

# The Assessment of Risk of Collision between Two Ships Avoiding Collision by Altering Course\*

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**Abstract.** Altering course is the most common and effective method employed by ships to avoid collision. Give-way vessels should take early and substantial action to avoid collision, the effectiveness of a course change is influenced by the distance between the two vessels. In some instances involving more than two vessels are present (special cases), the give-way vessel may not be able to alter course as early as possible. In other instances, the stand-on vessel may be required to take action due to the failure or inability of the give-way vessel to act. In the event of special cases and action by the stand-on vessel, it is important for navigating officers to be able to determine when a course change alone will avert a collision in order to plan for the worst case scenario. Thus, it is advantageous for navigating officers and masters to quickly and simply model the amplitude and effectiveness of course changes up to the distance between the two ships when course changes will no longer avert a collision. By using existing models of ship movement and maneuvering characteristics, a method will be presented to calculate the extent of risk of collision through reasoning process, provide a quantitative explanation of the effectiveness of course changes and identify the point at which course changes are no longer effective in collision avoidance. A worked example will illustrate the need for navigating officers to make early course changes by demonstrating the decreased effectiveness of course changes at small distances between ships.

**Keywords:** Risk of collision, Altering course, collision avoidance, Minimum angle alteration.

## 1 Introduction

A ship at sea can alter her course, change her speed, or alter both her course and change the speed simultaneously to avoid collision. Considering the performance of the main engine and response times while in transit at sea speed, in many instances altering course is the only viable option to execute timely action to avoid collision. The encounter of the ships is a process of approaching and reducing distance from an area of no or minimal potential risk of collision to an area of high potential risk of

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collision. When two vessels are meeting and a risk of collision exists according to the Rules (COLREGS, 1972), there are actions to be taken by the give-way and stand-on vessel. The give-way vessel has the responsibility to take early action to keep clear of the other vessel with a safe distance in an ample time (Zhao Yuelin, 2012). The stand-on vessel may take action if it becomes apparent that the give-way vessel is not taking appropriate action. However, the Rules do not give a quantitative explanation when action should be taken by the give-way (early and substantial action) or stand-on vessel (taking additional action) to avoid a collision.

This paper illustrates two concepts, range of collision avoidance courses  $\theta_s$  and the minimum alteration angle  $\Delta\varphi$  to assess the risk of collision of ships. A step by step reasoning such as one based on researcher A.S. Lenart's algorithm of the relationship between the speed and course of in case of a constant distance between two vessels, can be applied to the problem (A.S. Lenart, 1983). The result of applying an algorithm is useful to improving navigating officers' comprehension of the risk of collision, while illustrating the need for the give-way vessel need to take early action to avoid collision.

During the process of calculating and reasoning, the assumptions are:

- Own ship and target ship are thought to be an idealized ships;
- The give-way vessel takes altering course action with no change of speed;
- There is no delay from the rudder order to change of course;
- There is no influence to the speed of the ship from the change of the course.

## 2 Modeling and Calculation

### 2.1 Calculation of Action to Avoid Collision

In order to understand the algorithm, it is necessary to consider the coordination system to be used. The center of the coordinate system is the ship's directional axis of rotation with  $X$  axis represents the direction of true east, and the  $Y$  axis represents the direction of true north. The speed of own ship is  $V_o$  and course is  $\varphi$ , and the speed of target ship is  $V_t$ , the relative speed of target ship to own ship is  $V_r$ . The relationship between these elements are shown in equation (1) and equation (2).

$$V_o + V_r = V_t \quad (1)$$

$$\begin{cases} V_{ox} = V_o \sin \varphi \\ V_{oy} = V_o \cos \varphi \end{cases} \quad (2)$$

Here,  $V_{ox}$  is the component of own ship speed vector in the  $X$  direction, and  $V_{oy}$  is the component of own ship speed vector in the  $Y$  direction;  $V_{tx}$  is the component of target ship speed vector in the  $X$  direction, and  $V_{ty}$  is the component of target ship speed vector in the  $Y$  direction;  $V_{rx}$  is the component of target ship relative speed vector in the  $X$  direction, and  $V_{ry}$  is the component of target ship relative speed vector in the  $Y$  direction. Suppose that  $(X, Y)$  is the coordinate of target ship in the coordinate system

considered in front, and  $(X_0, Y_0)$  is the initial position of target ship, the coordinate of target ship position  $(X, Y)$  following with time  $t$  can be got with equation (3).

$$\begin{cases} X(t) = X_0 + V_{rx}t \\ Y(t) = Y_0 + V_{ry}t \end{cases} \quad (3)$$

Assuming  $D(t)$  that is the distance between own ship and target ship, the following relationship should exist as shown in equation (4).

$$D(t) = \sqrt{X^2(t) + Y^2(t)} \quad (4)$$

Derivate  $D(t)$  by  $t$  and get the following result:

$$D_{\min} = \left| \frac{XV_{ry} - YV_{rx}}{V_r} \right| \quad (5)$$

Where  $D_{\min}$  is the distance of closest point of approach (short for DCPA or CPA) between own ship and target ship. In order to keep a safe passing distance ( $D_s$ ) between own ship and target ship, it is necessary to let  $D_{\min} \geq D_s$ .

Square equation (4), and get equation (6).

$$V_{rx} = AV_{ry} \quad (6)$$

Then get,

$$A = \frac{X(t)Y(t) \pm D_{\min} \sqrt{(X^2(t) + Y^2(t) - D_{\min}^2)}}{X^2(t) - D_{\min}^2} \quad (7)$$

Where,

$$V_o = \frac{AV_{tx} - V_{ty}}{A \sin \varphi - \cos \varphi} \quad (8)$$

Thus, from equation (1), (2) and (7), it can be seen that,

$$V_o = \frac{AV_{tx} - V_{ty}}{A \sin \varphi - \cos \varphi} \quad (9)$$

Then,

$$V_{oy} = AV_{ox} - B \quad (10)$$

Where,

$$B = AV_{tx} - V_{ty} \tag{11}$$

From equation (11), it is the relationship of speeds, and change it into the relationship of distances, multiply both sides by time ( $\Delta t$ ) :

$$y = Ax - B\Delta t \tag{12}$$

As it can be seen, the most important factors affecting the risk of collision are distance of the closest point of approach (DCPA) and the time to closest point of approach (TCPA). When  $DCPA < D_s$  and  $TCPA > 0$  exist simultaneously, the risk of collision exists. If  $TCPA < 0$ , the two vessels will navigate in opposite directions, thus there is no risk of collision.

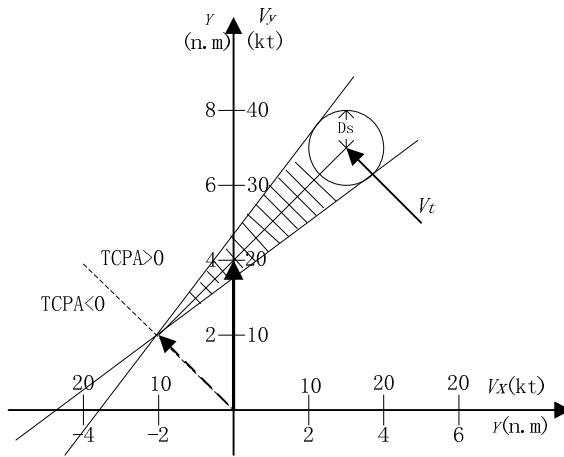


Fig. 1. The relationship between the speed and course (A.S. Lenart)

Equation (12) simply corresponds to the two lines when  $DCPA = D_s$ , which will intersect with each other as shown in figure 1. The area on the left side of intersection represents the situation when  $TCPA < 0$ , with no risk of collision to own ship, and the area on the right side of intersection representing the situation when  $TCPA > 0$ , where there is no guarantee the passage with a safe distance. Thus, if the vector of own ship is not located in the shaded area, the two ships will pass at a safe distance ( $D_s$ ).

### 2.2 Calculation of Range of Collision Avoidance Courses

When an own ship takes action to avoid collision with a target ship by altering course, all the ends of the own ship course vector will form a circle with the center in the middle of the ship and with radius  $V_o\Delta t$ . This circle will intersect with the shadow area as shown in figure 1, the figure is shown in figure 2. The shaded area in the circle indicates the ship's courses cannot safely navigate with, the angle is  $\theta$ , the unshaded portions of the circle indicate safe course options, the angle is  $(360-\theta)$ . Analysis of figure 2 leads the equation of the circle is:

$$X^2 + Y^2 = (V_o \Delta t)^2 \tag{13}$$

From equation (12) and (13),

$$\begin{cases} x = \frac{A(B\Delta t) \pm \sqrt{A^2(B\Delta t)^2 - (1+A^2)((B\Delta t)^2 - (V_o \Delta t)^2)}}{1+A^2} \\ y = \frac{\pm A\sqrt{A^2(B\Delta t)^2 - (1+A^2)((B\Delta t)^2 - (V_o \Delta t)^2)} - B\Delta t}{1+A^2} \end{cases} \tag{14}$$

In the equation (8), there are two solutions for A, so does B. So there are four solutions in equation (14), they are  $(x_i, y_i)$  ( $i=1,2,3,4$ ), which corresponds to the four intersections points in figure 2 from the lines of equation (11) and the circle of equation (13). Two of these four points are located in the area corresponding by  $TCPA < 0$ , which means no effect to passage with a safe distance, but the other two points are located in the area corresponding by  $TCPA > 0$ , the arc between them is the range which own ship course cannot navigate, and the other part of the circle is the courses that own ship can change to. It is not hard to calculate the length of the arc by equation (15).

$$s = \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2} \tag{15}$$

Corresponding degrees is,

$$\theta = 114.64 \arcsin\left(\frac{s}{2V_o \Delta t}\right) \tag{16}$$

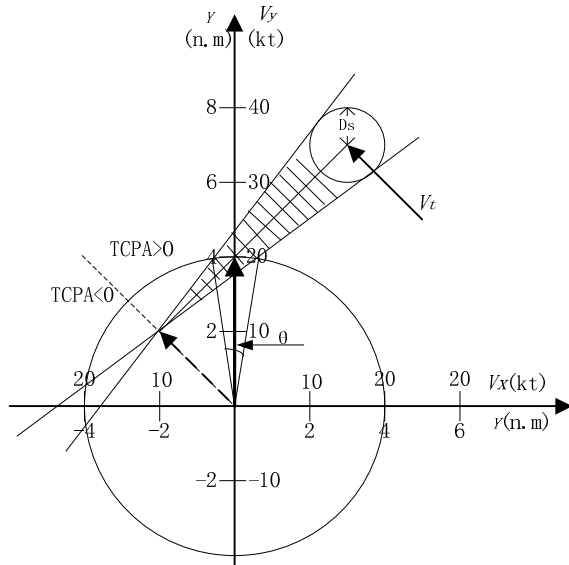


Fig. 2. Schematic diagram of  $\theta$  and  $\Delta\phi$

Where  $\theta$  is range of courses which own ship cannot alter course to, and the range which own ship can alter course to  $\theta_s$  is easy to get by  $\theta_s=360^\circ-\theta$ , also called range of collision avoidance courses. If the range of collision avoidance courses is bigger, it means that the difficulty to take action to avoid risk of collision by altering course is small, otherwise, is big.

### 2.3 Calculation of Minimum Alteration Angle

It is not difficult to get the coordinates of the two intersection points  $(x_1, y_1)$  and  $(x_3, y_3)$  on the side of  $\text{TCPA} > 0$  according to equation (14). In addition, when there is risk of collision between two vessels in sight of each in a crossing situation, the give-way vessel shall take action in ample time to avoid collision, and avoid crossing ahead of the stand-on vessel. So, the minimum alteration angle is from the initial course of own ship to the vector  $OB$ , and the speed vector  $OB$  is the new course of the ship. The minimum alteration angle  $\Delta\varphi$  can be calculated by the following equation (18).

$$\Delta\varphi = 57.32 \arcsin\left(\frac{x_i}{V_o \Delta t}\right) - \varphi \quad (17)$$

Here,  $\arcsin\left(\frac{x_i}{V_o \Delta t}\right)$  represents the new course, corresponding to the vector  $OB$ , changed degrees;  $\varphi$  is the initial course of own ship.

## 3 Calculation by an Example

Using researcher S.Lenart's example, the position of the target ship from own ship is (5 n mile, 5 n mile) with own ship and target ship speeds as indicated in table 1.

**Table 1.** Speed vectors of own ship and target ship

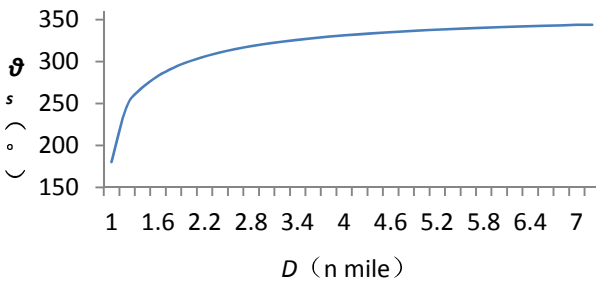
ITEM	V(kt)	X Axis(kt)	Y Axis(kt)
OS	20	0	20
TS	14.1	-10	10
$V_r$	14.1	-10	-10

As two ships approach each other, the range of collision avoidance courses and the minimum alteration angles are calculated shown in table 2.

**Table 2.** Results of  $\theta_s$ ,  $\Delta\varphi$  and *dif*

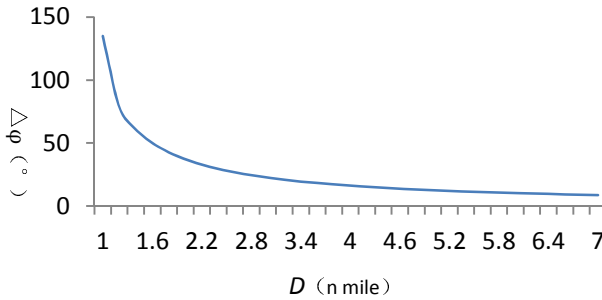
D ( n mile )	$\theta_s$ ( ° )	$\Delta\varphi$ (° )	dif
7.0	343.57237	8.798550	0.045632
6.5	342.29893	9.528705	0.049170
6.0	340.81045	10.39071	0.053304
5.5	339.04709	11.42373	0.058203
5.0	336.92438	12.68411	0.064099
4.5	334.31893	14.25592	0.071336
4.0	331.04284	16.27037	0.080437
3.5	326.79446	18.94415	0.092238
3.0	321.05469	22.66257	0.108181
2.5	312.84017	28.18370	0.131000
2.0	299.99558	37.24150	0.166679
1.5	276.37321	55.00827	0.232297
1.4	268.82391	60.92789	0.253267
1.3	259.42287	68.42934	0.279389
1.2	247.10641	78.43995	0.313593
1.1	229.23033	93.25132	0.363249
1.0	180	135	0.5

- The relationship between the range of collision avoidance courses and distance between two ships is shown in figure 3.



**Fig. 3.** Relationship between  $\theta_s$  and *D*

- The relationship between the minimum alteration angle and the distance between two ships is shown in figure 4.



**Fig. 4.** Relationship between  $\Delta\phi$  and  $D$

## 4 Conclusion

From the results of calculation in table 2 and the curve in figure 3 and figure 4, it can be seen that:

- When two ships are far away from each other, the range of collision avoidance courses decrease slowly and the minimum alteration angle increase slowly in a linear;
- As the two ships approach each other, especially when the distance is less than 2 times of safe passage distance, the range of collision avoidance courses decrease rapidly and the minimum alteration angle increase rapidly in a exponential way.

When the risk of collision exists when navigating on the sea, if you lost the best chance to alter course by reason of finding the target or taking action too late, the difficulty to keep clear of target ship and the minimum alteration course will rapidly increase, it will give us an explanation why should the give-way vessel need to take action in such an early time.

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