Determination of Jiles-Atherton Model Parameters Using Differential Evolution

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Abstract. Effective and robust method of determination of Jiles-Atherton model's parameters is one of the most significant problem connected with magnetic hysteresis loop modelling. Parameters of this model are determined during the optimisation process targeting experimental results of hysteresis loop measurements. However, due to appearance of local minima, the cognitive methods have to be applied. One of the most common method are evolutionary strategies. On the other hand, typical evolutionary strategies, such as $\mu+\lambda$ are expensive from the point of view of calculation time. To overcome this problem, differential evolution was applied. As a result, the calculation time for determination of Jiles-Atherton model's parameters was significantly reduced.

Keywords: magnetic hysteresis model, differential evolution.

1 Introduction

In spite of the fact, that Jiles-Atherton model of magnetization process [1, 2] was presented the first time in 1984, it is still very popular and utilized for crystalline and amorphous alloys [3–9]. Recent developments of Jiles-Atherton model are mainly focused on physical aspects [10–13] and engineering [14] applications.

However, all these works require robust, reliable and cost-effective methods of determination of Jiles-Atherton model's parameters. Since the beginning, for this task optimisation methods were used [15]. However, efficiency of gradient optimisation methods is significantly limited due to the fact, that typical target function exhibit local minima [16]. For this reason, cognitive method of global optimisation were

J. Awrejcewicz et al. (eds.), *Mechatronics: Ideas for Industrial Applications*,
Advances in Intelligent Systems and Computing 317, DOI: 10.1007/978-3-319-10990-9_2

used, such as genetic algorithms [17] or evolutionary strategies [18]. Among used solutions, the (μ+λ) strategy, together with simulating annealing and local gradient optimisation [19] is popular. However, such solution is extremely expensive from the point of view of calculation time.

This paper presents new approach to determination of Jiles-Atherton model parameters oriented on differential evolution algorithm. As a result the computation time was significantly limited without reduction of efficiency and robustness of determination of Jiles-Atherton model's parameters given for specific experimental results.

2 Principles of Jiles-Atherton Model of Magnetic Hysteresis

Modelling the magnetic hysteresis with Jiles-Atherton model covers two steps [2]: determination of anhysteretic magnetization M_{ah} and modelling the hysteresis by differential equation considering the sign of changes of magnetizing field H. This approach is recently criticized [10], however good agreement with experimental data can be achieved.

In Jiles-Atherton model anhysteretic magnetization for isotropic magnetic materials M_{ah_iso} is given by the Langevin equation [2]:

$$M_{ah\_iso} = M_s \left[\coth\left(\frac{H_e}{a}\right) - \left(\frac{a}{H_e}\right) \right] \tag{1}$$

where a is determined by the domain walls density in the magnetic material [2], whereas effective magnetizing field H_e is given as [1]:

$$H_e = H + \alpha \cdot M \tag{2}$$

where α determines interdomain coupling.

According to corrected Ramesh extension of Jiles-Atherton model for anisotropic, ferromagnetic materials, anhysteretic magnetization in anisotropic magnetic materials M_{ah_aniso} [20, 21] is given as:

$$M_{ah\_aniso} = M_s \left[\frac{\int_0^{\pi} e^{0.5(E(1)+E(2))} \sin\theta \cdot \cos\theta \cdot d\theta}{\int_0^{\pi} e^{0.5(E(1)+E(2))} \sin\theta \cdot d\theta} \right] \tag{3}$$

where

$$E(1) = \frac{H_e}{a}\cos\theta - \frac{K_{an}}{M_s \cdot \mu_0 \cdot a}\sin^2(\psi - \theta) \tag{4}$$

$$E(2) = \frac{H_e}{a}\cos\theta - \frac{K_{an}}{M_s \cdot \mu_0 \cdot a}\sin^2(\psi + \theta) \tag{5}$$

Presented equations are valid for uniaxial anisotropy, where K_{an} is the average energy density and ψ is the angle between direction of magnetizing field and the easy axis of magnetization due to the anisotropy. Other types of anisotropy were also considered [22], however, any form of anisotropic anhysteretic magnetization equation can be solved using antiderivatives. As a result, it have to be solved using numerical integration.

In Jiles-Atherton model, the hysteresis loop is determined by the irreversible magnetization M_{irr} [1]:

$$\frac{dM_{irr}}{dH} = \delta_M \frac{M_{ah} - M_{irr}}{\delta \cdot k} \tag{6}$$

where the parameter k quantifies average energy required to break pining site. In this equation parameter $\delta = +1$ for $\frac{dH}{dt} \geq 0$ and $\delta = -1$ for $\frac{dH}{dt} < 0$. Additional parameter $\delta_M = 0$ when $\frac{dH}{dt} < 0$ and $M_{an} - M > 0$ as well as when $\frac{dH}{dt} \geq 0$ and $M_{ah} - M < 0$. In other cases $\delta_M = 1$. Parameter δ_M guarantees that incremental susceptibility is always positive, what is physically judged [4, 17].

In the original Jiles-Atherton model, parameter k is constant [1, 2]. However, even Jiles and Atherton indicated, that this assumption is not judged from the physical point of view. Changes of parameter k are caused by changes of the average energy required to break pining site [23]. For this reason, J-A-S model's parameter k can be connected with the magnetic state of the material (described by magnetization M) by the following equation [24]:

$$k = k_0 + \frac{e^{k_2 \cdot (1 - |M| / Ms)} - 1}{e^{k_2} - 1} \cdot (k_1 - k_0) \tag{7}$$

where M_s is saturation magnetization, whereas k_0, k_1 and k_2 describe the function determining k. In given equation, parameter k_0 determines the minimal value of k, parameter k_1 determines the maximal value of k, and k_2 is shape parameter. For positive values of k_2 the $k(|M|/M_s)$ function is concave, and for negative value of k_2 this function is convex [25].

In the Jiles-Atherton model, the reversible magnetization M_{rev} is given by the equation [2]:

$$M_{rev} = c \cdot (M_{an} - M_{irr}) \tag{8}$$

where c is parameter describing magnetization reversibility. Finally, total magnetization M may be calculated from following ordinary differential equation (ODE) [4, 17]:

$$\frac{dM}{dH} = \frac{\delta_M}{1+c} \frac{M_{ah} - M}{\delta \cdot k - \alpha(M_{ah} - M)} + \frac{c}{1+c} \frac{dM_{ah}}{dH} \tag{9}$$

considering the initial state of demagnetizated material, where $H = 0$ and $M = 0$.

It should be highlighted, that accurate solving of Jiles-Atherton model's equation is not trivial. Anhysteretic magnetization should be calculated using Gauss-Kronrod approximation for cyclic functions [26], whereas for solving the differential equation (9), the 4th order Runge-Kutta [27] method is recommended.

3 Determination of Jiles-Atherton Model's Parameters Using Differential Evolution

To use optimization methods for determination of the Jiles-Atherton model's parameters, the target function has to be proposed. In presented investigation, the target function was given by the following equation:

$$F = \sum_{i=1}^{n} (B_{JA}(H_i) - B_{meas}(H_i))^2 \tag{10}$$

where B_{JA} were the results of the modelling for magnetizing field H_i and B_{meas} were the results of the experimental measurements respectively. It should be indicated that during the optimization process, target function F was calculated simultaneously for 3 hysteresis loops measured for different magnetizing fields. This enabled optimization focused on achieving the model's parameters suitable for wider range of the magnetizing field.

The differential evolution algorithm [28] is stochastic derivative-free method, designed for difficult non-linear non-convex optimization problems in continuous domain. As most of the other members of evolutionary algorithms family, differential evolution process group of solutions (called population of individuals). The algorithm is iterative – at each iteration t each solution from $t - 1$ is modified by mutation and crossover. The thing that is specific to differential evolution algorithm is a mutation operator. In the canonical version of the algorithm, a mutant v_i is generated by adding difference between two randomly selected solutions to the third randomly selected solution, i.e.:

$$v_i = x_{r0} + F \cdot (x_{r1} - x_{r2}) \tag{11}$$

where

$F \in (0, 1)$ is scale factor and it is a parameter of the algorithm.

There are several variants of this canonical schema. We used variant called differential evolution/local-to-best/1/bin, where the *i*-th mutant is a result of the sum of *i*-th solution, difference of two randomly selected solutions and difference of the best solution in current population and *i*-th solution, i.e.:

$$v_i = x_i + F \cdot (x_{best} - x_i) + F \cdot (x_{r1} - x_{r2}) \tag{12}$$

The model and search algorithm were implemented in R language [29]. We used differential evolution version implemented by Ardia et al. [30]. We accepted parameters of algorithm proposed by the implementation, i.e. 200 generations, 60 individuals, $F = 0.8$, $CF = 0.5$ (crossover probability).

4 Results of Modelling

During the experiment, the parameters of $B(H)$ magnetic hysteresis loops of Finemet $Fe_{73.5}Si_{13.5}Nb_3Cu_1B_9$ nanocrystalline alloy were determined using differential evolution algorithm. Since algorithm is stochastic, 25 independent runs were performed. Summary of the results is presented in Table 1, whereas Table 2 presents values of Jiles-Atherton parameters determined by the best solution found. These parameters were used to calculate theoretical magnetic hysteresis loop. The result if its comparison to measurements is depicted in Fig. 1. It can be seen, that acceptable agreement between experimental data and results of modelling was achieved.

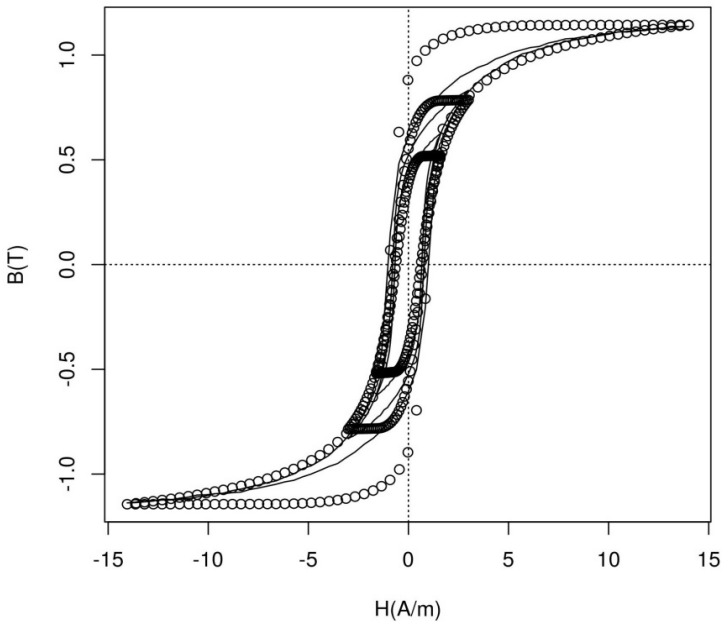

Fig. 1. Magnetic hysteresis loop of Finemet $Fe_{73.5}Si_{13.5}Nb_3Cu_1B_9$ nanocrystalline alloy: experimental results (lines) and results of modeling (circles)

Table 1 presents the values of Jiles-Atherton parameters determined during the experiment, whereas Table 2 presents the parameters describing efficiency differential evolution algorithm.

Table 1. Values of Jiles-Atherton parameters determined by differential evolution algorithm

Parameter		Value
A	A/m	1.464
k_0	A/m	330.5
k_1	A/m	0.556
k_2		–18.50
c		0.3906
M_s	A/m	$1.045\ 10^6$
α		$9.837\ 10^{-7}$
K_{an}	J/m^3	2791

Table 2. Summary of the results of 25 independent runs of differential evolution algorithm. An average, best and standard deviation of objective function value (equation 10) is reported together with an average time of one algorithm run

average	best	standard deviation of objective function value	time of calculations (h)
4.33	3.66	0.41	21

5 Conclusion

Presented results confirm, that differential evolution algorithm is interesting alternative for (μ+λ) evolutionary strategy algorithm used previously [5]. Time of calculation required for differential evolution algorithm is about 10 times lower than for (μ+λ) evolutionary strategy. On the other hand, both algorithms determine global minima of target function with accuracy sufficient for most of technical applications.

The support of the bilateral project APVV-Sk-PL-0043-12 is acknowledged from Slovak side. Polish side (Institute of Metrology and Biomedical Engineering) was supported by statutory founds within Polish-Slovak bilateral cooperation.

Calculations for the modelling were made in the Interdisciplinary Centre for Mathematical and Computational Modelling of Warsaw University, within grant G36-10.

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