

# The Effect of Overconsolidation and Particle Shape on the CPT End Resistance of Granular Soils

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**Abstract.** This paper examines the effect of overconsolidation and particle shape on the end resistance ( $q_c$ ) measured in a series of centrifuge Cone Penetration Tests (CPTs) conducted in three uniformly graded silica materials with distinct particle shape. For each soil type, the end resistances were measured for both normally and overconsolidated soil samples at centrifuge g-level of 100g. All samples were prepared and tested at two different relative density, and the over-consolidated samples were achieved by reducing the centrifuge g-level from 200g to 100g (with over-consolidation ratio, OCR=2). At a given relative density and stress level, the striking dependency of the CPTs end resistance ( $q_c$ ) on the particle shape can be observed. For a particular material, a tendency for  $q_c$  value to increase with the OCR was in evidence. An approach based on the spherical cavity expansion method was proposed to predict the  $q_c$  value of each soil, and particularly to investigate how the OCR and particle shape influence on the end bearing resistance. It was found that the predicted  $q_c$  are shown to match the measured data well, and the end bearing resistances were significantly affected by the critical friction angle and horizontal stress, which were closely related to the particle shape and overconsolidation ratio.

**Keywords:** Particle shape, Centrifuge CPT, Cavity expansion method, Over-consolidation, Friction angle.

## 1 Introduction

The cone penetration test is widely used to assess the end resistance of driven pile in the granular materials. studies have been extensively conducted, aiming to propose the correlations between the CPT end resistance and the soil property indices such as the relative density, vertical stress, and stress history [1], [7], [11], [18, 19, 20]. However, there are significant discrepancies among these correlations. It is generally found that an increase in lateral stress (or overconsolidation ratio, abbreviation as

OCR) can cause  $q_c$  value to increase for a particular sand at a given relative density, e.g., [18] proposed the following expression

$$q_c^{OC} / q_c^{NC} = 1 + 0.75 [ (k_o^{OC} / k_o^{NC})^{0.42} - 1 ] \quad (1)$$

where  $q_c^{OC}$  and  $q_c^{NC}$  are the cone tip resistance in overconsolidated and normally consolidated sand, respectively ;  $k_o^{OC}$  and  $k_o^{NC}$  are coefficient of the lateral earth pressure at rest in overconsolidated and normally consolidated sand, respectively. The Equation (1), derived from the chamber test data, provides a simple mean to evaluate the effect of stress history on the  $q_c$  value. However, [9] showed that Equation (1) overestimate the  $q_c^{OC} / q_c^{NC}$  by 20%-40% according to the bearing capacity theory. Further, [9] pointed out that the effect of high in-situ lateral stress caused by overconsolidation is offset by the low bearing capacity factors associated with a more localized failure surface based on the coupled DEM-BEM numerical analysis, and this eventually result in insignificant effect of OCR on the  $q_c$  value.

In spite of the above disagreement, the following expressions are currently widely applied to correlate the  $q_c$  value and relative density for overconsolidated sand [2], [11] as:

$$D_r = \frac{1}{C_0} \ln \left[ \frac{q_c}{C_1 (\sigma'_m)^{C_2}} \right] \quad (2)$$

where  $\sigma'_m$  is the mean norm stress; the  $C_0$  varies from 2.6 to 2.95 while  $C_1$  varies from 180 to 2051;  $C_2$  approximately equal to 0.5. The effect of loading history on  $q_c$  are embedded in  $\sigma'_m$ .

The Equation (2) was derived from the extensive chamber tests on ticino sand. However, errors may be expected to arise when it is applied to assess another sand deposit with dissimilar properties to Ticino sand. One such property is the soil particle shape, which is reported to greatly influence the mechanical characteristics such as compressibility and friction angle [4], [16]. Given the uncertainties surrounding the application of Equation (1)-(2) to quantify  $q_c$  values of over-consolidated sands with distinct particle shape, A laboratory centrifuge study were undertaken to examine the effect of particle shape and over-consolidation on the cone tip resistance. Three uniformly graded silica materials with various particle shapes are tested in drum centrifuge CPTs. The tests results are interpreted on the basis of cavity expansion method.

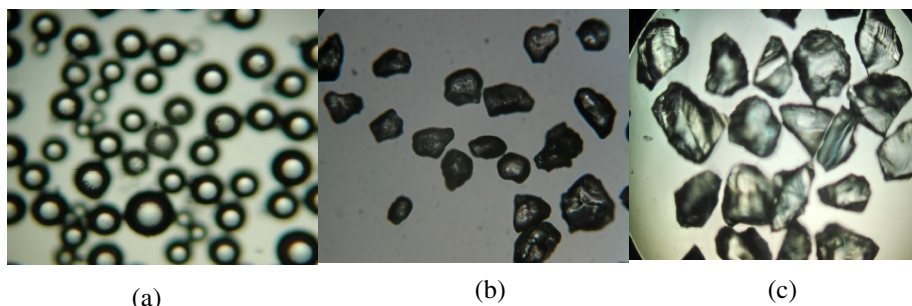
## 2 Granular Materials Investigated

Classification details of the three granular materials tested in this study are given in Table 1. All materials are uniformly graded (uniformity coefficient,  $C_u = d_{60}/d_{10} < 1.4$ ) with mean particle sizes ( $d_{50}$ ) ranging from 0.12mm to 0.45mm. The UWA-Sand is a

silica sand with a specific gravity ( $G_s$ ) of 2.65 while the two other “soils” comprise glass particles with  $G_s \sim 2.5$ . Images of particles from each material are presented in Figure 1. On inspection, the SGB particles are spherical, whereas the UWA-sand and GB soils are more angular and irregular. Three types of particle shape parameters (i.e. Roundness (R), Sphericity (S) and regularity ( $\rho$ ) = (R+S)/2) are provided in Table 1; further details concerning the measurement of these parameters can be found in [4] and [5].

**Table 1.** Index properties of three granular materials

Code	Gradation		Void ratio limits		Particle shape			$G_s$
	$d_{50}$ :mm	$C_u$ :	$e_{max}$	$e_{min}$	R	S	$\rho$	
UWA-Sand	0.15	1.4	0.81	0.52	0.7	0.62	0.66	2.65
SGB	0.12	1.05	0.75	0.51	1	1	1	2.52
GB	0.45	1.13	1.01	0.64	0.62	0.54	0.58	2.48



**Fig. 1.** Light microscope images of granular material used (a) SGB (b) UWA-sand and (c) GB

### 3 Centrifuge Cone Penetration Tests

#### 3.1 Sample Preparation and Test Programme

Cone Penetration Tests (CPTs) were performed on each soil sample at dense and loose state. The g-level was set at 100g when conducting cone penetration in the drum centrifuge located in the University of Western Australia (technical details of this drum centrifuge referred to [17]).

Loose and dense samples ( $Dr=0.2$  and  $Dr=0.8$ ) were prepared in centrifuge strong boxes. For loose samples, the soil was poured into the box using an electrically

controlled bar-type hopper. Consistent loose packing was produced by maintaining a soil drop height of a 20mm, hopper travel speed of 5 mm/s and a hopper aperture opening of 0.5 mm. Care was taken to ensure that the sand surface remained level throughout the complete process. For dense samples, another box was attached to the top of the strongbox to allow loose sand to be placed to a high level. Samples were then densified by placing the strongbox on the vibrating table to achieve the required density.

All samples were saturated slowly by application of a differential water pressure of 0.5kPa (maximum) to a screw hole located at the centre of the base of the strong box. A geofabric were also placed at the base of the soil sample to minimize the disturbance during the saturation. Light suction was then applied to each sample to prevent its collapse while being placed sideways in the centrifuge before centrifuge spinning began. The box samples were firstly spinning at a low g-level (i.e. 50g) and re-saturated for approximately 2 hours prior to commencement of the CPTs. A pause period of at least two hours was also allowed prior to the cone testing at the g-level of 100g. After completing the first round testing at g-level of 100g, the soils were spined to a higher g-level of 200g and re-saturated for another 2 hours. The over-consolidated soils (OCR=2) was then tested in identical circumstances by reducing the g-level from 200g to 100g. The cone has a diameter of 5mm and was installed at a (drained) rate of displacement of 0.1mm/s in the whole process of testing.

### 3.2 CPT Results

The CPT test results are given in Fig.2, where cone resistance  $q_c$  versus model penetration depth were plotted for each soil at different relative densities and OCR. Typically, the cone resistance increases approximately linearly with depth within the first 50mm and then reaches a stage of slowing in the rate of increase. However, no obvious maximum “limiting” end bearing resistance can be seen. At a given depth (vertical stress), as expected, the  $q_c$  values increase with the sample relative density.

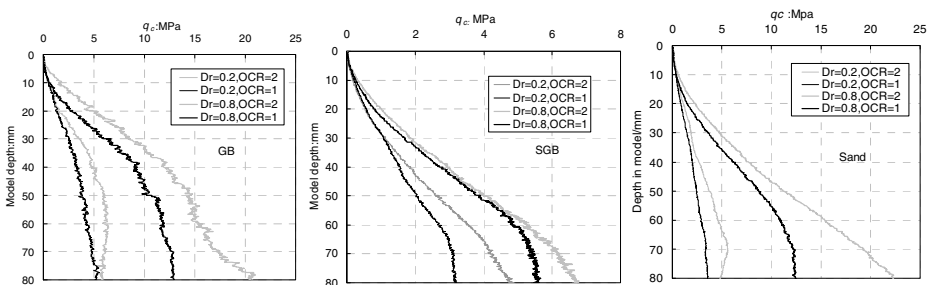


Fig. 2. CPT end resistances ( $q_c$ ) observed in centrifuge tests

Furthermore, on inspection of Fig.2, it appears that following points applies:

1. For the same relative density and the same vertical stress (overburden pressure from a given penetration depth), the cone end resistance measured in normally consolidated sands is significantly lower than that measured in overconsolidated sands with OCR=2. The difference of  $q_c$  values between the normally consolidated and overconsolidated sands typically varies from 1 to 5Mpa for dense samples ( $Dr=0.8$ ) and in the range of 1~3Mpa for loose samples ( $Dr=0.2$ ). Such differences for more angular UWA-sand and GB particles are higher than those for the more rounded SGB particles.
2. The marked dependence of the  $q_c$  values on the soil type are in evident. At a given relative density and OCR, The lowest values of  $q_c$  are acquired by SGB while the higher values are recorded by UWA-Sand and GB. In view of the similar effective particle size recorded for SGB and UWA-Sand, It is reasonable to attribute this striking difference in  $q_c$  values to variations in the particle shape. The  $q_c$  values appear to increase with the particle irregularity, regardless of whether the soils are normally consolidated or overconsolidated.

#### 4 Predicting the End Resistance Using Spherical Cavity Expansion Method

In view of the shape of failure model observed beneath a loaded pile tip in the granular soils, the spherical cavity expansion theory firstly proposed by [21] is widely accepted as analytical method for the prediction of the end bearing resistance of pile in the sands. Within the framework of spherical cavity expansion theory, the following equation suggested by [15] was normally adopted to link the limit cavity expansion pressure  $p_{limit}$  to the end bearing capacity  $q_c$

$$q_b = p_{limit}(1 + \tan \phi' \tan \alpha) \quad (3)$$

It is assumed that the soil immediately beneath the cone tip has been sheared to its ultimate state, so the friction angles  $\phi'$  should therefore be taken as critical state angle  $\phi'_{cv}$ . the value of  $\alpha$  equal to the that of cone angle, as being 60.

The limit expansion pressure for spherical cavity expansion in Equation (3) can be calculated by the expressions from [22], where the values for following input parameters are needed to carefully chosen :

- The in situ mean effective stress  $p_o$ , related to the  $\sigma'_v$  and  $K_o$
- The friction angle  $\phi'$  and dilation angle  $\psi$  for the sands
- The shear modulus  $G$  and poisson ratio  $\nu$ .

The evaluating procedure of these parameters for the three soils investigated in our study is expounded as follows:

(1) *Assessing the  $p_o$* : the mean effective stress  $p_o$  is expressed as  $\sigma'_v(1+2K_o)/3$ . Based on the extensive laboratory tests, [14] suggested the following expression to correlate the  $K_o$  with friction angle  $\phi'$  and OCR.

$$K_o = (1 - \sin\phi') (\text{OCR})^{\sin\phi'} \quad (4)$$

(2) *Assessing the  $\phi'$  and  $\psi$* : following the work of [3], the peak friction angle and dilation angle can be determined by linking critical state angle  $\phi'_{cv}$  with the soil relative density and mean effective stress at failure. i.e.:

$$\phi' = \phi'_{cv} + 3I_R \quad (5)$$

$$\psi = 3.75 I_R \quad (6)$$

where the  $I_R$  was defined as the relative dilatancy index and expressed as:

$$I_R = 5D_r - 1 \quad \text{for } p' < 150 \text{ kPa} \quad (7a)$$

$$I_R = D_r [10 - \ln p'] - 1 \quad \text{for } p' > 150 \text{ kPa} \quad (7b)$$

Preliminary direct shear box tests results [12] showed that the  $\phi'_{cs}$  is dependent on the soil particle shape, and a linear relationship between  $\phi'_{cs}$  and particle regularity  $\rho$  was suggested as:

$$\phi'_{cs} \text{ (degrees)} = 40 - 15.3\rho \quad (8)$$

This expression implies the SGB has the lowest value of  $\phi'_{cs}$  while the more angular GB acquired the highest value of  $\phi'_{cs}$  among three soils.

In view of the very significant increase in stress level beneath a cone as the vertical stress increases from  $\sigma'_v$  to  $q_c$ , [8] suggest that the mean effective stress in  $E_q$  (7b) should be taken as the geometric mean of the  $\sigma'_v$  and  $q_c$ , i.e.

$$p' \approx \sqrt{q_c \sigma'_v} \quad (9)$$

(3) *The practical evaluation of  $G$* : The shear modulus  $G$  usually can be derived from the small-strain testing. Many correlations has been proposed to express the  $G$  as a function of mean effective stress and either the void ratio or the relative density. The following correlation suggested by [13] proved valid for the silica sand.

$$G_o / p_a \sim S \exp(C_1 D_r) (p'_o / p_a)^n \quad (10)$$

where parameter  $S$  is about 600,  $C_1$  is 0.7 and  $n$  equal to 0.43.  $p_a$  is atmospheric pressure (100kPa). [15] suggested the value of  $S$  should been reduced for more compressible materials. [6] reported that the shear modulus for the SGB is approximately 50%

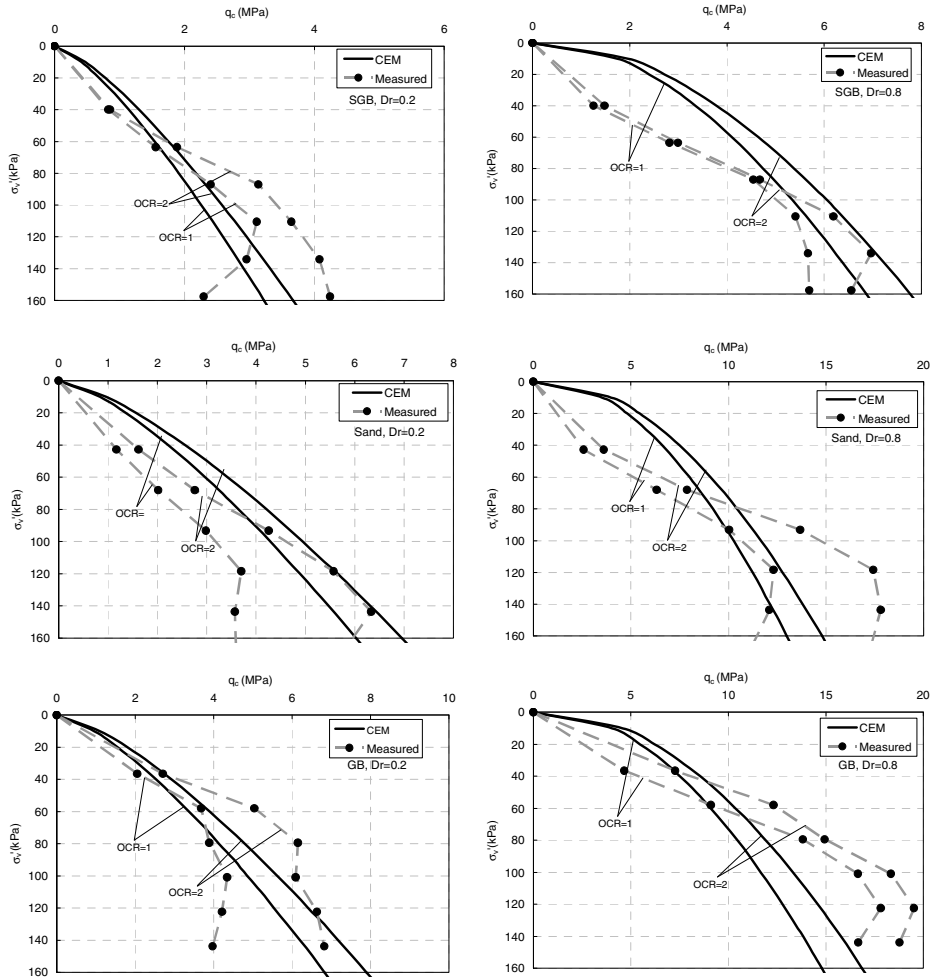
of the silica sand. Based on this, in the following calculation, the value of  $S$  in Equation (10) is taken as 300 for SGB. Furthermore, following the [4], who reported a decrease in particle regularity will lead to increase in the compressibility and decrease in the small-strain stiffness, the value of  $S$  is taken as 500 for GB with angular particles.

It needs to be emphasized that the cavity expansion solution is based on an idealized soil model, and the above correlations given for the shear modulus are limited by real behavior that values of the elastic parameters and the correlation index  $n$  are all affected by strain level. However, in spite of these limitations, this approach appears to be able to capture the significant effect of stress level on the traditional bearing capacity factor  $N_q$  and hence yield relatively realistic estimate of bearing capacity.

Taking the value of the passion ratio  $\nu$  as to be 0.2, an iteration calculation can be formed using the Equations (1) to (10) to predict the variations of  $q_c$  value with vertical stress for each type of soil at different relative density. Prediction of  $q_c$  obtained from the cavity expansion method are compared with the measures  $q_c$  values in Fig 3. A few points of note were implied on view of the figure 3.

- (a) Although with some scatter, overall, the predicted values of  $q_c$  are in good agreement with measured ones. The difference between the predicted and measured  $q_c$  values are more striking at low penetration depth (low vertical stress). These differences are mainly due to the higher dilation angle occurring in the low stress levels using the equation (6). Besides, the prediction leads to greater curvature of the end-bearing profiles.
- (b) The predictions conform that the  $q_c$  values in overconsolidated soils are typically 13% higher than those in normally consolidated soils. This is approximately consistent with the measured values. [10] showed the marketed dependency of  $q_c$  value on the in suit horizontal stress, which is directly related to the  $k_o$ . The equation (4) implies higher value of  $k_o$  recorded for overconsolidated sand, and therefore the higher horizontal stress is expected to act on the cone tip.
- (c) The predicted data indicated that the “ $q_c$  (OC) /  $q_c$  (NC)” is dependent on the soil type and appear to increase with the soil particle irregularity as SGB < UWA-sand < GB. This hierarchy may be attributed to the tendency for the  $\phi'_{cs}$  to increase with particle irregularity (see Eq.8) because the values of  $\phi'_{cs}$  greatly influence the values of  $\phi'$  and  $k_o$ , both of which can produce significant effect on the  $q_c$  value in the spherical cavity expansion method. The tendency for measured “ $q_c$  (OC) /  $q_c$  (NC)” to increase with the particle irregularity can also be observed when comparing data of the SGB with that of UWA-Sand. However, this tendency is indistinct between UWA-Sand and GB, which may be due to the effect of particle size.

It also can be seen from Fig.3 that the “ $q_c$  (OC) /  $q_c$  (NC)” is also dependent on the stress level, and the values of “ $q_c$  (OC) /  $q_c$  (NC)” increase with the depth. This dependency may arise due to the effect of stress level on the dilation angle and perhaps the stiffness of the soil. The equation (4) show the “ $k_o^{OC} / k_o^{NC}$ ,” varies only with the friction angle for the soils at particular OCR. In view of these , to diminish the effect



**Fig. 3.** The comparison of the measured  $q_c$  with the CEM predictions for both dense ( $Dr=0.8$ ) and loose ( $Dr=0.2$ ) samples

of dilation angle, the loose soils, of which friction angle approximately equal to the critical state angle, is examined to make a comparison of between “ $q_c$  (OC) /  $q_c$  (NC)” from the Centrifuge test, CEM and the Equation (1). The comparison is shown in Fig.4, where the CEM produce the approximately same value for one particular soil in the whole range of depth. The tendency for the “ $q_c$  (OC) /  $q_c$  (NC)” to increase with particle irregularity is clear in Fig.4, where both of the two predictions are lower than the measured ones. The best-fit expression to describe the variation of “ $q_c$  (OC) /  $q_c$  (NC)” with overconsolidation ratio and particle shape may be concluded as follows:

$$q_c(OC)/q_c(NC) = 0.7(OCR)^{1.5/\exp(0.5p)} \quad (11)$$



It needs to be emphasized that this equation only provides a preliminary and rough description of the effect of the particle shape and OCR on the  $q_c$  values, and the actual variation of the “ $q_c$  (OC) /  $q_c$  (NC)” with the depth (stress levels) has not been quantified exactly and needs to be studied further.

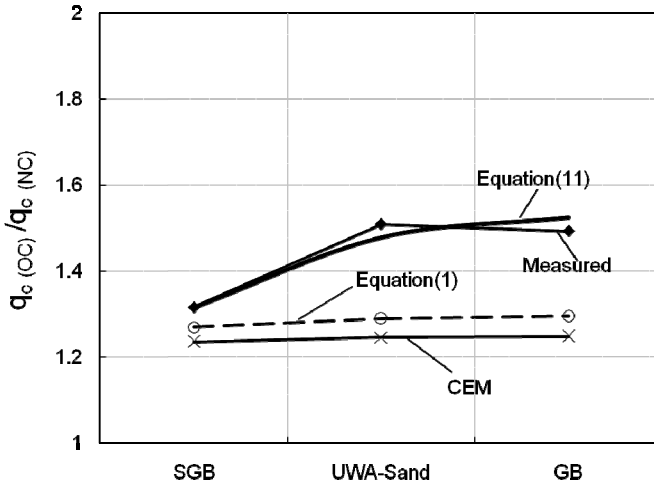


Fig. 4. Comparison of  $q_c$  (OC) /  $q_c$  (NC) using different methods

### 5 Conclusions

The effect of the particle shape and overconsolidation on the cone tip resistance is examined in the centrifuge tests. The  $q_c$  values in the overconsolidated soils is typically 13% higher than those in the normally consolidated soils. The ratio of the  $q_c$  in overconsolidated soils to that in normally consolidated ones is variable with the depth (stress levels) and particularly dependent on the particle shape. An approach based on the spherical expansion method is adopted to predict the  $q_c$  values. It can be concluded that the particle shape influence the  $q_c$  by the means of its effect on the critical state friction angle, and the over consolidated soils exert a higher in-situ horizontal stress surrounding the cone tip and therefore produce a higher  $q_c$  values. An equation (11) is finally proposed to approximately quantify the effect of the particle shape and overconsolidation on the cone tip resistance.

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