Stress Analysis of Stiffened Cylindrical Shells Under a Static Load

Paweł Biesiacki, Jan Awrejcewicz, Jerzy Mrozowski, and Jacek Jankowski

Lodz University of Technology, Poland
800045@edu.p.lodz.pl

Abstract. The presented work contains the numerical strain analysis of a ribbed shell of a rotary drum with two riding rings rigidly fixed to the drum shell. In the first stage of the conducted research a numerical model of a cylindrical shell was built with two stiffening rings of dimensions corresponding to the common constructions used in chemistry or the cement industry. The boundary conditions were set according to the mentioned machines working conditions and the model was subjected to loads corresponding to real data. The correctness of the numerical model was verified by a comparison with the results obtained in analytical calculations. The Fursow method was used in the analytical method. In the next step the model had been extended to include the longitudinal ribs added within the drum around the perimeter of the cylindrical shell. The ribs work as lifting flights. The bulk material was lifted by the flights during the rotation of the drum allowing for convective heat exchange between the material intended for drying and the air inside the shell. The bulk material was treated as a rigid body and the cylindrical shell of the drum was analyzed under its dead load. The effect of temperature was omitted due to the fact, that the drying was carried out at the temperature not exceeding 80 °C, hence the variability of Young's modulus was negligible. An analysis was performed with the use of numerical methods and commercial software ANSYS. The influence of the position of material loads on the stress and deformation reduction of the cylindrical shell and running ring was analyzed. As a result of the numerical simulations, the distribution of bending moments and the areas of greatest stress concentration and maximum strain were identified.

Keywords: ribbed cylindrical shells, stress tensor, strain tensor.

1 Introduction

The shells are widely used in various branches of industry as thin-walled constructions, especially when high stiffness and low weight are needed, for example in aerospace (aircraft constructions), marine (shipbuilding), building (roofs coverings) and chemical engineering (chemical equipment and machines). In this paper the strain's numerical analysis for a ribbed shell of a rotary drum with two riding rings rigidly fixed to the drum shell are presented. There are many monographs and papers (see for

J. Awrejcewicz et al. (eds.), *Mechatronics: Ideas for Industrial Applications*,
Advances in Intelligent Systems and Computing 317, DOI: 10.1007/978-3-319-10990-9_40

example [2–4, 6–7]), the theoretical basis and the simple cases of the shells loading are discussed. Shell buckling under dynamic loading has been described in [5]. It was also proved that the thin-walled structures are endangered with destruction, mostly due to the loss of stability, rather than to insufficient strength of the material. In [1] also the loss of stability for the cylindrical shell with initial imperfections and with applied axial load has been discussed. One of the ways to improve the shell's stiffness is the application of stiffening rings.

The rotary dryer as well as the rotary kiln are machines, in which the main part is a ribbed cylindrical shell. The calculation procedure of rotary kiln [9] as an shell radial deformation using the deformable finite elements method was presented in [9]. The results of the calculation have been compared with those obtained by Z. Banakiewicz by analytical means. On the ground of this comparison it can be stated that the strength analysis of a rotary drum by the deformable finite element method is decidedly more justified. The deformation problems for the case of the revolving boiler drum with two rolling rings fixed in a rigid way on the shell can be found in literature [10]. The two significant theories describing stresses and deformations of the shell: Goldenveizer, Novozhilow and Donnell have been discussed, and using a numerical method, a boiler drum free vibrations matter has been solved. In the work [11] a means of calculation of radial deformation of a tyre of rotary drum using the method of deformable finite elements was presented. The calculations have been carried out for the models of internodal loads expressed by the continuous load and point load. The influence of the kind of the contact between the tyre and the shell of the drum upon the character of tyre deformation and upon the magnitude of internal bending moments was analyzed. The results were compared with the results obtained on the analytical way by other authors. The finite element method (FEM) was applied to the nonlinear analysis of a cement rotary kiln [8]. The nonlinearity is due to contact conditions between the kiln body, tyres and foundations. The FEM was first used in a reduced model of the kiln in order to obtain the meshing criterion for the global model. Then, an overall FEM analysis was performed for the different operating and live loads at different positions of the rotary kiln.

2 The Fursow Method

The Fursow method is based on the Castigliano theory. Strenght calculations for the ring loaded with external forces system can be reduced to the determination of the statically indeterminate internal forces (axial force *N*, shear force *P* and bending moment *M*) for the ring's cross-section. The Fursow method allows one to obtain internal force in the ring in each section in dependence on the section angle. Mathematical equations are as follow [12]:

$$N = \frac{-1}{2\pi}\sum_{k=1}^{m} S_k + \sum_{k=1}^{m} S_k A_1(\varphi - \varphi_k) \tag{1}$$

$$P = \sum_{k=1}^{m} S_k A_4(\varphi - \varphi_k) \tag{2}$$

$$M = R\sum_{k=1}^{m} S_k A_1(\varphi - \varphi_k) \tag{3}$$

where: S_k – force, A_1–A_4 – auxiliary factors, R – outer radius, φ – section angle for calculations the internal forces, φ_k – the angles in the individual sections.

$$A_1 = \frac{1}{\pi}\sum_{n=2}^{\infty} \frac{\cos n\varphi}{n^2 - 1} \tag{4}$$

$$A_4 = -\frac{1}{\pi}\sum_{n=2}^{\infty} \frac{n\sin n\varphi}{n^2 - 1} \tag{5}$$

$$A_3 = \frac{1}{\pi}\sum_{n=2}^{\infty} \frac{\sin n\varphi}{n} \tag{6}$$

$$A_2 = -A_4 - A_3 \tag{7}$$

In the equations above n denotes number of applied S_k forces.

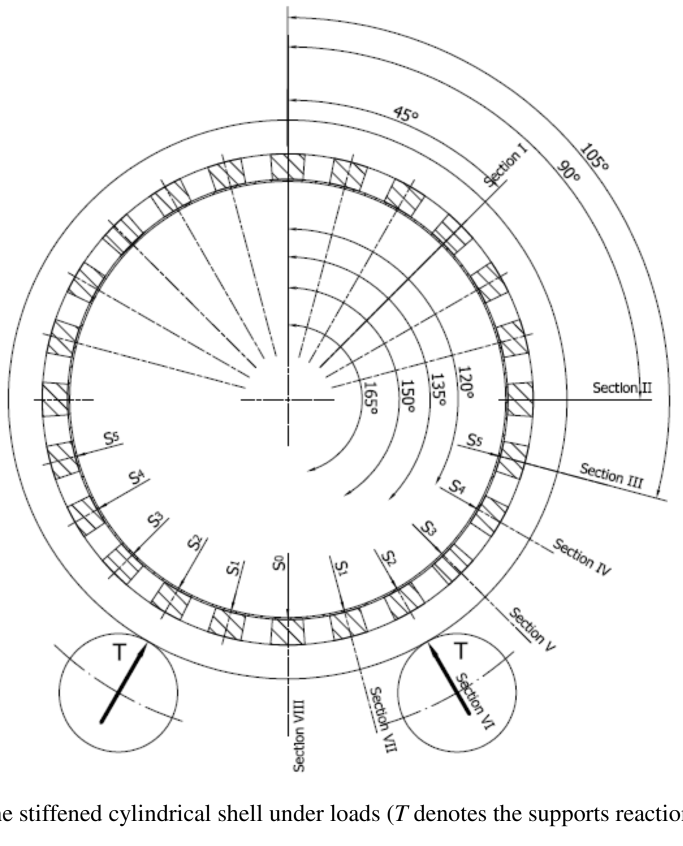

Fig. 1. The stiffened cylindrical shell under loads (T denotes the supports reaction forces)

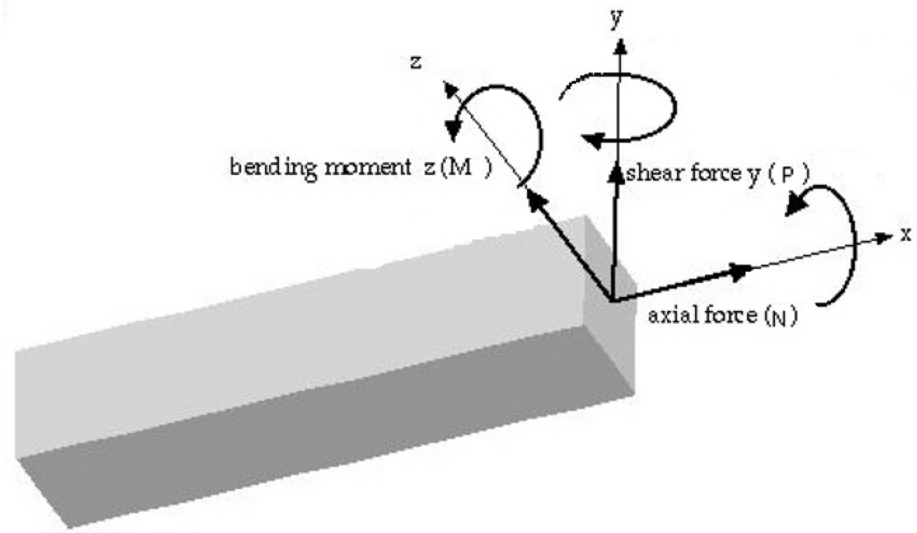

Fig. 2. The internal forces in the ring taken into account

3 Numerical Models

In order to carry out a reliable strength analysis of stiffened cylindrical shell two numerical models were constructed. The first one, devoid of strengthening ribs, served as a reference model. The second one, fitted out with flights, was more realistic. Geometric dimensions of the models were taken from real object. The radius (R) of cylindrical shell is 1.3 m and length of the cylindrical shell (L) is 10 m. The material properties were defined. Young modulus (E) is 200 GPa and Poisson's ratio (ν) is 0.3. A shell type of finite element (FE) was used to model a rotary dryer shell. This kind of FE has eight nodes and a double curvature with six degrees of freedom at each node. In the modeling of bricks forming the rings a solid element was used. This kind of FE has 20 nodes with three degrees of freedom at each node. The connection between drum dryer and the riding rings was modeled using the contact bonded elements. The boundary condition are presented in Fig. 3b. Basically, the contact nodes of the rings and the support rollers were fixed in six degrees of freedom (DoF). The analysed model was subject to the loads that come from the dead load of the machine and bulk material.

3.1 The Simplified Model

At the first stage the investigation on the simplified model was performed in order to assess the conformity of the analytical and numerical models (Fig. 3). Two cases – model with flights and model without flights was taken into account.

a) b)

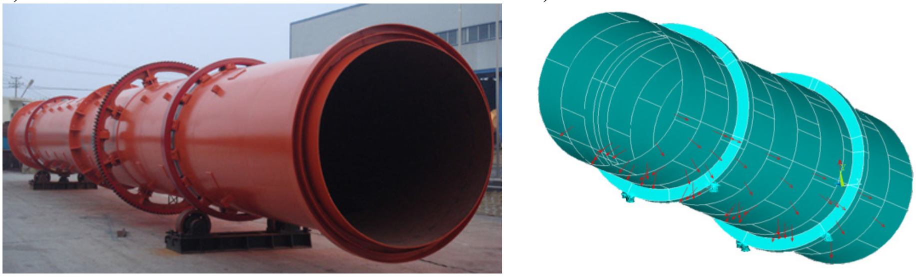

Fig. 3. The real view of the rotary dryer (a) and its corresponding numerical model (b). The cylindrical shell was subjected to load from the bulk material modeled as a the pressure about constant value corresponding to its weight (red arrows).

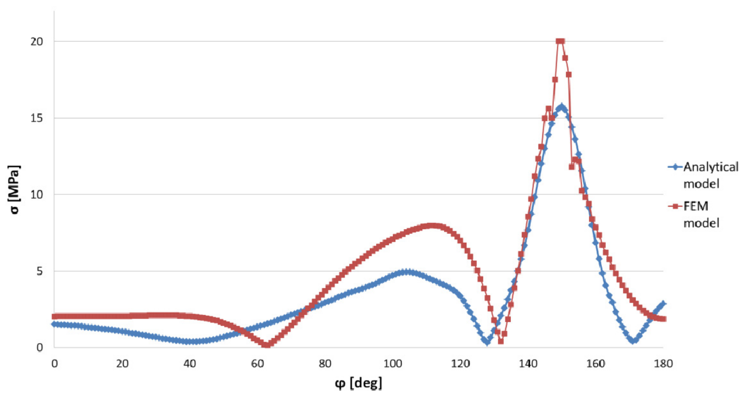

Fig. 4. The von Mises stress in the ring (σ) – analytical method vs FEM method

The numerical model was validated based on the ring stress distribution. The similar shape of the section angle-stress curves obtained for both models suggest their good accordance (Fig. 4).

3.2 Extended Model

Because the tests which were carried out using the simplified numerical model and the analytical model showed their good correspondence, in the second research stage an extended numerical model with additional flights was used (Fig. 5). During rotational movement of the rotary dryer the bulk material is lifted by the flights, and then, after exceeding the angle of repose, it slides down. Repose's angle for granular material has range from 0° to 90° and it refers to the maximum angle at which an object can rest on an inclined plane without sliding down. For modeling, critical angle of repose at the level of 45° was assumed. The lifted bulk material in the flights was modeled as the solid body weight according to the its density. Also in this case, the load from shifted bulk material was modeled as the pressure with constant value corresponding to its weight (blue arrows in Fig. 5b).

a) b)

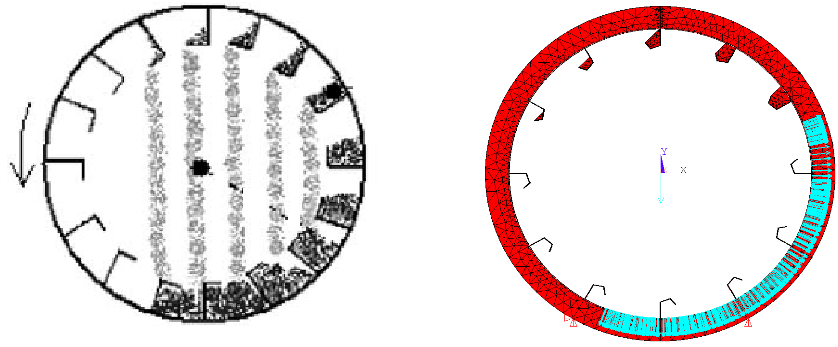

Fig. 5. A discrete slice of flighted rotary dryer: a) section view of rotary dryer with bulk material inside [13], b) numerical model with applied asymmetric load

The stress distribution for an arbitrary cross-section and a forming line (red line on the figures) was investigated. The simplifications applied to the extended model consisted of a lack of the reinforcements at the ends of the rotary dryer, caused their large deformations. Therefore, to obtain the most realistic results of analysis, a middle cross-section of the shell was chosen (Fig. 6a).

For the longitudinal analysis the line connecting the spots at the end of the shell with the highest stress magnitude was chosen (Fig. 6b).

a) b)

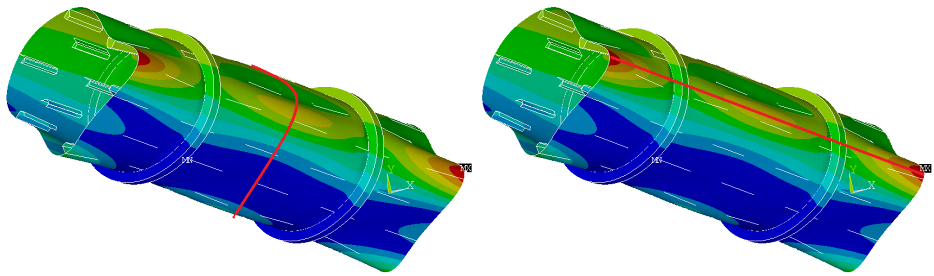

Fig. 6. Lines of the investigations of the stress distribution for cylindrical shell a) middle cross-section, b) longitudinal analysis

In following diagrams presented are the von Mises stress distributions in the ring, in the middle of the shell and along the forming line shell (Fig. 7, Fig. 9 and Fig. 10 respectively). The displacement of the middle of the shell is presented in Fig. 8.

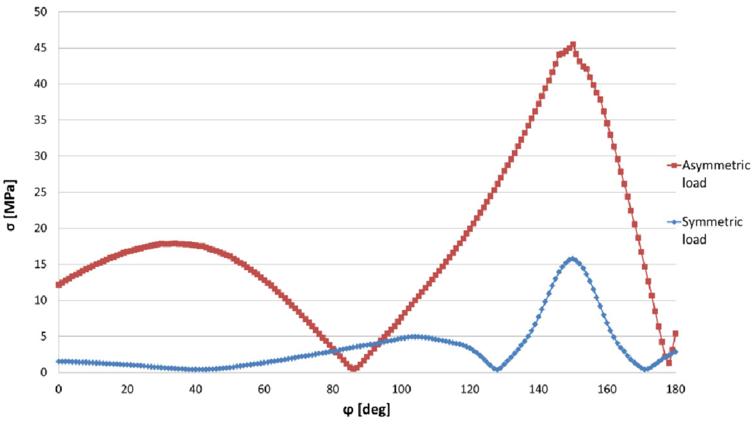

Fig. 7. The von Mises stress in the ring – symmetric vs. asymmetric load

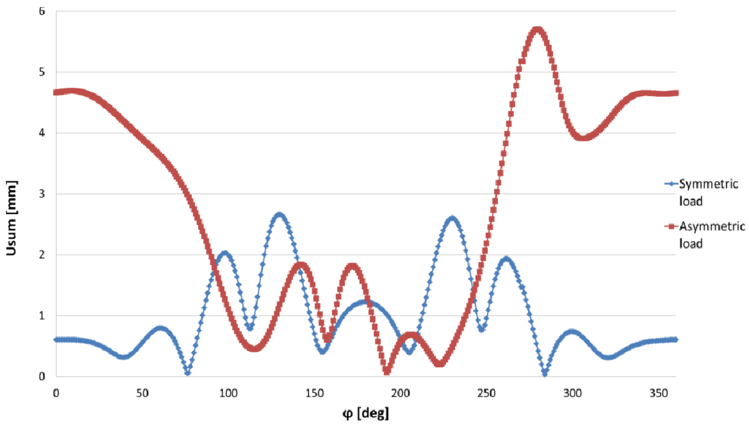

Fig. 8. The displacement in the middle of the shell

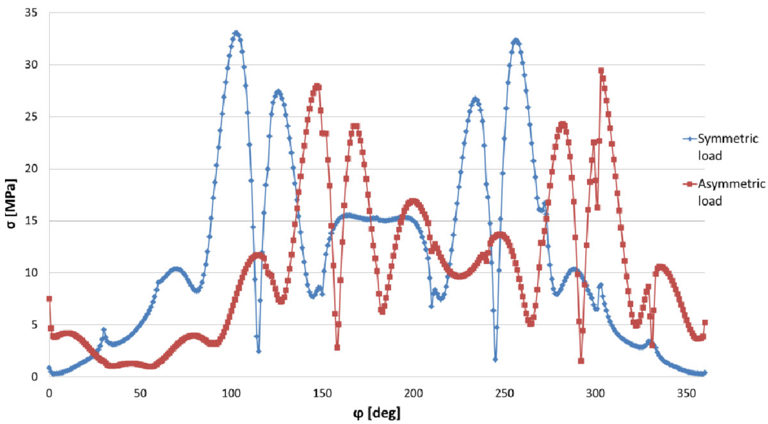

Fig. 9. The von Mises stress in the middle of the shell

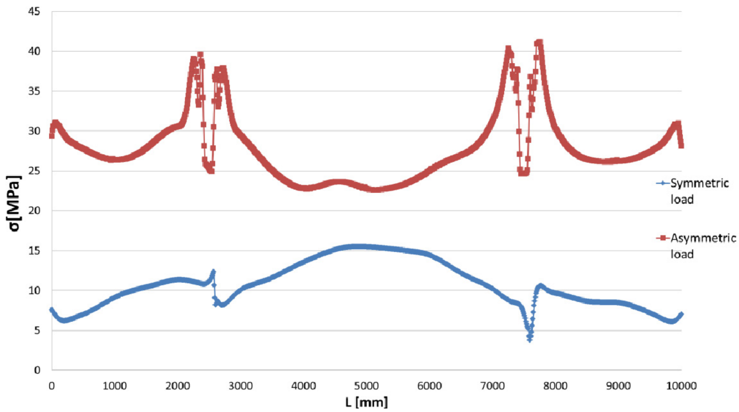

Fig. 10. The von Mises stress along the forming line

4 Conclusions

In this work a simple FEM model of the rotary ring was validated by analytical calculations using Fursow's method. The numerical model used for analysis allows one to successfully determine the areas of the greatest maximum stress and deformations. The research has shown that the loads are the greatest during the operation of the rotary dryer. That means that in order to safely design a rotary dryer asymmetric loads must be taken into account. The reduced stress values obtained during FEM analysis are smaller than the allowable ones. Maximum von Mises stresses in the analyzed ring under an asymmetric load have increased 233 % with respect to the symmetric load and were observed in the support areas. The maximum displacement of the stiffened cylindrical shell was detected in the center of the shell between the two rings.

References

1. Donnell, L.H., Wan, C.C.: Effects of Imperfections on Buckling of Thin Cylinders and Columns under Axial Compression. Journal of Applied Mechanics 17, 73–83 (1950)
2. Timoshenko, S., Woinowsky-Krieger, S.: Theory of Plates and Shells, 2nd edn., pp. 547–554. McGraw-Hill Book Co. (1959)
3. Timoshenko, S.P.: Theory of Elastic Stability, p. 439. McGraw-Hill Book Co., New York (1936)
4. Volmir, S.A.: Stability of deforming systems. Science (1967) (in Russian)
5. Volmir, S.A.: Nonlinear dynamic of plates and shells. Science (1972) (in Russian)
6. Girkmann, K.: Flächentragwerke. Book Co., Vienna (1946)
7. Flugge, W.: Stresses in shells. Springer-Verlag, Book Co., New York (1973)
8. Coz Díaz, J.J., Rodríguez Mazón, F., Garcıa Nieto, P.J., Suarez Dominguez, F.J.: Design and finite element analysis of a wet cycle cement rotary kiln. Journal in Finite Elements in Analysis and Design 39, 17–42 (2002)

9. Zachwieja, J.: Utilization of the deformable finite elements method for calculation of rotary klin shell radial deformation. Journal in Cement-Lime-Gypsum 7/69(2), 64–66 (2002)
10. Zachwieja, J.: Numerical analysis of a static load and vibrations of a rotational drum. Journal in Mechanic Scientific Papers, University of Technology and Life Sciences of Bydgoszcz 54(243), 329–345 (2004)
11. Zachwieja, J.: Radial deformation of tyres of rotary drums. Journal in Cement-Lime-Gypsum 8/70(1), 39–45 (2003)
12. Żurakowski, S., Hojarczyk, S.: Rotary kiln – designing and construction, WNT (1969)
13. Sheehan, M.E., Britton, P.F., Shneider, P.A.: A model for solids transport in flighted rotary dryers based on physical considerations. Journal of Chemical Engineering Science 60, 4171–4182 (2005)