Chapter 3
The Firewall Phenomenon

R.B. Mann

Abstract Black holes have presented us with some of the most baffling paradoxes in physics. From their original conception as dark stars, they have come to be understood as physical systems with their own thermodynamic behaviour. This same behaviour leads to paradoxical conflicts between some of the basic principles of physics whose resolution is not straightforward and that suggest a new structure—known as a firewall—may be present. This chapter provides an overview of the firewall problem, as it emerges from our understanding of black hole thermodynamics.

Keywords Information paradox · Black hole thermodynamics · Firewall

3.1 Introduction

Black Holes have presented us with paradoxical situations ever since their conceptualization in 1783 by the Reverend Michell [1]. Originally seeking a means for determining stellar masses by measuring the reduction in the speed of corpuscular light due to a given star's gravitational pull, Michell reasoned that the maximal effect measurable would be limited by the escape velocity from the star. This would have to be the speed of light, most recently measured by Bradley to be 301,000 km/s [2]. Any star more massive than this upper bound (500 times the mass of the sun assuming the same average density) would not permit light to escape from its surface. While no theoretical constraints for objects having speeds greater than c were known at the time, there were no empirical measurements indicating such objects existed either. Paradoxically, such stars would be *dark stars*, invisible to an outside observer, though they could be indirectly inferred from their gravitational influence on nearby luminous objects. The relationship between their mass and radius is given by the same relativistic value $R = \sqrt{2GM/c^2}$ for Schwarzschild black holes. Ironically, Michell's proposal for measuring the mass of a star by measuring its speed of light

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fails because light moves through space at constant speed regardless of the local strength of gravity.

It would take nearly two centuries before the paradoxes associated with dark stars—now referred to as black holes—would dawn upon the physics community at large. Their inexorable gravitational chokehold on matter turns from puzzle to paradox once the quantum information content of the matter is taken into account. At this point in time there is no consistent understanding of how quantum physics allows information to either be retained in or escape from a black hole.

Over the past two years this conundrum has received a new degree of scrutiny. It appears that a profound conflict between three core principles of physics—unitary, locality, and the equivalence principle—indicates that black holes may erect around themselves a new structure called a *firewall* [3]. The basic notion of a firewall is that of a chaotically violent surface of highly energetic quantum states. Whereas standard expectations from local gravitational physics would indicate that any detector (or observer) falling into a black hole would encounter nothing out of the ordinary, the reasoning behind the firewall argument implies that this encounter would be very damaging to pretty much any detection device.

The purpose of this chapter is to present the firewall argument in the context of its roots in black hole thermodynamics and the previously-understood paradoxes associated with this phenomenon. After a review of the notion of a black hole in Sect. 3.2, I will briefly describe the relationship between the laws of black hole mechanics and the laws of thermodynamics (Sect. 3.3). These laws in turn depend upon our understanding of quantum field theory in curved space-time and of pair creation, yielding in turn our basic understanding of black hole radiation (Sect. 3.4). This confluence of ideas led to what became known as the information paradox: the puzzle of how a thermally radiating black hole can be consistent with the unitary evolution quantum physics requires, discussed in Sect. 3.5. It was generally thought for a time that recent conjectures about duality between gravitational physics and gauge theories straightforwardly resolve the problem (at least in principle). However more detailed study of the information paradox indicates that the resolution of this problem is not at all straightforward [4], and that a new structure—known as a firewall—may be present. This strange phenomenon is discussed in Sect. 3.6, along with responses to this new perspective on black holes. A brief summary appears in Sect. 3.7.

3.2 Black Holes

The physical notion of a black hole is essentially the same as that contemplated by Michell: a region of space where the gravity is so strong that nothing can escape from it. If the region is spherical, then a particle will be trapped there if its kinetic energy is less than its gravitational potential energy

$$\frac{1}{2}mv^2 - \frac{GMm}{r} < \frac{1}{2}mc^2 - \frac{GMm}{r} < 0 \quad \Longrightarrow r < \frac{2GM}{c^2} \equiv r_+ \tag{3.1}$$

and so if the mass M is concentrated within a region smaller than r_+ it will trap all particles moving at subliminal speed—the object will be a black hole.

Relativistic considerations imply that this is a firm limit: the invariance of the speed of light for all observers indicates that all matter travels at subliminal speed. Hence (without taking quantum effects into account) a black hole will absorb all matter and emit nothing. It is a perfect absorber, whose physical temperature is zero.

The earliest and best known example of a black hole is the Schwarzschild solution

$$ds^2 = -c^2\left(1 - \frac{r_+}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{r_+}{r}} + r^2 d\Omega_2^2 \tag{3.2}$$

where $d\Omega_2^2 = d\theta^2 + (\sin\theta d\phi)^2$ is the standard line element on the sphere S^2. Curiously, the quantity r_+ plays the same limiting role as in Newtonian theory.

The metric appears to be singular at both $r = r_+$ and $r = 0$, but the former singularity is due simply to a coordinate choice. Writing

$$t = t_* \qquad r = r_+\left[W\left(\exp\left(\frac{r_*}{r_+} - 1\right)\right) + 1\right] \tag{3.3}$$

yields from (3.2)

$$ds^2 = -\frac{W\left(\exp\left(\frac{u-v}{2r_+} - 1\right)\right)}{W\left(\exp\left(\frac{u-v}{2r_+} - 1\right)\right) + 1} du dv + r_+^2\left[W\left(\exp\left(\frac{u-v}{2r_+} - 1\right)\right) + 1\right]^2 d\Omega_2^2 \tag{3.4}$$

where $(u, v) = ct_* \pm r_*$ and W is the Lambert-W function, defined via $W(y)\exp(W(y)) = y$. The horizon $r = r_+$ is at $r_* = -\infty$. The space-time smoothly continues through $r = r_+$.

A particle moving on a radial trajectory will have $d\theta/ds = d\phi/ds = 0$; if the particle moves at the speed of light (e.g. a photon) then $ds^2 = 0$. Hence from (3.4) it is easy to see that ingoing (outgoing) radial light rays follow lines $du/ds = 0$ ($dv/ds = 0$) or u = constant (v = constant). The metric (3.2) can be extended across $r = r_+$ along either of these null lines. Writing $(u', v') = (\exp(u/2r_+), -\exp(-v/2r_+)) = \sqrt{r/r_+ - 1}(\exp((r + ct)/2r_+), -\exp((r - ct)/2r_+))$ transforms (3.2) to

$$ds^2 = -\frac{4r_+^2 e^{-\left[W\left(-\left(\frac{u'v'}{e}\right)\right)+1\right]}}{W\left(-\left(\frac{u'v'}{e}\right)\right) + 1} du' dv' + r_+^2\left[W\left(-\left(\frac{u'v'}{e}\right)\right) + 1\right]^2 d\Omega_2^2 \tag{3.5}$$

which are referred to as Kruskal coordinates. A plot of the function $W(-x/e)$ indicates that it monotonically increases with increasing negative x and diverges at $x = 1$. Hence the metric is finite at $u'v' = 0$ (corresponding to $r = r_+$) but diverges at $u'v' = -1$ or $r = 0$. The Kretschmann scalar $R_{abcd}R^{abcd}$ diverges at this point and so this is a genuine curvature singularity. All geodesics either meet this singularity or else extend to infinite affine parameter—in this sense Kruskal coordinates are maximal.

The causal structure is more easily shown in a Penrose diagram, which maps the entire space-time to a finite region. For flat Minkowski space-time the metric becomes

$$\begin{aligned} ds^2 &= -c^2dt^2 + dr^2 + r^2 d\Omega_2^2 \\ &= (\sec^2 X_+ \sec^2 X_-)\left[dX_- dX_+ + \left(\frac{\tan X_+ - \tan X_-}{2\sec X_+ \sec X_-}\right)^2 d\Omega_2^2\right] \\ &= (\sec^2 X_+ \sec^2 X_-) d\tilde{s}^2 \end{aligned} \tag{3.6}$$

upon setting $\tan X_\pm = ct \pm r$. Since light rays obey $ds^2 = 0 = d\tilde{s}^2$ the causal relations between various regions are preserved in going from ds^2 to $d\tilde{s}^2$. The range of $X_\pm$ is between $\pm\frac{\pi}{2}$, and the entire space-time is mapped into a finite region, as shown at the top of Fig. 3.1.

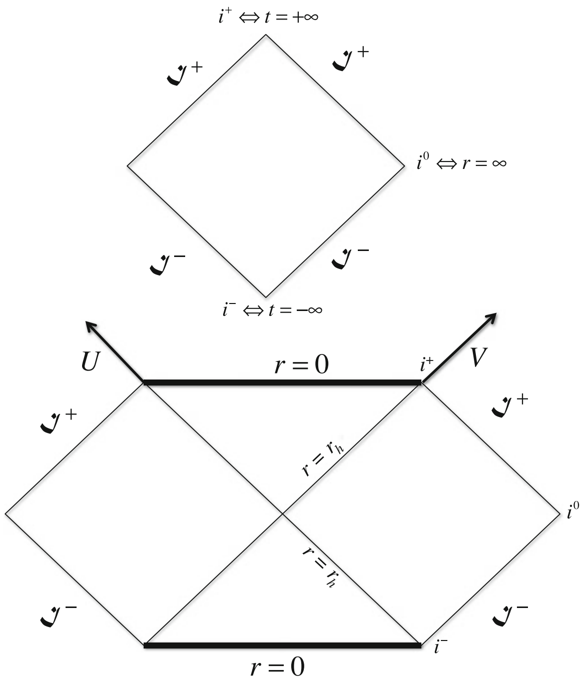


Fig. 3.1 Penrose Diagram The causal structure of Minkowski space-time (*top*) and Schwarzschild space-time (*bottom*). The coordinates U and V are depicted as well

More generally one maps a given spacetime manifold $\mathbb{M}$ with metric g_{ab} into a subset of a manifold $\tilde{\mathbb{M}}$ with metric $\tilde{g}_{ab}$. The conformal relation between the metrics is $\tilde{g}_{ab} = \Omega^2 g_{ab}$. The boundary of the image of $\mathbb{M}$ in $\tilde{\mathbb{M}}$ represents the 'points at infinity' in the original spacetime. Returning to the Schwarzschild metric and writing

$$\tan U = u' \qquad \tan V = v' \tag{3.7}$$

yields a metric conformal to (3.4), with the coordinates (U, V) playing the respective roles of $X_{\pm}$.

We see from the bottom part of Fig. 3.1 that whereas in Minkowski space-time all future-directed light rays can reach infinity $\mathcal{I}^+$ ('scri-plus'), in Schwarzschild space-time any future-directed light rays that cross $r = r_+$ will encounter the singularity, and hence so will all future-directed timeline curves. This is the idea of the trapped region a black hole induces. To make this notion more precise, we need to define a region to which particles are able to escape. From Fig. 3.1 this region should be the portion 'near infinity', i.e. at $\mathcal{I}^+$. So a black hole region $\mathbb{B}$, in mathematical terms, is defined as

$$\mathcal{B} = \mathbb{M} - \mathbb{I}^-(\mathcal{I}^+) \tag{3.8}$$

where $\mathbb{I}^-(A)$ denotes the chronological past of a region A. Hence a black hole is that part of space-time not in the past of the escape-region of light rays (not in the past of $\mathcal{I}^+$ or future null infinity).

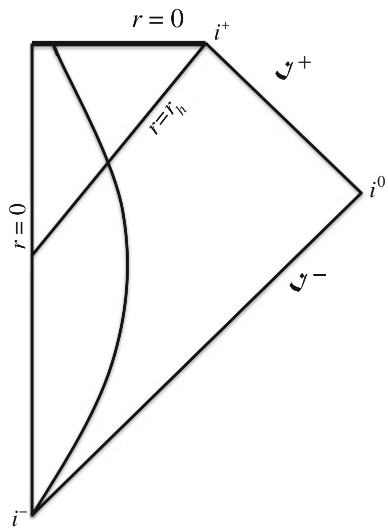
The event horizon $\mathbb{H}$ of the black hole is the boundary of $\mathbb{B}$. It is a null hypersurface (generally assumed to be at least once-differentiable) composed of future null geodesics without caustics that cannot be extended. In other words the expansion of the null geodesics comprising the horizon cannot become negatively infinite.

3.2.1 Gravitational Collapse

The Schwarzschild black hole (3.4) is very instructive for understanding the properties of black holes, but is physically unrealistic. Due to time-reversal symmetry, the singularity at $r = 0$ in the future has a counterpart in the past, yielding a 'white hole' structure at the bottom of the Penrose diagram in Fig. 3.1. The white hole $\mathbb{W}$ is defined as $\mathbb{W} = \mathbb{M} - \mathbb{I}^+(\mathcal{I}^-)$: it is the part of the manifold not in the future of the distant past of a time-reversed escape region. Just as a black hole is a total absorber, a white hole is a total emitter: nothing can enter it but anything can leave.

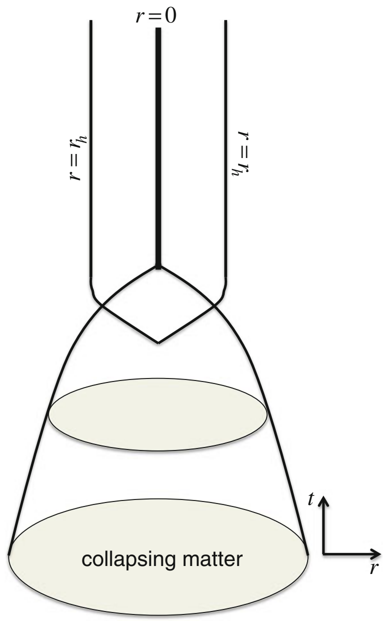
A more physically realistic solution to Einstein's equations matches a collapsing ball of dust (a form of stress-energy with density but no pressure) onto the metric (3.2), yielding a space-time that modelled the collapse of a star into a black hole. Known as the Oppenheimer-Snyder solution [5], it has since been generalized to many other cases. The general form of the Penrose diagrams for such spacetimes is given in Fig. 3.2. The left and bottom parts of the original space-time are no longer present, but the future event horizon remains. There is a point in time at which the

Fig. 3.2 Fluid Collapse The Penrose diagram of the collapse of a ball of dust. The boundary of the dust is given by the *curved line*



fluid collapses beyond which nothing can escape, even though the singularity has yet to form. This is given by the intersection of the diagonal line from i^+ with the vertical line in Fig. 3.2. This is the kind of black hole relevant to astrophysics. A depiction of the process in more familiar coordinates is given in Fig. 3.3.

Fig. 3.3 Gravitational Collapse Gravitational collapse of matter in more familiar coordinates



It is also relevant to considerations of black hole thermodynamics. The vacuum solution (3.2) is applicable everywhere outside of the fluid. Since it has an event horizon, the general properties of black hole radiation—and the conundrums they introduce—that are deduced from (3.2) will also be present for the collapse solution shown in Fig. 3.3.

3.2.2 Anti de Sitter Black Holes

An important class of solutions of particular relevance to string theory are solutions in which the space-time is asymptotic to a space-time of constant negative curvature. This latter space-time is known as anti de Sitter (AdS) spacetime, and is a solution to the Einstein equations

$$R_{ab} - \frac{1}{2}g_{ab}R - \Lambda g_{ab} = T_{ab} \tag{3.9}$$

with matter stress-energy tensor $T_{ab} = 0$ and cosmological constant $\Lambda = -(d-1)(d-2)/2\ell^2 < 0$ in d-dimensions. The most general vacuum metric in the static spherically symmetric case is

$$ds^2 = -c^2\left(\frac{r^2}{\ell^2} + k - \left(\frac{r_0}{r}\right)^{d-3}\right)dt^2 + \frac{dr^2}{\frac{r^2}{\ell^2} + k - \left(\frac{r_0}{r}\right)^{d-3}} + r^2 d\Omega_k^2 \tag{3.10}$$

where $d\Omega_k^2$ is the metric on a compact space Σ_k of constant curvature with sign k, $k = 1$ being the $(d-2)$-sphere, $k = 0$ being a torus, and $k = -1$ being a compact hyperbolic space (obtained via well-known identifications [6]. The metric (3.10) describes what is called a Schwarzschild-AdS black hole, with the constant of integration r_0 given by

$$r_0^{d-3} = \frac{16\pi GM}{(d-2)\mathcal{V}(\Sigma_k)} \tag{3.11}$$

where M is the mass of the black hole and $\mathcal{V}(\Sigma_k)$ is the volume of Σ_k (4π for a two-dimensional sphere). When $M = 0$ then $r_0 = 0$ and the metric (3.10) is that of anti de Sitter space-time.

Light cone (u, v) and Kruskal (u', v') coordinates are defined by

$$u, v = t \pm r_* = t \pm \int \frac{dr}{\frac{r^2}{\ell^2} + k - \left(\frac{r_0}{r}\right)^{d-3}} \qquad u' = e^{\kappa u} \quad v' = -e^{-\kappa v} \tag{3.12}$$

and repeating the procedure for the Schwarzschild case yields Fig. 3.4. There is no choice of conformal factor that allows both the asymptotic boundaries $\mathcal{I}$ and the singularity at $r = 0$ to be represented as straight lines [7], though it is common in the literature to do so.

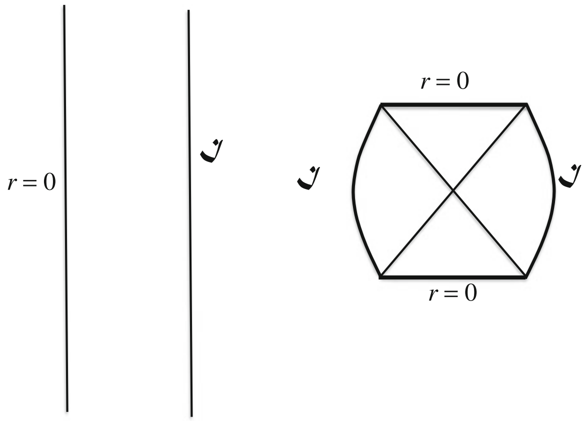


Fig. 3.4 Penrose Diagram for Schwarzschild-AdS Penrose diagrams for AdS (*left*) and the $k = 1$ Schwarzschild-AdS (*right*) space times

Note that asymptotic infinity is timelike in the Schwarzschild-AdS case. Any massive object projected away from the black hole will inevitably return to its starting point; the cosmological constant induces a confining potential for any massive particle, as an analysis of the geodesic equation shows. Light rays, however, can reach $r = \infty$ in finite time. It is common to put reflecting boundary conditions at $r = \infty$ so that light rays are 'confined' like massive objects. The negative cosmological constant thus prevents radiation emitted by the black hole from escaping to infinity, allowing the black hole to reach equilibrium with its Hawking radiation, provided it is large enough. In this sense the eternal black hole (3.10) depicted at the right of Fig. 3.4 is more physically relevant than its asymptotically flat counterpart in Fig. 3.1.

3.3 Black Hole Thermodynamics

The first hints of a fundamental relationship between gravitation, thermodynamics, and quantum theory came from studying black holes. While this subject is covered elsewhere in this volume, it is worth recapitulating the four laws of black hole mechanics [8]:

0th Law Surface gravity κ_+ is constant over the event horizon.

1st Law Differences in mass between nearby solutions are equal to differences in area times the surface gravity plus additional work-type terms

2nd Law The area $\mathcal{A}_+$ of the event horizon never decreases in any physical process provided the energy of matter is positive and space-time is regular

3rd Law No procedure can reduce the surface gravity to zero by a finite number of steps.

Henceforth setting for simplicity the constants G, c, and $\hbar$ to unity, if $\Lambda \neq 0$ then geometric arguments [9–12] indicate that the first law is

$$\delta\mathcal{M} = \kappa_+ \frac{\delta\mathcal{A}_+}{8\pi} + \sum_i (\Omega^i_+ - \Omega^i_\infty)\delta\mathcal{J}^i + V\delta P + \sum_j \Phi^j_+ \delta Q^j \tag{3.13}$$

where $\mathcal{M}$, $\mathcal{J}^i$ are respectively the mass and angular momenta of the D-dimensional black hole. The surface gravity κ_+ is obtained from

$$\kappa^2 = -\frac{1}{2}\nabla^a \xi^b \nabla_a \xi_b|_{r=r_+}$$

where ξ^a is a timelike Killing vector, always present for static black holes. For simplicity κ will be used to denote κ_+ henceforth. For each independent rotational plane, there is one $\mathcal{J}^i$, each with its corresponding conjugate angular velocity Ω_i; the quantities Ω^i_∞ allow for the possibility of a rotating frame at infinity [13]. The Φ^i_h are the potentials for the electric (and magnetic) $U(1)$ charges evaluated at the black hole horizon.

The inclusion of the $V\delta P$ term in (3.13) is a new addition, of considerable recent interest in black hole thermodynamics. Since a negative cosmological constant Λ induces a vacuum pressure, it seems reasonable to consider it as a thermodynamic variable [14] analogous to pressure in the first law [9, 10, 15]. The mass M is then understood as a gravitational version of chemical enthalpy. This is the total energy of a system including both its internal energy E and the energy PV required to "make room for it" by displacing its (vacuum energy) environment: $M = E + PV$. In other words, M is the total energy required to "create a black hole and place it in a cosmological environment". A new perspective on black hole thermodynamics thus emerges, leading to a different understanding of known processes and to the discovery of new phenomena. The thermodynamic correspondence with black hole mechanics is completed to include the familiar pressure/volume terms:

Thermodynamics		Black hole mechanics	
Enthalpy	H	Mass	M
Temperature	T	Surface gravity	$\frac{\kappa}{2\pi}$
Entropy	S	Horizon area	$\frac{A}{4}$
Pressure	P	Cosmological constant	$-\frac{\Lambda}{8\pi}$
First law	$dH = TdS + VdP + \cdots$	First law	$dM = \frac{\kappa}{8\pi}dA + VdP + \cdots$

(3.14)

where the black hole work terms are $\sum_i \Omega_i dJ_i + \Phi dQ$ for multiply rotating and charged black holes.

A number of interesting implications have been recently worked out, including the discovery that charged black holes behave as Van der Waals fluids [16, 17], the realization well-known first-order phase transition between radiation and large AdS black holes [18] can be understood as a "solid/liquid" phase transition, and the discoveries of *reentrant phase transitions* [12] and *triple points* [19] for Kerr-AdS black holes. The former refers to a situation in which a system can undergo a transition from one phase to another and then back to the first by continuously changing one thermodynamic variable, and the latter is a coalescence of small, medium, and large sized black holes into a single kind at a particular critical value of the pressure and temperature, analogous to the triple point of water. A recent review appears in [12].

3.4 Black Hole Radiation

The striking parallel between black hole mechanics and black hole thermodynamics raises the question as to the origin of this correspondence. Pivotal to this relationship is the derivation of black hole temperature, which necessarily relies on the incorporation of quantum physics into curved space-time settings [20]. There is not the space to review this vast subject here, so I shall sketch the situation to provide the necessary context for the information paradox and firewall argument.

3.4.1 Quantum Field Theory in Curved Spacetime

A scalar field D spacetime dimensions obeys the equation

$$\nabla^2\varphi - m^2\varphi = 0 \tag{3.15}$$

and can be decomposed into a *mode expansion*

$$\varphi(x) = \int d^3\,\boldsymbol{k}\left[\phi_{\boldsymbol{k}}(x)\hat{a}_{\boldsymbol{k}} + \phi^*_{\boldsymbol{k}}(x)\hat{a}^\dagger_{\boldsymbol{k}}\right]. \tag{3.16}$$

where the functions $\phi_{\boldsymbol{k}}(x)$ are called *mode functions*. They are each solutions to (3.15) and together form an orthonormal basis for the space of solutions in the sense that

$$\langle\langle\phi_{\boldsymbol{k}}(x), \phi_{\boldsymbol{k}'}(x)\rangle\rangle = -\langle\langle\phi_{\boldsymbol{k}}(x)^*, \phi_{\boldsymbol{k}'}(x)^*\rangle\rangle = \delta^3(\boldsymbol{k} - \boldsymbol{k}'), \quad \langle\langle\phi_{\boldsymbol{k}}(x), \phi_{\boldsymbol{k}'}(x)^*\rangle\rangle = 0, \tag{3.17}$$

with the inner product between fields defined as

$$\langle\langle\chi, \phi\rangle\rangle \equiv \int_\Sigma d\Sigma_a j^a\,(\chi, \phi) \tag{3.18}$$

where Σ_a is the volume element on Σ with unit normal n^a and the current

$$j^a\,(\chi, \phi) = -i\sqrt{g}g^{ab}(\chi^*\nabla_b\phi - (\nabla_b\chi^*)\phi) \tag{3.19}$$

is conserved ($\nabla_a j^a = 0$) provided both χ and ϕ) are solutions to (3.15). The conservation of j^a ensures that the inner product is independent of the choice of slice Σ. The operators $\hat{a}_{\boldsymbol{k}}$ are called *mode operators*, which obey the relations

$$[\hat{a}_{\boldsymbol{k}}, \hat{a}^{\dagger}_{\boldsymbol{k}'}] = \delta^3(\boldsymbol{k} - \boldsymbol{k}') \tag{3.20}$$

Particle creation is understood to be

$$\prod_{n=1}^{N} \left(\hat{a}^{\dagger}_{k_i}\right)^{n_{k_i}} |0\rangle = |n_{\boldsymbol{k}_1}, n_{\boldsymbol{k}_2}, \ldots\rangle \tag{3.21}$$

where the notation $n_{\boldsymbol{k}}$ refers to n quanta in the k-th frequency mode of the field. For example if the scalar field is replaced with an electromagnetic field, this would be n photons of wave vector k.

The state $|0\rangle$ is the lowest-energy state—the vacuum, having no particles—with the property

$$\hat{a}_{k_i}|0\rangle = 0 \quad \forall \chi \in \mathcal{Q}_+ \tag{3.22}$$

with $\mathcal{Q}_+$ the space of states of positive energy. In spacetimes that possess time translation symmetry (with timelike Killing vector ξ) it is defined to be the space of solutions such that

$$\xi^a \partial_a \phi_{\boldsymbol{k}}(x) = -i\omega_{\boldsymbol{k}} \phi_{\boldsymbol{k}}(x), \quad \xi^a \partial_a \phi^*_{\boldsymbol{k}}(x) = i\omega_{\boldsymbol{k}} \phi^*_{\boldsymbol{k}}(x), \tag{3.23}$$

where $\omega_{\boldsymbol{k}} > 0$. The *positive/negative frequency* modes have eigenvalues $\mp i\omega_{\boldsymbol{k}}$ respectively.

In a general curved spacetime there are many ways of carrying out the above construction, with no particular subspace singled out as a natural choice for the positive frequency space [21]. Different notions of positive frequency will yield different Fock space constructions that are unitarily inequivalent [22], and the vacuum state $|0\rangle$ with respect to one choice of $\mathcal{Q}_+$ will not necessarily be in the Fock space constructed from the vacuum $|\tilde{0}\rangle$ with respect to another choice $\tilde{\mathcal{Q}}_+$.

However there is a linear relation between different choices. For another choice $\tilde{\mathcal{Q}}_+$ any $\tilde{\phi} \in \tilde{\mathcal{Q}}_+$ is a linear combination $\tilde{\phi} = \phi + \chi^*$ for some $(\chi, \phi) \in \mathcal{Q}_+$. Consequently two complete sets of modes $\{\phi_{\boldsymbol{k}}, \phi^*_{\boldsymbol{k}}\}$ and $\{\chi_{\boldsymbol{k}}, \chi^*_{\boldsymbol{k}}\}$ with associated mode operators $\hat{a}_{\boldsymbol{k}}$ and $\hat{\mathrm{a}}_{\boldsymbol{k}}$ are, by completeness and orthonormality, related by

$$\chi_{\boldsymbol{k}}(x) = \int d^3\boldsymbol{k}' \left[\overline{\alpha}_{\boldsymbol{k}\boldsymbol{k}'} \phi_{\boldsymbol{k}'}(x) + \overline{\beta}_{\boldsymbol{k}\boldsymbol{k}'} \phi^*_{\boldsymbol{k}'}(x)\right],$$

where $\overline{\alpha}_{\boldsymbol{k}\boldsymbol{k}'} = \langle\langle \chi_{\boldsymbol{k}}(x), \phi_{\boldsymbol{k}'}(x)\rangle\rangle$ and $\overline{\beta}_{\boldsymbol{k}\boldsymbol{k}'} = -\langle\langle \chi_{\boldsymbol{k}}(x), \phi^*_{\boldsymbol{k}'}(x)\rangle\rangle$. This is called a *Bogoliubov transformation* [20] and the complex numbers $\overline{\alpha}_{\boldsymbol{k}\boldsymbol{k}'}$ and $\overline{\beta}_{\boldsymbol{k}\boldsymbol{k}'}$ are called *Bogoliubov coefficients*. Inverting this transformation yields

$$\phi_{\boldsymbol{k}} = \int d^3\,\boldsymbol{k}' \left[\overline{\alpha}^*_{\boldsymbol{k}'\,\boldsymbol{k}}\chi_{\boldsymbol{k}'}(x) - \overline{\beta}_{\boldsymbol{k}'\,\boldsymbol{k}}\chi^*_{\boldsymbol{k}'}(x)\right]$$

inducing in turn the following transformations

$$\hat{a}_{\boldsymbol{k}} = \int d^3\,\boldsymbol{k}' \left[\overline{\alpha}_{\boldsymbol{k}'\,\boldsymbol{k}}\hat{\mathrm{a}}_{\boldsymbol{k}'} + \overline{\beta}^*_{\boldsymbol{k}'\,\boldsymbol{k}}\hat{\mathrm{a}}^\dagger_{\boldsymbol{k}'}\right] \qquad \hat{\mathrm{a}}_{\boldsymbol{k}} = \int d^3\,\boldsymbol{k}' \left[\overline{\alpha}^*_{\boldsymbol{k}\,\boldsymbol{k}'}\hat{a}_{\boldsymbol{k}'} - \overline{\beta}^*_{\boldsymbol{k}\,\boldsymbol{k}'}\hat{a}^\dagger_{\boldsymbol{k}'}\right] \tag{3.24}$$

on the mode operators.

The commutation relations (3.20) of the mode operators imply

$$\begin{pmatrix} \overline{\alpha} & \overline{\beta} \\ \overline{\beta}^* & \overline{\alpha}^* \end{pmatrix} \begin{pmatrix} \overline{\alpha}^\dagger & -\overline{\beta}^{\mathrm{T}} \\ -\overline{\beta}^\dagger & \overline{\alpha}^{\mathrm{T}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

where the individual entries are to be interpreted as block matrices. From (3.24) we see that the Fock bases associated with these two mode expansions differ, leading to two different particle interpretations of the field excitations. Whenever any of the $\overline{\beta}$-coefficients are nonzero, positive frequency states get transformed into a combination of positive and negative frequency states, leading to particle production. Specifically, according to the particle interpretation based on the $\phi_{\boldsymbol{k}}(x)$ modes, particles are present in the vacuum of the $\chi_{\boldsymbol{k}}(x)$ mode expansion $|0\rangle_\chi$. The average number of particles present in mode $\boldsymbol{k}$ is given by

$${}_\chi\langle 0|N_k|0\rangle_\chi = {}_\chi\langle 0|\hat{a}^\dagger_{\boldsymbol{k}}\hat{a}_{\boldsymbol{k}}|0\rangle_\chi = \int d^3\,\boldsymbol{k}'\,|\overline{\beta}_{\boldsymbol{k}\,\boldsymbol{k}'}|^2.$$

In this sense, there is no invariant notion of particles in quantum field theory: as with simultaneity, particle interpretations are observer-dependent.

To cope with this, several key assumptions must be made.

1. All quantum states are defined on a spacelike slice Σ of 4-dimensional space-time, whose intrinsic curvature $R^{(3)}_{abcd}$ and extrinsic curvature K_{ab} are both everywhere small compared to the Planck length: $|R^{(3)}_{abcd}| \ll 1/l_p^2$, $|K_{ab}| \ll 1/l_p^2$.
2. There is some neighbourhood of Σ where the full space-time curvature R_{abcd} is also small: $|R_{abcd}| \ll 1/l_p^2$.
3. The wavelength λ of any quanta on Σ is much longer than the Planck length $\lambda \gg l_p$.
4. The stress-energy of all matter obeys positive energy conditions and the energy and momentum densities of the matter are small compared to the Planck density $(\tilde{1}/l_p^4)$.
5. For a least a certain interval of proper time τ, Σ evolves sufficiently smoothly (so that $dN/d\tau \ll 1/l_p$ and $dN^a/d\tau \ll 1/l_p$) into future slices that respect the preceding four properties.

The preceding conditions are sometimes referred to as the 'niceness' conditions [4], and are regarded as ensuring that semiclassical physics is valid (Fig. 3.5).

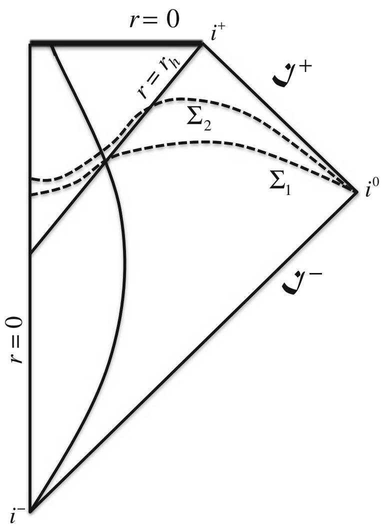


Fig. 3.5 Nice Slices Two different slices for the collapse diagram that satisfy the niceness conditions

3.4.2 Pair Creation

Generally black hole radiation can be understood as a phenomenon of pair-creation of particles due to the distortion of spacetime near the horizon. An intuitive argument illustrating how this can take place was recently given by Mathur [4]. Consider a choice of spacelike slices in which one spatial region evolves further in time than another, as shown in Fig. 3.6. This situation is permitted in generally covariant theories of gravity such as general relativity. Both slices satisfy the semiclassical (niceness) conditions. No particle creation occurs classically, but if a quantum field

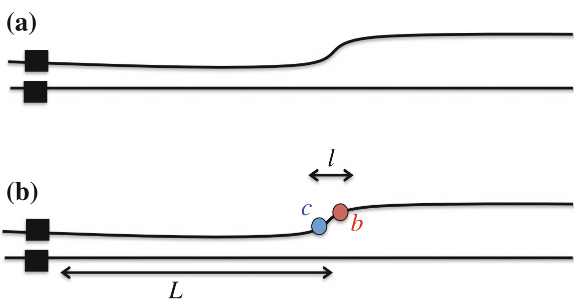


Fig. 3.6 Particle Creation due to Spacetime distortion The intrinsic geometry of the initial spatial slice (*horizontal line*) evolves forward in time differently on the *left* than on the *right*, with a concentration of (classical) matter symbolized by the box at the *left*. In **a** the evolution is fully classical and no pairs are created. In **b** the quantum field on the initial slice is in the vacuum, with the space-time distortion on the next slice creating a pair of quanta

is in the vacuum state (in Fig. 3.6b) then the created-pair state will be in the state

$$|\psi\rangle = (\alpha|0\rangle_c|0\rangle_b + \beta|1\rangle_c|1\rangle_b) + \cdots \tag{3.25}$$

where $|\alpha|^2 + |\beta|^2 = 1$; the ellipsis refers to multi-pair creation, neglected in the above. In Hawking radiation the created pair is maximally entangled, with $|\alpha| = |\beta| = 1/\sqrt{2}$, but we can understand what is going on for arbitrary entanglement. The quantum state of the entire system is

$$|\Psi\rangle \approx |\Phi\rangle_M \otimes \Big(\alpha|0\rangle_c|0\rangle_b + \beta|1\rangle_c|1\rangle_b\Big) \tag{3.26}$$

where $|\Phi\rangle_M$ is the quantum state of the matter, symbolized by the box at the left of Fig. 3.6. If $l \ll L$ then the influence of the matter on the created pair can be neglected (though in principle there is always some influence), and the full state is well approximated by the tensor product (3.26).

The locality assumption ensures that the state Ψ is a tensor product. Small departures from whatever physics yielding the (α, β) coefficients are permitted

$$|\Psi\rangle \approx \Big(\tilde{\alpha}|\Phi_0\rangle_M + \tilde{\beta}|\Phi_1\rangle_M\Big) \otimes \Big((\alpha+\epsilon)|0\rangle_c|0\rangle_b + (\beta-\epsilon)|1\rangle_c|1\rangle_b\Big) \tag{3.27}$$

but not states of the form

$$|\Psi\rangle \approx \Big((\tilde{\alpha}+\epsilon)|\Phi_0\rangle_M|0\rangle_c + (\tilde{\beta}-\epsilon)|\Phi_1\rangle_M|1\rangle_c\Big) \otimes \Big(\alpha|0\rangle_b + \beta|1\rangle_b\Big) \tag{3.28}$$

where for simplicity the matter is assumed to be a single qubit in one of two possible states $|\Phi_0\rangle_M$ or $|\Phi_1\rangle_M$. For the former case (3.27) the entanglement entropy is, upon tracing over the matter and state c,

$$\begin{aligned} S_{\text{ent}} &= -\operatorname{tr}_{c,M}[\rho \ln \rho] \\ &= -\Big(|\alpha|^2 \ln|\alpha| + |\beta|^2 \ln|\beta|\Big) \\ &\quad + 2\epsilon\Big(|\beta| \ln(2|\beta|^2) - |\alpha| \ln(2|\alpha|^2)\Big) \\ &\quad - \epsilon^2\Big(6 + 2\ln(|\alpha||\beta|)\Big) \end{aligned} \tag{3.29}$$

which has the value $S_{\text{ent}} = \ln 2 - \epsilon^2(6 - 2\ln 2) \approx \ln 2$ for maximal entanglement. However for the latter case (3.28) the entanglement entropy is

$$S_{\text{ent}} = -\operatorname{tr}_{c,M}[\rho \ln \rho] = 0 \tag{3.30}$$

since the state b is a direct product with the remaining states. So for $l_p \ll l \ll L$, the entanglement entropy is

$$\left| \frac{S_{\text{ent}}}{\ln 2} - 1 \right| \ll 1 \tag{3.31}$$

from pair creation due to space-time distortion when the semiclassal assumptions are valid.

The key distinction between radiation emitted from a black hole and that emitted from a hot material object (such as a lump of coal) is in how the emitted quanta are generated. A lump of coal emits radiation because the atoms near its surface are excited, and emit quanta as they fall to states of lower energy. A black hole, however, emits quanta that arise due to entangled pair creation from the distortion of space-time, with one partner in the pair remaining inside the black hole and thus inaccessible to observers outside. Put another way, hot material bodies emit radiation from their constituents whereas black holes pull entangled pairs of quanta out of the vacuum as the result of 'stretching' a region of a space-like slice. The situation is illustrated in Fig. 3.7.

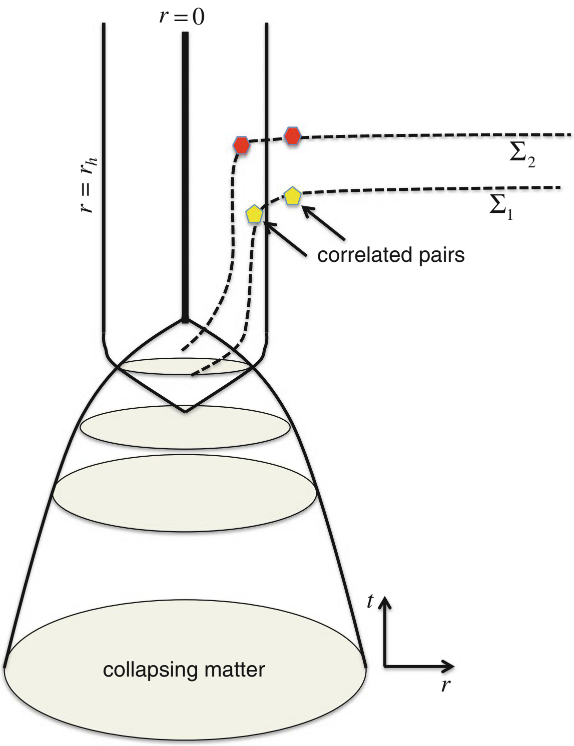


Fig. 3.7 Creation of Correlated Pairs outside a Black Hole Pairs of quanta (*symbolized* by the *shapes*) are created near the event horizon. The collapsing matter on the same corresponding spatial slices is very far from these pairs

The laws of black hole mechanics imply that the temperature T of a black hole is given by its surface gravity, $T = \kappa/2\pi$. The metric of any static black hole can be written as

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + r^2 d\Omega_2^2 \tag{3.32}$$

and

$$\kappa^2 = -\frac{1}{2}\nabla^a\xi^b\nabla_a\xi_b|_{r=r_+} = \frac{g(r)(f'(r))^2}{4f(r)}\Big|_{r=r_+} = \frac{g'_+ f'_+}{4} \tag{3.33}$$

yields the surface gravity at the horizon. Hawking's original calculation [23] was for a free scalar field propagating in a classical background spacetime describing gravitational collapse of matter to a Schwarzschild black hole. Prior to collapse the scalar is initially in its vacuum state. At late times, long after the black hole has formed, the positive frequency mode function corresponding to a particle state is traced backwards in time to determine its positive and negative frequency parts in the asymptotic past. The expected number of particles at infinity corresponds to emission from a perfect black body (of finite size) at the temperature $T = \kappa/2\pi$. Note that, other than in justifying use of the background space-time, nowhere are any gravitational field equations employed in this calculation. A collapse situation is well approximated by the eternal black hole (3.2) but with boundary conditions that are regular (Hadamard) only on the future horizon. This is called the Unruh state.

Despite this, there are a number of caveats and assumptions underlying the calculations of black hole temperature [4, 24].

A1 Invariant Hadamard states do not exist for all stationary black holes. The Kerr solution, describing a rotating black hole, is one such example, and as a consequence has a super-radiant instability [25–27].

A2 Asymptotically flat black holes will lose mass as they radiate, invalidating the late-time stationarity assumption. However the outgoing radiation will only carry an appreciable fraction of the mass over timescales $t \sim (M/M_P)^3$ [28].

A3 One of the most crucial assumptions is that the quantum state of the field is regular (Hadamard) at the horizon: its local behaviour at the horizon is the same as it would be in the Minkowski vacuum. This is an application of the equivalence principle, that locally (on sufficiently short time and distance scales) gravity and acceleration are indistinguishable. Freely-falling observers near the horizon should not see any unusual behaviour in high-energy processes. This is sometimes call the "no drama" assumption.

The no-drama assumption is somewhat paradoxical. An observer distant from a black hole formed from collapse who detects a mode at any finite frequency ω_f will realize that it has been redshifted. This must mean that it had a very large frequency in the past when it was propagating near the event horizon, of order $\omega = e^{\kappa t}\omega_f$ where t is the time it takes for the mode to reach the distant observer: the mode is blueshifted in the past. There is no past horizon (or other obvious physical effect) to provide a compensating effect for a mode propagating through the collapsing matter from the asymptotic past. Within a time scale of order $1/\kappa$ of the black hole's formation, the intermediate steps of the derivation implicitly involve propagation of trans-Planckian modes, modes that are much higher that the Planck frequency $\omega_P = \sqrt{c^5/\hbar G} \sim 10^{43}\ \mathrm{s}^{-1}$. This suggests lots of drama, since it is difficult to believe in the reliability of free-field theory (or any other known physical theory) at such high frequencies/energies [29].

Extensive study of this trans-Planckian problem [30–38] suggests that, despite the above, Hawking radiation is actually a low-energy phenomenon. Studies of quantized sound waves in a fluid undergoing supersonic flow indicate that a sonic analogue of the Hawking effect is present here as well. There is also a past-blueshift effect that renders invalid the continuum fluid equations that described the situation in the first place. However modifying these equations to yield a dispersion relation that is altered at ultra-high frequencies to alleviate this problem still yields sonic Hawking radiation. A variety of alternative models that significantly modify the continuum fluid equations at high energies have verified this, and a recent experiment with water waves [39] is in accord with the basic theoretical predictions.

The low-energy character of Hawking radiation appears to emerge from the behaviour of the field in the WKB regime [40]. Moving backward in time, the past-blueshift effect will bring the field into the WKB regime before it enters the trans-Planckian regime. The WKB approximation remains valid throughout the evolution provided the Hawking temperature is much smaller than the trans-Planckian scale (the scale at which the modified dispersion relation is relevant). The outgoing radiation does not come about because of any interaction with other degrees of freedom but rather is a consequence of the tidal disruption (or space-time distortion) of free-field evolution by stretching the wavelengths. The major ingredient of Hawking radiation appears to be a "tearing apart" of the waves into an outgoing (positive norm) component and its infalling (negative norm) partner (Fig. 3.8).

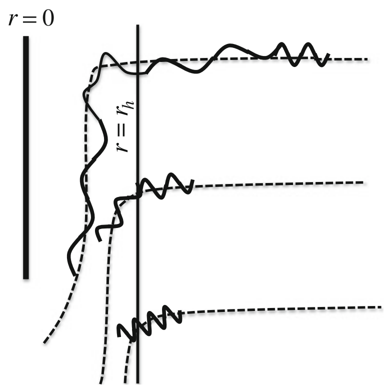


Fig. 3.8 Mode Stretching A Fourier mode created on some slice (represented by the *dashed line*) is stretched as it evolves to later slices. This eventually leads to a distorted waveform, resulting in particle creation

3.5 The Information Paradox

The no-drama assumption is tantamount to assuming the horizon of the black hole is 'information-free': that field modes with wavelengths $l_p \ll \lambda \lesssim M$ are described by curved space-time quantum field theory on the black hole background. We have seen that the notion of a particle is contingent on what the vacuum, or 'empty space' is taken to be. Modes with wavelengths λ smaller than the curvature scale R will yield differing definitions of particle quanta, but the difference between definitions will consist of about 1 quanta for wavelengths as large as the curvature scale, $\lambda \sim R$. For black holes $R \sim M$ and so different particle-definitions will differ by about 1 quanta for wavelength $\lambda \sim M$, but by a negligible number of quanta for wavelengths $\lambda < \tilde{k}M$, where $\tilde{k} \sim 10^{-1}$. Hence a robust notion of vacuum, or empty space, is well defined for such wavelengths: no modes are present for $l_p \ll \lambda < \tilde{k}M$.

In illustrating the information paradox, the assumptions A1–A3 will be assumed to hold, and so it will be sufficient to employ the metric (3.2). At late times this can describe a collapsing black hole, formed perhaps by a shell of matter or a ball of dust. Since this metric has a singularity at $r = 0$, spacelike slices must avoid this singularity or else the niceness conditions will not hold, undermining the calculations yielding black hole radiation.

The slicing choice will be taken to obey the following criteria [4]

Outside Σ_O Slices are at $t =$ constant for $r > 4M$; this portion is outside the black hole.

Inside Σ_I Slices are at $r =$ constant for $M/2 < r < 3M/2$; this portion is inside the black hole. This segment will be smoothly extended to $r = 0$ at early times before the singularity has formed.

Connect Σ_C The preceding two segments are joined by a smooth connecting segment $\mathcal{C}$; this portion crosses the event horizon. The spatio-temporal dimensions of this segment are both of size $\sim M$.

thereby respecting the niceness conditions. Nowhere does the slice $\Sigma(t, r, \mathcal{C}) = \Sigma_O \cup \Sigma_I \cup \Sigma_C$ go near the singularity, and the appropriate connecting segment can always be appropriately chosen.

Each slice is contingent on the choice of time, and it is essential that the slices smoothly evolve into each other (not merging or crossing). If $\Sigma_0 = \Sigma(t_0, r_0, \mathcal{C}_0)$ describes an initial slice, then a subsequent slice is $\Sigma_1 = \Sigma(t_1, r_1, \mathcal{C}_1) = \Sigma(t_0 + \delta t, r_0 + \delta r, \mathcal{C}_0 + \delta\mathcal{C})$. Increasing t_0 and r_0 respectively correspond forward evolution outside and inside the black hole. The geometry of the connecting segments can be taken to be the same for all slices provided $\delta r \ll M$ is sufficiently small. This has the consequence that the constant-r segments of the slices become increasingly longer since the constant-t parts outside the horizon are further in the future. The dashed lines in Fig. 3.7 are illustrative of this choice of slicing.

Foliating the space-time with these slices $\Sigma(t, r, \mathcal{C})$ along a unit timelike normal u^a (with zero shift vector) indicates that only the connecting segment $\mathcal{C}$ becomes stretched. The segment Σ_O advances forward in time with lapse function $N = \sqrt{1 - 2M/r}$, and the segment Σ_I will remain unchanged in its intrinsic geometry

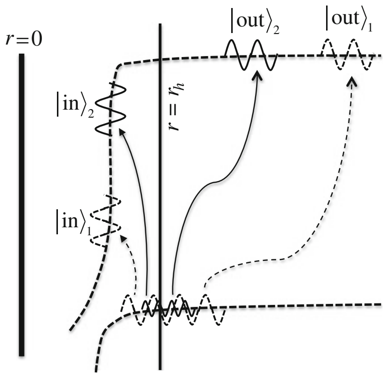


Fig. 3.9 Mode Evolution Long wavelength modes (*dashed*) and short wavelength modes (*solid*) created on the same slice generate created pairs differently. The long wavelength modes get distorted earlier, creating an entangled pair first ($|\text{outside}\rangle_1$, $|\text{inside}\rangle_1$). The short wavelength modes take longer to distort, and so the pairs they create ($|\text{outside}\rangle_2$, $|\text{inside}\rangle_2$) emerge later

provided δr is sufficiently small, though its length increases. The connecting segment Σ_C must therefore stretch since it has to evolve to cover the additional part of Σ_I and the connecting segment of the next slice.

The slicing is therefore time-dependent since it must cover both the outside and inside of the black hole; it therefore depends on the temporal coordinate r on the inside of the hole. The Kretschmann scalar and all other measures of curvature are small for all slices in this evolution. It is the time-dependent stretching of Σ_C, and not large-curvature effects, that yields particle production. This choice of slicing (and its resultant stretching) is a necessary consequence of the existence of the black hole. It is not an option in flat space-time: any such choice will necessarily force the slices to eventually become null and then timelike at some point in the evolution. Since spatial and temporal directions interchange roles for a black hole, the slices always remain spacelike. The stretching of the slices is localized to a region in the vicinity of the horizon: a field mode in this region will become increasingly stretched to longer wavelengths, generating particles for as long as the assumptions concerning the use of (3.2) are valid (Fig. 3.9).

The particle creation scenario proceeds along the following lines. The quantum state on the initial slice Σ_0 is that of the matter field $|\Phi(t, r)\rangle$ that will later form the black hole. This can be taken to be a sharply-peaked wavepacket that in the classical limit describes a shell of collapsing matter. Prior to formation of the black hole no particles are created since the entire slice is outside of the black hole. Upon formation of the black hole the quantum state is $|\Phi\rangle$. On some subsequent slice there will be sufficient stretching of region Σ_C to create a pair of quanta of sufficiently short wavelength $\lambda = 2\pi/k$. The matter state $|\Phi\rangle$ will be localized in Σ_I and so will be negligibly affected by this process. Consider for simplicity a single mode. The quantum state on this slice will therefore be

$$|\Psi\rangle_1 \approx \frac{1}{\sqrt{2}}|\Phi\rangle_{\mathrm{I}_1} \otimes \left(|0_k\rangle^+_{\mathrm{I}_1}|0_{-k}\rangle^-_{\mathrm{O}_1} + |1_k\rangle^+_{\mathrm{I}_1}|1_{-k}\rangle^-_{\mathrm{O}_1}\right) \tag{3.34}$$

where the subscripts I/O refer to inside/outside the horizon. An outside observer has no access to the states inside, and so must employ the reduced density matrix

$$\rho_{\mathrm{O}_1} = \mathrm{tr}_{\mathrm{I}}\Big[|\Psi\rangle\langle\Psi|\Big] = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \tag{3.35}$$

in describing the physics of the outside state. The entanglement entropy of the outside state with the state inside the black hole is

$$S_{\mathrm{ent}} = -\mathrm{tr}\left[\rho_{\mathrm{O}_1} \ln \rho_{\mathrm{O}_1}\right] = 2 \times \frac{1}{2} \ln 2 = 2 \ln 2 \tag{3.36}$$

On the next slice the process repeats. The inside state $|\psi\rangle_{\mathrm{I}}$ moves into region Σ_I and the matter state $|\Phi\rangle_{\mathrm{I}_1}$ moves deeper into this region, becoming the state $|\Phi\rangle_{\mathrm{I}_2}$. The outside state $|\psi\rangle_{\mathrm{O}}$ moves into region Σ_O. The additional stretching of region Σ_C creates a new pair, yielding the state

$$|\Psi\rangle_2 \approx \frac{1}{\sqrt{2}}|\Phi\rangle_{\mathrm{I}_2} \otimes \left(|0_k\rangle^+_{\mathrm{I}_1}|0_{-k}\rangle^-_{\mathrm{O}_1} + |1_k\rangle^+_{\mathrm{I}_1}|1_{-k}\rangle^-_{\mathrm{O}_1}\right) \otimes \left(|0_k\rangle^+_{\mathrm{I}_2}|0_{-k}\rangle^-_{\mathrm{O}_2} + |1_k\rangle^+_{\mathrm{I}_2}|1_{-k}\rangle^-_{\mathrm{O}_2}\right) \tag{3.37}$$

assuming the stretching region has negligible influence on the first pair of states. The reduced density matrix is now

$$\rho_{\mathrm{O}_2} = \mathrm{tr}_{\mathrm{I}}\Big[|\Psi\rangle\langle\Psi|\Big] = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{pmatrix} \tag{3.38}$$

and

$$S_{\mathrm{ent}} = -\mathrm{tr}\left[\rho_{\mathrm{O}_2} \ln \rho_{\mathrm{O}_2}\right] = 4 \times \frac{1}{4} \ln 4 = 2 \ln 2 \tag{3.39}$$

is entanglement entropy of the outside state on this next slice.

After n steps the quantum state is

$$|\Psi\rangle_n \approx \frac{1}{\sqrt{2}}|\Phi\rangle_{\mathrm{I}_n} \prod_{m=1}^{n} \left[\otimes\left(|0_k\rangle^+_{\mathrm{I}_m}|0_{-k}\rangle^-_{\mathrm{O}_m} + |1_k\rangle^+_{\mathrm{I}_m}|1_{-k}\rangle^-_{\mathrm{O}_m}\right)\right] \tag{3.40}$$

and the reduced density matrix is $\rho_{\mathrm{O}_n} = \mathrm{diag}(2^{-n}, 2^{-n}, \ldots, 2^{-n})$, yielding

$$S_{\text{ent}} = -\text{tr}\left[\rho_{O_n} \ln \rho_{O_n}\right] = -n \times \frac{1}{2^n} \ln 2^{-n} = n \ln 2 \qquad (3.41)$$

for the entanglement entropy.

The quantity n is an enormously large number, as can be seen from energy conservation. Suppose each quanta contains the same amount of energy in units of the Planck mass. For a mass M black hole, the energy per quanta is $E_Q = \sigma(M_p/M)M_p \ll M$ where σ is a parameter of order unity. The total mass of the black hole is $nE_Q = M$ implying $n = (M/M_P)^2/\sigma$. For a solar mass black hole $n \sim (2 \times 10^{30}/(2 \times 10^{-8}))^2 = 10^{76}$. There is no upper bound on n since in principle the mass of the black hole can be arbitrarily large, though one might argue that the largest black hole possible is constrained by the mass of the universe, $M_U = 10^{52}$ kg, giving $n \leq 10^{120}$.

Expression (3.41) contains the nub of the information paradox: the entanglement entropy of the radiation state $|\psi\rangle_O$ grows without bound as more pairs are created. Eventually it must terminate, of course, as the radiation cannot contain more energy than was in the initial quantum state $|\Phi\rangle$ (or the mass M of the black hole). Indeed the niceness conditions will (at least) fail to hold once $M \sim M_p$, since the Kretschmann scalar (for example) $K = 48M^2/r^6 \to M_p^2/l_p^6 = l_p^4$ becomes too large for semi-classicality to hold. Of course the assumptions leading to (3.41) do not hold in full generality. The mass of the black hole is not constant, the created pairs might interact with each other, and the state $|\Phi\rangle$ could have some interaction with the created pairs. Gravitational instantons, for example, yield tiny corrections proportional to $\exp\{-(M/M_P)^2\}$. Could such small corrections from these effects invalidate the argument in some way?

Unfortunately this proves not to be the case [4]. Suppose at the j-th step of the process, the full quantum state is not of the form (3.40) but is rather

$$|\tilde{\Psi}\rangle_j = |\tilde{\Psi}\rangle_{j-1}^{(+)}|\Xi\rangle_j^{(+)} + |\tilde{\Psi}\rangle_{j-1}^{(-)}|\Xi\rangle_j^{(-)} \qquad (3.42)$$

where

$$|\Xi\rangle_j^{(\pm)} = \frac{1}{\sqrt{2}}\left(\left|0_k\right\rangle_{I_j}^{+}\left|0_{-k}\right\rangle_{O_j}^{-} \pm \left|1_k\right\rangle_{I_j}^{+}\left|1_{-k}\right\rangle_{O_j}^{-}\right) \qquad (3.43)$$

and the state $|\tilde{\Psi}\rangle_{j-1}^{(+)}$ can be expressed as

$$|\tilde{\Psi}\rangle_{j-1}^{(\pm)} = \sum_{l,m} \alpha_{l,m}|\tilde{\tilde{\psi}}_l^{\pm}(\Phi, \mathrm{I})\rangle|\chi_m(\mathrm{O})\rangle = \sum_m \gamma_m|\tilde{\psi}_m^{\pm}(\Phi, \mathrm{I})\rangle|\chi_m(\mathrm{O})\rangle \qquad (3.44)$$

where $|\tilde{\psi}_m^{\pm}(\Phi, \mathrm{I})\rangle$ and $|\chi_m(\mathrm{O})\rangle$ are orthonormal bases for the respective inside and outside states, and a unitary transformation has been applied to obtain the second equality. The newly created state is spanned by $\Xi_j^{(\pm)}$. Assuming locality, the outside states $\left|\psi\right\rangle_{O_j}^{-}$ generated in earlier stages of the evolution are not affected by the proposed step (3.42); hence the basis $|\chi_m(O)\rangle$ remains unchanged.

The proposed correction (3.42) to the Hawking process is a deformation of the original process (3.40), in which $|\tilde{\Psi}\rangle_{j-1}^{(-)} = 0$. Unitarity implies that

$$\left|\ {}_{j-1}^{(+)}\langle\tilde{\Psi}|\tilde{\Psi}\rangle_{j-1}^{(+)}\ \right|^2 + \left|\ {}_{j-1}^{(-)}\langle\tilde{\Psi}|\tilde{\Psi}\rangle_{j-1}^{(-)}\ \right|^2 = 1 \tag{3.45}$$

since the $\Xi_j^{(\pm)}$ are orthonormal. If the corrections to (3.40) due to (3.42) are small, then

$$\left|\ {}_{j-1}^{(-)}\langle\tilde{\Psi}|\tilde{\Psi}\rangle_{j-1}^{(-)}\ \right| < \epsilon \tag{3.46}$$

where $\epsilon \ll 1$. To say that corrections to the Hawking radiation process are small is to say that one obtains the state $\Xi_j^{(+)}$ with high probability when the pair is created, and that the orthogonal state $\Xi_j^{(-)}$ is observed with low probability.

There are thus three subsystems at stage-j of the evolution: (i) the outside state $\{O_{j-1}\}$ of all previously emitted outside quanta, (ii) the inside state $|\tilde{\psi}(\Phi, I)\rangle_{j-1}$ consisting of the original matter state and previously-created inside partner quanta, modified perhaps by previous interactions, and (iii) the newly created pair described by the state $|\Xi\rangle_j$, spanned by the basis $|\Xi\rangle_j^{(\pm)}$. The reduced density matrix for this newly created pair is

$$\begin{aligned}\rho_{\Xi_j} &= \mathrm{tr}_{|\tilde{\psi}(\Phi,I)\rangle_{j-1},O_{j-1}}\left[|\tilde{\Psi}\rangle_j\ {}_j\langle\tilde{\Psi}|\right] \\ &= \begin{pmatrix} {}_{j-1}^{(+)}\langle\tilde{\Psi}|\tilde{\Psi}\rangle_{j-1}^{(+)} & {}_{j-1}^{(+)}\langle\tilde{\Psi}|\tilde{\Psi}\rangle_{j-1}^{(-)} \\ {}_{j-1}^{(-)}\langle\tilde{\Psi}|\tilde{\Psi}\rangle_{j-1}^{(+)} & {}_{j-1}^{(-)}\langle\tilde{\Psi}|\tilde{\Psi}\rangle_{j-1}^{(-)} \end{pmatrix} \\ &= \begin{pmatrix} 1-\epsilon_-^2 & \epsilon_{+-} \\ \epsilon_{+-}^* & \epsilon_-^2 \end{pmatrix} \end{aligned} \tag{3.47}$$

where by (3.46), $|\epsilon_{+-}| < \epsilon$ and $|\epsilon_-| < \epsilon$. The eigenvalues of this matrix are $\frac{1}{2}(1 \pm \sqrt{1+4(|\epsilon_{+-}|^2 - \epsilon_-^2})$ to this order, yielding

$$S(\Xi_j)_{\mathrm{ent}} = \left(|\epsilon_{+-}|^2 - \epsilon_-^2\right)\ln\left(\frac{e}{|\epsilon_{+-}|^2 - \epsilon_-^2}\right) + \cdots < \epsilon^2 \ln\epsilon < \epsilon \tag{3.48}$$

for the entanglement entropy of the newly created pair. The entropy of the joint subsystem $\{O_{j-1}, \Xi_j\}$ is therefore

$$S(\{O_{j-1}, \Xi_j\}) \geq |S(\{O_{j-1}\}) - S(\Xi_j)| = S(\{O_{j-1}\}) - \epsilon \tag{3.49}$$

using the strong subadditivity property of entropy.

Next, tracing over everything except the inside state I_j yields

$$\rho_{\mathrm{I}_j} = \mathrm{tr}_{|\tilde{\psi}(\Phi,\mathrm{I})\rangle_{j-1},\mathrm{O}_j}\left[|\tilde{\Psi}\rangle_j\, {}_j\langle\tilde{\Psi}|\right]$$
$$= \frac{1}{2}\begin{pmatrix} \left({}^{(+)}_{j-1}\langle\tilde{\Psi}| + {}^{(-)}_{j-1}\langle\tilde{\Psi}|\right)\left(|\tilde{\Psi}\rangle^{(+)}_{j-1} + |\tilde{\Psi}\rangle^{(-)}_{j-1}\right) & 0 \\ 0 & \left({}^{(+)}_{j-1}\langle\tilde{\Psi}| - {}^{(-)}_{j-1}\langle\tilde{\Psi}|\right)\left(|\tilde{\Psi}\rangle^{(+)}_{j-1} - |\tilde{\Psi}\rangle^{(-)}_{j-1}\right)\end{pmatrix}$$
$$= \frac{1}{2}\begin{pmatrix} 1+\Re(\epsilon_{+-}) & 0 \\ 0 & 1-\Re(\epsilon_{+-})\end{pmatrix} \tag{3.50}$$

and so

$$S(\mathrm{I}_j)_{\mathrm{ent}} = \ln 2 - 2[\Re(\epsilon_{+-})]^2 > \ln 2 - 2\epsilon^2 > \ln 2 - \epsilon \tag{3.51}$$

for the entanglement entropy of the inside partner of the newly created pair. Applying strong subadditivity to the system $\{\{\mathrm{O}_{j-1}\}, \mathrm{O}_j, \mathrm{I}_j\}$ gives

$$S(\{\mathrm{O}_j\})_{\mathrm{ent}} + S(\Xi_j)_{\mathrm{ent}} = S(\{\mathrm{O}_{j-1}\}, \mathrm{O}_j)_{\mathrm{ent}} + S(\mathrm{O}_j, \mathrm{I}_j)_{\mathrm{ent}} > S(\{\mathrm{O}_{j-1}\}) + S(\mathrm{I}_j)_{\mathrm{ent}} \tag{3.52}$$

or

$$S(\{\mathrm{O}_j\})_{\mathrm{ent}} > S(\{\mathrm{O}_{j-1}\}) + S(\mathrm{I}_j)_{\mathrm{ent}} - S(\Xi_j)_{\mathrm{ent}} > S(\{\mathrm{O}_{j-1}\}) + \ln 2 - 2\epsilon \tag{3.53}$$

using (3.48), (3.51).

The relation (3.53) is very important for the information paradox. It demonstrates that the entanglement entropy of the outgoing radiation always increases by at least $\ln 2 - 2\epsilon$ as each new pair is created. In other words, the increase of entanglement entropy is stable, and small corrections cannot accumulate to invalidate the result (3.41) [4].

This is unlike the situation for radiation emitted from normal matter, in which the matter/radiation interaction necessarily increases the dimensionality of the space of entangled states to leading order in the interaction. Each emission of radiation can be entangled with the emitting atom(s) in the matter in any one of a number of orthogonal states. The data of the state of the hot matter is shared amongst many quanta of radiation, making its original state difficult to extract; the actual correlations themselves change radically from emission to emission. In contrast to this, the stretching of space-time requires that the outgoing radiation is always entangled in the same way to leading order regardless of the state of the black hole. The actual correlations themselves do not change radically from emission to emission, but at best receive only small corrections, assuming semiclassical physics is valid at and near the horizon.

3.5.1 Implications of the Information Paradox

To summarize: if (a) the niceness conditions admit local Hamiltonian evolution and (b) the event horizon of the black hole is information-free (or alternatively freely-falling observers do not see any unusual behaviour in high-energy processes) then the end state of evaporation of the black hole is that of a remnant or a mixed state. These two conditions imply that any outgoing mode $\left|\vartheta\right\rangle$ whose wavelength λ is within the range $l_p \ll \lambda \lesssim M$ will predominantly be a vacuum state when expanded in a Fock basis near the horizon

$$\left|\vartheta\right\rangle = \alpha_0\left|0\right\rangle + \alpha_1\left|1\right\rangle + \alpha_2\left|2\right\rangle + \cdots \tag{3.54}$$

with $\sum_{j>0} |\alpha_j|^2 = \varepsilon \ll 1$, since otherwise the state at the horizon would not be a vacuum state. The evolution of $\left|\vartheta\right\rangle$ must therefore agree with the standard vacuum evolution to leading order such that any corrections are constrained by (3.46). The entanglement entropy S_{ent} therefore increases by $\ln 2 - 2\epsilon$ (with $\epsilon \sim \varepsilon$) during each step of the evolution. After n steps $S_{\rm ent} > n/2 \ln 2$ since $\epsilon \ll 1$. This process will continue until $n \sim (M/M_P)^2$, when the size of the black hole is about a Planck length L_p. At this point either the process stops, leaving behind a highly-degenerate remnant or the hole fully evaporates, leaving the outgoing radiation in a mixed state (it is entangled with nothing) violating unitarity.

What might resolve the information problem? Clearly what is needed is that the outgoing radiation at least contain the information of the matter state that forms the black hole. To see what this means in a simple example, consider the matter to be a shell collapsing to a black hole that is initially in the state $|\Phi\rangle = \alpha|\Phi_0\rangle + \beta|\Phi_1\rangle$ where $\{|\Phi_0\rangle, |\Phi_1\rangle\}$ are two possible orthogonal states of the shell. The information would escape the black hole if the pairs were created such that the evolution of the state were

$$|\Phi\rangle = \alpha|\Phi_0\rangle + \beta|\Phi_1\rangle \to \frac{1}{\sqrt{2}}\left(|\Phi_0\rangle\left|1_k\right\rangle^+_{\rm I_1} + |\Phi_1\rangle\left|0_k\right\rangle^+_{\rm I_1}\right) \otimes \left(\alpha\left|0_{-k}\right\rangle^-_{\rm O_1} + \beta\left|1_{-k}\right\rangle^-_{\rm O_1}\right) \tag{3.55}$$

since tracing over the inside will yield the density matrix of the pure state $\alpha\left|0_{-k}\right\rangle^-_{\rm O_1} + \beta\left|1_{-k}\right\rangle^-_{\rm O_1}$, which has all the information of the infalling matter. A resolution of the information paradox must implement this kind of evolution. However there are significant obstacles to overcome.

First, the proposed evolution (3.55) is a radical departure from that in (3.34) or even (3.42). It is *not* a small correction to the standard pair-creation process at the horizon, and so its implementation is not obvious. It would represent a radical departure from our understanding of the behaviour of quantum field theory in curved space-time as described in preceding sections, at least near the event horizon.

Second, there can be no other pair-creation processes accompanying (3.55). To see why, suppose that there are further steps to the evolution (prior and/or afterward) such that for the k-th mode

$$|\Phi\rangle = \alpha|\Phi_0\rangle + \beta|\Phi_1\rangle \to \frac{1}{\sqrt{2}}\Big(|\Phi_0\rangle\big|1_k\big\rangle^+_{\mathrm{I}_1} + |\Phi_0\rangle\big|0_k\big\rangle^+_{\mathrm{I}_1}\Big) \otimes \Big(\alpha\big|0_{-k}\big\rangle^-_{\mathrm{O}_1} + \beta\big|1_{-k}\big\rangle^-_{\mathrm{O}_1}\Big) \Big[\otimes\Big(\big|0_k\big\rangle^+_{\mathrm{I}_1}\big|0_{-k}\big\rangle^-_{\mathrm{O}_1} + \big|1_k\big\rangle^+_{\mathrm{I}_1}\big|1_{-k}\big\rangle^-_{\mathrm{O}_1}\Big)\Big]^n \tag{3.56}$$

after n steps. Even though the information forming the black hole comes out, the end point of the evolution is still either a mixed state or a remnant due to all of the other created pairs. It is not sufficient to modify the evolution so that the information comes out. It must be modified to prevent the growth in entanglement entropy from the extra created pairs. The number of quanta emitted from the black hole is somewhat larger than the number needed to retain the information of the infalling matter (the entropy of the emitted radiation is about 30 % larger than the horizon entropy $A/4$ [41]), and it is just as important that these additional quanta do not yield remnants or mixed states.

Third, purity of the outside state is not sufficient. An evolution of the form

$$|\Phi\rangle = \alpha|\Phi_0\rangle + \beta|\Phi_1\rangle \to \frac{1}{\sqrt{2}}\Big(\alpha|\Phi_0\rangle\big|1_k\big\rangle^+_{\mathrm{I}_1} + \beta|\Phi_0\rangle\big|0_k\big\rangle^+_{\mathrm{I}_1}\Big) \otimes \Big(\big|0_{-k}\big\rangle^-_{\mathrm{O}_1} + \big|1_{-k}\big\rangle^-_{\mathrm{O}_1}\Big) \tag{3.57}$$

yields a pure outside state, but this state retains no information about the (α, β) coefficients of the infalling matter. The challenge of resolving the information paradox is to construct an evolution in which the final state of the outside radiation is both pure and information-retaining. Pair-creation at the horizon does neither.

3.5.2 Complementarity

One idea that emerged as a means of reconciling black hole radiation with known quantum physics was complementarity [42, 43]. The idea here is that one cannot ask a physical theory to yield descriptions of observers that cannot exist—specifically observers that can make measurements both inside and outside of the black hole.

Consider a choice of spacelike slices describing an evaporating black hole. The original slice Σ contains matter that will go into forming the black hole, and the slices that go through the event horizon obey the niceness conditions. This breaks down at the point P (contained in the slice Σ_P) where the black hole finally evaporates (no remnant assumed), but one might expect that a proper understanding of quantum gravity will ameliorate this, leading to a well-defined physical description of the end point of evaporation. After evaporation, the slices Σ' obey the niceness conditions.

The problem presented by black hole evaporation is that the quantum state on Σ' must be described by unitary evolution from the quantum state on Σ. However the only way this can happen is if the quantum state on the part of the slice inside the black hole has no dependence on the initial state. This is effectively a 'bleaching' of the information: all distinctions between the initial states of infalling matter must be expunged before the state crosses the global event horizon. In other words, the evolution of the quantum state must proceed as follows

$$\begin{aligned}&\left|\Psi(\Sigma)\right\rangle \to \left|\Psi(\Sigma_P)\right\rangle = \left|\Xi(\Sigma_{\text{IN}})\right\rangle \otimes \left|\Upsilon(\Sigma_{\text{OUT}})\right\rangle \to \left|\Psi(\Sigma')\right\rangle \\ &\text{where } \left|\Psi(\Sigma')\right\rangle = U_2\left|\Upsilon(\Sigma_{\text{OUT}})\right\rangle = U_1\left|\Psi(\Sigma)\right\rangle\end{aligned} \tag{3.58}$$

with U_1 and U_2 unitary operators and with the Hilbert space of states on Σ_P likewise decomposing into a tensor product $\mathcal{H}_P = \mathcal{H}_{\text{IN}} \otimes \mathcal{H}_{\text{OUT}}$ with $\Xi(\Sigma_{\text{IN}}) \subset \mathcal{H}_{\text{BH}}$ and $\Upsilon(\Sigma_{\text{OUT}}) \subset \mathcal{H}_{\text{OUT}}$. The evolution between $\left|\Psi(\Sigma')\right\rangle$ and $\left|\Psi(\Sigma)\right\rangle$ is fully unitary and reversible, uninfluenced by $\left|\Xi(\Sigma_{\text{IN}})\right\rangle$.

Complementarity posits that the flaw in the above argument is in the assumption of the existence of $\left|\Psi(\Sigma_P)\right\rangle$. This is a quantum state that simultaneously describes both the interior and the exterior of a black hole. The claim is that any state of this nature has no operational meaning, since no "super-observers" exist that compare measurements both inside and outside the black hole. Rather any observer must choose a basis in which to work: either one describes particles beyond the horizon or the particles in the Hawking radiation, but not both. Indeed, the trans-Planckian problem suggests that large non-vanishing commutators exist between operators describing ingoing material and those describing outgoing Hawking radiation, and so correlations between inside and outside the hole lose any operational meaning.

The advent of the Anti de Sitter/Conformal Field Theory correspondence [44] provided further confidence for this perspective, suggesting that all information is indeed carried away by the Hawking radiation. The idea here is that the quantum states (and their evolution) of any gravity theory whose solutions are asymptotic to anti de Sitter (AdS) space-time are in 1–1 correspondence with those of a Conformal Field Theory (CFT). This conjecture has more recently has broadened to a proposed duality between gravitational and gauge theories (gauge/gravity duality) under more general asymptotic conditions and symmetries. The CFT (or dual gauge theory) is unitary and so cannot admit any information loss, and its duality with gravity indicates that the same must be true there as well. There is strong circumstantial evidence in favour of this kind of duality, and hence of the purity of radiation emitted by a black hole.

Of course gauge/gravity duality does not prove that information loss cannot occur. Rather it provides a new paradigm by which one might seek to understand the process of black hole formation and evaporation. If indeed a dual gauge theory can describe this process, then the onus is on this theory to explain either (a) which conditions are modified so that mixedness is avoided, (b) the formation of remnants, or (c) new physics in the gravity theory that either prevents black hole formation or modifies the state near the horizon. So far such a description has yet to be given.

So complementarity asserts that there is no logical contradiction in assuming that a distant outside observer sees all infalling information returned in Hawking-like radiation, and that the infalling observer experiences nothing unusual before or during horizon crossing. The thermality of Hawking radiation will be affected by interactions very near the horizon, and these presumably ensure that the net emission process is pure as seen by the outside observer. The physical model an outside observer employs will therefore postulate a boundary condition for all fields a few Planck distances away from the horizon. These include the brick wall model [45, 46], bounce models, and stretched horizon models [42]. A full quantum theory of gravity is expected to set this distance, but it can be input into the theory for the purpose of doing phenomenological calculations. This membrane/wall distance is dependent on the matter content of the theory (the number of fields, for example), and so constraining this distance to be consistent with observation will constrain the matter content of the theory, providing (in principle) an additional degree of falsifiability. From the perspective of the outside observer the membrane/wall absorbs infalling matter and then thermalizes it, unitarily re-radiating it as Hawking radiation via a process similar to the manner in which a normal body radiates. Complementarity implies that an observer falling into the black hole will see no such membrane or brick wall, in contrast to the outside observer for whom this virtual structure is quite real [42].

The postulates of black hole complementarity are as follows:

1. **Unitarity** The process of formation and evaporation of a black hole, as viewed by a distant observer, can be described entirely within the context of standard quantum theory. In particular, there exists a unitary S-matrix which describes the evolution from infalling matter to outgoing Hawking-like radiation.
2. **Semi-Classicality** Outside the stretched horizon of a massive black hole, physics can be described to good approximation by a set of semi-classical field equations. The semi-classical field equations are those of a low energy effective field theory with local Lorentz invariance.
3. **Placidity** A freely falling observer experiences nothing out of the ordinary when crossing the horizon, as expected from the equivalence principle—gravity is locally indistinguishable from acceleration. This is basically the 'information-free' condition mentioned earlier: it is exponentially unlikely for infalling observer to measure a quantum of energy $E \gg 1/r_+$.
4. **Thermality** To a distant observer, a black hole appears to be a quantum system with discrete energy levels. The dimension of the subspace of states describing a black hole of mass M, angular momentum J, and charge Q is is the exponential of the Bekenstein entropy $S(M, J, Q)$, so that the standard black hole thermodynamic relations are obeyed.

These postulates have been slightly modified from their original form, but contain the essential aspects of what black hole complementarity is based on. Black hole no-hair theorems implied that the membrane/wall must be virtual, as noted above; no known physics at the time complementarity was proposed could generate such a structure, though new ideas have been put forward recently [47]. But even though the degrees of freedom of the membrane/wall are virtual, they must be generated by some nonlocal effect. The reason is that if normal semiclassical physics is valid for small curvatures, then pair creation takes place via the Hawking process described above and not reflection/generation from a wall or membrane. This is a consequence of the niceness conditions, namely the assumption that one can always choose a set of slices through the black hole where curvatures are everywhere small and the vacuum is well-defined. If complementarity is valid then some new, nonlocal, physics must dominate—over scales of the horizon size—since semiclassical physics yielding pair creation is valid along the slices [48].

3.6 Firewalls

The Firewall argument asserts that the postulates of black hole complementarity are not self-consistent [3, 49]. Specifically one of the first three postulates must be incorrect, since assuming all three together yields a contradiction.

3.6.1 The Firewall Argument

This rather surprising claim follows from a fairly straightforward argument, put forward by Almheiri, Marolf, Polchinski and Sully (known as AMPS) [3]. The unitarity postulate #1 implies that radiation emitted from the black hole must not be in a mixed state, and so some process must convert the state (3.40) to

$$|\Psi\rangle_n \approx \frac{1}{\sqrt{2}}|\Phi\rangle_{\mathrm{I}_n}\prod_{m=1}^{n}\left[\otimes\left(\left|0_k\right\rangle^+_{\mathrm{I}_m}\left|0_{-k}\right\rangle^-_{\mathrm{O}_m}+\left|1_k\right\rangle^+_{\mathrm{I}_m}\left|1_{-k}\right\rangle^-_{\mathrm{O}_m}\right)\right] \to |\tilde{\Phi}\rangle_{\mathrm{I}_n}|\tilde{\Xi}\rangle_{\mathrm{O}_n} \tag{3.59}$$

after some large number n of steps, where the outside radiation state $|\tilde{\Xi}\rangle_{\mathrm{O}_n}$ is pure. At some point in time (called the Page time [28]) the entanglement entropy of the emitted radiation must reach a maximum, after which point there is more entropy in the radiation than there is in the black hole. The black hole continues to shrink in size and entropy, emitting successively fewer quanta. Consequently the number of states N_L accessible after the Page time [50] (the 'late' subspace) will be much smaller than the number N_E in the space of states prior to this (the 'early' subspace): $N_L \ll N_E$. Expanding the full outside radiation state $|\tilde{\Xi}\rangle_{\mathrm{O}_n}$ in an orthonormal basis $\{|\mathsf{j}\rangle_L\}$ of the late subspace yields

$$|\tilde{\Xi}\rangle_{\mathrm{O}_n} = \sum_{\mathsf{k}}^{N_L} |\psi_{\mathsf{k}}\rangle_E \otimes |\mathsf{k}\rangle_L \tag{3.60}$$

where $\{|\psi_1\rangle_E, |\psi_2\rangle_E, \ldots, |\psi_{N_E}\rangle_E\}$ span the early subspace. Consider the norm of the state $\mathcal{P}^{\mathsf{j}}|\tilde{\Xi}\rangle_{\mathrm{O}_n}$, where the operator $\mathcal{P}^{\mathsf{j}} \equiv P^{\mathsf{j}} - \hat{P}^{\mathsf{j}} = |\mathsf{j}\rangle_L\, {}_L\langle\mathsf{j}| - N_L|\psi_{\mathsf{j}}\rangle_E\, {}_E\langle\psi_{\mathsf{j}}|$. Expanding this out yields

$$
\begin{aligned}
\left|\left|\mathcal{P}^{\mathsf{j}}|\tilde{\Xi}\rangle_{\mathrm{O}_n}\right|\right|^2 &= \left|\left|\, |\mathsf{j}\rangle_L\, {}_L\langle \mathsf{j}|\tilde{\Xi}\rangle_{\mathrm{O}_n} - N_L|\psi_{\mathsf{j}}\rangle_E\, {}_E\langle\psi_{\mathsf{j}}|\tilde{\Xi}\rangle_{\mathrm{O}_n}\right|\right|^2 \\
&= \left|\left|\, |\mathsf{j}\rangle_L\, |\psi_{\mathsf{j}}\rangle_E - N_L|\psi_{\mathsf{j}}\rangle_E \sum_{\mathsf{k}}^{N_L} {}_E\langle\psi_{\mathsf{j}}|\psi_{\mathsf{k}}\rangle_E \otimes |\mathsf{k}\rangle_L\right|\right|^2 \\
&= |||\psi_{\mathsf{j}}\rangle_E||^2 \left|\left|\, |\mathsf{j}\rangle_L - N_L \sum_{\mathsf{k}}^{N_L} {}_E\langle\psi_{\mathsf{j}}|\psi_{\mathsf{k}}\rangle_E \otimes |\mathsf{k}\rangle_L\right|\right|^2 \\
&= |||\psi_{\mathsf{j}}\rangle_E||^2 \left[\left(1 - N_L\, {}_E\langle\psi_{\mathsf{j}}|\psi_{\mathsf{j}}\rangle_E\right)^2 + N_L^2 \sum_{\mathsf{k}\neq\mathsf{j}}^{N_L} |{}_E\langle\psi_{\mathsf{j}}|\psi_{\mathsf{k}}\rangle_E|^2\right]
\end{aligned}
\tag{3.61}
$$

Expanding the state $|\psi_{\mathsf{k}}\rangle_E = \sum_{\mathsf{k}=1}^{N_E} c_{\mathsf{ka}}|\mathrm{a}\rangle$ in an orthonormal basis $|\mathrm{a}\rangle$ of the early states yields

$$
\overline{c_{\mathsf{ja}}c^*_{\mathsf{kb}}} = \frac{1}{N_L N_E}\delta_{\mathsf{jk}}\delta_{\mathrm{ab}} \qquad \overline{c_{\mathsf{ja}}c^*_{\mathsf{kb}}c_{\mathsf{ic}}c^*_{\mathsf{ld}}} = \frac{1}{N_L^2 N_E^2}\left(\delta_{\mathsf{jk}}\delta_{\mathrm{ab}}\delta_{\mathsf{il}}\delta_{\mathrm{cd}} + \delta_{\mathsf{jl}}\delta_{\mathrm{ad}}\delta_{\mathsf{ik}}\delta_{\mathrm{bc}}\right) \tag{3.62}
$$

upon averaging over $|\tilde{\Xi}\rangle_{\mathrm{O}_n}$, assuming a uniform measure for the outside state. Consequently $\overline{{}_E\langle\psi_{\mathsf{j}}|\psi_{\mathsf{k}}\rangle_E} = \delta_{\mathsf{jk}}/N_L$ and $\overline{{}_E\langle\psi_{\mathsf{j}}|\psi_{\mathsf{k}}\rangle_E\, {}_E\langle\psi_{\mathsf{i}}|\psi_{\mathsf{l}}\rangle_E} = \delta_{\mathsf{jk}}\delta_{\mathsf{il}}/N_L^2 + \delta_{\mathsf{jl}}\delta_{\mathsf{ik}}/(N_L^2 N_E)$, yielding

$$
\begin{aligned}
\overline{\mathcal{E}} = \frac{\overline{\left|\left|\mathcal{P}^{\mathsf{j}}|\tilde{\Xi}\rangle_{\mathrm{O}_n}\right|\right|^2}}{|||\psi_{\mathsf{j}}\rangle_E||^2} &= 1 - 2N_L\, \overline{{}_E\langle\psi_{\mathsf{j}}|\psi_{\mathsf{j}}\rangle_E} + N_L^2 \sum_{\mathsf{k}}^{N_L} \overline{{}_E\langle\psi_{\mathsf{j}}|\psi_{\mathsf{k}}\rangle_E\, {}_E\langle\psi_{\mathsf{j}}|\psi_{\mathsf{k}}\rangle^*_E} \\
&= 1 - 2N_L\frac{\delta_{\mathsf{jj}}}{N_L} + \frac{N_L^2}{N_L^2N_E^2}\left(N_E^2\delta_{\mathsf{jj}} + N_E\sum_k \delta_{\mathsf{kk}}\right) = \frac{N_L}{N_E}
\end{aligned}
\tag{3.63}
$$

in the limit $N_E \gg N_L \gg 1$. Hence

$$
P^{\mathsf{j}}|\tilde{\Xi}\rangle_{\mathrm{O}_n} \approx \hat{P}^{\mathsf{j}}|\tilde{\Xi}\rangle_{\mathrm{O}_n} = |\psi_{\mathsf{j}}\rangle_E \otimes |\mathsf{j}\rangle_L \tag{3.64}
$$

and so it is possible to project onto any given subspace of the late radiation, up to a relative error of order N_L/N_E. The argument is essentially the same if grey-body factors are taken into account.

So for a distant observer after n steps the radiation $|\tilde{\Xi}\rangle_{\mathrm{O}_n}$ is near infinity and can be decomposed into a set of modes $\{|\mathsf{j}\rangle\}$. In particular, it is possible to project onto eigenspaces of the number operator in an observer-independent way, according to the semi-classicality assumption. These modes can be evolved backward in time toward the horizon—they will be of much higher frequency at these earlier times, but can be kept to the sub-Planckian regime if one does not evolve too far back. However the placidity assumption #3 implies that an infalling observer sees the vacuum near the horizon, and so the number operator of the radiation $|\tilde{\Xi}\rangle_{\mathrm{O}_n}$ must be zero, in contradiction to what the observer at infinity measures. This contradiction can be avoided if the infalling observer does not see a vacuum, but instead encounters a large number of high-energy modes: in other words, a firewall.

A regular horizon implies increasing entanglement, as shown in (3.53). Conversely, if entanglement is to decrease, then the state at the horizon cannot be the vacuum. This is the firewall argument in a nutshell.

An alternative version of the argument employs the strong subadditivity condition [47]. The radiation after n steps is $|\tilde{\Xi}\rangle_{O_n}$, with the next mode $|n{+}1\rangle_O$ emitted near the horizon. The former can be evolved backward in time near the horizon. Semiclassicality implies that

$$|\tilde{\Xi}\rangle'_{O_n} = |\tilde{\Xi}\rangle_{O_n} \qquad |n{+}1\rangle'_O = |n{+}1\rangle_O \tag{3.65}$$

where the primes refer to the states measured by the infalling observer. Since this observer sees a vacuum at the horizon, the state $|n{+}1\rangle'_O$ must be entangled with some state $|n{+}1\rangle'_I$ inside the horizon. Strong subadditivity then implies that (3.53) holds

$$S'(n{+}1)_{\text{ent}} > S'(n)_{\text{ent}} + \ln 2 - 2\epsilon \tag{3.66}$$

and the equivalence (3.65) implies

$$S(n{+}1)_{\text{ent}} > S(n)_{\text{ent}} + \ln 2 - 2\epsilon \tag{3.67}$$

which means the entanglement between the black hole and the radiation cannot decrease, in contradiction with the unitarity postulate, which implies that it must decrease after the halfway point.

So complementarity is incompatible with the local evolution that creates the pairs of Hawking quanta. Even though complementarity invoked non-locality to argue that the slices permitted by the niceness conditions are not valid, it requires that local semiclassical physics applies outside the membrane or stretched horizon. Non-local physics is therefore constrained to be inside the horizon. The firewall argument rules out this kind of complementarity and hence this kind of sharply-limited non-locality.

3.6.2 Responses to the Firewall Argument

The response of the physics community to the firewall argument was rapid, intense, and diverse, ranging from skepticism to ambivalence to endorsement.

Those endorsing the firewall argument have emphasized that standard arguments from quantum field theory in curved spacetime and quantum information should lead one to expect this result [51–63]. Standard semi-classical methods analyzing causal patches [59], string-creation [63], and freely-falling observers [60] have each been used to buttress the argument. Indeed numerical analysis of a particular class of models suggests a breakdown of effective field theory, in turn implying the existence of firewalls on black hole horizons [61]. It has even been suggested that alternatives to firewalls may suffer contradictions similar to those associated with time travel [52]. Rindler horizons have been argued to be immune (or at least not necessarily susceptible) to the firewall argument [64].

Nevertheless initial skepticism [65] was soon followed by a number of challenging responses to the firewall argument. A number were rebutted by Almheiri and collaborators [66]. Here I shall summarize some of the main responses.

3.6.2.1 Absorbing the Interior Hilbert Space

Since the outside modes must be entangled with both the early outside modes and with their inside pair-created partners (violating quantum limits on entanglement) then perhaps the interior Hilbert space of an old black hole is embedded in the larger Hilbert space of the early radiation. The claim is that the firewall phenomenon can occur only for an exponentially fine-tuned (and intrinsically quantum mechanical) initial state, analogous to an entropy decreasing process in a system with large degrees of freedom [67–72]. Alternatively, quantum computations required to do carry out the thought experiments undergirding the firewall argument take so long (a time exponential in the entropy of the black hole) that this prevents the experiments from being done [73, 74]. In other words, excitations exist at the horizon only if such quantum computations have been performed.

Both considerations run afoul of standard quantum mechanics. Assuming unitarity, an observer outside the black hole (Charlie) can extract a bit of information that will be entangled with a later outside pair-created bit. Another spacelike separated observer (Alice, say) can jump into the black hole later and extract information about both the inside and outside later-created pair, whilst Charlie can send the quantum state of the early bit to Alice. Alice will then possess information concerning three quantum bits, two of which are are maximally entangled with the third, which violates quantum mechanics.

More generally, operators associated with the early radiation will generically not commute with operators associated with the Hilbert space of an infalling observer if the the interior Hilbert space is embedded in the early radiation Hilbert space [66]. Consider the parity operator $(-1)^{N_e}$ of an early outside bit $e \subset E$

$$(-1)^{N_e} = \sigma^z \otimes I \tag{3.68}$$

written above in a basis factorized into the measured parity and everything else. Since the interior Hilbert space is a subset of the outside early radiation space, we can expand the parity operator of an inside bit $i \subset I$

$$(-1)^{N_i} = I \otimes S^0 + \sigma^x \otimes S^x + \sigma^y \otimes S^y + \sigma^z \otimes S^z \tag{3.69}$$

where the matrices S^λ are constrained by the requirement $(-1)^{N_i}(-1)^{N_i} = 1$. Suppose the parity of an early state $|\psi\rangle$ is positive, so that $(-1)^{N_e}|\psi\rangle = +|\psi\rangle$.

Then the expectation value of $(-1)^{N_e}$ for the state $(-1)^{N_i}|\psi\rangle$ is

$$\begin{aligned}&\langle\psi|(-1)^{N_i}(-1)^{N_e}(-1)^{N_i}|\psi\rangle\\ &\quad=\langle\psi|\sigma^z\otimes(S^0)^2+\sigma^x\sigma^z\sigma^x\otimes(S^x)^2+\sigma^y\sigma^z\sigma^y\otimes(S^y)^2\\ &\qquad+(\sigma^z)^3\otimes(S^z)^2|\psi\rangle+\text{cross terms}\\ &\quad=\langle\psi|\sigma^z\otimes\Big((S^0)^2-(S^x)^2-(S^y)^2+(S^z)^2\Big)|\psi\rangle+\text{cross terms}\end{aligned}$$

Upon averaging over all all possible operators S^λ (requiring $(-1)^{N_i}(-1)^{N_i}=1$) the cross-terms will average to zero since independent sign flips in parity are allowed. Each $(S^\lambda)^2$ term will average to the same value since these operators are generic and so their eigenvalues will be comparable in size. Hence $\langle\psi|(-1)^{N_i}(-1)^{N_e}(-1)^{N_i}|\psi\rangle$ averages to zero.

So if we start with an eigenstate of $(-1)^{N_e}$ and measure the parity $(-1)^{N_i}$, the expectation value flips from 1 to 0. Hence the eigenvalue changes with near-unit probability, implying the commutator of $(-1)^{N_i}$ and $(-1)^{N_e}$ is of order unity, and hence the commutator of early and interior operators is also of order unity. Hence if an infalling observer sees a vacuum (so that $\mathtt{a}|\psi\rangle=0$ where $\mathtt{a}$ is an annihilation operator in the Hilbert space of the infalling observer), then since the interior operators can be expanded in terms of the infalling operators, the early creation/annihilation operators will not commute with any of the operators $\mathtt{a}$, strongly perturbing the infalling vacuum and creating a firewall. This abolishes (or at least renders highly problematic) the notion that infalling observers see no firewall because the deviation from thermality is too small to detect [75].

3.6.2.2 Broadening Complementarity

One recent proposal posits that each observer has their own Hilbert space, with suitable overlap conditions [76–86]. This broadens the notion of complementarity insofar as there is no global Hilbert space. The idea is that space-time physics is described in terms of an infinite number of quantum systems, each of which encodes the physics as seen along a particular time-like trajectory, in a proper time dependent Hamiltonian [76]. Extending these ideas to a matrix theory model of black holes suggests that there are no high energy particles available that could constitute the firewall [79]. The vacuum entanglement that is a crucial feature of Hawking radiation is claimed not to be a feature of the physics described by matrix theory. Whether or not this proposal can be fully consistently implemented remains to be seen.

3.6.2.3 Non-locality

Prior to the advent of the Firewall argument, the idea that non-locality can and should play a role in resolving the information paradox was already being actively explored [87, 88]. Although generic nonlocality leads to causality paradoxes, perhaps there are regions (near a black hole, for example) where locality is not exact but only approximate. The idea here is to weaken the assumptions of the semi-classicality postulate #2 and introduce some form of mild (or non-violent) non-local physics [89–94].

Since if locality is exact outside the horizon any information transfer from the black hole to the radiation produces singular behaviour at the horizon [4, 49], it is necessary to weaken locality outside of the black hole. Each black hole would therefore have a "nonlocal zone" (about the size of the black hole itself) within which information transfer from the black hole to the outgoing radiation takes place. This information transfer is a transfer of the entanglement between the early radiation and black hole interior to entanglement between the early and late radiation. It requires an additional energy flux beyond that of the Hawking radiation [91, 92], and modulates the Hawking radiation in a sufficiently fine-grained manner so as to preserve the average properties of the Hawking flux. Leading order calculations in a model in which nonlocal metric perturbations couple to the stress tensor suggest in a two-dimensional model this might be possible [93].

The challenge such proposals face is that any scheme that physically separates transfer of energy from transfer of information runs into conflict with the Bekenstein-Hawking density of states $\exp[S]$ of the black hole. Consider a process in which quanta behind the horizon are transported to become outgoing quanta. Suppose a pair is created outside the black hole as in Eq. (3.34)

$$\begin{aligned}\frac{1}{\sqrt{2}}|\Phi\rangle_{\mathrm{I}} \otimes \left(\left|0_k\right\rangle_{\mathrm{I}}^{+}\left|0_{-k}\right\rangle_{\mathrm{O}}^{-}+\left|1_k\right\rangle_{\mathrm{I}}^{+}\left|1_{-k}\right\rangle_{\mathrm{O}}^{-}\right) \\ =\frac{1}{\sqrt{2}}\left(|\Phi_0\rangle+|\Phi_1\rangle\right) \otimes \left(\left|0_k\right\rangle_{\mathrm{I}}^{+}\left|0_{-k}\right\rangle_{\mathrm{O}}^{-}+\left|1_k\right\rangle_{\mathrm{I}}^{+}\left|1_{-k}\right\rangle_{\mathrm{O}}^{-}\right)\end{aligned} \tag{3.70}$$

where the Hilbert space has been separated into orthogonal states as in Eq. (3.55). As the outgoing mode moves through the nonlocal zone, some process will cause it to exchange information with a state in the interior so that

$$\begin{aligned}\frac{1}{\sqrt{2}}|\Phi\rangle_{\mathrm{I}} \otimes \left(\left|0_k\right\rangle_{\mathrm{I}}^{+}\left|0_{-k}\right\rangle_{\mathrm{O}}^{-}+\left|1_k\right\rangle_{\mathrm{I}}^{+}\left|1_{-k}\right\rangle_{\mathrm{O}}^{-}\right) \\ \rightarrow \frac{1}{\sqrt{2}}\left(|\Phi_0\rangle\left|1_k\right\rangle_{\mathrm{I}_1}^{+}+|\Phi_0\rangle\left|0_k\right\rangle_{\mathrm{I}_1}^{+}\right) \\ \otimes\left(\alpha\left|0_{-k}\right\rangle_{\mathrm{O}_1}^{-}+\beta\left|1_{-k}\right\rangle_{\mathrm{O}_1}^{-}\right)\end{aligned} \tag{3.71}$$

This would avoid the firewall problem just as it resolved the information paradox problem. However this approach will also entail the same difficulties noted in the previous section, and will allow the number of internal states of the black hole to exceed the entropy $S = A/4$ discussed in Sect. 3.2. The reason is that it is possible to interact with the outgoing bit as it moves through the nonlocal zone, say by introducing a phase

$$\begin{aligned} \frac{1}{\sqrt{2}}|\Phi\rangle_{\mathrm{I}} \otimes \left(\left|0_k\right\rangle_{\mathrm{I}}^{+}\left|0_{-k}\right\rangle_{\mathrm{O}}^{-} + \left|1_k\right\rangle_{\mathrm{I}}^{+}\left|1_{-k}\right\rangle_{\mathrm{O}}^{-}\right) \\ \rightarrow \frac{1}{\sqrt{2}}\left(|\Phi_0\rangle\left|1_k\right\rangle_{\mathrm{I}_1}^{+} - |\Phi_0\rangle\left|0_k\right\rangle_{\mathrm{I}_1}^{+}\right) \\ \otimes\left(\alpha\left|0_{-k}\right\rangle_{\mathrm{O}_1}^{-} + \beta\left|1_{-k}\right\rangle_{\mathrm{O}_1}^{-}\right) \end{aligned} \tag{3.72}$$

and so a larger set of interior states, beyond that given by $\exp[S]$ has been accessed by this process.

Another recent proposal involves using wormholes to transfer information beyond the horizon [95–99]. The idea is that the part of space-time connecting the right side of Fig. 3.1 ("our universe") and its left side (known as an Einstein Rosen bridge) is created by quantum correlations between the microstates of the black holes on each side. The conjecture is that any entangled pair of quantum states is connected a similar sort of space-time bridge or wormhole. These wormholes non-locally connect quantum states inside and outside of the horizon, allowing for information from the black hole to escape.

3.6.2.4 Exotic Objects

Of course if an event horizon never forms then a firewall can be avoided [100–102]. Could this be the resolution to the firewall problem?

One noteworthy attempt along these lines is the fuzzball proposal [103]. This approach grew out of several results that emerged from string theory that suggest the end-state of gravitationally collapsing matter is not a traditional black hole because the degrees of freedom of the hole distribute themselves throughout a horizon sized object referred to as a fuzzball. Particular examples of this kind of structure were obtained by considering various extremal black brane solutions to the low-energy string equations with multiple charges [104–117]. The basic idea is that as matter undergoes gravitational collapse, its (presumed) fundamentally stringy degrees of freedom distribute their momenta in such a way that the final solution has neither a horizon nor a singularity [108–110]. Instead of a black hole, matter undergoing gravitational collapse will quantum tunnel to a fuzzball: a complicated "hairy" structure that contains all of the degrees of freedom of the black hole. Hawking radiation would be due to emission from an ergoregion near the fuzzball. This radiation can carry information about the original state of matter because it is not entangled with any states inside a horizon because no such horizon exists [103].

The problem with the fuzzball proposal at the moment is its lack of generality. Notwithstanding the fact that first-order corrections suggest that perhaps fuzzballs can form from generic collapse [118, 119], the proposal only appears to work for particular brane configurations. However to resolve the firewall problem (and information paradox) *all* possible matter configurations must form a fuzzball structure.

Other recent speculative ideas along these lines include Grireballs [120], leaky horizons [121], and aether-like fluids whose atmosphere mimics Hawking radiation [122, 123]. All of these ideas must universally replace the generic collapse of matter into a black hole if they are to be viable candidates for eliminating the firewall. Of course if a remnant forms, this could also avoid a firewall; an explicit example in two dimensions was recently given [124].

3.6.2.5 Additional Degrees of Freedom

Some responses to the firewall argument have suggested it is lacking because additional degrees of freedom are present in quantum gravity that are otherwise unaccounted for [125–127] or not properly treated.

One such approach involves distinguishing virtual qubits (the entangled created pairs) from real qubits (that store the information inside the black hole) [128, 129]. The idea is that black hole information is stored both inside and outside the stretched horizon, yielding twice the usual black hole entropy and therefore extra room to arrange the quantum degrees of freedom so that paradoxical results are avoided. The apparent firewall obstruction can be removed, via a universal entanglement swap operation that transports all free quantum information from the interior of the black hole to its exterior. This swap can be created locally and in the near horizon region; however this firewall-removing operation cannot be use to transfer information from an infalling state into the outgoing radiation [130, 131].

3.6.2.6 Loopholes

A number of papers have been written contending that the existence of firewalls depends on a chain of reasoning that is incomplete, and that one or more loopholes exist that allow one to escape the conclusions of the argument.

It has been suggested that the space of physical quantum gravity states does not factorize into a tensor product of localized degrees of freedom, invalidating one of the assumptions of the firewall argument [132]. The idea here is that in any diffeomorphism invariant ultra-violet complete theory with an asymptotic region in which an algebra of observables can be defined (which presumably is a feature of quantum gravity), the Hamiltonian is a surface integral in this asymptotic region (or boundary). The boundary encodes all degrees of freedom, including those inside the horizon, and the algebra of boundary observables evolves into itself unitarily over time. Hence no boundary information can ever be lost, not even temporarily. This is argued to invalidate a key assumption of the firewall argument, which is that the early time Hawking radiation is in a mixed quantum state and gets purified later by the late time Hawking radiation to preserve unitarity. Rather there must be continuous purity, with the Hawking quanta always entangled with exterior degrees of freedom and never with interior ones. The Hilbert space does not factorize into exterior and interior state spaces, and so the 'partner behind the horizon' does not actually exist in a full quantum theory of gravity. Of course for this picture to be accepted, the details of the physical states and how they are encoded into the boundary needs to be made explicit.

Some effort has been put into seeing what happens if the horizon geometry undergoes quantum fluctuations [133]. The claim is that both the blackhole information paradox and firewalls originate from treating the geometry as strictly classical, and that it is an ill-posed problem to employ quantum fields in a classical curved space with a horizon. Instead, one should first integrate out fluctuations of the background geometry and then evaluate matter observables. Some models of shell collapse indicate that a firewall may or may not form depending on the ratio of the black hole entropy to the square of the number of coherently emitted particles [134, 135].

Additional evasive tactics have been proposed. Some have proposed that the firewall issue is purely quantum information theoretic and so should have an answer once we know exactly what computation we need to do [136]. Another argument posits that the firewall paradox is likely to be an artifact of using an effective theory beyond its domain of validity [137]. It has been suggested that a distillation-like process for extracting information needs to be clarified before one can conclude that black hole complementarity is not valid [138]; indeed this distillation process may back-react on the black hole, breaking cross-horizon entanglement and removing the firewall [139].

Another suggestion is that a firewall will not emerge if the energy cost of measurement on the early states (yielding information about the late states) is much smaller than the ultraviolet cutoff scale [140]. Perhaps it is necessary to modify the expected entanglement of states near a horizon [141] or to take macroscopic superpositions of black holes [142–144], or to introduce new causality requirements into physics [145].

The final state-proposal in which a generalization of quantum mechanics allows postselection on a final state at the black hole singularity, has been suggested as a resolution for the black hole information paradox [146] and for firewalls [147]. The idea here is that quantum information can escape from the black hole interior via postselected quantum teleportation [148]. The information moves forward in time to the singularity, backward in time from the singularity to the horizon, then forward in time from the horizon to future infinity. If suitable dynamical constraints are satisfied, this is equivalent to a causally ordered flow of information moving unitarily forward in time. However these constraints appear to be rigorously fulfilled only by fine tuning [147]. Furthermore, the final state projection postulate has been shown to be inadequate for abolishing firewalls [149].

Some have contented that the firewall follows from making assumptions about physics inside the stretched horizon that do not follow from the semiclassicality postulate #2. One claim [150, 151] is that firewalls are avoided if the degrees of freedom of the stretched horizon retain information for a sufficiently long time known as the scrambling time (the minimum time required for the information about the initial state to be lost without measuring a large fraction of all the degrees of freedom). Alternatively, if the semiclassicality postulate holds, firewalls are avoided, but at the price of introducing remnants [152].

Finally it was recently shown that Einstein's field equations do not admit a solution in which a Planck-density, Planck-scale firewall is just outside the event horizon [153]. Any shell located at the horizon of an astrophysical black hole must necessarily have a density many orders of magnitude lower than the Planck density. A recent analysis of the behaviour of photons from the cosmic microwave background falling into a black hole indicates that they form a "classical firewall" in the frame of a static observer near the horizon, but that this firewall has negligible effects on both freely infalling observers and the evolution of the black hole [154].

3.7 Summary

Black holes retain a powerful grip on both the physical universe and the human imagination. At a classical level they absorb all matter and energy they encounter, growing ever larger in the process. Our best understanding of quantum physics indicates that they thermally radiate like black bodies, undergoing phase transitions into other forms and eventually evaporating away.

But away to what? We have no self-consistent description of this process. Our present understanding suggests that either a radiating black hole eventually either cools down into an information-rich nugget or erects a firewall around itself. Neither scenario appears to be compatible with our understanding of physics. The problem is not so much with particular models of black hole radiation but rather with a clash of the basic principles of relativity and quantum physics.

It appears we must either give up the predictive power of quantum mechanics (unitarity), or the notion that gravity is locally indistinguishable from acceleration (the equivalence principle), or the view that a physical phenomenon is influenced directly only by its immediate surroundings (locality). Each of these principles is supported—directly and indirectly—by a wealth of experimentation. The physics community at the moment is quite divided on the resolution to this problem, and may be for some time to come.

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