# Clustering-Based Selection for Evolutionary Many-Objective Optimization

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**Abstract.** This paper discusses a selection scheme allowing to employ a clustering technique to guide the search in evolutionary many-objective optimization. The underlying idea to avoid the curse of dimensionality is based on transforming the objective vectors before applying a clustering and the selection of cluster representatives according to the distance to a reference point. The experimental results reveal that the proposed approach is able to effectively guide the search in high-dimensional objective spaces, producing highly competitive performance when compared with state-of-the-art algorithms.

## 1 Introduction

As problems with a large number of objectives become widespread in practice, the issue of dealing with many-objective problems has gained a significant attention in the evolutionary multiobjective optimization (EMO) community. Some researchers suggest to handle many-objective problems by modifying the Pareto dominance relation, assigning different ranks to nondominated solutions, using the decision maker's preferences during the search, incorporating the quality indicators or scalarizing functions into the fitness assignment, or reducing the problem's dimensionality whenever redundant objectives are identified. A good review of such approaches can be found in [1]. Despite the recent advances in solving many-objective problems, there are a number of disadvantages related to such approaches. In particular, the practical application of the hypervolume, which has nice mathematical properties, is limited due to the high computational cost. The use of a scalarizing fitness assignment necessitates a set of weight vectors to be provided in advance, being not always an easy task especially for high dimensions. The use of preference information during the search allows to find only certain regions of the Pareto front, whereas dimensionality reduction techniques are only suitable for problems having redundant objectives. Thus, the need for efficient and self-adaptive methodologies persists.

In this work, we focus on improving the scalability of Pareto-dominance based algorithms and argue that a clustering-based diversity maintenance can be successfully used for solving many-objective problems. For this purpose, we propose to perform a transformation on objective vectors, aimed at reducing the

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distances between solutions in high-dimensional spaces. So that a clustering can group those solutions, which are distant in the original space but can be viewed as representatives of similar regions of the fitness landscape. The necessary selection pressure is provided by minimizing the distances of population members to a reference point.

The remainder of this paper is organized as follows. Section 2 describes an evolutionary algorithm with the proposed selection scheme. Section 3 presents the results of a comparison study and discusses variants of the suggested selection. Section 4 concludes the work and outlines further research opportunities.

## 2 Algorithm

In the following, we present an evolutionary many-objective optimization algorithm with clustering-based selection (EMyO/C), considering an optimization problem of the form:

$$\underset{\boldsymbol{x}\in\Omega\subset\mathbb{R}^n}{\text{minimize:}}\ \boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x}))^{\mathrm{T}}, \tag{1}$$

where $m$ - is the number objective and $n$ - is the number of variables.

EMyO/C reflects a general framework of an evolutionary algorithm with the $(\mu + \lambda)$ selection scheme, where the mating selection, variation and environmental selection are successively applied during the generation. In EMyO/C, an initial population is randomly generated and a reference point, $\boldsymbol{z}$, initialized as: $\forall j \in \{1,\ldots,m\} : z_j = \min_{1\leq i\leq\mu} f_j(\boldsymbol{x}^i)$. In the mating selection, each population member is selected to the mating pool. The variation procedure relies on the DE operator, adopting the idea presented in [2]. For each individual in the mating pool, two different individuals are randomly selected. A difference vector, $\boldsymbol{v}$, is calculated using these individuals. To introduce additional variation, the polynomial mutation [3] is applied on the difference vector, $\boldsymbol{v}$. The resulting difference vector is restricted as follows:

$$v_j = \begin{cases} -\delta_j & \text{if } v_j < -\delta_j \\ \delta_j & \text{if } v_j > \delta_j \\ v_j & \text{otherwise} \end{cases} \tag{2}$$

where

$$\delta_j = \frac{ub_j - lb_j}{2} \quad \forall j \in \{1,\ldots,n\}, \tag{3}$$

$ub_j$ and $lb_j$ are the upper and lower bounds of the $j$-th variable, respectively. An offspring, $\boldsymbol{x}'$, is generated by mutating the parent individual, $\boldsymbol{x}$, as:

$$x'_j = \begin{cases} x_j + v_j & \text{if } rand < CR \\ x_j & \text{otherwise} \end{cases} \quad \forall j \in \{1,\ldots,n\}. \tag{4}$$

To ensure the offspring feasibility, it is repaired as:

$$x'_j = \min\{\max\{x'_j, lb_j\}, ub_j\} \quad \forall j \in \{1,\ldots,n\}. \tag{5}$$

The resulting offspring, $\boldsymbol{x}'$, is compared with its parent, $\boldsymbol{x}$. If there is a difference in at least one gene, then the offspring is evaluated and added to the offspring population. Otherwise, two other individuals are selected from the mating pool, and the above described steps - including computation of $\boldsymbol{v}$, mutation restriction, and creation of $\boldsymbol{x}'$ - are performed until an individual different from $\boldsymbol{x}$ is produced. This allows to avoid a situation in which the offspring identical to its parent is evaluated and added to the population. It should be noted that applying the polynomial mutation allows to produce new genotypes even when the whole population is converged to a single solution. Further, each time an offspring is evaluated, the components of the reference point are updated if there are smaller objective values. The environmental selection performs the nondominated sorting procedure [3] and selects a new population from the multiset of parents and offspring, according to the rank values. In a case where the last accepted front, $\mathcal{F}_l$, cannot be completely accommodated, the truncation procedure is performed to select $k$ best individuals as follows.

1. For each individual in $\mathcal{F}_l$, the Euclidean distance in the objective space to the reference point, $\boldsymbol{z}$, is calculated.
2. For each individual in $\mathcal{F}_l$, the objectives are translated as:

$$f_i = f_i - z_i \quad \forall i \in \{1, \dots, m\}. \tag{6}$$

3. Each individual in $\mathcal{F}_l$ is projected onto the unit hyperplane as:

$$f_i = f_i / \sum_{j=1}^{m} f_j \quad \forall i \in \{1, \dots, m\}. \tag{7}$$

4. Using projected individuals, $k$ clusters are formed as follows:
   Step 1 Initially, each individual is treated as a separate cluster $C = \{C_1, C_2, \dots, C_{|\mathcal{F}_l|}\}$.
   Step 2 If $|C| = k$, stop. Otherwise, go to Step 3.
   Step 3 For each pair of clusters, the distance between two clusters, $d_{12}$, is calculated as:
   $$d_{12} = \frac{1}{|C_1||C_2|} \sum_{i \in C_1, j \in C_2} d(i,j),$$
   where $d(i,j)$ is the Euclidean distance between individuals $i$ and $j$.
   Step 4 The pair of clusters having the smallest distance is merged.
   Go to Step 2.
5. In each cluster, a representative is selected and added to the new population, where a cluster representative is an individual having the smallest distance to the reference point.

It should be noted that the proposed selection procedure does not require any additional parameter, being completely adaptive and easy to implement. The complexity of the clustering algorithm is mainly governed by the number of points in $\mathcal{F}_l$, whereas it is polynomial in the number of objectives. This feature is especially attractive to solve problems with a large number of objectives.

## 3 Performance Assessment

To validate EMyO/C, it is compared with IBEA [4], MOEA/D [5], MSOPS [6], MSOPS2 [7], and HypE [8] on the DTLZ test suite [9] with 30 decision variables having between 2 and 20 objectives.

### 3.1 Performance Indicators and Statistical Comparison

The outcomes produced by the algorithms are assessed using the unary additive epsilon ($\epsilon+$) indicator [10], the hypervolume (HV) indicator [11], and the inverted generational distance (IGD) indicator [12]. To calculate the $\epsilon+$ and IGD indicators, for all problems, 1,000 uniformly distributed points along the Pareto front are generated. To calculate the HV indicator, the nadir point is used as a reference point. Solutions that do not dominate the nadir point are discarded. If there is no solution dominating the nadir point, then the hypervolume of the approximation set is equal to zero. Further, solutions used to calculate the hypervolume are normalized using the ideal and nadir points. When the number of objectives is more than 6 the hypervolume is approximated using $10^6$ uniformly sampled points, otherwise the hypervolume is computed exactly.

To provide the results with statistical confidence, the single-problem analysis is performed using the Wilcoxon rank-sum test. The multiple-problem analysis is performed on algorithms' ranks using the Friedman test to determine whether there is a significant difference among the results, and the Bonferroni procedure for a post-hoc statistical analysis to detect concrete differences among the algorithms [13]. All tests are performed at significance level of $\alpha = 0.05$.

### 3.2 Experimental Setup

EMOy/C is implemented and tested in Java™, whereas IBEA and HypE are used within the PISA [14] framework[1], MOEA/D is used within the jMetal [15] framework[2], and the implementations of MSOPS and MSOPS2 are taken from the author's web page[3].

For each algorithm, 30 independent runs are performed on each problem with a population size of $\mu = 300$, running for 500 generations. The other parameter settings for EMyO/C are: $CR = 0.15$, $\eta_m = 20$, and $p_m = 1/n$ ($n$ is the number of decision variables). The other algorithms use the default settings, except common parameters with EMyO/C, which use the same values to guarantee a fair comparison.

### 3.3 Performance Comparison

Table 1 presents the median values of the quality indicators and the statistical comparison of the algorithms. In the table, the small values of the $\epsilon+$ indicator

[1] available at `http://www.tik.ee.ethz.ch/pisa`
[2] available at `http://jmetal.sourceforge.net`
[3] available at `http://code.evanhughes.org`

**Table 1.** Median values of the epsilon (the lower the better), hypervolume (the higher the better), and IGD (the lower the better) indicators after 30 runs. The superscripts 1, 2, 3, 4, 5, and 6 indicate whether the respective algorithm performs significantly better than EMyO/C, IBEA, MOEA/D, MSOPS, MSOPS2, and HypE, respectively.

| | | EMyO/C | IBEA | MOEA/D | MSOPS | MSOPS2 | HypE |
|---|---|---|---|---|---|---|---|
| 2-objectives | | | | | | | |
| DTLZ1 | $\epsilon+$ | $0.002^{2,3,4,5,6}$ | $0.21^{3,4,5}$ | $7.696^{5}$ | $6.843^{5}$ | 14.489 | $0.034^{2,3,4,5}$ |
| | HV | $0.498^{2,3,4,5,6}$ | $0.136^{3,4,5}$ | 0 | 0 | 0 | $0.439^{2,3,4,5}$ |
| | IGD | $0.001^{2,3,4,5,6}$ | $0.121^{3,4,5}$ | $10.024^{5}$ | $9.099^{5}$ | 19.336 | $0.02^{2,3,4,5}$ |
| DTLZ2 | $\epsilon+$ | $0.005^{2,4,6}$ | $0.006^{4,6}$ | $0.002^{1,2,4,5,6}$ | $0.007^{6}$ | $0.004^{1,2,4,6}$ | 0.046 |
| | HV | $0.213^{4,6}$ | $0.213^{4,6}$ | $0.213^{1,2,4,6}$ | $0.212^{6}$ | $0.213^{1,2,3,4,6}$ | 0.203 |
| | IGD | $0.002^{2,4,6}$ | $0.005^{6}$ | $0.001^{1,2,4,5,6}$ | $0.002^{2,6}$ | $0.001^{1,2,4,6}$ | 0.009 |
| DTLZ3 | $\epsilon+$ | $0.005^{2,3,4,5,6}$ | $0.341^{3,4,5}$ | $22.478^{5}$ | $28.001^{5}$ | 40.545 | $0.12^{2,3,4,5}$ |
| | HV | $0.213^{2,3,4,5,6}$ | 0 | 0 | 0 | 0 | $0.13^{2,3,4,5}$ |
| | IGD | $0.002^{2,3,4,5,6}$ | $0.397^{3,4,5}$ | $27.579^{4,5}$ | $37.67^{5}$ | 48.984 | $0.061^{2,3,4,5}$ |
| DTLZ4 | $\epsilon+$ | $0.005^{2,4,5,6}$ | $0.005^{4,5,6}$ | $0.002^{1,2,4,5,6}$ | $0.009^{5,6}$ | 0.265 | $0.026^{5}$ |
| | HV | $0.213^{4,5,6}$ | $0.213^{1,4,5,6}$ | $0.213^{1,2,4,5,6}$ | $0.212^{5,6}$ | 0.051 | $0.208^{5}$ |
| | IGD | $0.002^{2,4,5,6}$ | $0.005^{5}$ | $0.001^{1,2,4,5,6}$ | $0.002^{2,5,6}$ | 0.226 | $0.005^{5}$ |
| DTLZ7 | $\epsilon+$ | $0.006^{2,4,5,6}$ | $0.018^{5,6}$ | $0.004^{1,2,4,5,6}$ | $0.011^{2,5,6}$ | 3.77 | $0.093^{5}$ |
| | HV | $0.336^{2,3,4,5,6}$ | $0.335^{4,5,6}$ | $0.336^{2,4,5,6}$ | $0.33^{5,6}$ | 0 | $0.293^{5}$ |
| | IGD | $0.002^{2,3,4,5,6}$ | $0.007^{4,5,6}$ | $0.002^{2,4,5,6}$ | $0.008^{5,6}$ | 2.945 | $0.136^{5}$ |
| 3-objectives | | | | | | | |
| DTLZ1 | $\epsilon+$ | $0.021^{2,3,4,5,6}$ | $0.276^{3,4,5,6}$ | $9.399^{5}$ | $4.025^{3,5}$ | 100.495 | $1.187^{3,4,5}$ |
| | HV | $0.806^{2,3,4,5,6}$ | $0.265^{3,4,5,6}$ | 0 | 0 | 0 | 0 |
| | IGD | $0.012^{2,3,4,5,6}$ | $0.187^{3,4,5,6}$ | $14.458^{5}$ | $5.884^{3,5}$ | 133.662 | $1.33^{3,4,5}$ |
| DTLZ2 | $\epsilon+$ | $0.051^{3,4,6}$ | $0.047^{1,3,4,6}$ | $0.087^{6}$ | $0.066^{3,6}$ | $0.043^{1,2,3,4,6}$ | 0.121 |
| | HV | $0.439^{3,4,6}$ | $0.441^{1,3,4,6}$ | $0.419^{6}$ | $0.437^{3,6}$ | $0.442^{1,2,3,4,6}$ | 0.397 |
| | IGD | $0.032^{2,3,4,6}$ | $0.061^{6}$ | $0.039^{2,6}$ | $0.034^{2,3,6}$ | $0.03^{1,2,3,4,6}$ | 0.077 |
| DTLZ3 | $\epsilon+$ | $0.053^{2,3,4,5,6}$ | $0.47^{3,4,5,6}$ | $20.974^{5}$ | $17.645^{5}$ | 244.304 | $6.717^{3,4,5}$ |
| | HV | $0.439^{2,3,4,5,6}$ | 0 | 0 | 0 | 0 | 0 |
| | IGD | $0.033^{2,3,4,5,6}$ | $0.553^{3,4,5,6}$ | $28.443^{5}$ | $27.796^{5}$ | 327.555 | $7.002^{3,4,5}$ |
| DTLZ4 | $\epsilon+$ | $0.053^{3,4,5,6}$ | $0.047^{1,3,4,5,6}$ | $0.081^{5,6}$ | $0.06^{3,5,6}$ | 0.634 | $0.098^{5}$ |
| | HV | $0.437^{3,4,5,6}$ | $0.441^{1,3,4,5,6}$ | $0.425^{5,6}$ | $0.436^{3,5,6}$ | 0.241 | $0.414^{5}$ |
| | IGD | $0.032^{2,3,4,5,6}$ | $0.06^{5,6}$ | $0.039^{2,5,6}$ | $0.034^{2,3,5,6}$ | 0.247 | $0.063^{5}$ |
| DTLZ7 | $\epsilon+$ | $0.051^{2,3,4,5,6}$ | $0.057^{3,4,5,6}$ | $0.167^{5,6}$ | $0.132^{3,5,6}$ | 5.044 | $0.428^{5}$ |
| | HV | $0.364^{2,3,4,5,6}$ | $0.361^{3,4,5,6}$ | $0.309^{5,6}$ | $0.312^{3,5,6}$ | 0 | $0.23^{5}$ |
| | IGD | $0.037^{2,3,4,5,6}$ | $0.092^{3,4,5,6}$ | $0.103^{4,5,6}$ | $0.158^{5,6}$ | 3.277 | $0.37^{5}$ |
| 5-objectives | | | | | | | |
| DTLZ1 | $\epsilon+$ | $0.075^{2,3,4,5,6}$ | $0.325^{3,4,5,6}$ | $7.619^{5,6}$ | $2.16^{3,5,6}$ | 102.255 | $15.715^{5}$ |
| | HV | $0.93^{2,3,4,5,6}$ | $0.409^{3,4,5,6}$ | 0 | 0 | 0 | 0 |
| | IGD | $0.069^{2,3,4,5,6}$ | $0.243^{3,4,5,6}$ | $11.502^{5,6}$ | $3.344^{3,5,6}$ | 159.717 | $18.451^{5}$ |
| DTLZ2 | $\epsilon+$ | $0.144^{3,4,6}$ | $0.131^{1,3,4,5,6}$ | $0.18^{6}$ | $0.159^{3,6}$ | $0.135^{1,3,4,6}$ | 0.377 |
| | HV | $0.705^{3,4,5,6}$ | $0.725^{1,3,4,5,6}$ | $0.651^{6}$ | $0.7^{3,5,6}$ | $0.696^{3,6}$ | 0.343 |
| | IGD | $0.164^{2,3,4,6}$ | $0.21^{4,6}$ | $0.203^{2,4,6}$ | $0.216^{6}$ | $0.159^{1,2,3,4,6}$ | 0.408 |
| DTLZ3 | $\epsilon+$ | $0.143^{2,3,4,5,6}$ | $0.584^{3,4,5,6}$ | $16.688^{5,6}$ | $11.724^{3,5,6}$ | 325.935 | $32.866^{5}$ |
| | HV | $0.703^{2,3,4,5,6}$ | 0 | 0 | 0 | 0 | 0 |
| | IGD | $0.164^{2,3,4,5,6}$ | $0.71^{3,4,5,6}$ | $20.895^{5,6}$ | $16.946^{3,5,6}$ | 464.898 | $32.464^{5}$ |
| DTLZ4 | $\epsilon+$ | $0.144^{3,4,5,6}$ | $0.13^{1,3,4,5,6}$ | $0.183^{6}$ | $0.158^{3,6}$ | $0.174^{6}$ | 0.213 |
| | HV | $0.703^{3,5,6}$ | $0.727^{1,3,4,5,6}$ | $0.666^{6}$ | $0.702^{3,5,6}$ | $0.662^{6}$ | 0.604 |
| | IGD | $0.168^{2,3,4,5,6}$ | $0.21^{3,4,6}$ | $0.246^{6}$ | $0.229^{3,6}$ | $0.18^{2,3,4,6}$ | 0.253 |
| DTLZ7 | $\epsilon+$ | $0.231^{4,5,6}$ | $0.257^{4,5,6}$ | $0.217^{4,5,6}$ | $0.46^{5,6}$ | 8.895 | $0.833^{5}$ |
| | HV | $0.323^{3,4,5,6}$ | $0.337^{1,3,4,5,6}$ | $0.271^{4,5,6}$ | $0.199^{5}$ | 0 | $0.217^{4,5}$ |
| | IGD | $0.261^{2,3,4,5,6}$ | $0.385^{3,4,5,6}$ | $0.413^{4,5,6}$ | $0.518^{5,6}$ | 4.913 | $0.648^{5}$ |
| 10-objectives | | | | | | | |
| DTLZ1 | $\epsilon+$ | $0.212^{2,3,4,5,6}$ | $0.334^{3,4,5,6}$ | $2.547^{5,6}$ | $1.151^{3,5,6}$ | 65.446 | $11.561^{5}$ |
| | HV | $0.484^{3,4,5,6}$ | $0.729^{1,3,4,5,6}$ | 0 | 0 | 0 | 0 |
| | IGD | $0.272^{3,4,5,6}$ | $0.304^{3,4,5,6}$ | $3.478^{5,6}$ | $1.766^{3,5,6}$ | 108.78 | $17.875^{5}$ |
| DTLZ2 | $\epsilon+$ | $0.249^{2,3,4,5,6}$ | $0.259^{3,5,6}$ | $0.289^{5,6}$ | $0.259^{3,5,6}$ | $0.364^{6}$ | 0.656 |
| | HV | $0.917^{3,4,5,6}$ | $0.923^{1,3,4,5,6}$ | $0.867^{5,6}$ | $0.911^{3,5,6}$ | $0.821^{6}$ | 0.348 |
| | IGD | $0.442^{2,3,4,6}$ | $0.481^{4,6}$ | $0.454^{2,4,6}$ | $0.49^{6}$ | $0.415^{1,2,3,4,6}$ | 0.679 |
| DTLZ3 | $\epsilon+$ | $0.411^{2,3,4,5,6}$ | $0.632^{3,4,5,6}$ | $5.628^{5,6}$ | $6.045^{5,6}$ | 209.395 | $25.339^{5}$ |
| | HV | $0.55^{2,3,4,5,6}$ | 0 | 0 | 0 | 0 | 0 |
| | IGD | $0.547^{2,3,4,5,6}$ | $0.849^{3,4,5,6}$ | $7.138^{4,5,6}$ | $8.762^{5,6}$ | 266.516 | $25.63^{5}$ |
| DTLZ4 | $\epsilon+$ | $0.267^{3,5,6}$ | $0.248^{1,3,4,5,6}$ | $0.322^{6}$ | $0.27^{3,5,6}$ | $0.282^{3,6}$ | 0.988 |
| | HV | $0.926^{3,4,5,6}$ | $0.931^{1,3,4,5,6}$ | $0.888^{6}$ | $0.916^{3,5,6}$ | $0.891^{6}$ | 0 |
| | IGD | $0.495^{3,4,5,6}$ | $0.475^{1,3,4,5,6}$ | $0.565^{6}$ | $0.545^{3,6}$ | $0.521^{3,4,6}$ | 1.087 |
| DTLZ7 | $\epsilon+$ | $0.727^{2,3,4,5,6}$ | $0.791^{4,5,6}$ | $0.76^{2,4,5,6}$ | $0.826^{5,6}$ | 13.077 | $0.832^{5}$ |
| | HV | $0.019^{5}$ | $0.183^{1,3,4,5,6}$ | $0.022^{5}$ | $0.033^{1,3,5}$ | 0 | $0.112^{1,3,4,5}$ |
| | IGD | $0.947^{3,4,5,6}$ | $0.97^{3,4,5,6}$ | $1.021^{4,5,6}$ | $1.622^{5}$ | 7.274 | $1.063^{4,5}$ |

| | | EMyO/C | IBEA | MOEA/D | MSOPS | MSOPS2 | HypE |
|---|---|---|---|---|---|---|---|
| 15-objectives | | | | | | | |
| DTLZ1 | $\epsilon+$ | $0.345^{5,6}$ | $0.336^{4,5,6}$ | $0.136^{1,2,4,5,6}$ | $0.424^{5,6}$ | 21.212 | $5.276^{5}$ |
| | HV | $0.007^{5,6}$ | $0.835^{1,4,5,6}$ | $0.918^{1,4,5,6}$ | $0.001^{5,6}$ | 0 | 0 |
| | IGD | $0.586^{4,5,6}$ | $0.32^{1,4,5,6}$ | $0.21^{1,2,4,5,6}$ | $0.77^{5,6}$ | 35.516 | $7.495^{5}$ |
| DTLZ2 | $\epsilon+$ | $0.31^{2,3,5,6}$ | $0.348^{5,6}$ | $0.332^{2,5,6}$ | $0.314^{2,3,5,6}$ | $0.37^{6}$ | 0.646 |
| | HV | $0.966^{3,4,5,6}$ | $0.968^{1,3,4,5,6}$ | $0.906^{5,6}$ | $0.948^{3,5,6}$ | $0.811^{6}$ | 0.515 |
| | IGD | $0.609^{2,4,6}$ | $0.662^{6}$ | $0.523^{1,2,4,6}$ | $0.613^{2,6}$ | $0.533^{1,2,4,6}$ | 0.752 |
| DTLZ3 | $\epsilon+$ | $0.396^{2,3,4,5,6}$ | $0.667^{4,5,6}$ | $0.501^{2,4,5,6}$ | $3.117^{5,6}$ | 65.966 | $11.838^{5}$ |
| | HV | $0.82^{2,3,4,5,6}$ | 0 | $0.701^{2,4,5,6}$ | $0.001^{2}$ | $0.001^{2}$ | $0.001^{2}$ |
| | IGD | $0.667^{2,4,5,6}$ | $0.939^{4,5,6}$ | $0.692^{2,4,5,6}$ | $4.907^{5,6}$ | 76.096 | $11.691^{5}$ |
| DTLZ4 | $\epsilon+$ | $0.334^{3,4,5,6}$ | $0.315^{1,3,4,5,6}$ | $0.369^{6}$ | $0.348^{3,5,6}$ | $0.361^{6}$ | 1.055 |
| | HV | $0.978^{2,3,4,5,6}$ | $0.973^{3,4,5,6}$ | $0.95^{6}$ | $0.957^{3,6}$ | $0.955^{3,6}$ | 0 |
| | IGD | $0.682^{3,4,5,6}$ | $0.651^{1,3,4,5,6}$ | $0.695^{6}$ | $0.703^{6}$ | $0.698^{6}$ | 1.275 |
| DTLZ7 | $\epsilon+$ | $0.772^{2,3,4,5,6}$ | $3.625^{5}$ | $0.793^{2,4,5,6}$ | $0.829^{2,5,6}$ | 14.944 | $0.841^{2,5}$ |
| | HV | $0.001^{5}$ | $0.083^{1,3,4,5,6}$ | $0.001^{1,5}$ | $0.016^{1,3,5,6}$ | 0 | $0.013^{1,3,5}$ |
| | IGD | $1.419^{2,4,5}$ | $1.57^{4,5}$ | $1.435^{2,4,5}$ | $3.333^{5}$ | 7.372 | $1.412^{2,3,4,5}$ |
| 20-objectives | | | | | | | |
| DTLZ1 | $\epsilon+$ | $0.111^{2,4,5,6}$ | $0.363^{5,6}$ | $0.063^{1,2,4,5,6}$ | $0.26^{2,5,6}$ | 2.239 | 1.751 |
| | HV | $0.697^{4,5,6}$ | $0.802^{4,5,6}$ | $0.997^{1,2,4,5,6}$ | $0.363^{5,6}$ | 0 | 0 |
| | IGD | $0.187^{2,4,5,6}$ | $0.351^{4,5,6}$ | $0.142^{1,2,4,5,6}$ | $0.43^{5,6}$ | 3.251 | 2.77 |
| DTLZ2 | $\epsilon+$ | $0.383^{2,6}$ | $0.434^{6}$ | $0.332^{1,2,5,6}$ | $0.337^{1,2,5,6}$ | $0.359^{1,2,6}$ | 0.638 |
| | HV | $0.98^{3,4,5,6}$ | $0.981^{3,4,5,6}$ | $0.899^{5,6}$ | $0.95^{3,5,6}$ | $0.813^{6}$ | 0.559 |
| | IGD | $0.7^{2,6}$ | $0.781^{6}$ | $0.607^{1,2,4,5,6}$ | $0.65^{1,2,6}$ | $0.633^{1,2,6}$ | 0.813 |
| DTLZ3 | $\epsilon+$ | $0.407^{2,4,5,6}$ | $0.99^{4,5,6}$ | $0.379^{2,4,5,6}$ | $2.048^{5}$ | 8.54 | $3.093^{5}$ |
| | HV | $0.975^{2,3,4,5,6}$ | $0.016^{4,5,6}$ | $0.83^{2,4,5,6}$ | 0 | $0^{4}$ | $0^{5}$ |
| | IGD | $0.702^{2,4,5,6}$ | $0.993^{4,5,6}$ | $0.688^{1,2,4,5,6}$ | $2.816^{5}$ | 13.802 | $2.86^{5}$ |
| DTLZ4 | $\epsilon+$ | $0.4^{4,5,6}$ | $0.376^{1,4,5,6}$ | $0.385^{1,4,5,6}$ | $0.406^{5,6}$ | $0.426^{6}$ | 0.927 |
| | HV | $0.99^{2,3,4,5,6}$ | $0.987^{3,4,5,6}$ | $0.969^{6}$ | $0.969^{6}$ | $0.972^{3,4,6}$ | 0.01 |
| | IGD | $0.793^{4,5,6}$ | $0.759^{1,3,4,5,6}$ | $0.772^{1,4,5,6}$ | $0.798^{5,6}$ | $0.807^{6}$ | 1.173 |
| DTLZ7 | $\epsilon+$ | $0.795^{2,3,4,5,6}$ | 10.519 | $0.805^{2,4,5}$ | $0.839^{2,5}$ | 11.53 | $0.803^{2,4,5}$ |
| | HV | 0.001 | $0.029^{1,3,4,5,6}$ | $0.001^{1,5}$ | $0.001^{1,3,5}$ | $0.001^{1}$ | $0.001^{1,3,5}$ |
| | IGD | $1.827^{2,4,5}$ | $3.204^{4,5}$ | $1.802^{2,4,5}$ | 3.806 | 3.892 | $1.765^{1,2,3,4,5}$ |

suggest that approximation sets produced by EMyO/C are relatively close to the true Pareto fronts. The HV values for EMyO/C are always greater than zero, suggesting that EMyO/C generates solutions being within the bounds of the Pareto fronts. Although the IGD indicator is non-Pareto compliant, the small values of IGD indicate the closeness to the Pareto front and adequate distributions of approximations obtained by EMyO/C.

From Table 1, it can be seen that EMyO/C dominates the other algorithms regarding the quality indicators on DTLZ1,3,7 problems with up to 10 objectives, besides IBEA and MOEA/D that give better results with respect to the $\epsilon+$ and HV indicators on DTLZ7 in some dimensions. DTLZ1 and DTLZ3 are multimodal problems with the linear and concave Pareto fronts, whereas the main characteristic of DTLZ7 is the disconnected Pareto front. Thus, EMyO/C appears to be capable of dealing with such problem properties in high-dimensional objective spaces. The competitive results for these problems in 15 and 20 dimensions confirm these observations. DTLZ2 and DTLZ4 do not present much difficulties in terms of the convergence. The best indicator values for these problems in dimensions higher than 3 are obtained by IBEA and MOEA/D, apart DTLZ2 with 5 and 10 objectives, being MSOPS the best algorithm with respect to IGD. The superior performance of IBEA regarding the $\epsilon+$ and HV indicators is not surprising, since its selection procedure relies on the concept of $\epsilon$-dominance. On the other hand, MOEA/D uses a set of uniformly distributed weight vectors that contributes to the high selection pressure and the uniform

**Table 2.** Mean ranks achieved by different algorithms. The superscripts 1, 2, 3, 4, 5, and 6 indicate whether the respective algorithm performs significantly better than EMyO/C, IBEA, MOEA/D, MSOPS, MSOPS2, and HypE, respectively.

| Indicator | EMyO/C | IBEA | MOEA/D | MSOPS | MSOPS2 | HypE |
|---|---|---|---|---|---|---|
| $\epsilon+$ | $1.73^{4,5,6}$ | $2.63^{5,6}$ | $3.10^{5,6}$ | $3.50^{5}$ | 5.27 | 4.87 |
| HV | $2.08^{3,4,5,6}$ | $2.10^{3,4,5,6}$ | 3.63 | 3.82 | 4.67 | 4.70 |
| IGD | $1.73^{4,5,6}$ | $2.97^{5,6}$ | $2.90^{5,6}$ | 3.97 | 4.77 | 4.67 |

distribution of solutions. Nevertheless, EMyO/C provides a highly competitive performance on all the considered problems.

The overall performance of the algorithms is compared by calculating ranks on each problem with respect to the quality indicators. It should be noted that testing DTLZ1-4,7 problems in 6 different dimensions there is a total of 30 distinct problems. Table 2 presents the mean ranks and statistical comparison. From the table, it can be seen that EMyO/C has the best mean ranks regarding all three indicators, though no difference is detected between EMyO/C and IBEA, as well as there is no difference between EMyO/C and MOEA/D regarding the $\epsilon+$ and HV indicators. These results emphasize the competitiveness of the proposed approach.

### 3.4 Selection on Different Shapes

We also investigate a generalized EMyO/C, which consists in controlling the shape of $\mathcal{F}_l$ for performing the clustering. It can be defined by the following transformation:

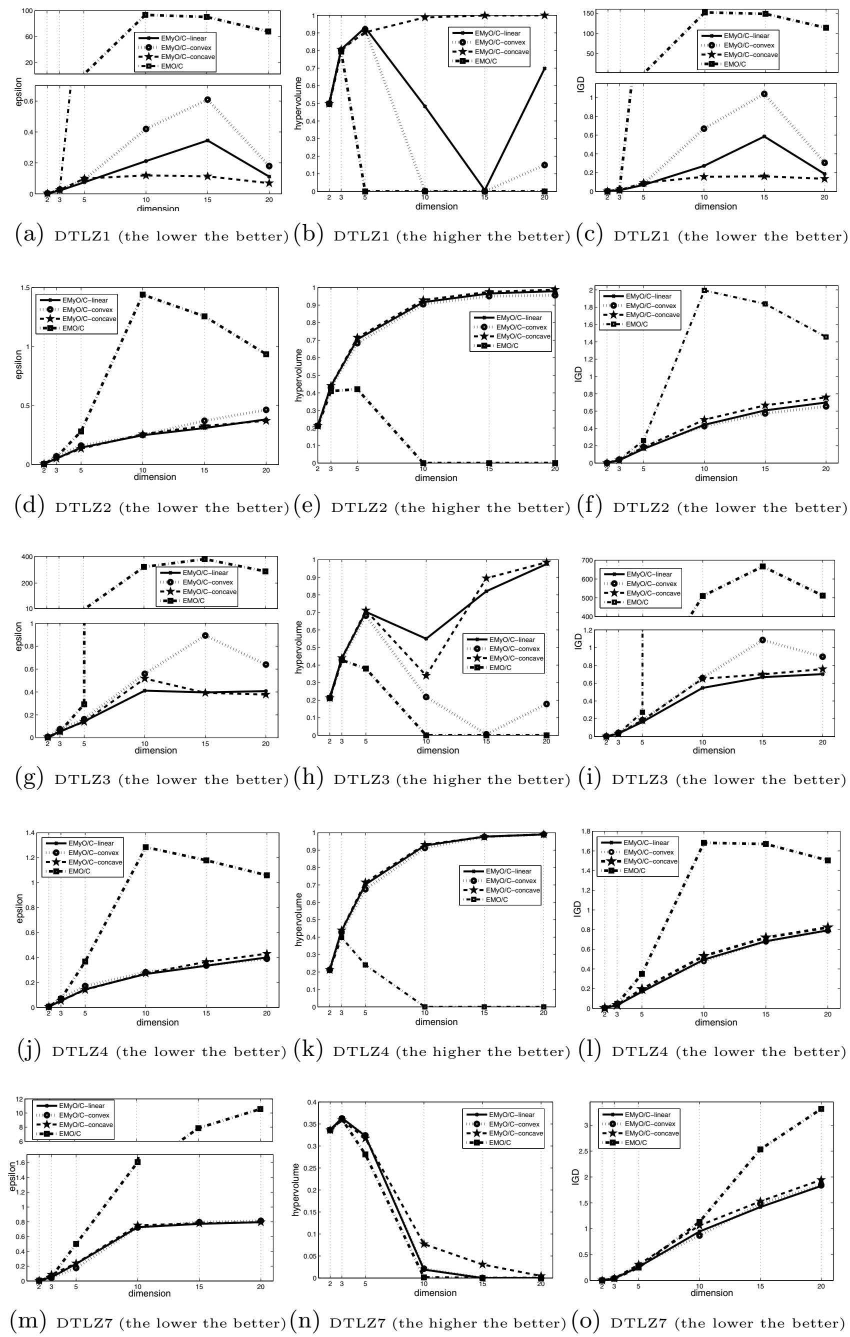
$$\bar{f}_i = f_i^p \quad \forall i \in \{1, \ldots, m\}, \tag{8}$$

where $p \in (0, \inf)$ is a parameter controlling the shape, $f_i$ is the value obtained in (7), and $\bar{f}_i \in (0, 1)$ is the resulting value. For $p > 1$ solutions in $\mathcal{F}_l$ are projected onto a convex shape, for $p < 1$ solutions in $\mathcal{F}_l$ are projected onto a concave shape. According to $p$, we define three variants of EMyO/C:

1. EMyO/C-linear ($p = 1$).
2. EMyO/C-convex ($p = 2$).
3. EMyO/C-concave ($p = 0.5$).

We run these three variants with the aforementioned settings, including an EMO algorithm referred as EMO/C. The difference between EMyO/C and EMO/C is that the latter performs clustering on the original objective vectors.

Figure 1 shows the graphical representation of the median values of the quality indicators obtained by the EMyO/C variants and EMO/C on the benchmark functions with varying dimensions. From the figure, it can be seen that EMO/C performs poorly on high-dimensional problems, having zero hypervolume and large values of the $\epsilon+$ and IGD indicators. All of the EMyO/C variants have quite similar performance on DTLZ2,4,7. However, the three variants perform differently on multimodal problems. EMyO/C-concave works better on DTLZ1,




(a) DTLZ1 (the lower the better) (b) DTLZ1 (the higher the better) (c) DTLZ1 (the lower the better)

(d) DTLZ2 (the lower the better) (e) DTLZ2 (the higher the better) (f) DTLZ2 (the lower the better)

(g) DTLZ3 (the lower the better) (h) DTLZ3 (the higher the better) (i) DTLZ3 (the lower the better)

(j) DTLZ4 (the lower the better) (k) DTLZ4 (the higher the better) (l) DTLZ4 (the lower the better)

(m) DTLZ7 (the lower the better) (n) DTLZ7 (the higher the better) (o) DTLZ7 (the lower the better)

**Fig. 1.** Performance comparison of EMyO/C-linear, EMyO/C-convex, EMyO/C-concave, and EMO/C on the DTLZ1-4,7 test problems. The plots present the median values of the epsilon (left-hand side), hypervolume (center), and IGD (right-hand side) indicators over 30 runs.

**Table 3.** Mean ranks achieved by the EMyO/C variants and EMO/C. The superscripts 1, 2, 3, and 4 indicate whether the respective algorithm performs significantly better than EMyO/C-linear, EMyO/C-convex, EMyO/C-concave, and EMO/C, respectively.

| Indicator | EMyO/C-linear | EMyO/C-convex | EMyO/C-concave | EMO/C |
|---|---|---|---|---|
| $\epsilon$+ | $1.63^{2,4}$ | 2.90 | $2.07^{4}$ | 3.40 |
| HV | $1.87^{2,4}$ | $2.92^{4}$ | $1.50^{2,4}$ | 3.72 |
| IGD | $1.47^{2,3,4}$ | $2.57^{4}$ | $2.43^{4}$ | 3.53 |

whereas EMyO/C-linear and EMyO/C-concave produce different performance in different dimensions. The three variants perform better on DTLZ1 with $m = 20$ than with $m = 15$ due to the smaller number of distance parameters in the former. The obtained results reveal that performing the clustering on different shapes not only affects the distribution of solutions but the convergence and entire performance of the algorithm. Thus, controlling the parameter $p$ can be beneficial for search.

Finally, Table 3 presents the mean ranks and statistical comparison for the performed experiments. It can be seen that the EMyO/C variants are statistically better than EMO/C regarding all three indicators, except for EMyO/C-convex concerning the $\epsilon$+ indicator. EMyO/C-linear gives the best results with respect to the $\epsilon$+ and IGD indicators, whereas EMyO/C-concave performs the best with regard to the HV indicator.

## 4 Conclusions

In this paper, we proposed a clustering-based selection scheme to guide the search in high-dimensional objective spaces. The experimental results obtained on problems with up to 20 dimensions reveal that the proposed scheme is capable of dealing with many-objective problems, producing a highly competitive performance when compared with the state-of-the-art algorithms. Furthermore, we discussed different variants of the proposed approach, showing their advantages and the relevance of the proposed approach.

As future work, we intend to extend the proposed selection to the domain of GA-based algorithms, developing an effective parent selection mechanism. Further, the self-adaptation of the parameter controlling the shape is another promising direction, as well as calculating the distances in different spaces can bring new opportunities.

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