A New Definition of Evaluation/Defuzzification of an Interval Type-2 Fuzzy Set

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Abstract. In this paper we propose a new evaluation/defuzzification formula for an Interval Type-2 Fuzzy Quantity (IT2 FQ), that is an Interval Type-2 Fuzzy Set (IT2 FS) defined by two Type-1 Fuzzy Quantities (T1 FQs) having membership functions that may be neither convex nor normal. We start from a parametric formula to evaluate them and we propose to call the IT2 FQ value their average. To compare the results we obtain changing the parameters, we use the final output of an example of Interval Type-2 Fuzzy Logic System (IT2 FLS).

Keywords: Fuzzy sets, fuzzy quantities, interval type-2 fuzzy sets, evaluation.

1 Introduction

Type-2 fuzzy sets and systems generalize type-1 fuzzy sets and systems so that more uncertainty can be handled. When fuzzy sets enter in scientific world, one of critics is due to the fact that the membership function of a Type-1 Fuzzy Set (T1 FS) has no uncertainty associated with it. This fact seems to contradict the word "fuzzy". In 1975 Prof. Lotfi A. Zadeh [20] proposed more sophisticated kinds of fuzzy sets, he called Type-2 Fuzzy Sets (T2 FSs). A T2 FS lets us incorporate uncertainty about the membership function into fuzzy set theory, and is a way to address the above criticism of T1 FS heads-on. The membership function of a T2 FS is three-dimensional, where the third dimension is the value of the membership function at each point on its two-dimensional domain which is called its footprint of uncertainty (FOU). Interval Type-2 Fuzzy Sets (IT2 FSs) are particular T2 FSs in which third dimension value is constant (e.g., 1). This means that no new information is contained in the third dimension of an IT2 FS and only the FOU is used to describe it. An IT2 FS is completely described by two T1 FSs whose membership functions are the lower and upper bounds of its FOU.

After the wide number of applications of Type-1 Fuzzy Logic Systems (T1 FLSs), even the Interval Type-2 Fuzzy Logic Systems (IT2 FLSs) started and

P. Grzegorzewski et al. (eds.), *Strengthening Links between Data Analysis & Soft Computing*,
Advances in Intelligent Systems and Computing 315, DOI: 10.1007/978-3-319-10765-3_5

found a lot of interesting and successful applications in signal processing, fingerprints detection and in Computing With Words fields. The researches on IT2 FLSs had a wide impulse by Prof. Jerry Mendel and others researchers works [12,13,14,15]. The final output of an IT2 FLS is an IT2 FS and thus one needs methods for the evaluation/defuzzification of an IT2 FS. Karnik and Mendel [11] proposed a defuzzification method based on an algorithm that evaluates an IT2 FS taking the average of the centroids of T1 FSs embedded in the FOU zone.

This paper goes in the same direction and proposes a parametric evaluation/defuzzification formula for an Interval Type-2 Fuzzy Quantity (IT2 FQ), that is an IT2 FS defined by two Type-1 Fuzzy Quantities (T1 FQs) whose membership functions may be neither convex nor normal. We start from a parametric formula for the evaluation of the two T1 FQs and we propose to call the IT2 FQ value their average. This approach allows us, by changing the set of parameters, to recover the T1 FQs evaluations proposed by Fortemps and Roubens [8,3], Yager and Filev [18,19], Anzilli and Facchinetti [3] and Center of Gravity (COG). To illustrate how our method works, we apply it to the final output of an example of IT2 FLS and compare the numerical results we obtain changing the set of parameters. In Section 2 and Section 3 we introduce the concepts of IT2 FS and IT2 FQ. In Section 4 we give an example of IT2 FLS and in section 5 we present the evaluation model for an IT2 FQ and apply it to the defuzzification of the final output of the IT2 FLS.

2 Interval Type-2 Fuzzy Sets

We give a short presentation of T2 FSs and IT2 FSs (for detail see [15]).

Definition 1. *A T2 FS $\tilde{A}$ in the universe of discourse X is characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u)$ where $x \in X$ and $u \in J_x \subseteq [0,1]$, i.e.*

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u))\,;\ \forall x \in X\,, \forall u \in J_x \subseteq [0,1]\}$$

in which $0 \le \mu_{\tilde{A}}(x,u) \le 1$. J_x is a closed interval of real numbers. A T2 FS $\tilde{A}$ can also be represented as $\tilde{A} = \int_{x\in X}\int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u)$.

Definition 2. *If all $\mu_{\tilde{A}}(x,u) = 1$ then $\tilde{A}$ is called an IT2 FS.*

An IT2 FS $\tilde{A}$ can be considered as a special case of a T2 FS and it can be expressed as $\tilde{A} = \int_{x\in X}\int_{u\in J_x} 1/(x,u)$. J_x is called the primary membership of x.

The footprint of uncertainty (FOU) of an IT2 FS $\tilde{A}$ is defined by $\mathrm{FOU}(\tilde{A}) = \bigcup_{x\in X} J_x$. The FOU is a complete description of an IT2 FS. The upper membership function $\mu^U_{\tilde{A}}$ and the lower membership function $\mu^L_{\tilde{A}}$ of an IT2 FS $\tilde{A}$ are defined as the two type-1 membership functions that bound the FOU. Thus $J_x = [\mu^L_{\tilde{A}}(x), \mu^U_{\tilde{A}}(x)]$ for all $x \in X$. In the following an IT2 FS $\tilde{A}$ will be denoted by $\tilde{A} = (A^L, A^U)$, where A^L and A^U are the T1 FSs with membership functions $\mu_{A^L} = \mu^L_{\tilde{A}}$ and $\mu_{A^U} = \mu^U_{\tilde{A}}$, respectively.

The intersection and the union of two IT2 FSs $\tilde{A}$, $\tilde{B}$ are defined as the IT2 FSs given by

$$\tilde{A} \sqcap \tilde{B} = \int_{x \in X} \int_{u \in [\mu^L_{\tilde{A} \sqcap \tilde{B}}(x), \mu^U_{\tilde{A} \sqcap \tilde{B}}(x)]} 1/(x,u)$$

$$\tilde{A} \sqcup \tilde{B} = \int_{x \in X} \int_{u \in [\mu^L_{\tilde{A} \sqcup \tilde{B}}(x), \mu^U_{\tilde{A} \sqcup \tilde{B}}(x)]} 1/(x,u)$$

with $\mu^L_{\tilde{A}\sqcap\tilde{B}}(x) = T(\mu^L_{\tilde{A}}(x), \mu^L_{\tilde{B}}(x))$, $\mu^U_{\tilde{A}\sqcap\tilde{B}}(x) = T(\mu^U_{\tilde{A}}(x), \mu^U_{\tilde{B}}(x))$, $\mu^L_{\tilde{A}\sqcup\tilde{B}}(x) = S(\mu^L_{\tilde{A}}(x), \mu^L_{\tilde{B}}(x))$ and $\mu^U_{\tilde{A}\sqcup\tilde{B}}(x) = S\mu^U_{\tilde{A}}(x), \mu^U_{\tilde{B}}(x))$, where T is the t-norm operator and S is the t-conorm operator.

3 Interval Type-2 Fuzzy Quantities

We now introduce the concept of T1 FQ (see [3,4]) and the definition of IT2 FQ.

Definition 3. *Let N be a positive integer and let $a_1, a_2, \ldots, a_{4N}$ be real numbers with $a_1 < a_2 \leq a_3 < a_4 \leq a_5 < a_6 \leq a_7 < a_8 \leq a_9 < \cdots < a_{4N-2} \leq a_{4N-1} < a_{4N}$. We call type-1 fuzzy quantity*

$$A = (a_1, a_2, \ldots, a_{4N};\ h_1, h_2, \ldots, h_N,\ h_{1,2}, h_{2,3}, \ldots, h_{N-1,N}) \tag{1}$$

where $0 < h_j \leq 1$ for $j = 1, \ldots, N$ and $0 \leq h_{j,j+1} < \min\{h_j, h_{j+1}\}$ for $j = 1, \ldots, N-1$, the fuzzy set defined by a continuous membership function $\mu : \mathbb{R} \to [0,1]$, with $\mu(x) = 0$ for $x \leq a_1$ or $x \geq a_{4N}$, such that for $j = 1, 2, \ldots, N$

(i) μ is strictly increasing in $[a_{4j-3}, a_{4j-2}]$, with $\mu(a_{4j-3}) = h_{j-1,j}$ and $\mu(a_{4j-2}) = h_j$,
(ii) μ is constant in $[a_{4j-2}, a_{4j-1}]$, with $\mu \equiv h_j$,
(iii) μ is strictly decreasing in $[a_{4j-1}, a_{4j}]$, with $\mu(a_{4j-1}) = h_j$ and $\mu(a_{4j}) = h_{j,j+1}$,

and for $j = 1, 2, \ldots, N-1$

(iv) μ is constant in $[a_{4j}, a_{4j+1}]$, with $\mu \equiv h_{j,j+1}$,

where $h_{0,1} = h_{N,N+1} = 0$. Thus the height of A is $h_A = \max_{j=1,\ldots,N} h_j$.

We observe that in the case $N = 1$ the T1 FQ defined in (1) is fuzzy convex, that is every α-cut A_α is a closed interval. If $N \geq 2$ the T1 FQ defined in (1) is a non-convex fuzzy set with N humps and height $h_A = \max_{j=1,\ldots,N} h_j$.

Definition 4. *We call Interval Type-2 Fuzzy Quantity (IT2 FQ) an IT2 FS $\tilde{A}$ such that $\mu^L_{\tilde{A}}$ and $\mu^U_{\tilde{A}}$ are membership functions of T1 FQs.*

If $\tilde{A}$ is an IT2 FQ we denote by A^L the T1 FQ with membership function $\mu_{A^L} = \mu^L_{\tilde{A}}$ and by A^U the T1 FQ with membership function $\mu_{A^U} = \mu^U_{\tilde{A}}$ (see Figure 2). In the following an IT2 FQ $\tilde{A}$ will be denoted by $\tilde{A} = (A^L, A^U)$.

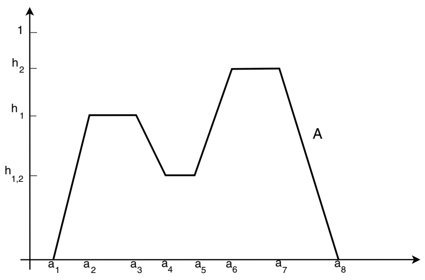


Fig. 1. Piecewise linear T1 FQ ($N = 2$)

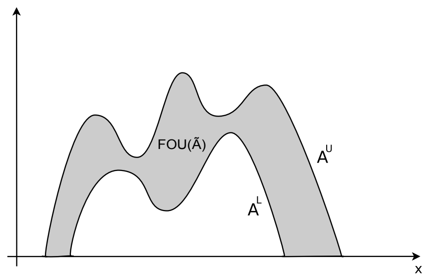


Fig. 2. IT2 FQ $\tilde{A} = (A^L, A^U)$

4 An Example of Interval Type-2 Fuzzy Logic Systems

Suppose we have an Interval Type-2 Fuzzy Logic Systems (IT2 FLS) with p inputs, $x_1, \dots, x_p$ and one output y. Consider its rule-block characterized by M rules where the m-th rule has the form

$$R_m : \quad \text{IF } x_1 \text{ is } \tilde{F}_{1m} \text{ and } \dots \text{ and } x_p \text{ is } \tilde{F}_{pm} \text{ THEN } y \text{ is } \tilde{G}_m \qquad m = 1, \dots, M$$

where $\tilde{F}_{im}$, $\tilde{G}_m$ are IT2 FSs. Note that $\tilde{F}_{im}$ is the linguistic label associated with i-th antecedent in the m-th rule and $\tilde{G}_m$ is the linguistic label associated with the output variable in the m-th rule. Let us define $\tilde{F}_m = \prod_{i=1}^{p} \tilde{F}_{im}$, $m = 1, \dots, M$. The output $\tilde{G}_m^*$ of each rule is the IT2 FS given by $\tilde{G}_m^* = \tilde{F}_m' \circ (\tilde{F}_m \to \tilde{G}_m)$, where $\circ$ is the sup-star composition operator. The final output $\tilde{G}^*$ is the IT2 FS obtained as $\tilde{G}^* = \bigsqcup_{m=1}^{M} \tilde{G}_m^*$.

We consider a singleton IT2 FLS, that is a IT2 FLS with crisp input $x' = (x_1', \dots, x_p')$. We assume that Mamdani implications are used, $T = \min$, $S = \max$. For each rule $m = 1, \dots, M$ we compute the firing interval $[\mu_{\tilde{F}_m}^L(x'), \mu_{\tilde{F}_m}^U(x')]$ as

$$\mu_{\tilde{F}_m}^L(x') = \min_{i=1,\dots,p} \mu_{\tilde{F}_{im}}^L(x_i') , \qquad \mu_{\tilde{F}_m}^U(x') = \min_{i=1,\dots,p} \mu_{\tilde{F}_{im}}^U(x_i') .$$

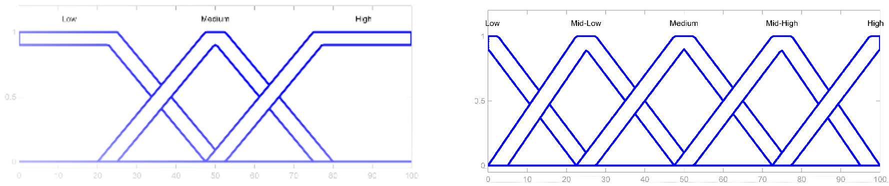
For $m = 1, \dots, M$ the output IT2 FS of rule m, $\tilde{G}_m^* = (\tilde{G}_m^{*\,L}, \tilde{G}_m^{*\,U})$, is calculated as

$$\mu_{\tilde{G}_m^*}^L(y) = \min\left\{\mu_{\tilde{F}_m}^L(x'), \mu_{\tilde{G}_m}^L(y)\right\} , \qquad \mu_{\tilde{G}_m^*}^U(y) = \min\left\{\mu_{\tilde{F}_m}^U(x'), \mu_{\tilde{G}_m}^U(y)\right\} .$$

The final output IT2 FS $\tilde{G}^* = (\tilde{G}^{*L}, \tilde{G}^{*U})$ is obtained as $\tilde{G}^* = \bigsqcup_{m=1}^{M} \tilde{G}_m^*$, that is

$$\mu_{\tilde{G}^*}^L(y) = \max_{m=1,\dots M} \mu_{\tilde{G}_m^*}^L(y) , \qquad \mu_{\tilde{G}^*}^U(y) = \max_{m=1,\dots M} \mu_{\tilde{G}_m^*}^U(y) .$$

To show our defuzzification method we consider a very simple example of IT2 FLS with two inputs and one output. The example is the type-2 translation of a client financial risk tolerance model illustrated in [5, p.130], with a little difference on output granularity. “Financial service institutions face a difficult task in evaluating clients risk tolerance. It is a major component for the design of an investment policy and understanding the implication of possible investment options in terms of safety and suitability. Here we present a simple model of client's risk tolerance ability (RT), which depends on his/hers annual income (AI) and total net worth (TNW)". Suppose the financial experts agree to describe the input variables AI and TN by the linguistic terms {L (Low), M (Medium), H (High)} and the output variable RT by the linguistic terms {L (Low), LM (Low-Medium), M (Medium), MH (Medium-High), H (High)}. Each granule is an IT2 FS in which the domains are : $U_1 = \{x \times 10^3\,; 0 \leq x \leq 100\}$ for input AI, $U_2 = \{y \times 10^4\,; 0 \leq y \leq 100\}$ for input TN and $U_3 = \{z\,; 0 \leq z \leq 100\}$ for output RT. The real numbers x and y represent euros in thousands and hundred of thousands, correspondingly, while z takes values on a psychometric scale from 0 to 100 measuring risk tolerance. All the granules are described by triangular or trapezoidal IT2 FSs, as shown in Fig. 3.



(a) Input variables AI and TN   (b) Output variable RT

Fig. 3. Input and output variables of IT2 FLS

We assume that the financial experts selected the rules:

R_1: IF AI is L and TN is L THEN RT is L
R_2: IF AI is L and TN is M THEN RT is ML
R_3: IF AI is L and TN is H THEN RT is ML
R_4: IF AI is M and TN is L THEN RT is ML
R_5: IF AI is M and TN is M THEN RT is M
R_6: IF AI is M and TN is H THEN RT is MH
R_7: IF AI is H and TN is L THEN RT is MH
R_8: IF AI is H and TN is M THEN RT is MH
R_9: IF AI is H and TN is H THEN RT is H

If we set crisp inputs by $x = 38$ and $y = 70$, the final output is the IT2 FQ $\tilde{G}^* = (\tilde{G}^{*L}, \tilde{G}^{*U})$ shown in Fig. 4, where $\tilde{G}^{*L}$ and $\tilde{G}^{*U}$ are T1 FQs (see (1)) given by

$$\begin{aligned} G^{*L} &= (5.00, 11.91, 39.73, 43.00, 57.00, 64.20, 84.60, 95.00; 0.31, 0.47, 0.18) \\ G^{*U} &= (0.00, 11.86, 39.32, 43.41, 56.59, 63.86, 85.27, 100.00; 0.53, 0.65, 0.36)\,. \end{aligned} \tag{2}$$

5 Evaluation of Interval Type-2 Fuzzy Quantities

In [4] we propose a way to approximate a T1 FQ by an interval. Our proposal starts from Grzegorzewski's papers in which the author defines and finds the approximating interval of a fuzzy number. Starting from a distance between two fuzzy numbers and observing that any closed interval is a fuzzy number, the author defines the approximating interval of a fuzzy number as the interval of minimum distance. The distance he uses is based on the distance between intervals introduced by Trutschnig et al. [16]. This idea needs that each α-cut is an interval, that is the fuzzy set has to be convex. Hence, we cannot follow the same approach for non convex fuzzy quantities. To overcome this obstacle we noticed that Grzegorzewski's procedure may be regarded as the study of the minimum of the variance between the α-cuts family identifying a fuzzy number and a generic interval. This new way to look at the problem may be useful for non convex fuzzy quantities too.

Proposition 1. *Let A be the T1 fuzzy quantity defined in* (1) *with height h_A. Then for each $\alpha \in [0, h_A]$ there exist an integer n_α, with $1 \leq n_\alpha \leq N$, and $A_1^\alpha, \ldots, A_{n_\alpha}^\alpha$ disjoint closed intervals such that $A_\alpha = \bigcup_{i=1}^{n_\alpha} A_i^\alpha = \bigcup_{i=1}^{n_\alpha} [a_i^L(\alpha), a_i^R(\alpha)]$, where we have denoted $A_i^\alpha = [a_i^L(\alpha), a_i^R(\alpha)]$, with $A_i^\alpha < A_{i+1}^\alpha$ (that is $a_i^R(\alpha) < a_{i+1}^L(\alpha)$). Thus n_α is the number of intervals producing the α-cut A_α.*

From decomposition theorem for T1 FSs and using previous result, we get

$$A = \bigcup_{\alpha \in [0,h_A]} \alpha \, A_\alpha = \bigcup_{\alpha \in [0,h_A]} \alpha \bigcup_{i=1}^{n_\alpha} A_i^\alpha = \bigcup_{\alpha \in [0,h_A]} \bigcup_{i=1}^{n_\alpha} \alpha \, A_i^\alpha$$

and thus the T1 FQ is identified by the intervals $\{A_i^\alpha;\ i = 1 \ldots, n_\alpha,\ 0 \leq \alpha \leq h_A\}$.

Definition 5. *We say that $C^*(A) = [c_L^*, c_R^*]$ is an approximation interval of the T1 FQ A with respect to p, f, θ if it minimizes the weighted mean of the squared distances*

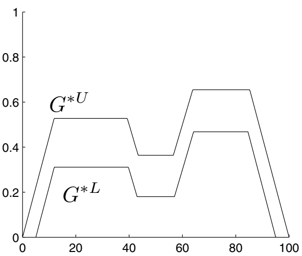


Fig. 4. Output IT2 FQ $\tilde{G}^* = (\tilde{G}^{*L}, \tilde{G}^{*U})$ of IT2 FLS

$$D^{(2)}(C;A) = \frac{1}{\int_0^{h_A} f(\alpha)\,d\alpha} \int_0^{h_A} \sum_{i=1}^{n_\alpha} d_\theta^2(C, A_i^\alpha)\, p_i(\alpha)\, f(\alpha)\, d\alpha$$
$$= \frac{1}{\int_0^{h_A} f(\alpha)\,d\alpha} \int_0^{h_A} \sum_{i=1}^{n_\alpha} \left[\left(mid(C) - mid(A_i^\alpha)\right)^2 + \theta(\alpha)\left(spr(C) - spr(A_i^\alpha)\right)^2 \right] p_i(\alpha)\, f(\alpha) d\alpha$$

among all the intervals $C = [c_L, c_R]$, where, for each level α, the weights $p(\alpha) = (p_i(\alpha))_{i=1,\ldots,n_\alpha}$ satisfy the properties $p_i(\alpha) \geq 0$ and $\sum_{i=1}^{n_\alpha} p_i(\alpha) = 1$, the weight function $f : [0,1] \to [0, +\infty[$ is such that $\int_0^{h_A} f(\alpha)\,d\alpha > 0$ and $\theta : [0,1] \to]0,1]$ is a function that indicates the relative importance of the spreads against the mids ([10,16]).

We have denoted by $mid(I) = (a+b)/2$ and $spr(I) = (b-a)/2$ the middle point and the spread of the interval $I = [a,b]$.

Theorem 1. *[4] The approximation interval $C^*(A) = C^*(A; p, f, \theta) = [c_L^*, c_R^*]$ of the T1 FQ A with respect to p, f, θ is given by*

$$c_L^* = \frac{\int_0^{h_A} \sum_{i=1}^{n_\alpha} mid(A_i^\alpha)\, p_i(\alpha)\, f(\alpha)\, d\alpha}{\int_0^{h_A} f(\alpha)\, d\alpha} - \frac{\int_0^{h_A} \sum_{i=1}^{n_\alpha} spr(A_i^\alpha)\, p_i(\alpha)\, f(\alpha)\, \theta(\alpha)\, d\alpha}{\int_0^{h_A} f(\alpha)\, \theta(\alpha)\, d\alpha}$$
$$c_R^* = \frac{\int_0^{h_A} \sum_{i=1}^{n_\alpha} mid(A_i^\alpha)\, p_i(\alpha)\, f(\alpha)\, d\alpha}{\int_0^{h_A} f(\alpha)\, d\alpha} + \frac{\int_0^{h_A} \sum_{i=1}^{n_\alpha} spr(A_i^\alpha)\, p_i(\alpha)\, f(\alpha)\, \theta(\alpha)\, d\alpha}{\int_0^{h_A} f(\alpha)\, \theta(\alpha)\, d\alpha}.$$

Definition 6. *We call evaluation of the T1 FQ A with respect to p, f, θ and $\lambda \in [0,1]$ the real number*

$$V^{\lambda,\theta}(A) = \phi_\lambda(C^*(A)),$$

where ϕ_λ is defined by $\phi_\lambda(I) = (1-\lambda)a + \lambda b = mid(I) + (2\lambda - 1)spr(I)$ for any interval $I = [a,b]$ and $\lambda \in [0,1]$ is a pessimistic/optimistic parameter. Thus

$$V^{\lambda,\theta}(A) = \frac{\int_0^{h_A} \sum_{i=1}^{n_\alpha} mid(A_i^\alpha)\, p_i(\alpha)\, f(\alpha)\, d\alpha}{\int_0^{h_A} f(\alpha)\, d\alpha} + (2\lambda - 1)\frac{\int_0^{h_A} \sum_{i=1}^{n_\alpha} spr(A_i^\alpha)\, p_i(\alpha)\, f(\alpha)\, \theta(\alpha)\, d\alpha}{\int_0^{h_A} f(\alpha)\, \theta(\alpha)\, d\alpha}.$$

This general formula includes, for suitable choices of parameters λ, p and f, the evaluations proposed by Fortemps and Roubens [8,3], Yager and Filev [18,19], Anzilli and Facchinetti [3] and Center of Gravity (COG), as shown in Table 1.

Definition 7. *We define the value of the IT2 FQ $\tilde{A} = (A^L, A^U)$ as*

$$V^{\lambda,\theta}(\tilde{A}) = (V^{\lambda,\theta}(A^L) + V^{\lambda,\theta}(A^U))/2.$$

As an application, we now compute the evaluation of the final output $\tilde{G}^* = (\tilde{G}^{*L}, \tilde{G}^{*U})$ given in (2) (see Fig. 4) using different methods. First, we evaluate the T1 FQs $\tilde{G}^{*L}$ and $\tilde{G}^{*U}$ and then we obtain the value of the IT2 FQ $\tilde{G}^*$ as $V(\tilde{G}^*) = (V(G^{*L}) + V(G^{*U}))/2$. The numerical results are shown in Table 2. The "Interval Type-2 Fuzzy Logic Toolbox" [6] produces 52 as centroid.

Table 1. Set of parameters

Evaluation	λ	$p_i(\alpha)$	$f(\alpha)$
Fortemps and Roubens	1/2	$1/n_\alpha$	n_α
Yager and Filev	1/2	$spr(A_i^\alpha)/\sum_{j=1}^{n_\alpha} spr(A_j^\alpha)$	1
Anzilli and Facchinetti	1/2	$spr(A_i^\alpha)/\sum_{j=1}^{n_\alpha} spr(A_j^\alpha)$	n_α
COG	1/2	$spr(A_i^\alpha)/\sum_{j=1}^{n_\alpha} spr(A_j^\alpha)$	$2\sum_{j=1}^{n_\alpha} spr(A_j^\alpha)$

Table 2. Evaluation of IT2 FQ $\tilde{G}^* = (\tilde{G}^{*L}, \tilde{G}^{*U})$

Evaluation	$V(G^{*L})$	$V(G^{*U})$	$V(\tilde{G}^*)$
Fortemps and Roubens	56.48	53.57	55.03
Yager and Filev	58.28	54.48	56.38
Anzilli and Facchinetti	56.49	53.53	55.01
COG	53.44	51.49	52.47

6 Conclusion

In this paper we introduce a different type-reduction method for IT2 FLSs. We consider only the T1 membership functions that bound the Output FOU zone and for its defuzzification we present a general formula completely different from centroid proposed by Karnik and Mendel for two reasons. First of all it is obtained working on an α-cuts approach while centroid works on x-axis. Moreover it is presented in a parametric formulation leaving a wide set of freedom. This opportunity has allowed us to obtain not only other methods already known, not only other completely new but the centroid too. We have obtained this general formula starting from an idea of the interval nearest to T1 FQ respect to a general functional suggested by the distance proposed by Trutschnig et al. [16]. Now we are working on a more general way to approximate T1 FQs based on a triangular fuzzy set and in the following on trapezoidal fuzzy sets. These works are in preparation.

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