Differences between Moore and RDM Interval Arithmetic

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Abstract. The uncertainty theory solves problems with uncertain data. Often to perform arithmetic operations on uncertain data, the calculations on intervals are necessary. Interval arithmetic uses traditional mathematics in the calculations on intervals. There are many methods that solve the problems of uncertain data presented in the form of intervals, each of them can give in some cases different results. The most known arithmetic, often used by scientists in calculations is Moore interval arithmetic. The article presents a comparison of Moore interval arithmetic and multidimensional RDM interval arithmetic. Also, in both Moore and RDM arithmetic the basic operations and their properties are described. Solved examples show that the results obtained using the RDM arithmetic are multidimensional while Moore arithmetic gives one-dimensional solution.

Keywords: interval arithmetic, uncertainty theory, fuzzy arithmetic, granular computing, computing with words.

1 Introduction

Interval arithmetic was deemed as necessary with the development of the theory of uncertainty [1]. It was realized that the use of uncertain parameters and uncertain data is very important for the description of reality in the form of a mathematical model. Interval arithmetic is used in scientific fields such as uncertainty theory [1], grey systems [4], granular computing [8], fuzzy systems [3,7], to determine the uncertain data and modeling of uncertain systems. The most common and most frequently used interval arithmetic is Moore arithmetic [5,6,8]. A number of limitations and the drawbacks has been found in the Moore interval arithmetic [2,9,14] such as: the excess width effect problem, dependency problem, difficulties of solving even simplest equation problem, interval equation's right-hand side problem, absurd solutions and request to introduce negative entropy into the system problem. In Moore arithmetic basic operations on intervals $A = [\underline{a}, \overline{a}]$ and $B = [\underline{b}, \overline{b}]$ are realized by formulas (1).

$$\begin{aligned}
[\underline{a}, \overline{a}] + [\underline{b}, \overline{b}] &= [\underline{a} + \underline{b}, \overline{a} + \overline{b}] \\
[\underline{a}, \overline{a}] - [\underline{b}, \overline{b}] &= [\underline{a} - \overline{b}, \overline{a} - \underline{b}] \\
[\underline{a}, \overline{a}] \cdot [\underline{b}, \overline{b}] &= [\min(\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}), \max(\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b})] \\
[\underline{a}, \overline{a}] / [\underline{b}, \overline{b}] &= [\underline{a}, \overline{a}] \cdot [1/\overline{b}, 1/\underline{b}] \ \text{ if } 0 \notin [\underline{b}, \overline{b}]
\end{aligned} \tag{1}$$

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The alternative for Moore arithmetic can be multidimesional RDM interval arithmetic. The idea of multidimensional RDM arithemtic was developed by A. Piegat [10,11,12,13]. Abbreviation RDM stands for Relative Distance Measure where given value x from interval $X = [\underline{x}, \overline{x}]$ is described using RDM variable α_x, $\alpha_x \in [0,1]$, as shown in (2).

$$x = \underline{x} + \alpha_x(\overline{x} - \underline{x}) \tag{2}$$

In notation RDM the interval $X = [\underline{x}, \overline{x}]$ is described in the form (3).

$$X = \{x : x = \underline{x} + \alpha_x(\overline{x} - \underline{x}),\ \alpha_x \in [0,1]\} \tag{3}$$

The RDM variable α_x gives possibility to obtain any value between left border $\underline{x}$ and right border $\overline{x}$ of interval X. For $\alpha_x = 0$ the value from interval X equals $\underline{x}$ and the variable $\alpha_x = 1$ gives $\overline{x}$. Lets consider value $x \in [2,4]$, in RDM notation the value x is written as $x = 2 + 2\alpha_x$, where $\alpha_x \in [0,1]$. Fig. 1 shows the interval $X = [\underline{x}, \overline{x}]$ and the meaning of the RDM variable α_x in case $\underline{x} \le \overline{x}$.

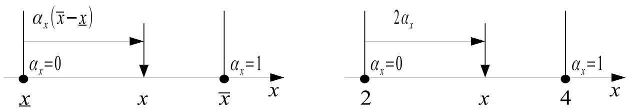

Fig. 1. The interval $X = [\underline{x}, \overline{x}]$ and the meaning of the RDM variable $\alpha_x \in [0,1]$, $\underline{x} \le \overline{x}$

2 Operations in RDM Interval Arithmetic

In RDM arithmetic the following operations are definied: addition, subtraction, multiplication and division. Depending on the numbers of variables in the calculations, the obtained solution is in multidimensional space, in Moore arithmetic the solution are in 1-dimensional space.

Let X and Y are two intervals: $X = [\underline{x}, \overline{x}] = \{x : x = \underline{x} + \alpha_x(\overline{x} - \underline{x}),\ \alpha_x \in [0,1]\}$ and $Y = [\underline{y}, \overline{y}] = \{y : y = \underline{y} + \alpha_y(\overline{y} - \underline{y}),\ \alpha_y \in [0,1]\}$.

Addition in RDM

$$X + Y = \{x + y : x + y = \underline{x} + \alpha_x(\overline{x} - \underline{x}) + \underline{y} + \alpha_y(\overline{y} - \underline{y}),\ \alpha_x, \alpha_y \in [0,1]\} \tag{4}$$

Subtraction in RDM

$$X - Y = \{x - y : x - y = \underline{x} + \alpha_x(\overline{x} - \underline{x}) - \underline{y} - \alpha_y(\overline{y} - \underline{y}),\ \alpha_x, \alpha_y \in [0,1]\} \tag{5}$$

Multiplication in RDM

$$X \cdot Y = \{xy : xy = [\underline{x} + \alpha_x(\overline{x} - \underline{x})] \cdot [\underline{y} + \alpha_y(\overline{y} - \underline{y})],\ \alpha_x, \alpha_y \in [0,1]\} \tag{6}$$

Division in RDM

$$X/Y=\{x/y : x/y = [\underline{x} + \alpha_x (\overline{x} - \underline{x})] / \left[\underline{y} + \alpha_y \left(\overline{y} - \underline{y}\right)\right], \alpha_x, \alpha_y \in [0,1]\}, \text{if } 0 \notin Y \quad (7)$$

For intervals $X = [\underline{x}, \overline{x}]$ and $Y = \left[\underline{y}, \overline{y}\right]$ and the base operations $* \in \{+, -, \cdot, /\}$ span is an interval defined as (8), operation / is defined only if $0 \notin Y$.

$$s(X * Y) = [\min\{X * Y\}, \max\{X * Y\}] \quad (8)$$

Example 1. To show multidimensionality of solution in RDM arithmetic and 1-dimensional solution in Moore arithmetic, we will consider operations such as addition, subtraction, multiplication and division of two intervals $A = [1, 3]$ and $B = [3, 4]$.

The first solution in 1-dimension space by Moore arithmetic will be presented by equations (9).

$$\begin{aligned} &A + B = [1, 2] + [3, 4] = [4, 6] \\ &A - B = [1, 2] - [3, 4] = [-3, -1] \\ &A \cdot B = [1, 2] \cdot [3, 4] = [3, 8] \\ &A/B = [1, 2]/[3, 4] = [1, 2] \cdot [1/4, 1/3] = [1/4, 2/3] \end{aligned} \quad (9)$$

To find solutions by RDM arithmetic we should write intervals in RDM notation using RDM variable α_a and α_b, where $\alpha_a \in [0, 1]$ and $\alpha_b \in [0, 1]$, formula (10).

$$\begin{aligned} &A = [1, 2] = \{a : a = 1 + \alpha_a,\ \alpha_a \in [0, 1]\} \\ &B = [3, 4] = \{b : b = 3 + \alpha_b,\ \alpha_b \in [0, 1]\} \end{aligned} \quad (10)$$

Obtained solutions are presented in equations (11).

$$\begin{aligned} &A + B = \{a + b : a + b = 4 + \alpha_a + \alpha_b,\ \alpha_a, \alpha_b \in [0, 1]\} \\ &A - B = \{a - b : a - b = -2 + \alpha_a - \alpha_b,\ \alpha_a, \alpha_b \in [0, 1]\} \\ &A \cdot B = \{ab : ab = 3 + 3\alpha_a + \alpha_b + \alpha_a\alpha_b,\ \alpha_a, \alpha_b \in [0, 1]\} \\ &A/B = \{a/b : a/b = (1 + \alpha_a)/(3 + \alpha_b),\ \alpha_a, \alpha_b \in [0, 1]\} \end{aligned} \quad (11)$$

To show the illustration of solution obtained by RDM arithmetic we should find border values of the results, Table 1.

Illustration of the solutions obtained by RDM arithmetic presents Fig. 2.

Spans of the 3-dimensional solutions (12), (13), (14) and (15) are the same as intervals calculated by Moore arithmetic (9).

$$s(A + B) = \left[\min_{\substack{\alpha_a \in [0,1] \\ \alpha_b \in [0,1]}} (4 + \alpha_a + \alpha_b),\ \max_{\substack{\alpha_a \in [0,1] \\ \alpha_b \in [0,1]}} (4 + \alpha_a + \alpha_b)\right] = [4, 6] \quad (12)$$

$$s(A - B) = \left[\min_{\substack{\alpha_a \in [0,1] \\ \alpha_b \in [0,1]}} (-2 + \alpha_a - \alpha_b),\ \max_{\substack{\alpha_a \in [0,1] \\ \alpha_b \in [0,1]}} (-2 + \alpha_a - \alpha_b)\right] = [-3, -1] \quad (13)$$

Table 1. Results of the basic operations for two intervals $A = [\underline{a}, \overline{a}] = [1, 2]$ and $B = [\underline{b}, \overline{b}] = [3, 4]$ for border values of RDM-variables $\alpha_a \in [0, 1]$ and $\alpha_b \in [0, 1]$

α_a	0	0	1	1
a	1	1	2	2
α_b	0	1	0	1
b	3	4	3	4
$a+b$	$\underline{a}+\underline{b}$	$\underline{a}+\overline{b}$	$\overline{a}+\underline{b}$	$\overline{a}+\overline{b}$
	4	5	5	6
$a-b$	$\underline{a}-\underline{b}$	$\underline{a}-\overline{b}$	$\overline{a}-\underline{b}$	$\overline{a}-\overline{b}$
	-2	-3	-1	-2
ab	$\underline{a}\underline{b}$	$\underline{a}\overline{b}$	$\overline{a}\underline{b}$	$\overline{a}\overline{b}$
	3	4	6	8
a/b	$\underline{a}/\underline{b}$	$\underline{a}/\overline{b}$	$\overline{a}/\underline{b}$	$\overline{a}/\overline{b}$
	1/3	1/4	2/3	1/2

$$s(AB) = \left[\min_{\substack{\alpha_a \in [0,1] \\ \alpha_b \in [0,1]}} (3 + 3\alpha_a + \alpha_b + \alpha_a\alpha_b), \max_{\substack{\alpha_a \in [0,1] \\ \alpha_b \in [0,1]}} (3 + 3\alpha_a + \alpha_b + \alpha_a\alpha_b) \right] = [3, 8] \tag{14}$$

$$s(A/B) = \left[\min_{\substack{\alpha_a \in [0,1] \\ \alpha_b \in [0,1]}} [(1 + \alpha_a)/(3 + \alpha_b)], \max_{\substack{\alpha_a \in [0,1] \\ \alpha_b \in [0,1]}} [(1 + \alpha_a)/(3 + \alpha_b)] \right] = [1/4, 2/3] \tag{15}$$

As it can be seen in Fig. 2 spans are only partial information pieces about full 3-dimensional result granules.

3 Properties of RDM and Moore Interval Arithmetic

Commutativity

Both Moore arithmetic and RDM arithnetic are commutative. For any intervals X and Y equations (16) and (17) are true.

$$X + Y = Y + X \tag{16}$$

$$X \cdot Y = Y \cdot X \tag{17}$$

Associativity

Also it is easy to show that both interval addition and multiplication in Moore arithmetic and RDM arithnetic are associative. For any intervals X, Y and Z there are true equations (18) and (19).

$$X + (Y + Z) = (X + Y) + Z \tag{18}$$

$$X \cdot (Y \cdot Z) = (X \cdot Y) \cdot Z \tag{19}$$

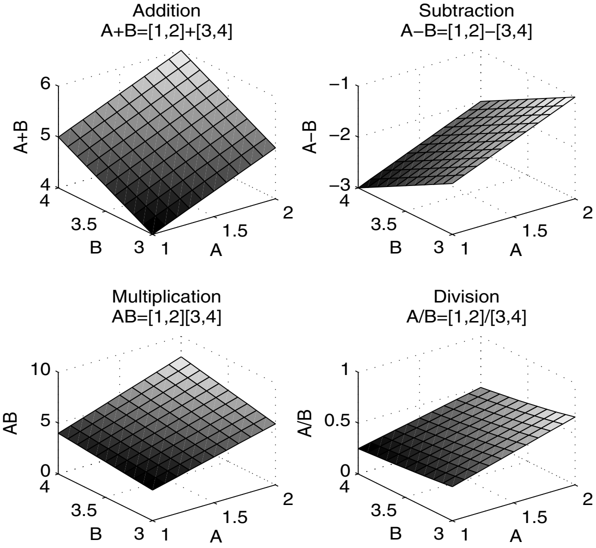

Fig. 2. Example of addition, subtraction, multiplication and division in RDM arithmetic of two intervals $A = [1, 2]$ and $B = [3, 4]$, 3-dimensional result

Neutral elements of addition and multiplication

In the conventional and RDM arithmetic there exist additive and multiplicative neutral elements such as degragative intervals 0 and 1 for any interval X, as shown in equations (20) and (21).

$$X + 0 = 0 + X = X \tag{20}$$

$$X \cdot 1 = 1 \cdot X = X \tag{21}$$

Inverse elements

In RDM arithmetic $-X = -[\underline{x}, \overline{x}] = \{-x : -x = -\underline{x} - \alpha_x (\overline{x} - \underline{x}),\ \alpha_x \in [0, 1]\}$, is an additive inverse element of interval $X = [\underline{x}, \overline{x}] = \{x : x = \underline{x} + \alpha_x (\overline{x} - \underline{x}),\ \alpha_x \in [0, 1]\}$, so:

$$X - X = \{x - x : x - x = \underline{x} + \alpha_x (\overline{x} - \underline{x}) - \underline{x} - \alpha_x (\overline{x} - \underline{x}),\ \alpha_x \in [0, 1]\} = 0. \tag{22}$$

An multiplicative inverse element of $X = \{x : x = \underline{x} - \alpha_x (\overline{x} - \underline{x}),\ \alpha_x \in [0, 1]\}$, if $0 \notin X$, in RDM arithmetic is $X = \{x : x = \underline{x} - \alpha_x (\overline{x} - \underline{x}),\ \alpha_x \in [0, 1]\}$:

$$X/X = \{x/x : x/x = [\underline{x} + \alpha_x (\overline{x} - \underline{x})] / [\underline{x} - \alpha_x (\overline{x} - \underline{x})],\ \alpha_x \in [0, 1]\} = 1. \tag{23}$$

In Moore arithmetic an additive inverse element for interval X does not exist, as equation (24) shows.

$$X - X = [\underline{x}, \overline{x}] - [\underline{x}, \overline{x}] = [\underline{x}, \overline{x}] + [-\overline{x}, -\underline{x}] = [\underline{x} - \overline{x}, \overline{x} - \underline{x}] \tag{24}$$

Equation (24) only for $\underline{x} = \overline{x}$ (if width of X is 0) equals $[0, 0]$.

In Moore arithmetic a multiplicative inverse element of interval X, $0 \notin X$, does not exist, shown in equation (25), except degenarate intervals where width equals zero.

$$X/X = X \cdot (1/X) = [\underline{x}, \overline{x}] \cdot [1/\overline{x}, 1/\underline{x}] = \begin{cases} [\underline{x}/\overline{x}, \overline{x}/\underline{x}] \text{ for } \underline{x} > 0 \\ [\overline{x}/\underline{x}, \underline{x}/\overline{x}] \text{ for } \overline{x} < 0 \end{cases} \tag{25}$$

Subdistributive law

The subdistributive law (26) in RDM arithmetic holds,

$$X(Y + Z) = XY + XZ. \tag{26}$$

Proof. For any three intervals described in RDM notation: $X = [\underline{x}, \overline{x}] = \{x : x = \underline{x} + \alpha_x (\overline{x} - \underline{x}),\ \alpha_x \in [0, 1]\}$, $Y = [\underline{y}, \overline{y}] = \{y : y = \underline{y} + \alpha_y (\overline{y} - \underline{y}),\ \alpha_y \in [0, 1]\}$ and $Z = [\underline{z}, \overline{z}] = \{z : z = \underline{z} + \alpha_z (\overline{z} - \underline{z}),\ \alpha_z \in [0, 1]\}$, we have:

$$\begin{aligned}
&X(Y + Z) = [\underline{x}, \overline{x}] \left([\underline{y}, \overline{y}] + [\underline{z}, \overline{z}]\right) \\
&= \{x(y + z) : x(y + z) = [\underline{x} + \alpha_x (\overline{x} - \underline{x})] \left[\underline{y} + \alpha_y \left(\overline{y} - \underline{y}\right) + \underline{z} + \alpha_z (\overline{z} - \underline{z})\right], \\
&\alpha_x, \alpha_y, \alpha_z \in [0, 1]\} \\
&= \{xy : xy = [\underline{x} + \alpha_x (\overline{x} - \underline{x})] \left[\underline{y} + \alpha_y \left(\overline{y} - \underline{y}\right)\right],\ \alpha_x, \alpha_y \in [0, 1]\} \\
&+ \{xz : xz = [\underline{x} + \alpha_x (\overline{x} - \underline{x})] [\underline{z} + \alpha_z (\overline{z} - \underline{z})],\ \alpha_x, \alpha_z \in [0, 1]\} \\
&= [\underline{x}, \overline{x}] [\underline{y}, \overline{y}] + [\underline{x}, \overline{x}] [\underline{z}, \overline{z}] = XY + XZ.
\end{aligned}$$

□

In Moore arithmetic the subdistributive law holds only in the form (27).

$$X(Y + Z) \subseteq XY + XZ. \tag{27}$$

Cancellation law

The cancellation law for addition of intervals (28) holds for both Moore and RDM arithmetic.

$$X + Z = Y + Z \Rightarrow X = Y \tag{28}$$

Proof. Concerns RDM arithmetic. For any intervals $X = \{x : x = \underline{x} + \alpha_x (\overline{x} - \underline{x}),\ \alpha_x \in [0, 1]\}$, $Y = \{y : y = \underline{y} + \alpha_y (\overline{y} - \underline{y}),\ \alpha_y \in [0, 1]\}$ and $Z = \{z : z = \underline{z} + \alpha_z (\overline{z} - \underline{z}),\ \alpha_z \in [0, 1]\}$ in RDM notation using inverse element of interval Z and associativity we have:

$$X + Z = Y + Z$$

$$[\underline{x}, \overline{x}] + [\underline{z}, \overline{z}] = [\underline{y}, \overline{y}] + [\underline{z}, \overline{z}]$$

$$\begin{aligned}
&\{x + z : x + z = \underline{x} + \alpha_x (\overline{x} - \underline{x}) + \underline{z} + \alpha_z (\overline{z} - \underline{z}), \alpha_x, \alpha_z \in [0, 1]\} \\
&= \{y + z : y + z = \underline{y} + \alpha_y \left(\overline{y} - \underline{y}\right) + \underline{z} + \alpha_z (\overline{z} - \underline{z}), \alpha_y, \alpha_z \in [0, 1]\}
\end{aligned} \tag{29}$$

Adding an inverse interval $-Z = \{-z : -z = -\underline{z} - \alpha_z (\overline{z} - \underline{z}),\ \alpha_z \in [0, 1]\}$, to both sides of equation (29), we obtain:

$$\{x : x = \underline{x} + \alpha_x (\overline{x} - \underline{x}), \alpha_x \in [0, 1]\} = \{y : y = \underline{y} + \alpha_y \left(\overline{y} - \underline{y}\right), \alpha_y \in [0, 1]\}$$

$$[\underline{x}, \overline{x}] = [\underline{y}, \overline{y}]$$
$$X = Y$$

□

Example 2 shows that multiplicative cancellation does not hold in interval arithmetic, from equation $XZ = YZ$ we cannot imply $X = Y$.

Example 2. Let us give three intervals: $X = [1, 3]$, $Y = [2, 3]$ and $Z = [-1, 1]$. Using Moore arithmetic values of multiplication XZ and YZ are equal (30), but X and Y are different intervals.

$$\begin{aligned} XZ &= [1,3][-1,1] = [-3,3] \\ YZ &= [2,3][-1,1] = [-3,3] \end{aligned} \tag{30}$$

To find a product by RDM arithmetic we write intervals X, Y and Z in RDM notation with RDM variable (31).

$$\begin{aligned} X &= [1,3] = \{x : x = 1 + 2\alpha_x,\ \ \alpha_x \in [0,1]\} \\ Y &= [2,3] = \{y : y = 2 + \alpha_y,\ \ \alpha_y \in [0,1]\} \\ Z &= [-1,1] = \{z : z = -1 + 2\alpha_z,\ \ \alpha_z \in [0,1]\} \end{aligned} \tag{31}$$

The solutions with RDM variable α_x, α_y and α_z are presented in (32).

$$\begin{aligned} XZ &= [1,3][-1,1] = \{xz : xz = (1 + 2\alpha_x)(-1 + 2\alpha_z), \alpha_x, \alpha_z \in [0,1]\} \\ YZ &= [2,3][-1,1] = \{yz : yz = (2 + \alpha_y)(-1 + 2\alpha_z),\ \ \alpha_y, \alpha_z \in [0,1]\} \end{aligned} \tag{32}$$

To find graphical illustration of solution, the border values should be computed, Table 2 and Table 3.

Table 2. Multiplication results of fwo intervals $XZ = [\underline{x}, \overline{x}]\,[\underline{z}, \overline{z}] = [1,3][-1,1]$ for border values of RDM-variables $\alpha_x \in [0,1]$ and $\alpha_z \in [0,1]$

α_x	0	0	1	1
x	1	3	1	3
α_z	0	1	0	1
z	-1	1	-1	1
xz	$\underline{xz}$	$\underline{x}\overline{z}$	$\overline{x}\underline{z}$	$\overline{xz}$
	-1	1	-3	3

Fig. 3 and Fig. 4 show fully 3-dimensional results of interval multiplication XZ and YZ, the span of solution in both cases are equal $[-3, 3]$, but the solution surfaces are different.

Comparing solutions obtained by Moore and RDM arithmetic we see that in Moore arithmetic multiplication interval $Z = [-1, 1]$ by different intervals $X = [1, 3]$ or $Y = [2, 3]$ $(X \neq Y)$ gives the same results $XZ = YZ$ and the differences in multiplication XZ and YZ are not noticeable. Analizing the solution obtained by RDM arithmetic Fig. 3 and Fig. 4 show that the results of multiplication

Table 3. Multiplication results of two intervals $YZ = [\underline{y}, \overline{y}]\,[\underline{z}, \overline{z}] = [2,3][-1,1]$ for border values of RDM-variables $\alpha_y \in [0,1]$ and $\alpha_z \in [0,1]$

α_y	0	0	1	1
y	2	2	3	3
α_z	0	1	0	1
z	-1	1	-1	1
YZ	$\underline{y}\underline{z}$	$\underline{y}\overline{z}$	$\overline{y}\underline{z}$	$\overline{y}\overline{z}$
	-2	2	-3	3

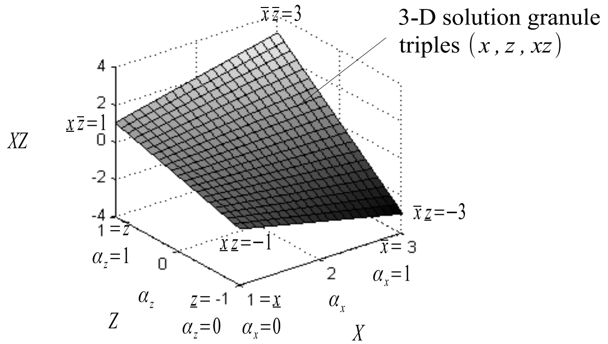

Fig. 3. 3-dimensional result of interval multiplication $XZ = [1,3][-1,1]$ with use of RDM interval arithmetic with span $[-3,3]$

intervals XZ and YZ are different, $XZ \neq YZ$. The span $[-3,3]$ are equal in both multiplications but the surfaces of solutions have different shapes.

Example 2 also shows that Moore arithmetic does not give a full solution, the solution obtained by Moore method is 1-dimensional and describes only the span. Example 3 shows that the results of operation made by Moore arithmetic depend on the form of equation.

Example 3. Let us consider the results of Moore and RDM interval arithmetic for nonlinear equation (33) where $A = [0,2]$.

$$C = A - A^2 \tag{33}$$

Equation (33) can take a form (34) and (35).

$$C = A(1 - A) \tag{34}$$

$$C = (A - 1) + (1 - A)(1 + A) \tag{35}$$

Calculating value C from equation (33), (34) and (35) for $A = [0,2]$ using Moore arithmetic we obtain different results (36), (37) and (38).

$$C_1 = [0,2] - ([0,2])^2 = [-4,2] \tag{36}$$

$$C_2 = [0,2](1 - [0,2]) = [-2,2] \tag{37}$$

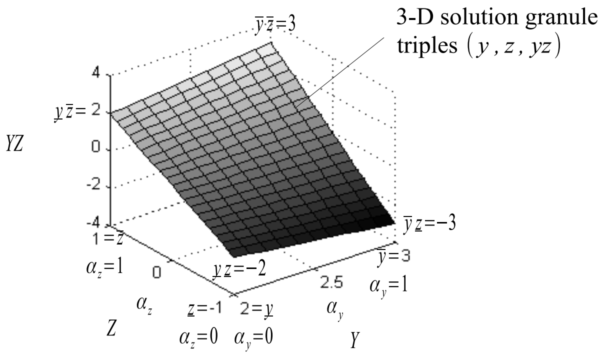

Fig. 4. 3-dimensional result of interval multiplication $YZ = [2, 3][-1, 1]$ with use of RDM interval arithmetic with span $[-3, 3]$

$$C_3 = ([0, 2] - 1) + (1 - [0, 2])(1 + [0, 2]) = [-4, 4] \tag{38}$$

Which solution is correct?

Solving equations (33), (34) and (35) by RDM arithmetic the inetrval $A = [0, 2]$ in notation RDM takes the form (39).

$$A = [0, 2] = \{a : a = 2\alpha_a,\ \alpha_a \in [0, 1]\} \tag{39}$$

The solution obtained by RDM arithmetic for different forms of the equation (33) gives the same results (40), (41) and (42).

$$C = A - A^2 = \{c : c = 2\alpha_a - 4\alpha_a^2,\ \alpha_a \in [0, 1]\} \tag{40}$$

$$\begin{aligned} C &= A(1 - A) = \{c : c = 2\alpha_a(1 - 2\alpha_a), \alpha_a \in [0, 1]\} \\ &= \{c : c = 2\alpha_a - 4\alpha_a^2, \alpha_a \in [0, 1]\} \end{aligned} \tag{41}$$

$$\begin{aligned} C &= (A - 1) + (1 - A)(1 + A) \\ &= \{c : c = (2\alpha_a - 1) + (1 - 2\alpha_a)(1 + 2\alpha_a), \alpha_a \in [0, 1]\} \\ &= \{c : c = 2\alpha_a - 4\alpha_a^2, \alpha_a \in [0, 1]\} \end{aligned} \tag{42}$$

The solution calculated by RDM arithmetic has only one RDM variable $\alpha_a \in [0, 1]$ so is 1-dimensional and has the form (43).

$$C = \left[\min_{\alpha_a \in [0,1]} (2\alpha_a - 4\alpha_a^2),\ \max_{\alpha_a \in [0,1]} (2\alpha_a - 4\alpha_a^2)\right] = [-2, 1/4] \tag{43}$$

4 Conclusions

The paper compares the Moore and RDM interval arithmetic. The results obtained with Moore arithmetic are one-dimensional, the RDM arithmetic gives a multidimensional solution. In some cases the solutions in Moore arithmetic depend on the form of the equation, so it suggests that Moore arithmetic cannot correctly solve more complicated problems. The RDM arithmetic for different forms of the equation gives the same results. The Moore arithmetic gives only the span, not a full solution, except one-dimensional problems where the solution is an interval.

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