the auxiliary support of the parametric 3D model. At this level of undergraduate studies, students still lacked the necessary proficiency to generate automated 2D outputs from the parametric model. However, a full automated process is perfectly feasible for the system proposed.

The assembly of the panels to complete a whole fragment of unfolded conical surface had to be executed in two stages: (i) the first phase was the assembly of panels until they reach the transport limit size, (ii) the second phase, after transportation, was the completion of each unfolded surface before setting them up in the definite position. This work was carried out on the floor, without the need to operate on the two sides of the surface, thanks to the use of steel t-nuts hammered into the plywood panels.



Fig. 11 *Left*: auxiliary sectioning planes (in vertical position) to determine the shapes of the wedges. *Middle*: photograph of the definite position of the wedges after setting out with the total station and the full-scale templates. *Right*: installation of nylon cable ties to link the wooden surfaces to the wedges

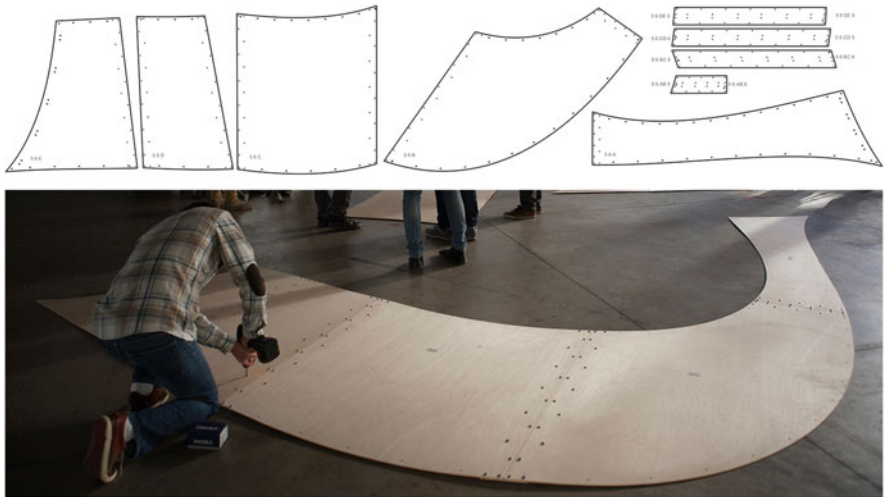


Fig. 12 *Top*: detail of the different pieces/panels to make up the unfolded surface. *Bottom*: photograph of the second phase of the panels assembly

The final task was the installation and linkage of the conical surfaces. The steps taken for this purpose can be summarised as follows:

- Transportation of the unfolded surfaces from the assembly point to the installation place. Special attention had to be paid in order not to subject the surfaces to curvature radii under the limit supported by the wood. To ensure the integrity of the panels, they were transported trying to keep the original planarity until the final bending time.
- Bending of the surfaces. Once the surface was at the installation place, bending would be executed in the following way; first, the feet of the surfaces must be



Fig. 13 *Left*: transportation of the unfolded surface to the installation place. *Middle*: bending of the surfaces. *Right*: final bending of the surface and installation of ties

taken to the definite position at the same time as the upper part of the surface must be raised and held to avoid inappropriate deformations (Fig. 13).

- Installation of the nylon cable ties. To conclude the process, and once the surface has been bended and held by hand, cable ties must be applied by starting with the linkage of the surface and the wedges on the floor, and then, at the connection with the previously installed surface by starting with the keystone of the common arch.

Conclusion

The theorem of the intersection of quadrics studied, applied to rotational conical surfaces under the conditions stated in this work, together with their parameterisation, constitute an efficient generation law for the creation of longitudinal spaces.

The geometrical principles that govern the process provide the set of surfaces with very stable structural behaviour; a far cry from the weak properties of each of the isolated surfaces which make up the whole. This renders the use of any auxiliary structure unnecessary since the set operates as a single self-supporting surface. Hence, although the material needed to carry out this kind of installation is subject to only minor strength capacities, the main requirement is flexibility to support the minimum existing bending radius. In this work, a single layer of only five-millimetre-thick wooden laminate panels perfectly matched the needs of the project.

The simplicity of conception, fabrication and execution, in addition to high structural stability, draws interest not only to the solution achieved in this project, but also to the potential applicability of this concept to other uses; other applications on ephemeral architecture, non-longitudinal constructions by extension of the generation and growth laws to 2D and 3D laws for the design of spaces with a greater freedom of form, double-layer structures, as well as the generation of components that can be adapted for the materialization of other kinds of design surfaces (Fig. 14).

(continued)

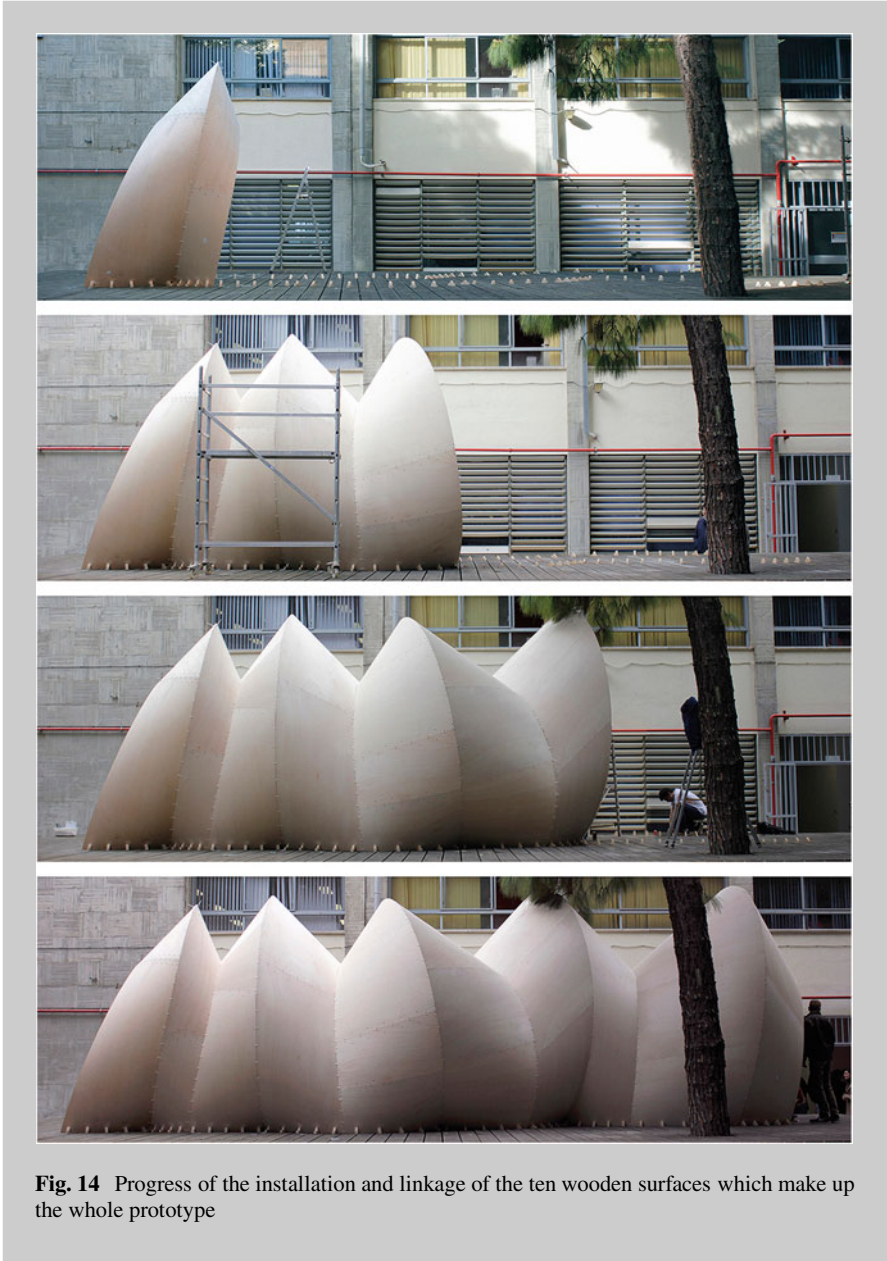


Fig. 14 Progress of the installation and linkage of the ten wooden surfaces which make up the whole prototype

Acknowledgements We would like to thank Fab Lab Sevilla and the schools of Building Engineering and Architecture of the University of Seville, as well as the group of students and lectures from these schools for their support and collaboration throughout the development,

fabrication and assembly of this project. Thanks to the sponsors which funded the fabrication of the gallery, especially to Luz Diaz for the photography and Visualmov for the production of the documentary short film about the fabrication and assembly (<http://vimeo.com/88452240>).

References

- Bentscheff, I., Gengnagel, C.: Towards teaching generative design in architecture. In: *Advances in Architectural Geometry*, pp. 113–128. TU Wien, Vienna (2010)
- Gentil-Baldrich, J. M.: *Sobre la intersección de las cuádras de revolución de ejes paralelos*. Universidad de Sevilla – Departamento de Expresión Gráfica Arquitectónica, Spain (1997)
- Meredith, N., Kotronis, J.: Self-detailing and self-documenting systems for wood fabrication: the Burj Khalifa. In: *Advances in Architectural Geometry*, pp. 185–198. TU Wien, Vienna (2012)
- Pottmann, H., Asperl, A., Hofer, M., Kilian, A.: *Architectural Geometry*. Bentley Institute Press, Exton (2007)
- Taibo-Fernández, A.: *Geometría Descriptiva y sus aplicaciones*. Tomo II. Tébar Flores, Madrid (1983)
- Weinstock, M.: *The Architecture of Emergence : The Evolution of Form in Nature and Civilisation*. Wiley, Chichester (2010)