First Results on Height System Unification in North America Using GOCE

M.G. Sideris, E. Rangelova, and B. Amjadiparvar

Abstract

We study the impact of GOCE on the North American height system unification by assessing different factors: the performance of the GOCE global geopotential models, the models' omission error and its effect on the computed mean height datum offsets, and the effect of the biased local gravity data. Depending on the distribution of the data points, the omission error of the third release time-wise GOCE model used up to degree and order 180 contributes 13–15 cm to the computed mean offset of CGVD28 in Canada and only 2 cm to the mean offset of NAVD88 in the USA. The effect of the biased local gravity anomalies on the datum offsets is assessed by means of Stokes's integration with the original and residual kernels in a regional simulation scenario. This effect is found to be negligible when GOCE geopotential models are used in the computation of the geoid heights.

Keywords

GOCE • Height system unification • Vertical datum

1 Introduction

North America is characterized by two official height datums: CGVD28 (Canadian Geodetic Vertical Datum of 1928; Cannon 1929) in Canada and NAVD88 (North American Vertical Datum of 1988; Zilkoski et al. 1992) in the USA and Mexico. In addition, orthometric heights from the last, unofficial adjustment Nov07 of the first-order levelling network in Canada are available and used primarily for validation of the regional gravimetric and global geoid models (Véronneau, personal communication). The levelling networks in Canada and the USA are considered outdated, with large systematic errors accumulated from coast to coast (e.g., NAVD88 and to a lesser degree Nov07) and large regional distortions (CGVD28). Benchmarks are subject to rapid accuracy degradation due to the crustal motion from postglacial rebound, earthquakes, subsidence, frost heave, and local instabilities of the monuments.

These continental-size, levelling-based height datums will soon be replaced by geoid-based national and international height datums (Véronneau et al. 2006; NGS 2008). The zero height level surface of such a modern height datum will be defined by the regionally representative value $W_o = 62{,}636{,}856.00\ \mathrm{m^2/s^2}$ of the geopotential determined at North American tide gauges (Roman and Weston 2012). The datum will be realized by means of a high resolution and accuracy gravimetric geoid model based on combined GRACE/GOCE global geopotential models and computed with improved local gravity and topographic data sets.

By providing improved medium wavelengths of the Earth's gravity field (ESA 1999), the GOCE mission initiated new studies of the regional and global unification of the existing over one hundred height systems worldwide. All datums can be connected using the global level surface defined by a GOCE-derived geoid model. In addition, GOCE provides a means for assessing systematic errors in the regional and national height datums, e.g., the large east-west tilt of NAVD88 in Canada. The determination of the North

M.G. Sideris • E. Rangelova (✉) • B. Amjadiparvar
Department of Geomatics Engineering, Schulich School of Engineering, University of Calgary, 2500 University Drive NW, Calgary, AB, Canada T2N 1N4
e-mail: evrangel@ucalgary.ca

U. Marti (ed.), *Gravity, Geoid and Height Systems*, International Association of Geodesy Symposia 141,
DOI 10.1007/978-3-319-10837-7_28,

American height datum offsets with respect to a global level surface is needed for analyzing the effect of the unified height systems on gravity and topography data sets used in regional geoid modeling, and for homogenization of these terrestrial data.

In this study, we follow the Geodetic Boundary Value Problem (GBVP) approach developed by Rummel and Teunissen (1988) for height system unification. Related to this approach, we study the effect of the omission error of the GOCE geopotential model on the height datum offsets. In addition, we study the magnitude of the so-called indirect bias term in the observation equation of the geoid height derived by means of the GBVP approach.

2 The GBVP Approach

It is assumed that the whole Earth (including the oceans) is covered by J non-overlapping vertical datum zones Ω^j, $j = 0, \ldots, J$. At least one point P in each Ω^j should be given with its GNSS ellipsoidal height h_P and its orthometric height H^j_P. One arbitrary vertical datum zone Ω^o defined by the geopotential value W_o can be chosen as a (global) reference equipotential surface with respect to which the geopotential differences $\delta W^j_o = W_o - W^j_o$ and the offsets $\delta N^j = \delta W^j_o/\gamma$ can be computed, where W^j_o defines the reference potential of the datum zone Ω^j and γ is the normal gravity on the ellipsoid.

The solution to the GBVP at point P in terms of the anomalous potential T is

$$T_P = \frac{\delta GM}{R} + \frac{R}{4\pi}\iint_{\Omega} St\left(\psi_{PQ}\right)\left\{\Delta g^j_Q + 2\frac{\delta W^j_o}{R}\right\} d\Omega_Q, \tag{1}$$

where $\Delta g = \Delta g^j + 2\delta W^j_o/R$ is the gravity anomaly in the global datum, computed from the gravity anomaly

$$\Delta g^j = g_P - \frac{\partial g}{\partial h} H^j - \gamma_{P_o} \tag{2}$$

in the local datum zone Ω^j; g_P is the measured gravity at the topographic surface, $(\partial g/\partial h)H^j$ is the reduction to the zero height level in Ω^j, and γ_{P_o} is the normal gravity on the ellipsoid at point P_o. The term $2\delta W^j_o/R$ in Eq. (1) is interpreted as the "free-air" reduction used for reducing the gravity anomaly Δg^j from the local zero height level to the reference surface defined by the W_o value. R is the mean Earth's radius, δGM is the difference in the geocentric gravitational constant GM of the geoid and GM^e of the normal ellipsoid, and $St(\psi_{PQ})$ is Stokes's function computed with the spherical distance ψ_{PQ}.

Equation (1) is inserted in Bruns's equation

$$N^j_P = \left(T_P - \Delta W_o + \delta W^j_o\right)/\gamma, \tag{3}$$

and if δW^j_o is assumed to be a constant over the datum zone Ω^j, the geoid height is expressed as

$$\begin{aligned} N^j_P = {} & \left(\frac{\delta GM}{R\gamma} - \frac{\Delta W_o}{\gamma}\right) + \frac{\delta W^j_o}{\gamma} \\ & + \frac{R}{4\pi\gamma}\iint_{\Omega} St\left(\psi_{PQ}\right)\Delta g^j_Q \, d\Omega_Q \\ & + \sum_{i=1}^{J} \frac{1}{2\pi\gamma}\delta W^i_o \iint_{\Omega^i} St\left(\psi_{PQ}\right) d\Omega^i_Q. \end{aligned} \tag{4}$$

The constant term (Heiskanen and Mortiz 1967)

$$N_o = \delta GM/(R\gamma) - \Delta W_o/\gamma \tag{5}$$

contains $\Delta W_o = W_o - U_o$, computed with the potential of the normal ellipsoid U_o. The third term in Eq. (4) is Stokes's integral with the local gravity anomalies Δg^j as input:

$$N^j_{P_{Stokes}} = \frac{R}{4\pi\gamma}\iint_{\Omega} St\left(\psi_{PQ}\right)\Delta g^j_Q \, d\Omega_Q. \tag{6}$$

The fourth term in Eq. (4) is the so-called indirect bias term. With $S^i_P = (1/2\pi)\iint_{\Omega^i} St\left(\psi_{PQ}\right) d\Omega^i_Q$, the indirect bias term can be written as

$$\begin{aligned} N^{ind}_P &= \frac{1}{\gamma} S^j_P \delta W^j_o + \sum_{i=1, i\neq j}^{J} \frac{1}{\gamma} S^i_P \delta W^i_o \\ &= S^j_P \delta N^j + \sum_{i=1, i\neq j}^{J} S^i_P \delta N^i. \end{aligned} \tag{7}$$

This equation shows that the geoid height in the datum zone Ω^j is affected by the offsets of all datum zones. This excludes the reference datum zone Ω^o, for which the offset is zero by definition. With Eqs. (5)–(7), Eq. (4) can be written as

$$N^j_P = N_o + \delta N^j + N^j_{P_{Stokes}} + N^{ind}_P, \tag{8}$$

where δN^j is the datum offset. The local geoid height in the left hand side of Eq. (8) can be computed also as a difference of the GNSS ellipsoidal height and the orthometric height:

$$N^j_P = h_P - H^j_P \tag{9}$$

and used in the observation equation of the geoid height in the datum offset computational scheme in Fig. 1.

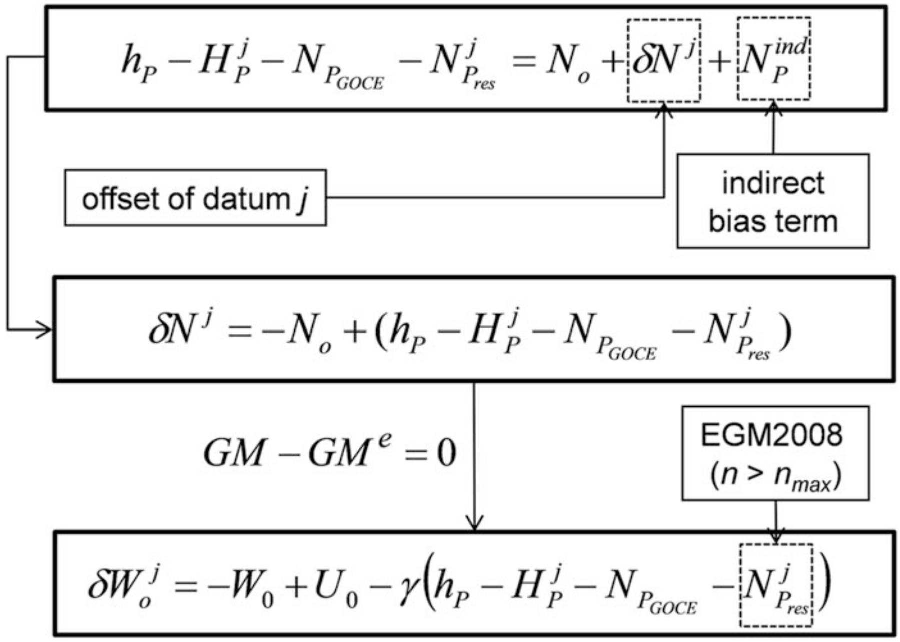

Fig. 1 Computational scheme of the vertical datum offset and geopotential difference

The local gravity anomalies in Eq. (2) used in Stokes's integration are biased because of the bias of the orthometric heights with respect to the reference datum. The use of the GOCE geoid can mitigate the effect of these biases and can simplify the computations in Eq. (8) and Fig. 1. The geoid height can be computed from a GOCE geopotential model and Stokes's integral in Eq. (4) with $\Delta g_{res}^j = \Delta g^j - \Delta g_{GOCE}$ to account for the omission error of the GOCE model. Therefore,

$$N_{P_{Stokes}}^j = N_{P_{GOCE}} + N_{P_{res}}^j. \quad (10)$$

The residual gravity anomalies Δg_{res}^j, which contain the gravity signal with frequencies higher than the maximum degree and order of the GOCE geopotential model $n_{\max}$, integrated by the residual Stokes's kernel (Gerlach and Rummel 2013)

$$St^{res}(\psi) = St(\psi) - St^{n_{\max}}(\psi) \quad (11)$$

provide $N_{P_{res}}^j$. The same kernel must also be used in Eq. (7).

It will be shown in Sect. 4.1 that the indirect bias term is below 1 cm in North America for degree larger than or equal to 70 ($n_{\max} \geq 70$). Therefore, the indirect bias term can be omitted in Eq. (8) for practical computations. In this case, the datum offset δN^j is computed as a (weighted) mean of the differences between the geometric geoid heights in Eq. (9) and the gravimetric geoid heights at the GNSS benchmarks. Figure 1 shows how the geopotential difference δW_o^j is computed. It also shows that the effect of the GOCE model omission error can be assessed by means of the EGM2008 geoid (Pavlis et al. 2012) for $n > n_{\max}$.

3 Data

The Canadian GNSS benchmarks data set consists of 2,579 data points obtained from the Geodetic Survey Division (GSD) of Natural Resources Canada. From this data set, 308 points are extracted that are benchmarks with orthometric heights computed in the Nov07 adjustment using the levelling data after 1981. In addition, CGVD28 normal-orthometric heights and NAVD88 orthometric heights are available for these GNSS benchmarks. The geodetic coordinates are determined in ITRF2005, epoch 2006. The 308 GNSS benchmarks in Fig. 2 represent well the coverage of the Canadian levelling network over the mainland, and they are used for computing the mean offset of the zero height levels of CGVD28, NAVD88 and Nov07. In addition, 95 GNSS tide gauge stations on the Atlantic and Pacific coasts are added to the GNSS benchmarks. The USA data set in Fig. 3 consists of 18,399 GNSS benchmarks with orthometric heights in NAVD88 and geodetic coordinates in ITRF2005, epoch 2006.0.

For height system unification with a centimetre accuracy, it is required that the GNSS ellipsoidal heights, orthometric heights and geoid heights (i) be given in one reference frame

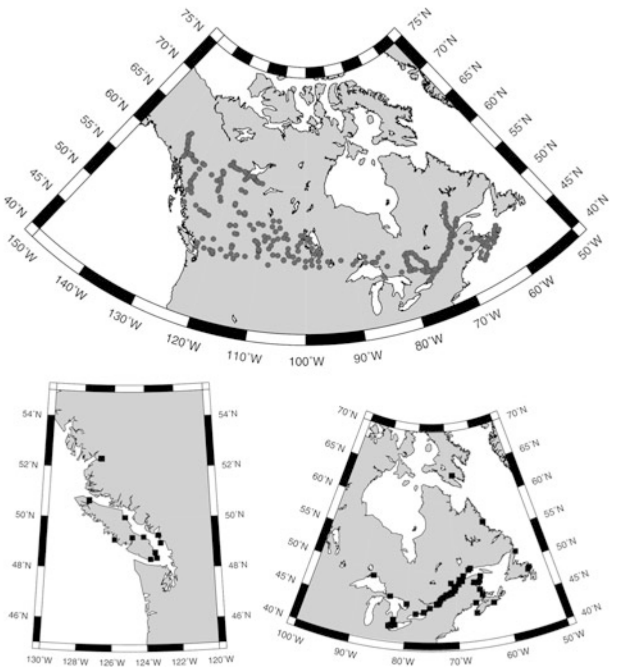

Fig. 2 A subset of 308 Canadian GNSS benchmarks and 95 tide gauges

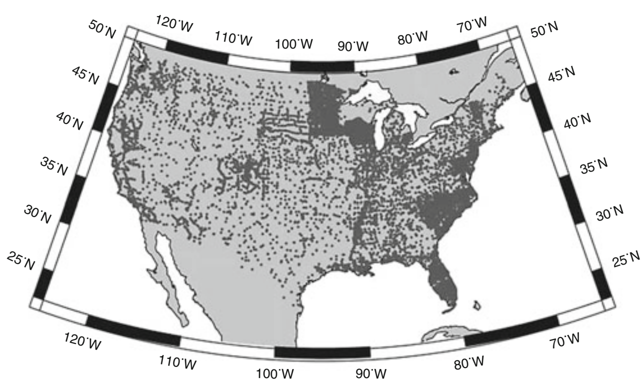

Fig. 3 GNSS benchmarks in the USA

and epoch, (ii) refer to the same reference ellipsoid, (iii) be in the same tidal system, and (iv) refer to the same epoch in areas with significant crustal motion and mass displacement. Our data sets satisfy the first three requirements. The fourth requirement cannot be easily fulfilled because it is not feasible to unify the epoch of the levelling measurements.

4 Analysis of Results

First, we discuss the effect of the indirect bias term in a simulation study for North America using rounded offset estimates of −30 cm for CGVD28 and −50 cm for NAVD88 (Amjadiparvar et al. 2012) with respect to the equipotential surface defined by $W_o = 62{,}636{,}856.00\ \mathrm{m^2/s^2}$.

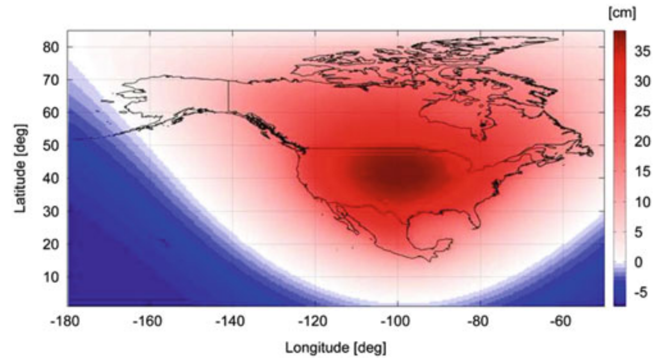

Fig. 4 Indirect bias term computed with the original Stokes's kernel

4.1 Effect of the Indirect Bias Term

The indirect bias term in Eq. (8) was evaluated with residual Stokes's kernels with $n_{\max} = 70$, 120, 150 and 200 related to the spectral resolution of existing satellite-only geopotential models; see Eq. (11). Figure 4 shows the indirect bias term over North America computed with the original Stokes's kernel. The maximum effect of 38 cm is located over the USA. Figure 5 shows the indirect bias term computed with the residual kernels. The maximum effect is less than 1 cm for all $n_{\max}$ values. Therefore, the indirect bias term can be omitted if a global geopotential model with $n_{\max} \geq 70$ is used.

4.2 Mean Datum Offsets

The computation of the mean datum offsets was performed with the GOCE model go_cons_gcf_2_tim_r3 (TIM3, Pail et al. 2011). Table 1 shows that this model has one of the smallest standard deviations of the geoid height differences at the Canadian and USA GNSS benchmarks among the 11 evaluated satellite-only geopotenial models. The difference between the standard deviations for TIM3 and EGM2008 shows the omission error of the GOCE model for its maximum degree and order (D/O) 250: 18.4 cm in Canada and 14.6 cm in the USA.

The mean datum offsets of CGVD28, NAVD88 and Nov07 are shown in Fig. 6. The offsets in Canada (CAN) are computed with the GNSS benchmarks (GNSS/BMs), GNSS tide gauges (GNSS/TGs) and the combined data set GNSS/BMs + GNSS/TGs. Two global geopotential models were used to compute the geoid heights: TIM3 up to D/O 180 and TIM3 expanded by means of EGM2008 from D/O 181 to 2190. The difference in the offsets computed by means of the two models provides an estimate of the effect of the omission error of TIM3 for the particular data point distribution. It can be expected that because the GNSS/BMs sample the

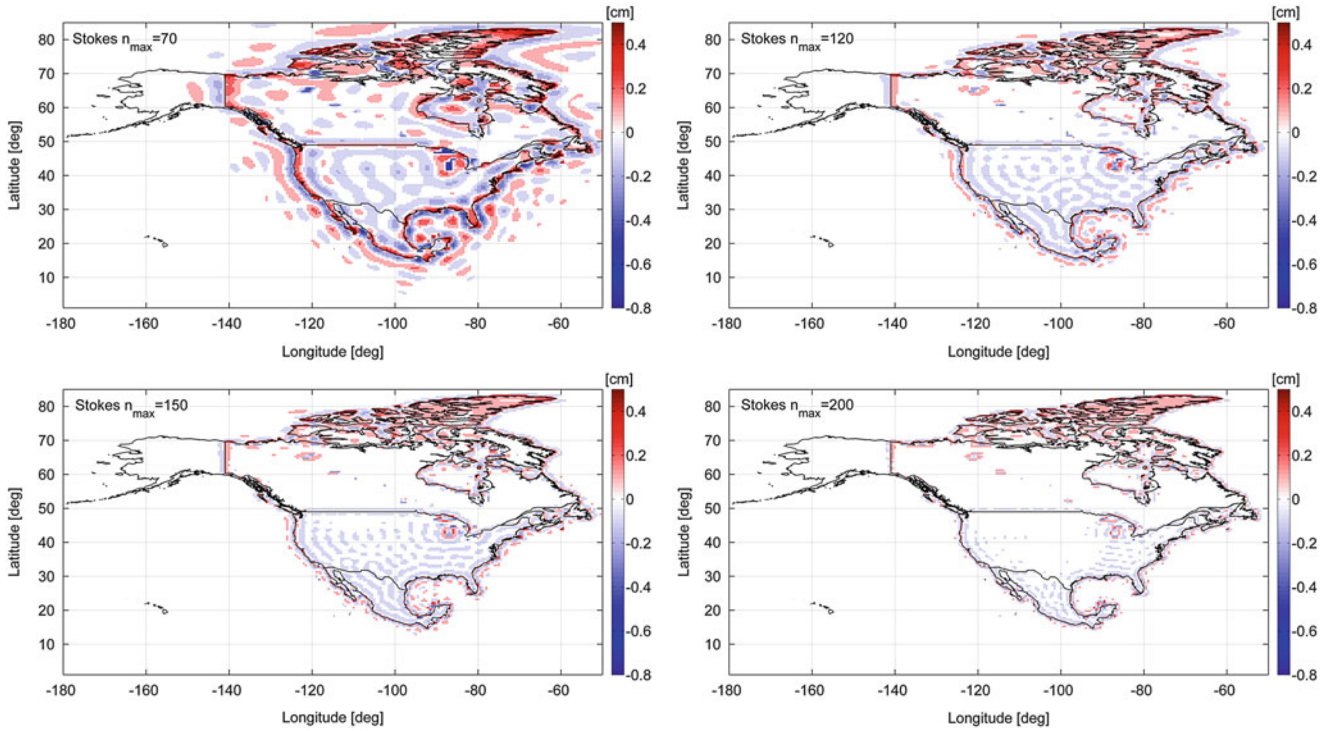

Fig. 5 The indirect bias term in North America computed with the residual Stokes's kernel with $n_{\max} = 70$, 120, 150 and 200

Table 1 Standard deviations of the geoid height differences for 11 satellite-only global geopotential models and EGM2008 evaluated in Canada and the USA; unit is cm

Model	D/O	Canada	USA
go_cons_gcf_2_dir_r3	240	32.6	45.0
go_cons_gcf_2_tim_r3	250	32.8	44.5
go_cons_gcf_2_dir_r2	240	35.1	46.3
go_cons_gcf_2_tim_r2	250	33.9	45.1
go_cons_gcf_2_tim_r1	224	36.1	47.0
eigen-6s	240	35.3	46.6
Goco03s	250	32.4	44.3
Goco02s	250	33.6	44.8
Goco01s	224	35.7	46.5
itg-grace2010s	180	43.9	57.3
eigen-5s	150	49.6	62.5
EGM2008	2190	14.2	30.4

landmasses better than the GNSS/TGs, the omission error will have a smaller effect on the offsets. This can be seen for all three offsets in Fig. 6. The difference of 13 cm between the values computed with TIM3 and TIM3 + EGM2008 using CAN GNSS/BMs is smaller than the corresponding difference of 15–21 cm computed with CAN GNSS/TGs. The USA NAVD88 offset is less affected by the omission error: the difference between the offsets computed with TIM3 and TIM3 + EGM2008 is only 2 cm. This small value is due to the much more regular and dense data coverage of the USA landmasses.

In addition to the GOCE model omission error, the systematic errors in the minimum constrained national levelling networks affect the computed offsets. NAVD88 has a very large 1.5 m tilt from the Atlantic to the Pacific coast of Canada, most likely due to the accumulation of errors in the first-order levelling network. This large tilt results in a large offset of the Canadian NAVD88 of −82 cm. The USA NAVD88 has accumulated errors in east-west and north-south directions (Wang et al. 2012) that compensate each other in the computed offset (e.g., Amjadiparvar et al. 2012). The USA NAVD88 offset is −48 cm. Nov07 is a minimum constrained datum but has a much smaller tilt because of the care taken by GSD to minimize the propagation of the levelling errors in the network adjustment. Nov07 is also defined by the mean sea level at the same fundamental tide gauge station at Rimouski, Québec, Canada. Although NAVD88 and Nov07 are defined by the same unique datum station, the computed mean offsets may not agree because NAVD88 and Nov07 are realized through different networks with different systematic levelling errors. The computed Nov07 offset is −46 cm.

The mean offset of the over-constrained CGVD28 (defined by the mean sea level at five tide gauges on the Atlantic and Pacific coasts) is affected by the regional distortions due to the piece-wise densification of the first-order lev-

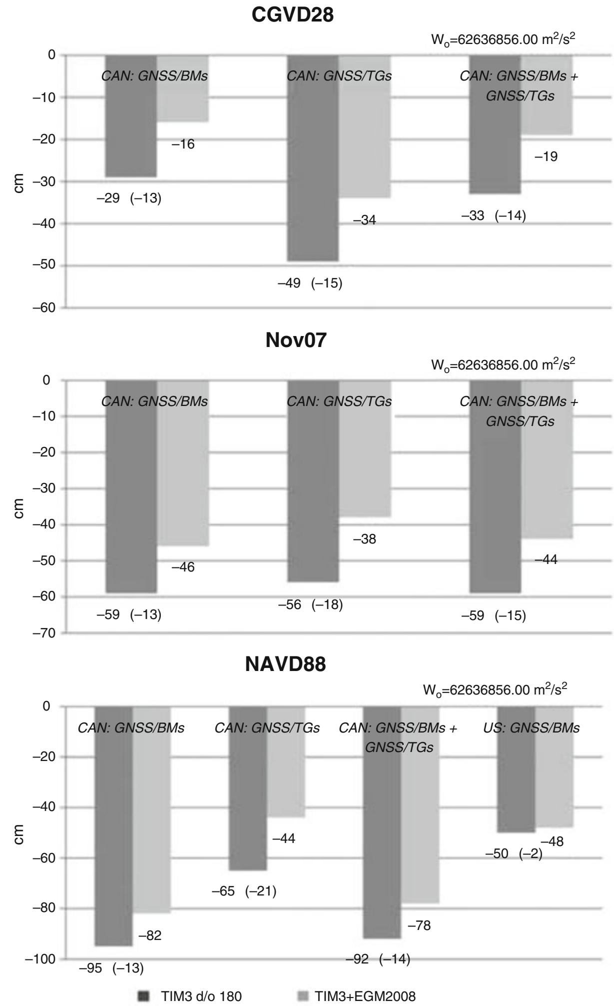

Fig. 6 Mean offsets of CGVD28, NAVD88 and Nov07 with respect to the level surface $W_o = 62{,}636{,}856.00\ \mathrm{m^2/s^2}$, in cm

elling network over the years. These unknown distortions are sampled to a certain extent by the GNSS/BMs, but not by the GNSS/TGs. On the other hand, it has been shown that the mean sea level sampled by the Canadian tide gauges is approximately 20 cm above the reference surface of $W_o = 62{,}636{,}856.00\ \mathrm{m^2/s^2}$ for the Pacific coast and approximately 40 cm below this surface for the Atlantic coast (Hayden et al. 2013) due to the dynamic ocean topography not taken into account in CGVD28. These differences will be reflected in the mean CGVD28 offset of −34 cm computed by means of the GNSS/TGs (see Fig. 6).

Finally, the effect of the commission error of the GOCE model of D/O 180 on the computed mean offsets is studied. This error together with the standard deviations of the GNSS ellipsoidal heights and orthometric heights comprise the stochastic information for the computation of the Nov07 offset. The inclusion of this stochastic information results in a 4 cm difference in the computed offset.

Conclusions

In this study, we presented the first results of the unification of the height datums in North America using a third generation GOCE geopotential model. Our investigations show that in North America the third generation GOCE models can be used up to degree and order 180. For this resolution, the model's performance is similar to EGM2008. The model's omission error, however, is significant. In a follow-on study, we will model the residual geoid signal using local gravity and topography information with a focus on the GNSS tide gauge stations. We will use these high-resolution geoid heights to study the regional biases of CGVD28 along the coasts with respect to the $W_o = 62{,}636{,}856.00\ \mathrm{m^2/s^2}$ level surface.

Furthermore, we have shown that the indirect bias term can be omitted if a GOCE model of degree and order 180 is used in the datum offset computations. This is because the use of a residual Stokes's kernel, which is employed since a global geopotential model provides the long wavelengths of the gravity anomalies and the geoid, diminishes the significance of the biased local gravity anomalies in the datum offset computations. As a consequence, the computational procedure is simplified enormously.

Acknowledgements This work is a contribution to the project STSE—GOCE+ Height System Unification with GOCE, funded by the European Space Agency. The Geodetic Survey Division of Natural Resources Canada, and the National Geodetic Survey, NOAA, USA, are acknowledged for the provided data sets and discussions. The satellite models are obtained from the International Centre for Global Earth Models http://icgem.gfz-potsdam.de/ICGEM/.

References

Amjadiparvar B, Rangelova EV, Sideris MG, Véronneau M (2012) North American height datums and their offsets: the effect of GOCE omission errors and systematic leveling effects. J Appl Geod 7(1):39–50. doi:10.1515/jag-2012-0034

Cannon JB (1929) Adjustment of the Precise Level Net of Canada 1928. Publication No. 28, Geodetic Survey Division, Earth Sciences Sector, Natural Resources Canada, Ottawa, Canada

European Space Agency, ESA (1999) Gravity field and steady-state ocean circulation mission. Report for mission selection of the four candidates Earth Explorer missions, ESA SP-1233(1)

Gerlach C, Rummel R (2013) Global height system unification with GOCE: a simulation study on the indirect bias term in the GBVP approach. J Geod 87:57–67. doi:10.1007/s00190-012-0579-y

Hayden T, Rangelova E, Sideris MG, Véronneau M (2014) Contribution of tide gauges for the determination of W_o in Canada, In: U. Marti (ed.), Gravity, Geoid and Height Systems, International Association of Geodesy Symposia 141, Springer International Publishing Switzerland 2014

Heiskanen WA, Mortiz H (1967) Physical geodesy. WH Freeman, San Francisco, CA. Reprint, Technical University, Graz, Austria, 1999

National Geodetic Survey, NGS (2008) The National Geodetic Survey ten-year plan: mission, vision, and strategy 2008–2018. Silver Spring, MD, U.S. Dept. of Commerce, National Oceanic and Atmospheric Administration, National Ocean Service

Pail R, Bruinsma S, Migliaccio F, Foerste C, Goiginger H, Schuh WD, Hoeck E, Reguzzoni M, Brockmann JM, Abrikosov O, Veicherts M, Fecher T, Mayrhofer R, Krasbutter I, Sanso F, Tscherning CC (2011) First GOCE gravity field models derived by three different approaches. J Geod 85:819–843

Pavlis NK, Holmes SA, Kenyon SC, Factor JK (2012) The development and evaluation of the Earth Gravitational Model 2008 (EGM2008). J Geophys Res 117, B04406. doi:10.1029/2011JB008916

Roman D, Weston ND (2012) Beyond GEOID12: implementing a new vertical datum for North America. FIG Proceedings 2012, Rome, Italy, 6–10 May 2012. http://www.fig.net/pub/fig2012/papers/ts04b/TS04B_weston_5691.pdf

Rummel R, Teunissen P (1988) Height datum definition, height datum connection and the role of the geodetic boundary value problem. Bulletin Géodésique 62:477–498

Véronneau M, Duval R, Huang J (2006) A gravimetric geoid model as a vertical datum for Canada. Geomatica 60(2):165–172

Wang YM, Saleh J, Li X, Roman DR (2012) The US Gravimetric Geoid of 2009 (USGG2009): model development and evaluation. J Geod 86:165–180. doi:10.1007/s00190-011-0506-7

Zilkoski D, Richards J, Young G (1992) Results of the general adjustment of the North American vertical datum of 1988. Surv Land Inform Syst 52(3):133–149