

Chapter 6

Standard Practice of Statistics

6.1 Introduction

Most commonly used statistical methods, interval estimation (confidence intervals), estimation, hypothesis tests, and significance tests have as justification their properties under repeated sampling. This is the frequency interpretation of statistical methods and is the basis for much (most) of the statistical methods commonly applied to data in research and practice. This chapter is based on Table 2.1 in Royall. What we have shown in Chaps. 3–5 is that many of the standard statistical results are misleading.

6.2 Frequentist Statistical Procedures

6.2.1 Estimation

- **Procedure**

An estimator $t(X)$ depending on a random variable X

- **Property of Procedure**

Determined by the distribution of X

The expected value of $t(X)$ is θ

The standard error of $t(X)$ is σ

$$E[t(X)] = \theta, \quad \text{var}[t(X)] = \sigma^2$$

- **Observation** Realized value of a random variable X

$$X = x, \text{ assumed to be generated by } f_\theta$$

- **Result of Procedure:** Fixed by observing $X = x$

An estimate, $t(x)$
an estimate of variability, $\sigma(x)$

- **“Usual” Interpretation:** Property used to interpret results
 1. The observation x provides evidence that θ is near $t(x)$.
 2. The smaller $\sigma(x)$, the stronger the evidence.

6.2.2 Confidence Intervals

- **Procedure**

An interval $[\ell(X), u(X)]$ depending on a random variable X

- **Property of Procedure**

Determined by the distribution of X

1. We are 95 % confident that the random interval $[\ell(X), u(X)]$ will contain θ .
2. The confidence coefficient is

$$P[\ell(X) \leq \theta \leq u(X)] = 0.95$$

- **Observation:** Realized value of a random variable X

$X = x$, assumed to be generated by f_θ

- **Result of Procedure:** Fixed by observing $X = x$

An interval $[\ell(x), u(x)]$

- **“Usual” Interpretation:** Property used to interpret results
 1. The observation x provides evidence that θ is in the interval $[\ell(x), u(x)]$.
 2. Large confidence coefficient means strong evidence.

6.2.3 Hypothesis Testing

- **Procedure**

A test $\delta(X)$ depending on a random variable X

- **Property of Procedure**

Determined by the distribution of X

1. The Type I error probability is α .
2. The Type II error probability is β .

- **Observation:** Realized value of a random variable X

$$X = x, \text{ assumed to be generated by } f_\theta$$

- **Result of Procedure:** Fixed by observing $X = x$. A selected hypothesis:

$$H_1 \text{ if } \delta(x) = 1, H_0 \text{ if } \delta(x) = 0$$

- **“Usual” Interpretation:** Property used to interpret results

1. The observation x provides evidence in favor of the selected hypothesis.
2. Small α and β mean strong evidence.

6.2.4 Significance Tests

- **Procedure**

A statistic $t(X)$ depending on a random variable X

- **Property of Procedure**

Determined by the distribution of X

The probability of extreme values of t

- **Observation:** Realized value of a random variable X

$$X = x, \text{ assumed to be generated by } f_\theta$$

- **Result of Procedure:** Fixed by observing $X = x$

1. A P -value defined by $P[t(X) \geq t(x)]$.
2. The P -value is calculated under the assumption that H_0 is true.

- **“Usual” Interpretation:** Property used to interpret results.

1. The observation x provides evidence against the null hypothesis.
2. The smaller the P -value the stronger the evidence.