
HRG2009: New High Resolution Geoid Model for Croatia

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Abstract

Using an updated Earth's gravity field data set widely available and applying the least square collocation calculation technique, a new Croatian geoid model HRG2009 was calculated. Significant accuracy improvements have been achieved regarding the previous national geoid model HRG2000; four times better resolution and 69% better absolute accuracy of the model. The following data sets were used: great number of new measured terrestrial gravity data and data obtained from satellite altimetry for the area of the Adriatic Sea, along with geoid undulations at discrete points measured by GNSS/leveling on almost 500 stations all over the mainland, furthermore long and mediumwave field structures taken from the latest detailed global geopotential model EGM2008 and high frequencies field structures modeled with the help of $3'' \times 3''$ Shuttle Radar DEM's. Absolute accuracy assessment made on 59 GNSS/leveling control stations, that were not used in calculations, have resulted with 69% improvement of new national geoid model regarding the previous one, with standard deviation of 3.5 cm on overall computation area. It is significantly more reliable surface as compared to the earlier HRG2000 solution with standard deviation of 11.4 cm.

The new geoid surface has been used for different purposes, primary in the precise height definition using modern GNSS technology. Therefore, the Croatian Positioning System CROPOS was upgraded in 2011 with the new service which enables the real time transformation of ellipsoidal heights to the (normal) orthometric heights using HRG2009 geoid GRID and Trimble Transformation Generator software for more than 440 surveying and geoinformation companies in Croatia.

Keywords

External accuracy • Least square collocation • New national geoid

1 Introduction

Several significant studies (Denker et al. 2009; Bašić and Hećimović 2006; Grgić et al. 2010) have preceded the computation of the new national geoid HRG2009.

Firstly, the analysis (Liker et al. 2010) of recent global geopotential models based on CHAMP (Prange et al. 2010), GRACE (Mayer-Gürr et al. 2010) and GOCE (Pail et al. 2011) mission as well as testing of EGM2008 model (Denker et al. 2008) have been conducted. Secondly, great number of additional gravity data for the area of interest was gathered. Furthermore, a $3'' \times 3''$ DTM model from SRTM data for the calculation of the Earth's gravity field topographic effects was developed (Bašić and Buble 2007). Regarding terrestrial data, it was of great significance that the Basic Gravimetric Network (Barišić et al. 2008) was established that was used together with EUVN and EUVN_DA data (Grgić et al.

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2009) for independent control of the HRG2009. And finally, the analysis of new and previous national height datum was made, as well as the establishment of more than 500 GNSS/levelling points regularly distributed over the Croatian territory for the purpose of geoid orientation and independent control of previous HRG2000 geoid model.

The determination of the local high-resolution gravity field is divided in three parts regarding the different wavelengths of the observed gravity field structures. The long-wavelength part is derived from global geopotential model (EGM2008 in this case), the medium-wavelength part originates in terrestrial point gravity field observations, and the short-wavelength part is taken from the high-resolution digital terrain model. In the simple remove-restore technique, the reduced observations are thus written as linear functional of the anomalous gravity potential (Bašić 1989):

$$x_i = L_i(T) - L_i(T_{EGM}) - L_i(T_{RTM}) + n_i \quad (1)$$

The least square collocation (Moritz 1980) results in predictions $L_j(\tilde{T}')$. To obtain the desired results, the effect of the anomalous masses and the effect of the geopotential model needs to be added later on in the “restore” procedure:

$$L_j(\tilde{T}) = L_j(\tilde{T}') + L_j(T_{EGM}) + L_j(T_{RTM}) \quad (2)$$

We decided to use the least square collocation because it has shown to be most suitable with regard to the relatively small area of the territory of Croatia, so that large and extensive numerical operations could be carried out more easily in just one-step due to their flexibility in handling irregularly distributed heterogeneous data. In addition, we preferred to have the error estimates of predicted quantities. The calculation area covers the entire territory of Croatia spreading between the area 42–46.6° in latitude, and 13.0–19.5° in longitude.

The calculation was carried out in a regular grid of 30'' × 45'' resolution (approximately 1 × 1 km), which is four times better resolution than the one of the previous national geoid HRG2000 (Bašić et al. 1999). Consequently, the number of calculating residual points increased from 72297 with HRG2000 to 288113 with HRG2009 solution.

2 Analysis

In Table 1, the main statistical indicators of the original gravity anomalies, EGM2008 and RTM effects as well as residual field are presented.

In performing the residual terrain modelling, the following grids have been used: (a) the DTM of 4'' × 5'' (approximately 120 × 110 m), obtained from 3'' × 3'' SRTM data, covering the area from 40° to 48° in latitude, and 10°

Table 1 Statistical indicators of gravity anomalies in the *remove* procedure, in [mGal]

N = 29330	Δg_{GRS80}	$\Delta g_{EGM2008}$	Δg_{RTM}	Δg_{RES}
Mean	11.58	15.45	-4.14	0.276
Standard deviation	29.20	28.55	13.18	5.491
Min	-130.71	-102.79	-142.69	-14.994
Max	166.47	163.12	62.58	14.996

to 22° in longitude, (b) the coarse 1' × 1' grid of relief height covering the bigger area from 36° to 52° in latitude and from 5° to 27° in longitude, and (c) 5' × 5' RTM reference grid of the same area as the coarse one.

The effect of the applied “remove” procedure is evident in decreasing standard deviation, which drops from 29.20 mGal for the observed anomalies to 5.49 mGal for the residuals ($\Delta g_{GRS80} - \Delta g_{EGM2008} - \Delta g_{RTM}$). A significant reduction of the mean value from 11.58 mGal to 0.28 mGal (good centered data) can also be recognized ($1 \text{ mGal} = 10^{-5} \text{ ms}^{-2}$).

A-priori information about the variation of the local gravity field is introduced through the empirical covariance function calculated using 29330 residual gravity anomalies (Fig. 1, right). In this study the variance of the empirical covariance function has the value of only 30.03 mgal² and the first zero-value occurs already at 9 km distance (covariance graph on Fig. 1 right).

For the purpose of correcting the absolute orientation of the calculated geoid surface, a significant number (495) of GNSS/levelling points distributed across Croatia has been used. The statistical indicators are presented in Table 2, where an apparent residual bias effect is present again, but it should be noted that the value of the mean $N_{RES} = -1.024 \text{ m}$, most likely originates from the discrepancy between the used EGM2008 model and the definition of national vertical datum that relates to five tide gauges.

3 Quality Assessment

The assessment of the HRG2009 quality was done in two ways. Firstly, its internal accuracy was evaluated through the analysis conducted on 495 points, on which measured GNSS/levelling undulations were compared with calculated undulations from HRG2009 geoid model. Internal accuracy analysis has shown a remarkably high mutual compatibility, giving the standard deviation of 2.7 cm and the mean difference almost zero (Table 3). These internal control indicators obtained with GNSS/levelling undulations used for computation of new geoid surface, verified well-chosen methodology and conducted computation, implicating on high reliability of the new geoid solution of 2–3 cm over most of the Croatian mainland (Fig. 2, left).

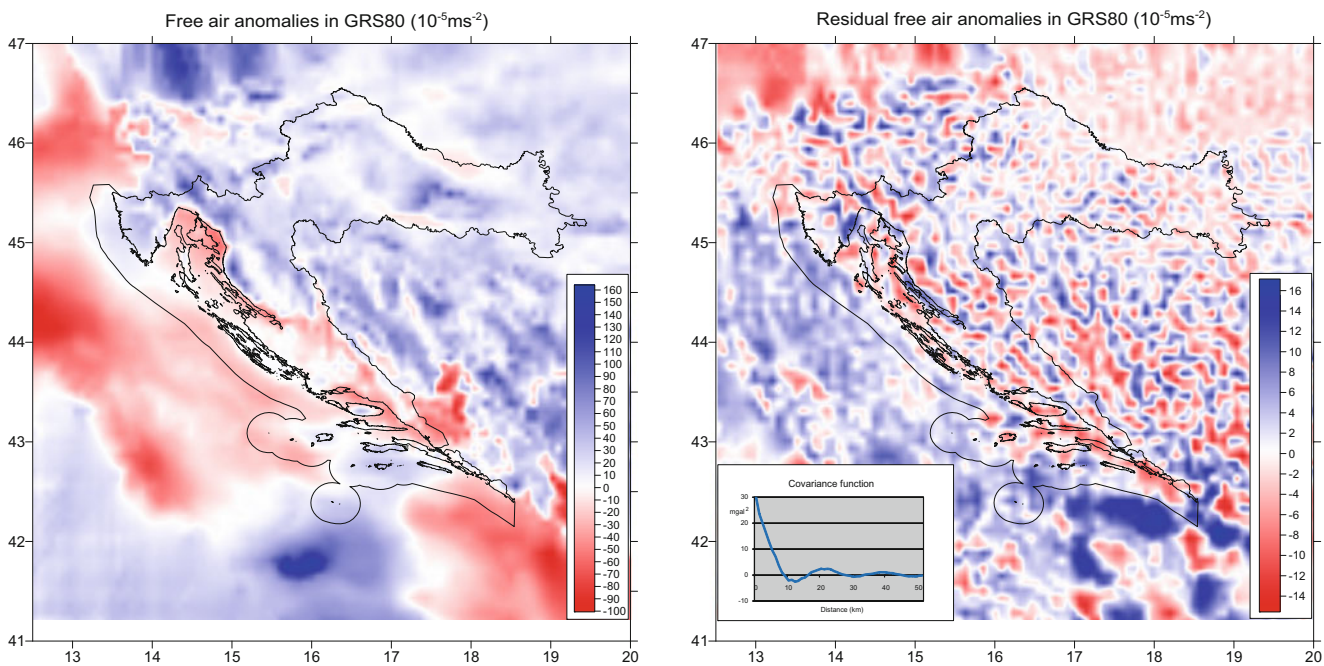


Fig. 1 Gravity anomaly field obtained in *remove* procedure: (left) free air anomalies (right) residual free air anomalies

Table 2 Indicators of the geoid reduction effect in 495 GNSS/leveling points, in [m]

	N_GNSS/LEV.	N_EGM2008	N_RTM	N_RES
Mean	44.548	45.438	0.134	-1.024
Standard deviation	1.100	1.081	0.030	0.062
Min	40.414	41.429	0.089	-1.271
Max	46.666	45.517	0.247	-0.899

Beside the internal accuracy, we made an external independent quality assessment on 59 stations that were not used in the computation procedure (Bašić 2009). Comparison was made between 59 GNSS/leveling undulations measured for external geoid control and calculated HRG2009 undulations for these stations. This comparison confirmed that there was an enviable good absolute accuracy of the new national geoid, especially considering the obtained standard deviation of 3.5 cm (Table 3) and the mean difference of almost zero, absolutely confirming high reliability of this solution (Fig. 2, right).

The improvement of the HRG2009 with respect to the previous HRG2000 geoid model is analysed on 59 control stations which were neither used in HRG2000 computations so unambiguous comparison could be made. External control conducted on 59 GNSS/levelling points gave the standard deviation of 11 cm for HRG2000 solution while for new geoid solution HRG2009 it amounts 3.5 cm. Therefore, the improvement of the new national geoid HRG2009 with respect to the previous geoid model resulted with 69 % better compatibility to the Earth’s surface.

Table 3 Internal (495 points) and external (59 points) accuracy indicators, in meters

	Min	Max	Average	Standard deviation
Internal control				
HRG2009–495 GNSS/lev.	-0.071	0.059	-0.004	0.027
Absolute (external) control				
HRG2009–59 GNSS/lev.	-0.078	0.058	-0.012	0.035
HRG2000–59 GNSS/lev.	-0.275	0.242	-0.024	0.114

New national high-resolution geoid model HRG2009 (Bašić 2009) is shown on Fig. 3. Its average geoid undulation equals 44.151 m with standard deviation of 2.055 m, while minimum and maximum are 36.807 and 50.262 m respectively.

4 HRG2009 Versus EGG2008

The comparison between HRG2009 and EGG2008 (Denker et al. 2008) geoid model has been done for all points of 30'' × 45'' grid, within the solution area for Croatia, meaning 42–46.6°N and 13–19.5°E. The comparison has shown the mean surface difference of 24.1 cm, which is consequent of different height datum surfaces, i.e. Croatian geoid model refers to HVRS71 mean sea level surface,

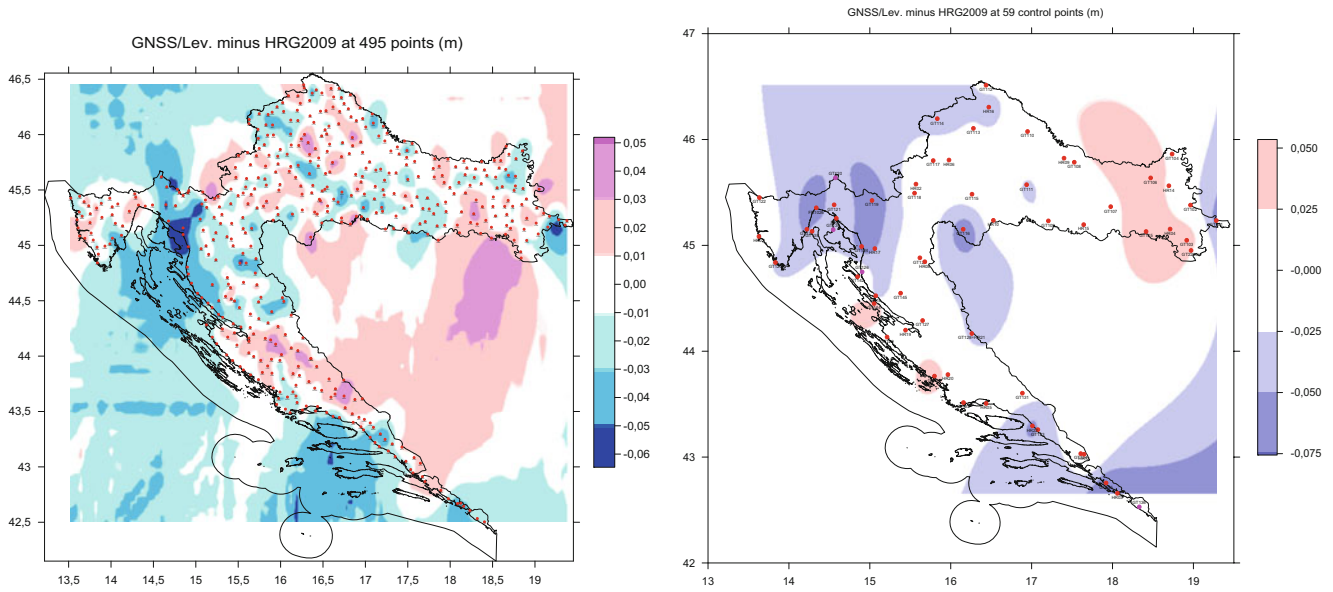


Fig. 2 Quality of the HRG2009 geoid: (left) internal accuracy, (right) absolute accuracy

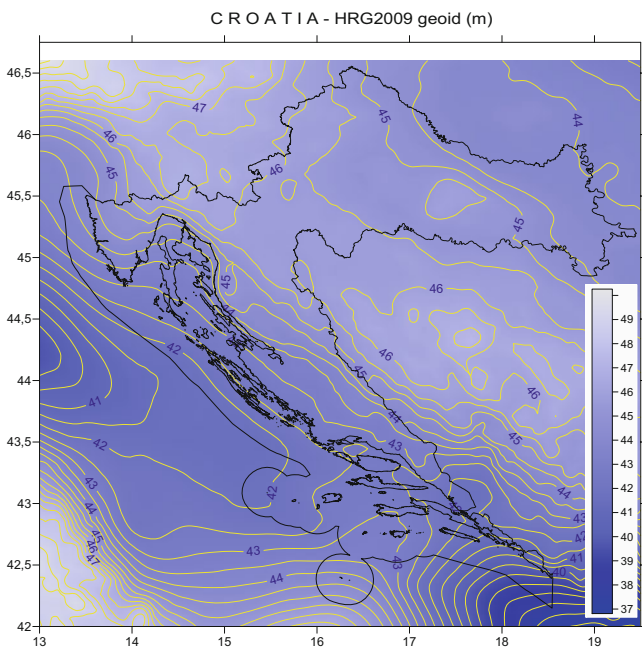


Fig. 3 New high-resolution geoid model HRG2009

while EGG2008 refers to Amsterdam MSL. Furthermore, the standard deviation is 17.0 cm, while the range varies from the minimum of -1.214 m to the maximum of 0.452 m (Denker and Bašić 2011). Greater disagreements can be found outside the Croatian borders, due to different data used, different methods, as well as border effects.

The comparison between EGG2008 and 495 GNSS/lev. points was made and it has shown good mutual matching after removing the mean difference of an 22.8 cm (Table 4).

Table 4 Quasigeoid differences regarding the EGG2008, in [m]

Quasigeoid differences	Min	Max	Average	Standard deviation
GNSS/lev.–EGG2008	-0.175	0.095	-0.228	0.039
HRG2009–EGG2008	-0.155	0.064	-0.223	0.026

After the mean difference of 24.1 cm (height datum difference) had been removed, HRG2009 was compared with EGG2008 over the 495 GNSS/leveled points. The results obtained gave very good indicators for land areas, with the standard deviation of 2.6 cm (Table 4).

There are several areas with differences between 10 and 15 cm, but they are all on the borders with Monte Negro, Italy close to Trieste, and the area of Rijeka and Bakar bay.

Quality assessment of the EGG2008 made on Croatian territory has confirmed good reliability of European quasigeoid on a larger part of territory. Neglecting problematic border areas discussed above, EGG2008 geoid model fits to the continental part within $3\text{--}5$ cm.

5 HRG2009 Implementation in Online GNSS Service

By the Decision on determining the official geodetic datum and map projections of Republic of Croatia (8 August 2004) passed by Croatian State Geodetic Administration, regulations on the transfer from the old to the new geodetic national datum have been determined. Consequently, in collaboration with Faculty of Geodesy unique transformation model T7D based on GRID transformation procedure has

been developed. It consists of 7 parameter transformation and $30'' \times 45''$ regular grid of predicted distortion for height and positional coordinates and it provides both transformation within ± 0.06 m on the national land area.

The implementation of the Decision has resulted further in implementing the new geoid HRG2009 within the T7D model to the online service of national positioning system-CROPOS for real time height transfer.

The online service is in its full operation since January 03, 2011, which allows CROPOS users to select easily CROPOS_VRS_HTRS96 service online on their instruments and obtain orthometric heights in real time in the new official height datum HVRS71, through implemented HRG2009. Also, the same service allows immediate online transformation for horizontal positioning. For that matter, user is provided by ellipsoidal coordinates in new national datum HTRS96 with option to use the new official map projection HTRS96/TM and display them directly on the field.

Prior to release, CROPOS_VRS_HTRS96 service was successfully tested on 604 control points through out the whole Croatian territory.

Conclusion

As shown in this study, a new national high-resolution geoid solution HRG2009 has been calculated, showing an accuracy of few (2–3) cm for the predominant part of Croatia, especially in land areas. At the same time, regional quasigeoid EGG2008 fits very well at GNSS/leveling points over a large part of continental Croatia, varying just 3–5 cm, apart from few problematic areas, mostly on the borders to Monte Negro and Italy, where consistency varies from 10 to 15 cm. Due to these facts, along with the newest GOCE results, there is a clear need for a more precise overlap of the European and Croatian geoid model to be achieved in the forthcoming years. Better fitting new solutions are needed both at regional and local levels. As the new GOCE data are expected to be of even better resolution, the integration of the Croatian geoid model with GOCE data is expected to be further developed and improved.

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