Chapter 1
Multiple Criteria Decision Analysis

Abstract A probabilistic approach to multicriteria decision problems can take in due account the uncertainty that is inevitably present in preference evaluations. Translating the preference measurements according to different criteria into probabilities of being chosen as the best alternative has two advantages. First, it makes comparable preference evaluations that come in entirely different scales. Besides, it opens probabilistic ways to automatically combine the evaluations according to the multiple criteria.

Keywords Criterion · Multicriteria decision · Preference relation

1.1 Preference Relations

The multi-criteria decision problem consists of deriving a unique preference relation between certain alternatives from evaluations of these alternatives according to a set of different criteria.

To evaluate alternatives according to a criterion means, by considering some attribute, to associate a value to each of them. According to each criterion, a different preference relation can be stated, and the problem is to derive from them a unique combination of such preference relations.

To establish a preference relation means to rank the alternatives in such a way as to be able to say if each of them is preferable to each other or not.

1.2 Classes of Decision Problems

A particular class of decision problems is that of choice. Instead of a full ranking, what is then sought is to determine among the alternatives that which is the best for a particular purpose. This is a multi-criteria decision problem if there are different criteria according to which it is possible to compare the alternatives.

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Another case of decision problems is that of sorting, which consists of selecting from among some predetermined classes the most suitable to place an alternative. This can be seen as a choice problem in which what is chosen is, for each alternative, the most suitable from among a small number of classes. In this sorting problem, a preference between the alternatives is established whenever the predetermined classes, where such alternatives are classified, are previously ranked.

Each class is previously identified by a small set of representative alternatives. In the case of multi-criteria analysis, the vector of the evaluations of each of the class representative alternatives using the multiple criteria forms what is called a class reference profile.

1.3 Probabilities of Choice

In a probabilistic framework, attention is given to subjective aspects of the decision problem that make it impossible to evaluate the alternatives precisely. In the following chapters, an approach to take into account the presence of uncertainty in the assessments of preference and thereby to generate rules for ranking or sorting the alternatives based on probabilities of choice is presented for each decision problem.

The fact that the main interest of the decision maker—and often the sole interest—is to choose the best alternative offers a path to simplify the probabilistic modeling of the problem. In such an approach, the vectors of values of the attributes of interest give way to vectors of probabilities for presenting the best value for these attributes. Even if a ranking of all the options is desired, a better idea of the possibilities of ranks' inversion can be provided if the final ranking is derived from probabilities of being the best alternative.

Additionally, the importance of the different criteria for the choice becomes clearer if the corresponding evaluations are given in terms of probabilities of being the best according to each of them. Moreover, with all the evaluations given in the same terms, the problem of combining evaluations generated by employing different measurement standards is eliminated.

The next two chapters prepare the presentation of this probabilistic approach. After being fully developed in Chaps. 4–8, it is applied in the three last chapters in specific contexts.

1.4 Applications of the Probabilistic Approach

This probabilistic approach is applied, for instance, to the evaluation of risks, helping to detect the risks that are of higher priority. In this case, the application consists of combining risk ratings according to different sources of risk. The probabilistic composition can be applied to combine the scores of risk according to the factors of Failure Modes and Effects Analysis (FMEA): severity, frequency, and

difficulty of detection of the modes of failure to generate one-dimensional risk priority probabilities.

In this context, the probabilistic approach has the advantage of allowing for flexible rules for aggregating risks generated by different factors. To increase discrimination between modes of failure related to those factors for which the scores are concentrated in a small interval, the probabilistic distributions may be modeled with a range varying with the observed range. However, to give the evaluators the option of spacing the scores only to discriminate according to factors that they judge more relevant, the probabilistic approach allows for modeling the distribution of the evaluations according to the three factors with the same range, determined by extremes previously established.

Another field of application is the assessment of productivity. The probabilistic approach can be applied in the context of evaluating the efficiency of production units employing compositions of sets of inputs to generate sets of outputs. In the probabilistic approach to this problem, the criteria can be the output/input ratios for the different pairs of input-output.

The decision may also be based on maximizing each output variable and minimizing each input variable separately. Then, a criterion will be associated with each input and each output. In this last form of modeling, the probabilities of preference according to each criterion to be computed will be, respectively, those of maximizing the revenue from the sale of each product and of minimizing the cost of the acquisition of each resource. Different treatments can then be applied to the aggregation of the two separate sets of evaluations, according to inputs and according to outputs.

The uniformization provided by the probabilistic transformation may be used to extend the possibility of application of capacities. In fact, to combine evaluations according to different criteria by the integral of Choquet with respect to a capacity in the set of criteria, the evaluations enter the computation ordered according to the values taken. This does not make sense unless the evaluations are set in a same framework. The transformation into probabilities of being the best provides this common framework.

The transformation into probabilities of being the best also has an effect of approximating to identical values the preferences for the alternatives less preferred. This feature makes the probabilistic approach useful in other contexts. For instance, it can be used to provide rough measurements to the decision attributes in applications of Rough Sets Theory with Dominance There, it allows for reducing contradictions and extracting simpler decision rules.