Gaussian Mixture Model Based Non-Local Means Technique for Mixed Noise Suppression in Color Images

Maria Luszczkiewicz-Piatek

University of Lodz, Faculty of Mathematics and Computer Science,
Department of Applied Computer Science, Banacha 22, 90-238 Lodz, Poland
mluszczkiewicz@math.uni.lodz.pl

Abstract. In this paper a new approach to reduction of a mixture of Gaussian and random impulse noises in color images is presented. The proposed filtering scheme is based on the application of the Bilateral Filter in order to address the problem of impulse noise reduction by determining the rate of region homogeneity, for calculating the weights needed for the Non-Local Means (NLM) averaging operation. Gaussian Mixture Model approach is applied for determining similarity between local image regions. The proposed solution is capable to successfully suppress the mixed noise of various intensities, at a lower computational cost than NLM method, due to the adaptive choice of size of search window for similar local neighborhoods. Experimental results prove that the introduced design yields better results than the Non-Local Means and Anisotropic Diffusion techniques in the case of color images contaminated by strong mixed Gaussian and impulsive noise.

1 Introduction

Over the recent years, many spectacular technological achievements have revolutionized the way information is acquired and handled. Nowadays, more than ever, there is an exponentially growing number of color images being captured, stored and made available on the Internet and therefore the interest in color image enhancement is rapidly growing. Quite often, color images are corrupted by various types of noise introduced by malfunctioning sensors in the image formation pipeline, electronic instability of the image signal, faulty memory locations in hardware, aging of the storage material, transmission errors and electromagnetic interferences due to natural or man-made sources. The problem of the noise reduction is one of the most frequently performed image processing operation, as the enhancement of images or video streams corrupted by noise, is essential stage in order to facilitate further image processing steps.

Let us note, that there exist large collection of efficient methods designed to remove Gaussian noise. The significant leap was made by proposing Non-Local Means Filter (NLM) [1]. This idea is based on the estimation the original image using weighted mean along similar local patches. This filter is very efficient when

R.S. Choraś (ed.), *Image Processing & Communications Challenges 6*,
Advances in Intelligent Systems and Computing 313, DOI: 10.1007/978-3-319-10662-5_10

applied for restoring images corrupted by Gaussian noise, but fails in the presence of distortions introduced by impulsive noise. Similarly, the problem of impulse noise removal was also explored by many approaches e.g. [3,4,7].

Let us consider that although methods proved to be very effective in removing Gaussian noise generally fail when impulse noise is present. Moreover, methods successfully dealing with impulse noise usually lose their effectiveness when applied for other noise type removal.

In recent years a few filters were proposed for removing of a mixture of Gaussian and impulse noises [5,6,8], although such noises can take place quite often. One of the approaches to this problem is ROAD statistics [2] applied for detecting noisy pixels combined with Bilateral Filter [13] for Gaussian noise removal, resulting in Trilateral Filter (TriF) and its variations e.g. [7].

This work is focused on restoration of images corrupted by mixed Gaussian and impulse noise using the concept of the adaptive weighted averaging (i.e. NLM approach), where assigned weights help to determine the outlying observations, decreasing their influence on the filtering result based on the region homogeneity assessment, evaluated using Bilateral Filter approach.

The reminder of this paper is organized as follows. Section 2 introduces the concept of region homogeneity assessment using Bilateral Filter approach. In Section 3 the similarity between image patches using Gaussian Mixture Model is introduced. Non-Local Means filtering scheme is introduced in Section 4. Experimental setup and results are presented and discussed in Sections 5 and 6 accordingly.

2 Region Homogeneity

The proposed solution, based on NLM approach, averages image pixels on the basis of the similarity of their surrounding. However, if it would be the only criterion for evaluating new pixel value, only the influence of the Gaussian noise will be suppressed. Let us note, that even if the pixels, whose local neighborhood is similar to the local neighborhood of the pixel which is currently being processed are similar, the calculated average, can be significantly influenced if averaged pixels are corrupted by impulse noise. Therefore, the proposed approach overcomes this drawback by identifying those pixels which are outlying from their local surrounding. Thus, even if their local neighbourhood is similar to that of the processed pixel, they should be considered less important during the averaging process. This is achieved by the application of the *homogeneity maps* [10] based on Bilateral Filter approach. In details, each pixel is represented in the weight map, as the associated coefficient reflecting its similarity to the color of neighboring pixels with the relation to the spatial distance between them. In consequence, this approach assigns significantly larger weights to pixels belongings to large color regions, than to small ones, often reflecting unimportant details, artifacts or impulse noise. Thus, this approach provides information if analyzed pixel is significantly different than its neighbours, indicating that it can be corrupted by e.g. impulse noise or it is edge pixel.

In details, the weights assigned to the pixel at position (x, y) are computed according to the following scheme:

$$w_{x,y} = \frac{1}{n} \sum_{(i,j)\in W} \exp\left(-\frac{\| \boldsymbol{c}_{x,y} - \boldsymbol{c}_{i,j} \|}{h}\right)^{k_1} \cdot \exp\left(-\frac{d_{i,j}}{\delta}\right)^{k_2}, \quad (1)$$

where $\boldsymbol{c}_{i,j}$ and $\boldsymbol{c}_{x,y}$ denote the color pixels at positions (i, j) and (x, y) respectively, h is the color difference scaling parameter, $d_{i,j}$ is the Euclidean distance between the pixel at position (i, j) and (x, y), which is the center of the filtering window W and δ is a spatial normalizing parameter equal to the diameter of the square filtering window. The number of pixels n in W was set to be equal to 10% of the total number of pixels in the image and we assumed $k_1 = k_2 = 2$. For the color difference evaluation the $CIEDE2000$ color similarity measure [11] was used.

Fig. 1 presents the homogeneity maps, evaluated using the formula 1, for exemplary original images corrupted with mixture of Gaussian ($\sigma = 10$) and impulse ($p = 0.1$) noise. The lower values indicate that pixel neighbourhood is non-homogenous in terms of color.

Fig. 1. The color homogeneity maps (right in each pair) for original images corrupted with mixture of Gaussian ($\sigma = 10$) and impulse noise (p=0.1) (left in each pair)

3 Gaussian Mixture Modeling

Next step of the proposed methodology is to determine the similarity between pixel region on the basis of their Gaussian Mixture Models (GMM). Each local neighborhood is represented by GMM parameters. These parameters are compared in order to determine the region similarity.

The very important decision concerning the color image data modeling is the choice of the color space suitable for the retrieval experiments. In this paper the results were evaluated using the $CIE\ La^*b^*$ color space[12].

The first step in evaluating region similarity is to construct the histogram $H(x, y)$ in the $a-b$ chromaticity space defined as $H(x, y) = N^{-1}\sharp\{a_{i,j} = x, b_{i,j} = y\}$, where $H(x, y)$ denotes a specified bin of a two-dimensional histogram with a component equal to x and b component equal to y, the symbol $\sharp$ denotes the number of samples in a bin and N is the number of color image pixels.

The next stage of the presented technique is the modeling of the color histogram using the Gaussian Mixture Models (GMM) and utilizing the

Expectation-Maximization (EM) algorithm for the model parameters estimation, [13].

Let us assume the following probabilistic model: $p(x|\Theta) = \sum_{m=1}^{M} \alpha_m p_m(x|\theta_m)$,
which is composed of M components and its parameters are defined as: $\Theta = (\alpha_1, \ldots \alpha_M, \theta_1, \ldots, \theta_M)$, with $\sum_{m=1}^{M} \alpha_m = 1$. Moreover, each p_m is a function of the probability density function which is parameterized by θ_m. Thus, the analyzed model consists of M components with M weighting coefficients α_m. Finally after derivations shown in [13] the model parameters are defined as:

$$\alpha_m^{k+1} = N^{-1} \sum_{i=1}^{N} p(m|x_i, \Theta^k), \quad \mu_m^{k+1} = \frac{\sum_{i=1}^{N} x_i \cdot p(m|x_i, \Theta^k)}{\sum_{i=1}^{N} p(m|x_i, \Theta^k)}, \tag{2}$$

$$\upsilon_m^{k+1} = \frac{\sum_{i=1}^{N} p(m|x_i, \Theta^k)(x_i - \mu_m^{k+1})(x_i - \mu_m^{k+1})^T}{\sum_{i=1}^{N} p(m|x_i, \Theta^k)} \quad , \tag{3}$$

where μ and υ denote the mean and variance, m is the index of the model component and k is the iteration number. The E (Expectation) and M (Maximization) steps are performed simultaneously, according to (2) and (3) and in each iteration, as the input data we use parameters obtained in the previous one, until convergence. The number of components are initially set to 20 and decreased in each iteration if associated weight α_m is less than 0.01.

4 Non-Local Means Algorithm

Let us assume the following image formation model $F(x, y) = I(x, y) + n(x, y)$, where x, y represents the pixel coordinates, I is the original image, F is the noise infested image, and $n(x, y)$ is the additive noise component respectively. The noise component is white, Gaussian noise distributed with zero mean and variance σ^2. The goal of the denoising is to obtain an estimate $\widehat{I}(x, y)$ from $I(x, y)$ using the observed image $F(x, y)$. The NLM algorithm computes the estimate of the original image according to the following equation: $\widehat{I}(x, y) = \frac{\sum_{k,l \epsilon I} W_{x,y}(k,l) F(k,l)}{\sum_{k,l \epsilon I} W_{k,l}}$. Finally, the evaluated filter output $\widehat{I}(x, y)$ is a weighted average of the image pixels. Let us underline that the NLM approach does not restrict the weighted average to only a local neighborhood of the processed pixel, but pixels of the entire image can be taken into account. The weights W are computed based on the differences of the two local regions centered at coordinates x, y and k,l as follows:

$$W_{x,y}(k, l) = \exp \frac{-\|F(\Omega_{x,y} - \Omega_{k,l})\|_a^2}{\varphi^2}, \tag{4}$$

where $\Omega_{x,y}$ and $\Omega_{k,l}$ are two windows centered at x, y and k, l respectively, $\| \cdot \|_a^2$ is the Gaussian weighted Euclidean distance, a is the standard deviation of the Gaussian kernel and φ is a parameter controlling the degree of filtering, chosen experimentally, [1]. Let us note that the number of arithmetic operations needed is quite large. The solution proposed in this paper adaptively chooses the size of region S of searching of similar local neighborhoods $\Omega_{x,y}$ and $\Omega_{k,l}$ on the basis of the pixel nature (e.g. whether it is impulse noise corrupted or it is edge pixel).

5 Experimental Setup

In order to suppress noise using the proposed *GMM-NLM* technique firstly the homogeneity map should be calculated using the scheme described in previous Section 2. Then the obtained values are analyzed. If the homogeneity map value $w(x,y)$ associated with the image pixel $F(x,y)$ is low it indicates that this pixel can be either influenced by impulse noise or is edge pixel. In both cases low value indicates that pixel is distinctively different that its surrounding. For noise suppression purposes it is important to differentiate between pixels which are impulse noise infested (need to be corrected) and belongs to edge (need to be preserved). Thus, the *GMM-NLM* method examines the surrounding $\Upsilon_{x,y}$ of that pixel in homogeneity map in form of comparison between median value of the surrounding and the analyzed value. In case of these two values are distinctively different the search window $S_{x,y}$ in which the image patches $\Omega_{k,l}$ are compared to $\Omega_{x,y}$, is enlarged in order to find and take into account more similar local regions. On the contrary, if analyzed image pixel belongs to the edge, the enlargement of the search region $S_{x,y}$ will not improve the denoising results, because distant pixels, separated by the detected edge, possibly will not have similar neighborhoods to that of the analyzed pixel. Such approach is motivated by computation savings, i.e. less local regions to visit and compare.

The search window $S_{x,y}$ ($s_{x,y} \times s_{x,y}$) size is calculated on the basis of the formula $s_{x,y} = round(\xi \cdot r \cdot w(x,y) + \eta + 3)$ where ξ is the smaller of the width and length dimensions of the analyzed image, r is the scale ratio (here assumed $r = 5\%$), $w(x,y)$, $w\epsilon(0,1)$ is the homogeneity coefficient evaluated using Eq.1, η is coefficient related to nature of pixel local non-homegeneity (i.e. $\eta = \kappa \cdot [w(x,y) - \tilde{\Upsilon}_{x,y}]$, for $\kappa = 10$). Then, the comparison of the local neighborhoods $\Omega_{k,l}$ for the image pixels in the search region $S_{x,y}$ to local neighborhood $\Omega_{x,y}$ is evaluated. The size of the image patch $\Omega_{x,y}$ ($\o_{x,y} \times \o_{x,y}$) is also related to homogeneity maps values $w_{x,y}$, such that $\o_{x,y} = round(10 \cdot w_{x,y} + 3)$. Each local neighborhood Ω is is modeled using *GMM* technique described in Section 6 producing vector of model parameters compared by *Earth Mover's Distance* [14]. The measured *EMD* distance is defined as a minimum amount of work needed to transform one histogram into the other. As this method operates on signatures and their weights using GMM, we assigned as signature values the *mean* of each component and for the *signature weight* the weighting coefficient of each Gaussian in the model. The nature of the *EMD* measure, accepting slight shifts in color histogram rather than bin-by-bin comparison, is well suited in case of distortions introduced by Gaussian noise. Having the local patches similarities calculated, the new estimate of the pixel value $\widehat{I}(x,y)$ is evaluated on the basis of the formula 4 with region similarities calculated using approach introduced in this Section, where $\varphi = s$.

6 Noise Suppression Results

In order to test the proposed methodology the mixture of Gaussian and impulse noise were applied to the set of original images shown in Fig. 2. The efficiency of this approach was evaluated in terms of the visual quality of the restored image and also in terms of objective quality measures. Additionally, the proposed solution effectiveness was compared with the denoising NLM [1] and $Anisotropic\ Diffusion$ [15] methods.

Firstly, the proposed filtering scheme was tested on the standard test images: GIRL, LENA and PEPPERS (Fig. 2) corrupted with mixed Gaussian and impulse noise (salt & pepper in each channel). The test images were contaminated by: mixed Gaussian and impulsive noise of $\sigma = 10$ and $p = 0.1$, $\sigma = 20$ and $p = 0.2$, $\sigma = 30$ and $p = 0.3$, where p denotes the contamination probability. The restoration capabilities of the proposed solution in terms of visual quality for mixture noise of $\sigma = 0.1$ and $p = 0.1$ can be seen in Fig. 3. Let us note that significant noise suppression can be observed along with edge preserving. This Fig. also illustrates the ineffectiveness of the Non-Local Means technique when impulse noise is present.

(a) GIRL (b) LENA (c) PEPPERS

Fig. 2. Color test images

Fig. 4 illustrates the noise suppression capabilities of the proposed $GMM-NLM$ technique for mixture of Gaussian noise ($\sigma = 20$ and $\sigma = 30$) and impulse noise of various intensity $p = \{0.1, 0.2, 0.3\}$ in comparison with the original image $PEPPERS$.

The noise removal capabilities of the proposed solution was tested using the Peak Signal to Noise Ratio ($PSNR$) and the Mean Absolute Error (MAE). The Peak Signal to Noise Ratio is given as $PSNR = 20 \cdot \log_{10}\left(\frac{MAX_I}{\sqrt{MSE}}\right)$, where MSE (Mean Squared Error) is defined by $MSE = \frac{\sum_{i=1}^{N}\sum_{k=1}^{3}[O_k(x_i)-J_k(x_i)]^2}{3N}$, where k denotes $k-th$ color channel of analyzed pixel, O_k is original image, J denotes filtered version of original image and N is total number of image pixels. $PSNR$ measure is the impulse noise suppression of analyzed filtering method. The Mean Absolute Error (MAE) is used to estimate filter ability to preserve fine details and is calculated using the following formula: $MSE = \frac{\sum_{i=1}^{N}\sum_{k=1}^{3}[O_k(x_i)-J_k(x_i)]}{3N}$. The $PSNR$ and MAE values calculated for mixture of Gaussian ($\sigma = \{10, 20, 30\}$) and impulse noise ($p = \{0.1, 0.2, 0.3\}$) for test images shown in Fig. 2 are summarized in Fig. 5.

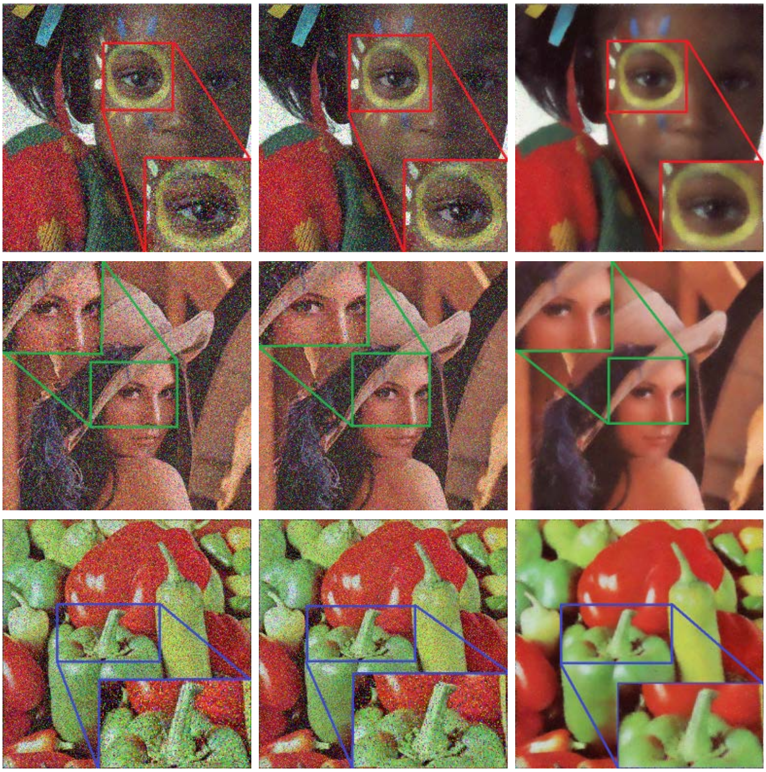

Fig. 3. The comparison between original images corrupted with mixture Gaussian ($\sigma = 20$) and impulse noise (p=0.2) (left) and its denoised versions usin Non-Local Means technique (middle) and proposed GMM-NLM method (right)

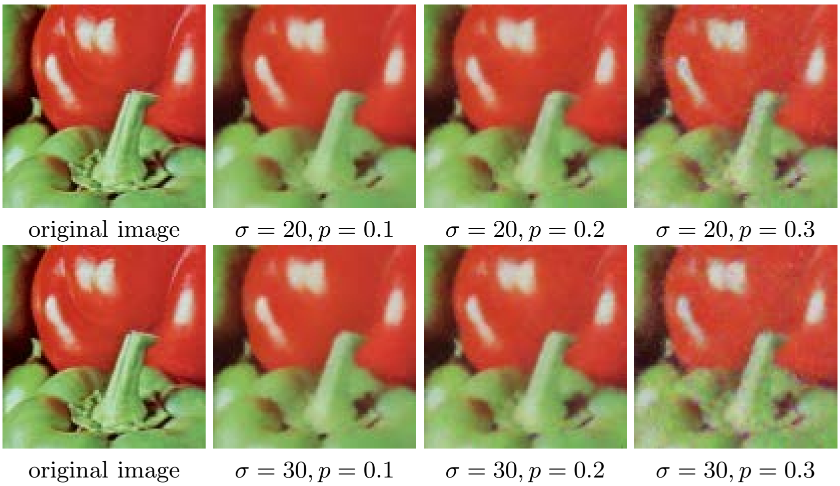

Fig. 4. The noise suppression results evaluated for mixture of Gaussian noise ($\sigma = 20$ in upper row and $\sigma = 30$ for bottom row) and impulse noise ($p = \{0.1, 0.2, 0.3\}$)

LENA						
	PSNR			MAE		
p−σ	10	20	30	10	20	30
0.1	25.07	24.79	23.60	9.03	9.44	11.43
0.2	24.79	24.70	23.30	9.56	9.95	12.34
0.3	24.60	24.26	22.48	10.01	10.82	14.00

PEPPERS						
	PSNR			MAE		
p−σ	10	20	30	10	20	30
0.1	24.76	23.67	21.82	8.32	9.67	13.06
0.2	24.22	23.42	21.40	9.15	10.57	14.45
0.3	23.81	22.92	20.77	10.14	11.85	16.37

GIRL						
	PSNR			MAE		
p−σ	10	20	30	10	20	30
0.1	25.31	24.33	22.46	8.66	9.73	12.61
0.2	25.89	24.10	22.15	9.49	10.59	14.05
0.3	24.44	23.57	21.36	10.43	11.94	16.30

Fig. 5. The $PSNR$ and MAE values calculated for mixture of Gaussian ($\sigma = \{10, 20, 30\}$) and impulse noise ($p = \{0.1, 0.2, 0.3\}$) for test images shown in Fig. 2

	LENA						PEPPERS					
	PSNR			MAE			PSNR			MAE		
(p, σ)	GMM	NLM	AD	GMM	NLM	AD	GMM	NLM	AD	GMM	NLM	AD
(0.1, 10)	25.074	19.51	25.24	9.03	9.22	9.16	24.76	18.54	24.33	8.32	10.49	10.69
(0.2, 20)	24.70	20.22	24.01	9.95	11.76	11.42	23.42	19.48	22.52	10.57	13.21	14.22
(0.3, 30)	22.48	21.09	22.67	14.08	13.93	14.43	20.717	20.84	20.83	16.37	16.59	18.06

Fig. 6. Comparison of $PSNR$ and MAE values evaluated for proposed solution, Non-Local Means [1] method and Anisotropic Diffusion [15]

The effectiveness of the proposed solution was also tested in comparison with Non-Local Means (NLM) method. In this approach the control parameters depend on the noise intensity and therefore they were experimentally chosen to provide the best possible noise suppression results. Proposed method efficiency was also compared with Anisotropic Diffusion (AD), [15]. The comparison results in terms of $PSNR$ and MAE were summarized in Fig. 6.

7 Conclusions

In this paper a new noise reduction method that combines the Non-Local Means averaging (for Gaussian noise removing) scheme with the region homogeneity assessment (for impulse noise suppression) is introduced. Our new noise reduction method, called GMM-NLM, outperforms significantly the NLM scheme in case of mixture noise, at a lower computational cost due to the adaptive choice of size of search window for similar local neighborhoods and the size of each image patch. Also, the noise parameters do not need to be estimated during the image processing. Moreover, this approach can decrease the computation cost in comparison with classical NLM approach. The proposed method was also compared with $Anisotropic\ Diffusion$ technique yielding better results.

Acknowledgment. This work was supported by the Polish National Science Center (NCN) under the Grant: DEC-2012/05/B/ST6/03428.

References

1. Buades, A., Coll, B., Morel, J.M.: A non-local algorithm for image denoising. In: IEEE Conf. on Computer Vision and Pattern Recognition, USA, vol. 2, pp. 60–65 (2005)
2. Garnett, R., Huegerich, T., Chui, C., Wenjie, H.: A universal noise removal algorithm with an impulse detector. IEEE Transactions on Image Processing 14(11), 1747–1754 (2005)
3. Smolka, B.: Peer group switching fillter for impulse noise reduction in color images. Pattern Recognition Letters 31(6), 484–495 (2010)
4. Nikolova, M.: A variational approach to remove outliers and impulse noise. J. Math. Imag. Vis. 20(1), 99–120 (2004)
5. Tang, K., Astola, J., Neuvo, Y.: Nonlinear multivariate image filtering techniques. IEEE Transactions on Image Processing 4(6), 788–798 (1995)
6. Lukac, R., Smolka, B., Plataniotis, K.N., Venetsanopoulos, A.N.: Vector sigma lters for noise detection and removal in color images. Journal of Visual Communication and Image Representation 17(1), 1–26 (2006)
7. Dong, Y., Chan, R., Xu, S.: A detection statistic for random-valued impulse noise. IEEE Trans. Image Process. 16(4), 1112–1120 (2007)
8. Lamichhane, B.P.: Finite Element Techniques for Removing the Mixture of Gaussian and Impulsive Noise. IEEE Transactions on Signal Processing 57(7), 2538–2547 (2009)
9. Tomasi, C., Manduchi, R.: Bilateral filtering for gray and color images. In: Sixth International Conference on Computer Vision, pp. 839–846 (1998)
10. Luszczkiewicz, M., Smolka, B.: Application of bilateral filtering and Gaussian mixture modeling for the retrieval of paintings. In: Proc. of ICIP, pp. 77–80 (2009)
11. Sharma, G., Wu, W., Dalal, E.N.: The CIEDE2000 color-difference formula: implementation notes, supplementary test data, and mathematical observations. Color Research and Application 30(1), 21–30 (2005)
12. Łuszczkiewicz-Piątek, M.: Which Color Space Should Be Chosen for Robust Color Image Retrieval Based on Mixture Modeling. In: S. Choras, R. (ed.) Image Processing and Communications Challenges 5. AISC, vol. 233, pp. 55–64. Springer, Heidelberg (2014)
13. Bilmes, J.: A Gentle Tutorial on the EM Algorithm and its Application to Parameter Estimation for Gaussian Mixture and Hidden Markov Models. University of Berkeley, ICSI-TR-97-021 (1997)
14. Rubner, Y., Tomasi, C., Guibas, L.J.: The Earth Mover Distance as a Metric for Image Retrieval. International Journal of Computer Vision 40(2), 99–121 (2000)
15. Perona, P., Malik, J.: Scale-space and edge detection using anisotropic diffusion. IEEE Transactions on Pattern Analysis and Machine Intelligence 12(7), 629–639 (1990)