

Chapter 13

Cellular Automaton Manipulator Array

Ioannis Georgilas, Andrew Adamatzky and Chris Melhuish

Abstract We present a cellular automaton architecture for massive-parallel manipulation tasks. The cellular-automaton manipulator is an array of actuators, which interact locally with each other and generate coordinated manipulation forces for precise translation of the manipulated object. The cellular-automaton actuator arrays behave as an excitable medium, where initial perturbation leads to propagation of excitation waves. The excitation waves are physically mapped onto the hardware actuation waves. We analyse different types of excitation and manipulation patterns and physical implementations of the actuating surface.

13.1 Introduction

Traditionally industrial manipulation tasks are performed in an autonomous manner with the use of large, usually 6 Degrees of Freedom manipulators, that can move objects lighter than the manipulating robot itself. Such manipulators are either single units or small groups of units. In the groups of units collaborative tasks are achieved by planning the spatial trajectories of individual units in advance, often before start of the manipulation. Although the vast majority of manipulation tasks are still completed by these robots a new type of manipulation emerged: micro-scale manipulation and assembly. This new type of manipulation require high precision, optimum force application, non-prehensile handling and concurrent manipulation of several objects during the work cycle. Classic manipulation approaches struggle to satisfy these requirements. We therefore explore a specialised massively parallel hardware operating like a smart manipulating surface [13]. A manipulating surface is

I. Georgilas (✉) · A. Adamatzky · C. Melhuish
University of the West of England, Coldharbour Lane, Bristol, UK
e-mail: ioannis.georgilas@uwe.ac.uk

C. Melhuish
Bristol Robotics Laboratory, Coldharbour Lane, Bristol, UK
e-mail: Chris.Melhuish@brl.ac.uk

A. Adamatzky
e-mail: andrew.adamatzky@uwe.ac.uk

an array of simple actuators, each of them has a small power output, that collectively transport, orient and position objects whose masses and sizes are higher compared to the power output generated by a single actuator. Each individual actuator is relatively inexpensive, and a modular structure of the parallel manipulator would allow for mass production and scalability.

One of the key technologies utilised to control the motion of the objects for these systems are arrays of air jets [11, 14, 22, 24]. Other approaches include mechanical wheel based arrangements [23, 26], sound based solutions [31] and electromechanical actuators to excite membranes [12, 27]. Most of these manipulation methods are using the underlying physical phenomena in an intelligent way to control the manipulation, i.e. flow interactions [16], and anisotropic friction control [30]. Various control systems have been proposed mainly on the concept of closed-loop control, with feedback provided either from a camera [15] or other form of light information, i.e. photodiodes [11].

The main issue in all these control methods is a scalability of the task. Controlling the vast number of actuators is computationally plausible as shown by simulation examples [17] but still expensive and usually comes at the cost of precision [8]. The manipulation task becomes even more complicated when multiple objects must be processed simultaneously and the controller must synchronise the spatio-temporal trajectories of the objects.

A strong alternative to address the problem of scalability of control is the use of lattice automata as the underlying controllers of the manipulation array. Specifically each individual actuator's state is controlled by the state of the automaton. The intelligence and control is achieved by the emergence of order in the lattice. Furthermore the system scales uniformly since the computation cost of the state-machine for each actuator remains the same irrespective of the number of actuators. The use of lattice automata as robot controllers is not new and have been utilised to control path planning for mobile robot platforms [4, 5, 29]. The proposed here use of the lattice automata differentiates from previous implementations in the effort to enhance and bring forward the synergies necessary to complete the required manipulation task.

We discuss firstly what characteristics of lattice automata are ideal for this type of control, and what 'modes' of operations optimally can achieve this, and by which set of state rules. Also some details on the importance of the hardware and how it improves the synergy will be given. We evaluate the behaviour and operation of the lattice performance in a series of simulation and real-world experiments. The qualitative and quantitative performance of the experimental prototype of cellular-automaton actuator array is analysed.

13.2 Cellular Automata Controller

Actuators arranged in a two-dimensional array is one of most optimal ways to achieve modularity, controllability and fault tolerance. If we want an actuator array to be autonomous, i.e. not controlled by a host computer, we should allow each elementary

actuating unit to have some computing power, in principle some local sensing, and be able to interact with its immediate actuating neighbours. By using local interaction the actuators can establish a coordinated action and manipulate objects, which are substantially larger in size than a single actuator, leading to epiphenomenal ‘task’ behaviour. Cellular automata would be ideal controller for the arrays of actuators. This is because a cellular automaton is an array of finite state machines, or cells, which update their states in discrete time depending on states of their immediate neighbours. Thus we can assign a unique cell of a cellular automaton to an elementary unit of an actuating array. The topology is preserved. All cells of a cellular automaton update their states by the same rule, all units of the actuator array act by the same mechanics.

A requirement for focused force application suggests us that most convenient manipulation rules should be based on propagating patterns, either omni-directional waves or travelling localizations [3]. Mobile self-localizations can be described as waves or wave-fragments (gliders, wavelets), the manipulation abilities of which have been previously demonstrated in [21]. The use of wave-fragments and their synchronisation signals have been analysed in [18], where the concept of metachronal waves [7] was applied in lattice automata-controlled hardware.

The lattice automaton we are proposing to utilise is the 2^+ medium [1, 6], a 3-state (excited (+), refractory (-), resting(·)) cellular automaton with well defined mobile self-localizations. The cell-state transition rule of the 2^+ medium is given in Eq. 13.1: a resting cell excites if it has exactly two excited neighbours; an excited cells becomes refractory; a refractory cell returns to a resting state.

$$x^{t+1} = \begin{cases} +, & x^t = \cdot \text{ and } \sum_{y \in u(x)} \chi(y, +) = 2 \\ -, & x^t = + \\ \cdot, & \text{otherwise} \end{cases} \tag{13.1}$$

where $\chi(y, +) = 1$ if $y = +$ and 0, otherwise.

In the 2^+ medium both omnidirectional waves and wave-fragments can be created using simple initial conditions. Both types of waves can travel in the cardinal directions, which is an extra benefit for manipulation tasks, since it allows for clear directional vectors for the manipulated objects. The basic manipulation element is the wave-fragment, or glider, consisting of an excited head (two excited cells) and a refractory tale (two refractory cells), Fig. 13.1 The ternary nature of the automaton is convenient for the hardware interface. Each state of the automaton can be directly mapped onto a state of actuator motor: off, clockwise spin, and counter-clockwise spin.

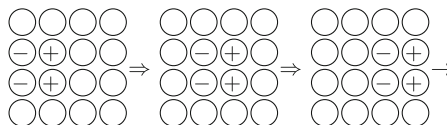


Fig. 13.1 Typical travelling localisation in 2^+ -medium [2]. The localisation propagates eastward. The excitation wave-front is followed by refractory tail. Excited sites are shown by ‘+’, refractory by ‘-’

Table 13.1 Combination scenarios of physical contacts between actuator surfaces and manipulated object

Cilia in...	Scenario 1	Scenario 2	Scenario 3	Scenario 4 ^a
Object	–	+	–	+
Surface	–	–	+	+

Presence of cilia on object or surface is indicated by ‘+’, absence by ‘–’

^aScenario 4 has not been investigated

13.3 Hardware Layer Role

The hardware architecture of a cellular automaton controller is proposed in [20]. The architecture draws its inspiration from the ciliary motion of *Paramecium caudatum*. Cilia are also used to exploit anisotropic friction [25, 32] and thus to generate force fields to move the object. Although some research teams have introduced this idea of programable force-fields with anisotropic friction using vibratory motion [28, 30], our method differentiates on the mechanism of generating the friction, hence the force. We propose the coordinated vibratory motion of cilia structures controlled by lattice automata.

To fully evaluate the applicability of the proposed method different hardware architectures are investigated. The differentiation factor of these architectures is the location of the cilia-like structures. Table 13.1 gives us the potential combinations.

Scenario 4 in Table 13.1 is being presented only for completeness. Although it is possible to create a system with cilia in both the object and the manipulation surface, it will be a system of complex physical interactions between the cilia on actuators’ surfaces and the cilia on the manipulated object.

13.4 Experimental and Simulation Results

Each of the scenarios presented in Table 13.1, except Scenario 4, was tested in hardware or computer simulation to evaluate the performance of the lattice automata in control. In hardware tests we used the prototype described in [19, 20]. The prototype is an array 8×8 oscillating motors, covered with a silicon membrane. The array is 110 × 110 mm in size. Rotation direction of each motor is controlled independently on other motors. We used overhead video camera to measure motion of the manipulated objects.

Computer simulation of the actuator array was made using MATLAB and APRON—(A)rray (P)rocessing envi(RO)nement [9], a real-time simulation platform for working with and debugging two-dimensional arrays of data and rapidly prototyping array based algorithms. For some of the simulation experiments, where the focus was on the physics-based analysis of manipulation, an environment for physics simulation was utilised [10].

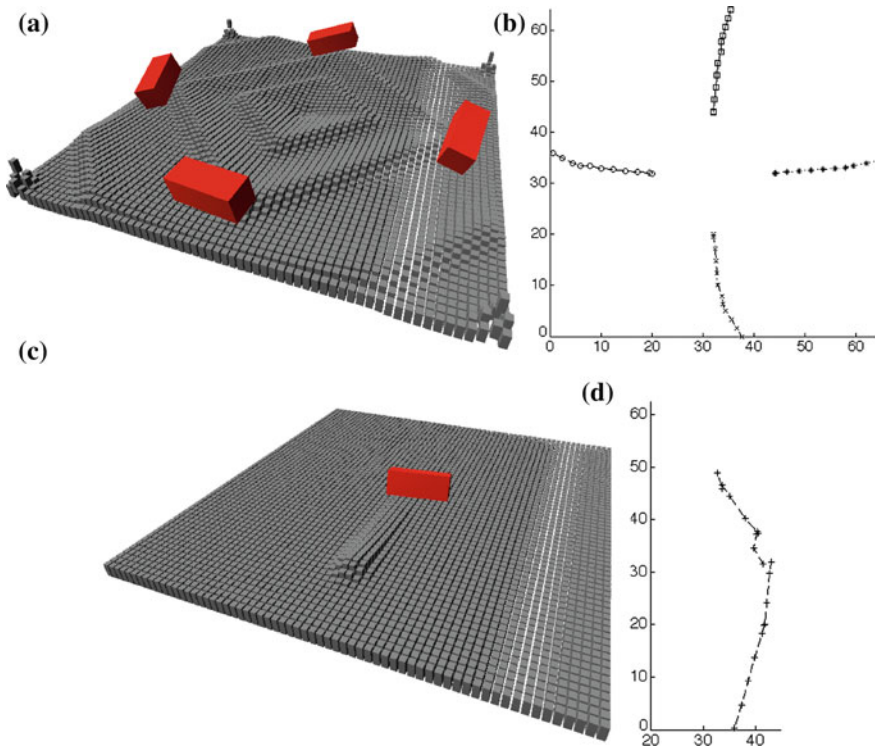


Fig. 13.2 Manipulation with waves. Simulation frames of manipulating objects with omnidirectional wave (a) and travelling localisation (c). Trajectories of objects translated by these waves are shown in (b) and (d). x and y -axis in simulator distance units. Simulation is implemented in APRON [9]

13.4.1 Scenario 1: No Cilia

In the no-cilia scenario two sets of simulation experiments were carried out and the behaviour of omnidirectional and wave fragments was investigated. In the initial experiment, firstly four objects were displaced using omnidirectional waves, Fig. 13.2a, and secondly a single object using linearly propagating wavelets, Fig. 13.2c. In the latter experiment only one object was used given the narrow focus of the glider. The trajectories of the objects were recorded. They are shown in Fig. 13.2b, d.

In the laboratory experiments with hardware prototype the object shown in Fig. 13.3a was manipulated by the actuator array. The object is a 40×20 mm hollow, plastic, rectangular box. We did not execute laboratory experiments with omnidirectional waves because in such type of manipulation objects move along coupled trajectories which is not suitable for a precise manipulation; also, the manipulated objects ‘hopped’ above the surface, see more details in Sect. 13.5. In the laboratory

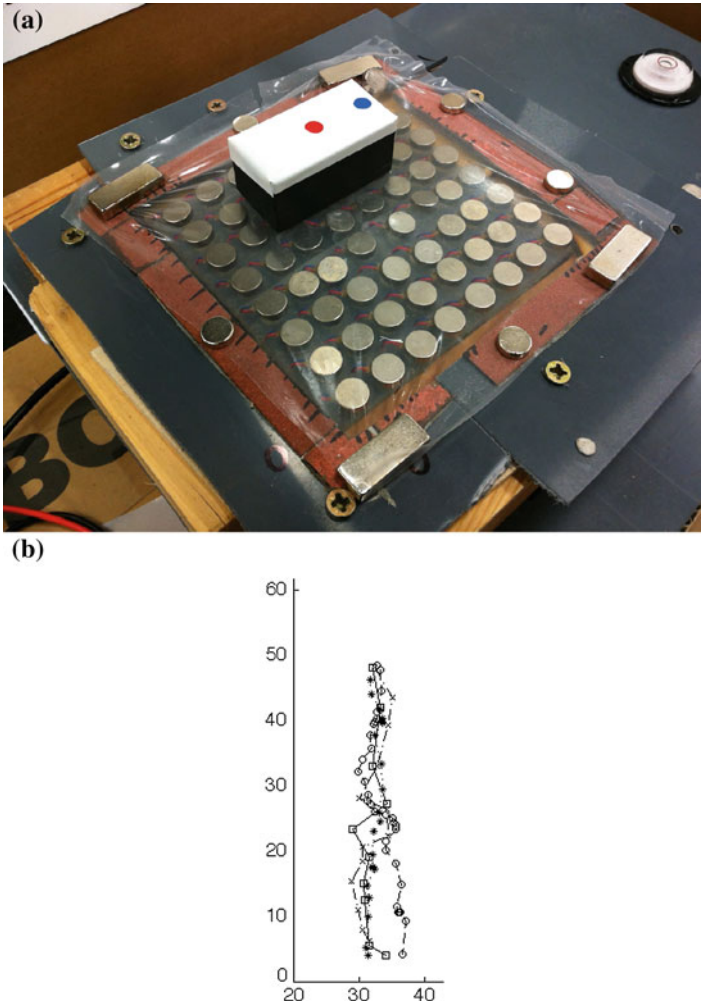


Fig. 13.3 Neither actuator array nor the manipulated object are equipped with cilia. **a** Photograph of a hardware prototype of 8×8 actuator array with the manipulated object on *top*. **b** Four experimental trajectories; x and y axes adjusted to simulator distance units

experiments on manipulating with travelling localisations four different trajectories were recorded, see Fig. 13.3b. Although each trajectory is slightly different due to mechanical variations of the prototype, the overall pattern is as predicted in the simulation experiment.

As can be observed for the omni-directional waves the ‘broadcast’ mode is typical: waves traveling all across the lattice render the trajectories of the objects coupled. Moreover, as seen from the screen shot in Fig. 13.2a, the delivered energy causes vertical displacement of the objects, a ‘hopping’ effect. On the other hand, the wavelet

manipulation operates as expected, the object moves in a linear fashion. The slight parabolic motion can be attributed to the specific dynamics of the simulation engine.

13.4.2 Scenario 2: Object with Cilia/Surface Without Cilia

In the second scenario an object with cilia was manipulated by the actuator array without cilia. We selected a toothbrush as a manipulated object due to its intriguing geometry, see Fig. 13.4a, b and the availability of a ciliated object of standardised construction. The toothbrush was 35×12 mm in size. A photo of the experimental prototype is shown in Fig. 13.4c. The cilia, or bristles, of the toothbrush create an anisotropic friction that allows the surface to manipulate the object.

Two sets of experiments have been conducted. The first set dealt with linear motion of the toothbrush, the target is in the same line as the toothbrush's initial position. In the second set of experiments, we introduced a change of the toothbrush's direction path is necessary: the target was positioned at a 90° angle from the line of direction of the toothbrush. This cornering action was achieved by a combination of glider motions and turning patterns. The turning patterns were represented in cellular automaton controller as two excited + and two refractory - cells in a crossed configuration.

Two trajectories were recorded for each set. In Figs. 13.5 and 13.6 the overlap of the trajectories, with rotation tracked by the directional vectors, can be seen along with the start position of the toothbrush and the target. In Figs. 13.5 and 13.6 the trajectory and rotation of toothbrush body is separately plotted for ease of analysis.

By analysing the data of the trajectories we can identify some interesting patterns. Although there are variations, both sets of trajectories are similar with most of the same characteristic motifs. These variations can be attributed to specific hardware and software aspects. Three key aspects are identified:

- **Drifting movement of the toothbrush.** The toothbrush head, because of its special geometry, tends to move in a drifting fashion, oscillating from side to side.
- **Motor array variations.** Although the highest specifications were used to design and construct the prototype some variations still exist. Differences in geometry of the modules and inevitable differences in motor operation (manufacturer's specifications) result in small variations of performance from cell to cell.
- **Algorithm and CA propagation.** The implementation of the lattice automaton algorithm affects operations in two distinctive ways. First, the pattern used to turn the toothbrush might affect its trajectory in an unwanted manner. Secondly, the lattice refresh interval (200m in the experiments) affects movement patterns, indicating that there are other spatio-temporal dynamic phenomena at play impacting the operation of the system.

The different reasons for the patterns recognised in the experiments, allow to draw some interesting experience regarding the system both in hardware and software terms. The first two, drifting movement and motor variations, are related to the

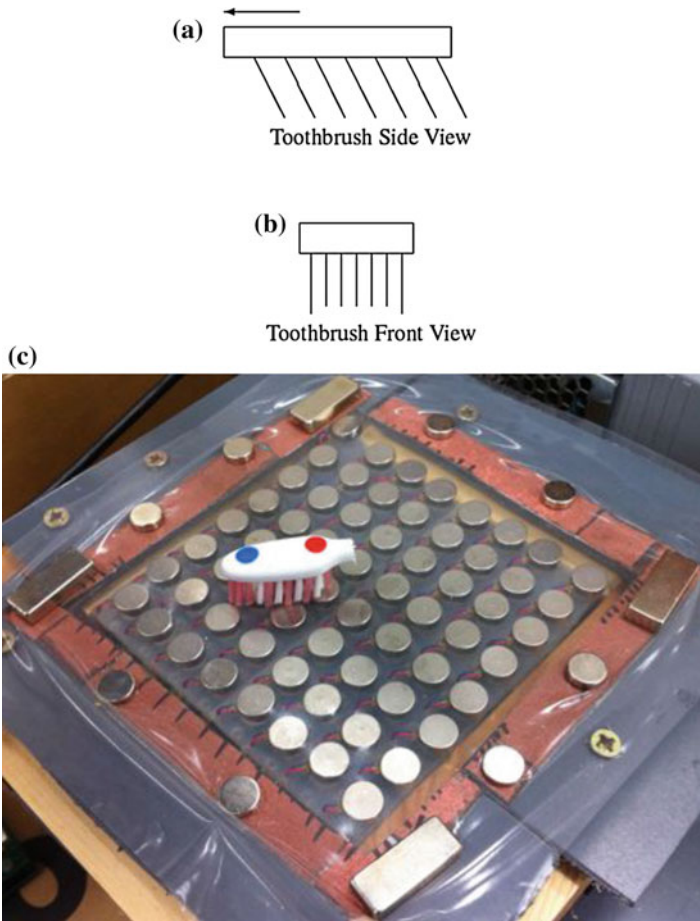


Fig. 13.4 Manipulated object has cilia but actuator array has none. **a** Side and **b** front view of object's cilia geometry responsible for the anisotropic friction, **c** Photograph of the parallel manipulator with an ciliated object on *top*

hardware being used, toothbrush head geometry and prototype variant. In order to emphasise the robustness of the proposed conveyor system the decision was taken to accept mechanical variations within specific tolerances. This way the system proves that it can compensate for those variations using the intelligent underlying control algorithm. Due to the systemic occurrences of both phenomena, it is possible to map those mechanical imperfections and incorporate them in the lattice automata controller making the system more robust.

The third reason is related to the implementation of the automaton controller. Simple linear gliders in 2^+ medium seem to provide a good linear object propagation while the proposed turning patterns address to a certain degree the 90° turns required for the selected trajectories. Both the gliding and the turning patterns perform

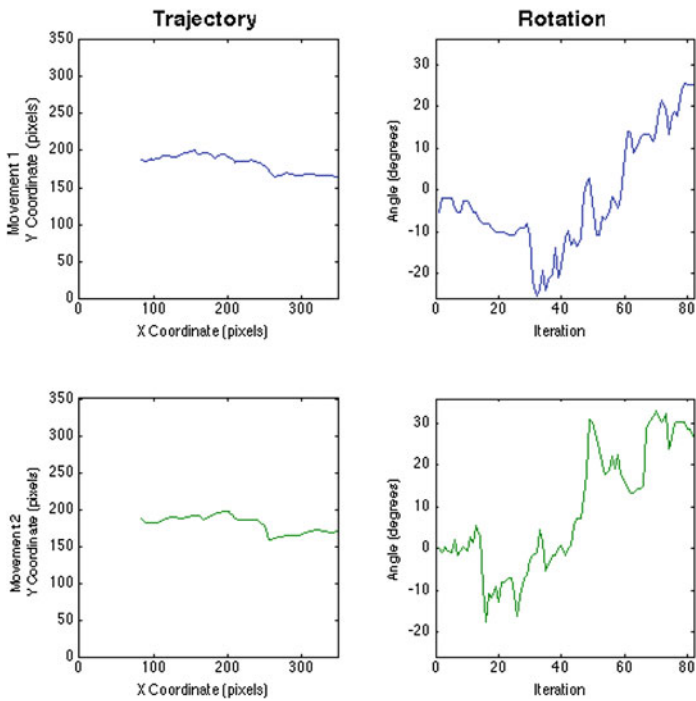
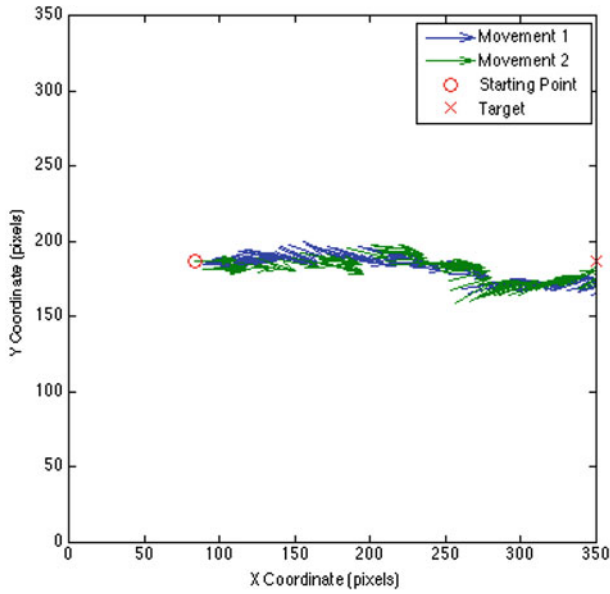


Fig. 13.5 Set of Linear Movements

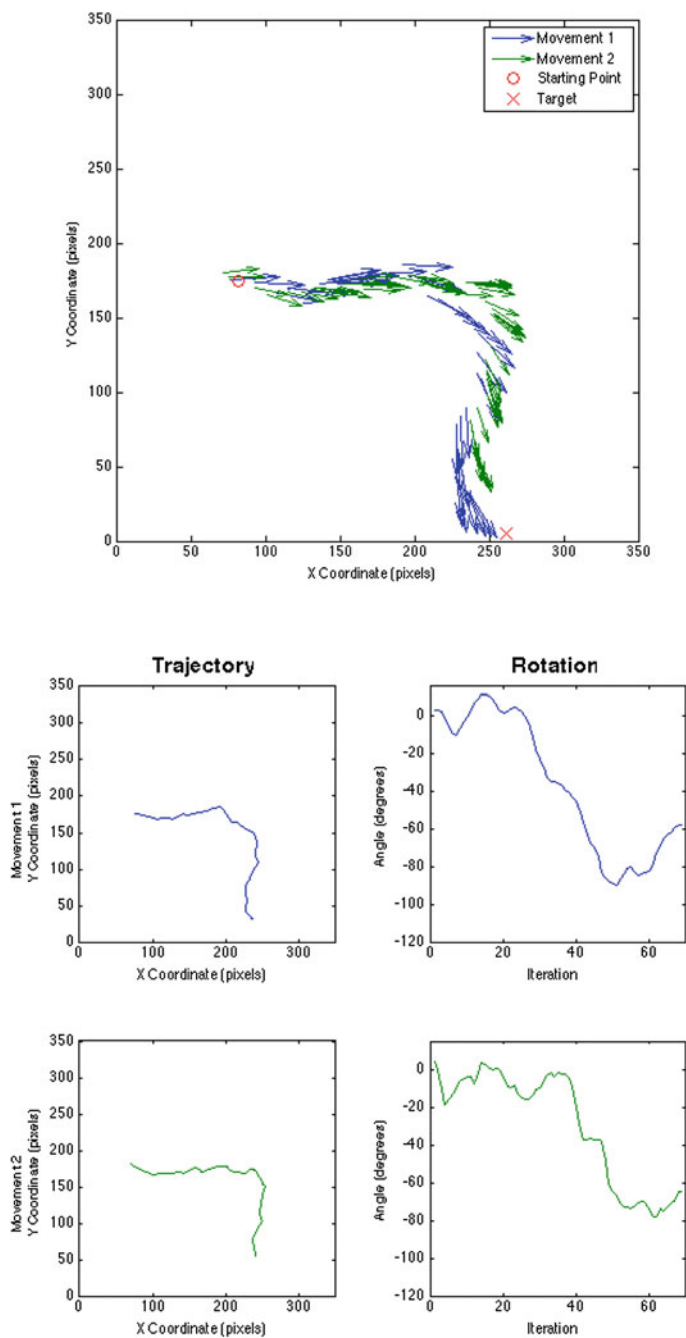


Fig. 13.6 Set of Corner (90°) Movements

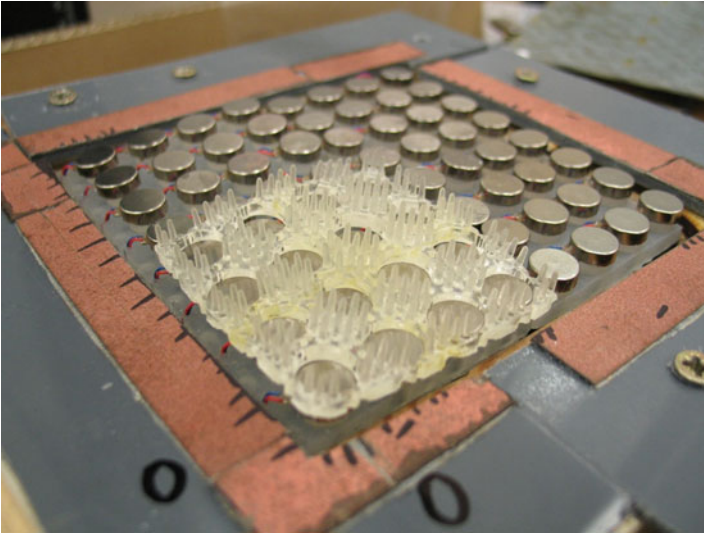


Fig. 13.7 Photograph of the bristles fabricated to emulate natural cilia. This design is used as an alternative to the membrane design described in [20]

satisfactorily in compensating for the hardware’s weaknesses. Undoubtedly, determining the iteration (generation) step is a crucial parameter affected by the dynamics of the system.

13.4.3 Scenario 3: Object Without Cilia/Surface with Cilia

The third scenario is tested using simulation experiments based on the modelled behaviour of the ciliated surface. A small hardware version of the proposed system has been fabricated for verification, see Fig. 13.7. The model is based in the vortex-like behaviour of a single motor and the resulting force field as exerted by the cilia structures is shown in Fig. 13.8a.

In this scenario, the task was to move a square object from the lower right corner of the lattice to the centre of it. This corresponds to the natural task encountered in microorganisms for the transportation of food to their ‘mouth’ pore [18]. The importance of synchronisation signals was investigated with the application of three different signals. Specifically the signals tested are (a) Single motor actuation, Fig. 13.8b; (b) Random motor actuation, Fig. 13.8c; and (c) Metachronal wave motor actuation, created by linearly traveling localisation, Fig. 13.8d, f. This selection of signal was made to demonstrate the necessity of proper synchronisation and co-action of actuators in the manipulation tasks.

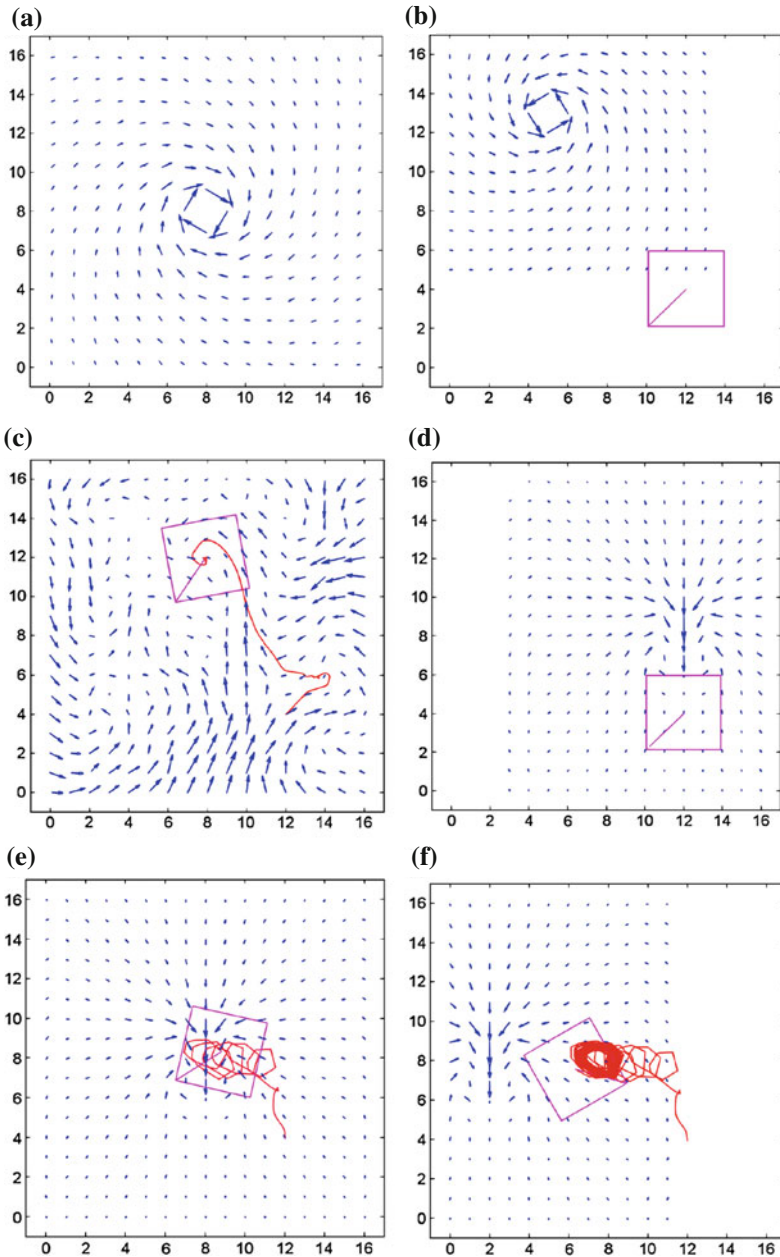


Fig. 13.8 Simulation frames from MATLAB with APRON generated control signals. The vectors of the force field, the rectangular object with a rotation indication line and the trajectory are depicted. **a** The vortex force-field created by a single motor. **b** The single motor force field ‘pulling’ the object towards the centre of the lattice (frame 60). **c** The trajectory of the object under a randomly generated force field (frame 400). **d–f** Frames 0, 60 and 400 of the trajectory under the metachronal wave signal. Object is placed at coordinates (12,4). Axis are simulation based units

Analysing the trajectories from the different control signals we can deduce some interesting results regarding the cooperative attributes of the various control signalling (Fig. 13.8). In the single motor approach, the motor trying to ‘pull’ the object towards the centre fails even to start moving the object (Fig. 13.8a, b). Investigating the exhibited forces we find that the single motor is not able to exert the friction between the object and the surface. This is a demonstration that a single ‘cilium’ fails to achieve the task and some form of cooperative action needs to take place. The random motor approach overcomes the lack of power, since it does move the object from its initial position. Nonetheless, the object is moving along a random path (Fig. 13.8c). Furthermore, it might be locked in local attractors that will not coincide with the intended target. Hence, the use of multiple ‘cilia’ is necessary to produce manipulation, but random ‘beating’, i.e. signalling, does not create controllable behaviour. Finally, the 2^+ -medium glider, seen as metachronal wave, moves the object towards the target (Fig. 13.8d, f).

13.5 Conclusions

In this chapter we demonstrate the use of lattice automata as distributed control systems for manipulation applications. We have shown that controllable manipulation can be reached via interaction between cells/actuators and also between physical surfaces of the actuator units and the manipulated object. In all three scenarios, as shown in computer modelling and laboratory experiments, travelling localisations (gliders, solitons, wave-fragments) are proved to be ideal manipulating patterns. Further studies are required to select the best morphology of the control patterns in cellular automata, which can lead to robust and precise manipulation. Actuator arrays discussed in the chapter are open-loop manipulators: actuating units do not sense the manipulated object. In our future research we aim to develop closed-loop manipulators, where actuators are equipped with sensors allowing them to feel the manipulated object.

References

1. Adamatzky, A.: Dynamical universal computation in excitable lattices. In: Margenstern, M. (ed.) MCU, vol. 2, pp. 194–213. IUT, Metz (1998)
2. Adamatzky, A.: Computing in Non-linear Media and Automata Collectives. Institute of Physics Publishing, Bristol (2001)
3. Adamatzky, A., Costello, B.D.L., Asai, T.: Reaction-Diffusion Computers. Elsevier, New York (2005)
4. Adamatzky, A., De Lacy Costello, B., Melhuish, C., Ratcliffe, N.: Experimental reaction-diffusion chemical processors for robot path planning. *J. Intel. Robot Syst.* **37**(3), 233–249 (2003). doi:[10.1023/A:1025414424756](https://doi.org/10.1023/A:1025414424756)
5. Adamatzky, A., Melhuish, C.: Phototaxis of mobile excitable lattices. *Chaos. Soliton. Fract.* **13**(1), 171–184 (2002). doi:[10.1016/S0960-0779\(00\)00233-2](https://doi.org/10.1016/S0960-0779(00)00233-2)

6. Adamatzky, A.I.: Controllable transmission of information in excitable media: The 2+ medium. *Adv. Mater. Opt. Electron.* **5**(3), 145–155 (1995). doi:[10.1002/amo.860050303](https://doi.org/10.1002/amo.860050303), <http://dx.doi.org/10.1002/amo.860050303>
7. Aiello, E., Sleight, M.A.: The metachronal wave of lateral cilia of mytilus edulis. *J. Cell Biol.* **54**(3), 493–506 (1972)
8. Ashley-Rollman, M., Pillai, P., Goodstein, M.: Simulating multi-million-robot ensembles. In: *Proceedings—IEEE International Conference on Robotics and Automation*, pp. 1006–1013 (2011)
9. Barr, D.R., Dudek, P.: Apron: a cellular processor array simulation and hardware design tool. *EURASIP J. Adv. Signal Process.* **2009**, 9 (2009)
10. Barr, D.R., Walsh, D., Dudek, P.: A smart surface simulation environment. In: *IEEE International Conference on Systems, Man, and Cybernetics (SMC)*, 2013, pp. 4456–4461. IEEE (2013)
11. Berlin, A., Biegelsen, D., Cheung, P., Fromherz, M., Goldberg, D., Jackson, W., Preas, B., Reich, J., Swartz, L.E.: Motion control of planar objects using large-area arrays of mems-like distributed manipulators. Xerox Palo Alto Research Center CA/USA, Presented at *Micro-mechatronics* (2000)
12. Bohringer, K.F., Bhatt, V., Donald, B., Goldberg, K.: Algorithms for sensorless manipulation using a vibrating surface. *Algorithmica* **26**(3–4), 389–429 (2000). (New York)
13. Bohringer, K.F., Donald, B.R., Mihailovich, R., MacDonald, N.C.: Sensorless manipulation using massively parallel microfabricated actuator arrays. In: *Proceedings on IEEE International Conference on Robotics and Automation*, 1994, pp. 826–833. IEEE (1994)
14. Boutoustous, K., Laurent, G., Dedu, E., Matignon, L., Bourgeois, J., Le Fort-Piat, N.: Distributed control architecture for smart surfaces. In: *2010 IEEE/RSJ International Conference Intelligent Robots and Systems, IROS'10*, pp. 2018–2024, Taipei (2010)
15. Delettre, A., Laurent, G., Le Fort-Piat, N.: 2-dof contactless distributed manipulation using superposition of induced air flows. In: *2011 IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS'11*, pp. 5121–5126. San Francisco, CA (2011). doi:[10.1109/IROS.2011.6048251](https://doi.org/10.1109/IROS.2011.6048251)
16. Delettre, A., Laurent, G.J., Fort-Piat, L., Varnier, C., et al.: 3-dof potential air flow manipulation by inverse modeling control. In: *IEEE International Conference on Automation Science and Engineering (CASE)*, 2012, pp. 930–935. IEEE (2012)
17. Dhoutaut, D., Piranda, B., Bourgeois, J.: Efficient simulation of distributed sensing and control environments. In: *Proceedings—2013 IEEE International Conference on Green Computing and Communications and IEEE Internet of Things and IEEE Cyber, Physical and Social Computing, GreenCom-iThings-CPSCom 2013*, pp. 452–459 (2013)
18. Georgilas, I., Adamatzky, A., Barr, D., Dudek, P., Melhuish, C.: Metachronal waves in cellular automata: Cilia-like manipulation in actuator arrays. In: *Nature Inspired Cooperative Strategies for Optimization (NICSO 2013)*, pp. 261–271. Springer International Publishing (2014)
19. Georgilas, I., Adamatzky, A., Melhuish, C.: Manipulating objects with gliders in cellular automata. In: *2012 IEEE International Conference on Automation Science and Engineering (CASE)*, pp. 936–941. IEEE (2012)
20. Georgilas, I., Adamatzky, A., Melhuish, C.: Towards an intelligent distributed conveyor. In: *Advances in Autonomous Robotics* pp. 457–458 (2012)
21. Georgilas, I., Adamatzky, A., Melhuish, C.: Manipulating with excitations: Waves or gliders? In: *Workshop notes of the ICRA Workshop in Unconventional Approaches to Robotics, Automation and Control Inspired by Nature*, vol. *International Conference in Robotics and Automation (ICRA)*. Karlsruhe (2013)
22. Laurent, G.J., Delettre, A., Zeggari, R., Yahiaoui, R., Manceau, J.F., Fort-Piat, N.L.: Micropositioning and fast transport using a contactless micro-conveyor. *Micromachines* **5**(1), 66–80 (2014). doi:[10.3390/mi5010066](https://doi.org/10.3390/mi5010066), <http://www.mdpi.com/2072-666X/5/1/66>
23. Luntz, J., Messner, W., Choset, H.: Distributed manipulation using discrete actuator arrays. *Int. J. Rob. Res.* **20**(7), 553–583 (2001)

24. Moon, H., Luntz, J.: Distributed manipulation of flat objects with two airflow sinks. *IEEE Trans. Robot.* **22**(6), 1189–1201 (2006). doi:[10.1109/TRO.2006.882921](https://doi.org/10.1109/TRO.2006.882921)
25. Mróz, Z., Stupkiewicz, S.: An anisotropic friction and wear model. *Int. J. Solids Struct.* **31**(8), 1113–1131 (1994)
26. Murphey, T., Burdick, J.: Feedback control methods for distributed manipulation systems that involve mechanical contacts. *Int. J. Robot Res.* **23**(7–8), 763–781 (2004)
27. Setter, E., Bucher, I.: Flexural vibration patterning using an array of actuators. *J. Sound. Vib.* **330**(6), 1121–1140 (2011)
28. Umbanhowar, P., Vose, T., Mitani, A., Hirai, S., Lynch, K.: The effect of anisotropic friction on vibratory velocity fields. In: *Proceedings of 2012 IEEE International Conference on Robotics and Automation*, pp. 2584–2591 (2012)
29. Vazquez-Otero, A., Faigl, J., Munuzuri, A.: Path planning based on reaction-diffusion process. In: *2012 IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS'12*, pp. 896–901 (2012)
30. Vose, T., Turpin, M., Dames, P., Umbanhowar, P., Lynch, K.: Modeling, design, and control of 6-dof flexure-based parallel mechanisms for vibratory manipulation. *Mech. Mach. Theory* **64**, 111–130 (2013)
31. Vose, T., Umbanhowar, P., Lynch, K.: Vibration-induced frictional force fields on a rigid plate. In: *2007 IEEE International Conference on Robotics, ICRA'07*, pp. 660–667. Rome (2007)
32. Zmitrowicz, A.: Mathematical descriptions of anisotropic friction. *Int. J. Solids Struct.* **25**(8), 837–862 (1989)