

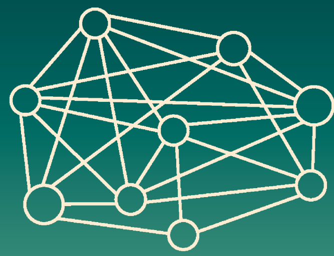

Re-Thinking Time at the Interface of Physics and Philosophy

The Forgotten Present

Albrecht von Müller
Thomas Filk
Editors

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The Forgotten Present

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Editor's Preface to This Volume

Our most immediate and intimate experience of the world, the experience of a present is not accounted for in physics, and even modern philosophy tends to avoid this subject. This is in sharp contrast to classical philosophy: the treatises of Aristotle and Augustine of Hippo on time and the notion of "now" belong to the deepest discussions of this subject even until today. While Einstein might have hoped that "the present" will find its place in the theory of relativity, he later, in a discussion with Carnap, expressed his disappointment that he was not able to realize this hope.

In October 2006 and in May 2010, the Parmenides Foundation organized two workshops dedicated to the subject of "The Present." In both cases, scientists from Physics and Philosophy presented and discussed their ideas about how a theory of "the present" may look like or how it can be incorporated into the existing theories. The participants as well as the subjects were mainly from physics and philosophy, and the workshop aimed to foster the exchange between these two disciplines about the concept of "time."

This volume is not meant to be a Proceedings volume. Many of the participants of the workshops contributed with articles related to their presentations at the workshops; however, we also contacted other scientists, who for various reasons could not take part in the workshops, and invited them to contribute to this volume. In this way, we hope to have managed to collect a remarkable series of articles from many renowned scientists working in this field.

It was not easy to order these articles in a sequential structure, mainly because the subject has been addressed from various perspectives. However, we think that the overall arrangement starting from more mathematical and, in particular, relational concepts of space and time and developing towards more philosophical ideas helps the reader to find his or her way through the various ideas.

As editors of this volume, we refrain from giving our own view on this subject here, as it could only be biased and we also contributed articles to this volume. The first by one of us, Albrecht von Müller, offers a fundamental revision of the notion of time in which the phenomenon of the present moves to its center. In a sense, the reader may consider this article as describing the opinion of the editors, which replaces an extended summary here in the preface. The next two articles by Andrej Nikonov and Thomas Filk emphasize relational concepts of space and time in general. To a certain degree, they describe a physical model of the general philosophical ideas outlined in the article of Albrecht von Müller (this holds in particular for the contribution by Andrej Nikonov).

The two articles by Dustin Lazarovici and Domenico Giulini deal with more mathematical aspects of classical (in the sense of non-quantum) space–time. The article by Domenico Giulini addresses the fundamental problems related to the mathematical description and the operational meaning of space–time in relativity, while the article by Dustin Lazarovici deals with the problem of locality of interactions in a relativistic theory as well as nonlocal extension and their relation to the notion of a "Now."

Quantum aspects of space–time are addressed in the article by Mohammad Bahrami, Angelo Bassi, Sandro Donadi, Luca Ferialdi, and Gabriel León. Even though space–time is still treated as a classical "background," the collapse of quantum states is attributed to certain collapse-centers distributed in space–time. Irreversibility, the "arrow of time," is built into the model right from the beginning, and the collapse may be a physical correlate to what we call a "Now."

The contributions by Basil Hiley and Teijinder P. Singh emphasize a more algebraic approach towards concepts of time and the present. In the case of Basil Hiley's article, the point of departure is an algebraic theory of "processes" inspired by Grassmann algebras, while Teijinder P. Singh starts from non-commutative geometries.

An almost continuous bridge to the more philosophically oriented articles are two contributions about the role of probability in physics, in particular in quantum physics, and its relation to the notions of time and "now." While the article by Philippe Blanchard is still very mathematical and emphasizes the non-commutativity of observables and its relation to non-boolean logic, the contribution of Michael Drieschner deals with the relation of probability to the concepts of future and past. This article also contains a nice survey of historical approaches to the concept of time and the present.

The contribution of Michael Esfeld concentrates on the philosophical notion of presentism (the only form of reality is the present—as opposed to eternalism). In contrast to claims from physics (in particular general relativity), the article argues that presentism cannot be proven to be wrong, neither by physical nor by metaphysical arguments.

Finally, in his article, Hartmann Römer touches upon the difference between the physical (outer) time and the experienced (inner) time. The conscious perception of the observer plays an important role for the existence of inner time and "now," and quantum mechanics (and maybe already thermodynamics) requires this conscious observer.

We would like to thank all scientists who contributed to this volume as well as all participants of our workshops for their presentations and stimulating input.

Pullach, Germany Albrecht von Müller
Freiburg, Germany Thomas Filk
Spring 2014

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The Forgotten Present

A Philosophical Invitation to Rethink Time and Reality and to Discover the Autogenetic Nature of Our Universe

Albrecht von Müller

Abstract This essay proposes a radical re-thinking of time and reality. "With Kant beyond Kant" it is argued that all theories are based on categorial foundations. These are interrelated symmetry breakings that enable, but by enabling also constrain everything that we can think thereafter. Both the enigmatic character of quantum physics (QP) and its incompatibility with general relativity theory (GRT) are rooted in our unawareness of these categorial underpinnings. Metaphorically speaking, this unawareness results in a categorial "facticity imprisonment" of our thinking. We inadvertently reduce reality to its factual footprints and time to its sequential structure. Both are correct and important aspects of time respectively reality. But they provide only a partial picture. In order to overcome the rift between the two foundational theories of modern physics, it is necessary to unearth the different categorial underpinnings of the two theories, and to develop a richer, overarching categorial framework.

Facts are just the traces of the actual taking place of reality, left behind on the co-emerging canvas of local spacetime. The actual taking place of reality occurs still in the primordial, still non-local form of time, for which the notion "timespace of the present" (TSP) is introduced. Interestingly enough, Albert Einstein already complained about the "painful, but inevitable abandonment" of the present in physics vis-à-vis Rudolf Carnap (Carnap's intellectual bibliography. In: Schilpp PA (ed) The Philosophy of Rudolf Carnap. Open Court Publishing, La Salle, pp 3–84, 1963). A necessity to abandon the present exists, however, only as long as time is reduced to its linear-sequential aspect, i.e., as long as the present is erroneously reduced to a *point-like now*.

By recognizing the non-local TSP as the primordial form of physical time, the sequential structure of time becomes a derivative feature—and with this step a radically novel way to interpret QP and its relation to GRT become feasible. The two theories address different chrono-ontological portraits of reality. QP addresses the actual taking place of reality, i.e., the actual "coming into being" of facts, as it

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occurs still in the TSP. Classical and relativistic physics, instead, address already the factual portrait of time and reality.

Quantum physical reduction is the bridge from the first to the latter format of time and reality. But this can't be understood as long as one recognizes only the second chrono-ontological portrait, the factual aspect of time and reality. There exists, however, also a transition in the opposite direction, i.e., from the factual portrait of time and reality back to their "statu nascendi" portrait. In the singularities of GRT the local spacetime fabric melts away—driven by the strong self-referentiality of gravity. In this way, quantum physical reduction and the singularities of GRT turn out to describe inverse transitions: Into and out of the local spacetime format of time and reality.

In the novel account, also our human experience of a present can be seen in a radically novel light. It no longer needs to be derogated as a subjective confabulation, distorting the correct perception of physical time. Instead, it turns out that our perception of the present is the most advanced *adaptation of cognitive evolution to the actual taking place of reality*—as it occurs still in the TSP. This new view is strongly supported by considering Darwinian evolution. In neurobiological terms, the experience of the present—and its twin, the phenomenon of explicit self-awareness—are the two most demanding and "costly" endeavors of the human brain. Wouldn't they bring us to a more accurate, and, thus, more powerful appreciation of time and reality, they would have never developed in the first place or they would have, at least, been swept away by evolutionary selection pressures.

The novel conceptual framework becomes possible by unearthing the categorial foundations of our theories, i.e., by recognizing their nontrivial structure of these foundations and by appreciating their crucial role for all subsequent theorizing. For Kant the basis of his epistemological considerations was Newtonian physics. Modern physics progressed far beyond that—but hitherto it had not taken its own categorial foundations into account. Only by unearthing them, and by making them part of our theories, it will be possible to overcome the impasse of modern physics. The theory of an autogenetic universe offers the conceptual framework for that.

Introductory Remark

The hypothesis introduced and elaborated in the following section is as simple as it is radical (Fig. 1):

> We live in an autogenetic universe. Autogenesis means that something unfolds out of itself, within itself, and toward itself. The latter occurs when in human thinking, our universe starts to become aware of itself.
>
> Unfortunately, modern science is erected on categorial foundations that are incompatible with the phenomenon of autogenesis and its conceptual counterpart, the structure of strong self-referentiality.
>
> The autogenetic universe doesn't unfold within local spacetime. The emergence of the latter is part of its unfolding.
>
> Constellatory self-unfolding is the most fundamental, all-pervasive feature of an autogenetic universe. Facticity is an important, but derivative aspect of reality within it.

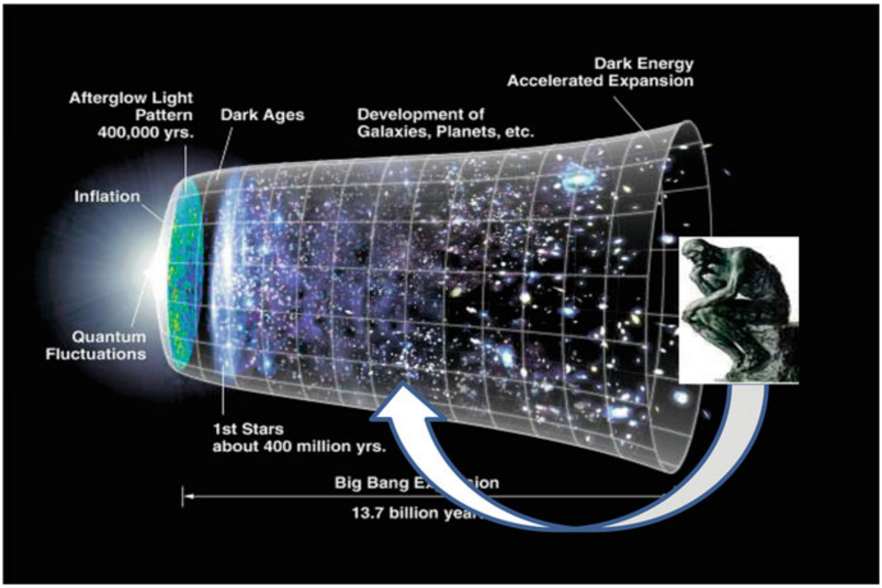


Fig. 1 An Autogenetic Universe. NASA/WMAP Science Team. *Timeline of the universe*. Ca. 2006. [Modified by Ryan Kaldari 2010]. http://en.wikipedia.org/wiki/File:CMB_Timeline300_no_WMAP.jpg (09.12.14). Rodin, Auguste. *Le Penseur*. Bronze. H. 180 cm, B. 98 cm, D. 145 cm. Museé Rodin. Paris 1903. [Erstentwurf 1880]. http://www.musee-rodin.fr/fr/collections/sculptures/le-penseur (09.12.14)

Ever since an early, immediately post-Socratic symmetry breaking our thinking has become increasingly entrapped in a one-sided, structurally deprived notion of time and reality. Aristotle's unique achievement was to pave the way for almost the entire classical portrait of time and reality in a single stroke. The downside of this fantastic achievement was that over time—and especially with the establishment of modern science—this portrait turned increasingly into a rigid and self-immunizing filter against all other forms of conceptualizing time and reality. Due to the self-immunizing character of this early symmetry breaking, people increasingly understood an important but partial and derivative aspect of time—namely, its sequential structure—as the essence of time. *Uno actu*, they overlooked the primordial form and function of time: to offer the platform or scene in which all of reality can actually "take place"—and by this become part of local spacetime.

According to the novel account of time and reality, most of the modern science is characterized by an unrecognized "facticity imprisonment." The huge price that we pay for this cognitive deprivation is the inability to cope with two closely intertwined phenomena: strong self-referentiality (in a Gödel sense) and "autogenesis," i.e., the pre-causal, constellatory self-unfolding through which something "becomes what it is." The observation of quantum physical reduction was the first instance in which modern science came so close to the very "fabric of reality" that this primordial feature of reality—its ubiquitous autogenetic unfolding—became undeniable. What makes quantum physics so enigmatic and controversial is that in trying to explain what we observe, we prematurely draw on the categorial apparatus that belongs only to the factual aspect of reality: the linear-sequential structure of time, causality, clear-cut separability of observer and observandum, and a Boolean predication structure. However, this categorial framework does not allow us to understand what we see: The autogenesis of reality. Consequently, quantum physics appeared to be a huge, unresolved enigma since inception.

If this argument holds water, overcoming the present impasse in modern physics requires *digging even deeper and unearthing the categorial foundations of our present theories*. Only by going this "philosophical extra-mile" it becomes possible to understand what is actually addressed by QP and general relativity theory (GRT) and how they can be united in an overarching framework that consists of three complementary portraits interrelated in the topological structure of Borromean rings. The task of this paper is to give an initial overview of the main steps in this fundamental re-thinking of both our notions of time and of reality.

Three points should be mentioned briefly in order to avoid misunderstandings.

First, this paper is essentially an "essay" in the most literal sense, i.e., an *attempt* to give a brief, yet synoptic overview on a radically new account of time and reality. The usual scientific credibility boosters, such as a flood of supportive footnotes, do not make much sense in such an effort. Instead, it must mainly rely on the endogenous convincing power of the presented constellation of arguments. For really foundational frameworks there exist no external truth criteria. They must obviously cover all of the known empirical facts, but this is not sufficient. A foundational framework is essentially a self-contained conceptual entity: It gains its stability and consistency—somewhat comparable to fullerenes in chemistry—only from the (ideally) elegantly simple configuration of its constitutive arguments.

The second point is closely related to the first. This paper frequently draws on one specific module of the *visual reasoning language* EIDOS. The task of EIDOS is to support the human brain in dealing with complexity by combining ratiomorphic clarity or analytical precision with a synoptic representation of an issue in its entirety. The first quality is the essence of rationality, which can be understood as the ability of achieving increasingly detailed analytical distinctions. But this essentially Boolean mode of connecting asymptotically well-defined propositions never allows for appreciating an issue in its entirety, i.e., in its ongoing constellatory self-unfolding. Appreciating the latter requires a completely different way of thinking, i.e., connecting mental content. This complementary mode of thinking is also based on certain rules and principles, although these are very different in comparison to those that apply in Boolean predication. This paper introduces the notion of a Logic of Constellations (LoC) for this complementary set of rules and principles regarding how to connect mental content.

When seen from the theory of an autogenetic universe (TAU), rationality is the cognitive counterpart of the factual aspect of reality. In contrast, LoC allows us to appreciate phenomena characterized by strong self-referentiality and autogenetic unfolding. In its full philosophical sense, reason can be understood as a smooth interplay between both modes of thinking: Ratiomorphic concatenation of (asymptotically) well-defined arguments and the constellatory self-unfolding of meaning—as described and appreciated by LoC.

Reason never contradicts ratiomorphic insights—but goes far beyond them. As a reaction to exploding complexity and the increasingly faster pace of change, our epoch is characterized by a dramatic hypertrophy of ratiomorphic thinking efforts. Unfortunately, this development has been accompanied by an almost complete marginalization of reason. The resulting, dangerous imbalance can only be overcome by re-discovering and fostering the complementary mode of thinking that is encapsulated in the LoC. In addition to the theoretical ambition of offering a more elegant and consistent account of time and reality, the TAU also has a major practical goal: to reinvigorate and advance the hitherto most advanced achievement of cognitive evolution: the quintessentially human faculty of reason.

The third point that should be mentioned is that the new account of time and reality introduced here differs radically from presentism (which claims that only the present is real) or the opposite position, eternalism (which claims that our subjective experience of the flow of time is essentially an illusion). Both approaches are still entrapped in perceiving time as just a sequential order of events. In the first case, one single moment—the point-like present—is highlighted at the expense of the rest. In the second, exactly inverse view, time is instead geometrized. It becomes a kind of fourth spatial dimension of a block universe and does so at the expense of a complete denigration of a meaningful notion of the present. Seen from the new perspective developed here, the time–space of the present (TSP) constitutes an aspect of time in its own right, and it is, as mentioned, even the "primordial" form of time, in a most literal sense.

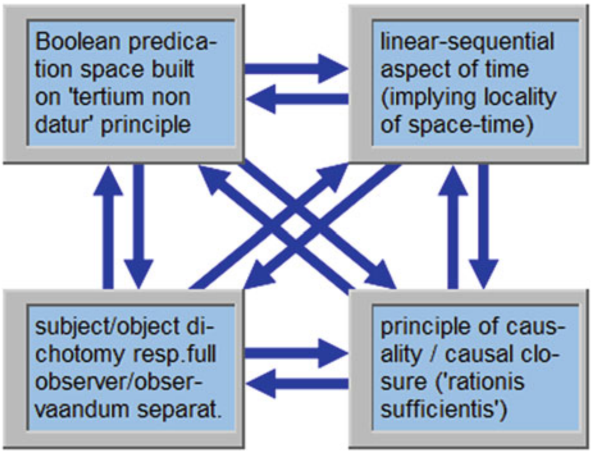
1 Existence and Role of Categorial Apparatus

Categories are the most fundamental structures in our cognition. They enable, but—by enabling—also constrain all subsequent thinking processes. Immanuel Kant's *Critique of Pure Reason* [1] drew attention to the most fundamental impact that these basic features of our appreciation of space and time have on our understanding of our world.

For Kant, sequential time and three-dimensional local space were two indispensable "forms" without which we could not make sense of reality in our thinking. Today we know that this characterization is narrowly linked to Newtonian physics, and, thus, too rigid in a technical sense. But the basic thrust of drawing attention to these underlying pre-formations—which, in modern terms, could be called symmetry breakings—remains absolutely valid.

In the following, the concept of the category is used in a quite specific, terminological sense. It addresses a small number of interrelated symmetry breakings in our cognitive approach to time and reality. These are so fundamental to our thinking that we usually are not aware of them. The claim is further that these symmetry breakings

Fig. 2 The F apparatus



don't occur in isolation but as interrelated features of the formation of a "categorial apparatus."[1] A categorial apparatus can be compared to spectacles through which we appreciate time and reality. As with all good glasses, we usually don't see them, but through them. There is a "classical" categorial apparatus which constitutes the "factual aspect" of reality (in the following called "F apparatus"). It consists of four fully interdependent components (Fig. 2).

The four constituents of this F apparatus are interdependent in the following way: Without the sequential structure of time, we could not use the concept of causality in any meaningful manner. The same holds true the other way around: Imagine a film of a watch that has been chopped up into its individual pictures, shuffled arbitrarily, and then glued together again. Without causality it would be impossible to tell whether the displayed timepiece works smoothly or not. The same holds true for Boolean predication. If we lived in a reality in which the truth of a proposition could coexist with its direct negation (i.e., if the principle of *tertium non datur* could be violated), we could not postulate a clear-cut sequential order of time or reliable causal structures. Lastly, the separability of subject and object is linked to the other three components in a similar way. If the observer and the observandum can't be separated in a clear-cut manner, the separation of cause and effect is also lost—and together with the sequential structure of time, and so forth.

The common denominator of all four constituents of the F apparatus is *clear-cut separability*. If this is given up in one of the four "corners" of the categorial apparatus, it cannot be maintained in any of the others. This raises the question about the general structure of a categorial apparatus: A categorial apparatus consists of four "niches" that must be filled with compatible and even mutually supporting thought patterns. These four niches are:

[1]This was first shown in the author's PhD thesis, von Müller, Albrecht (1983), *Zeit und Logik* (Time and Logic), Baur Verlag, Munich.

- a specific *structure of a predication space*,
- a specific *structure of time*,
- a specific *relation between events*,
- a specific *basic epistemological setting*.

In the case of the F apparatus these four niches are filled by Boolean logic, the linear-sequential notion of time, the structure of causality (in the sense of a sufficient cause), and the clear-cut dichotomy between observer and observandum, respectively subject and object.

The claim is that one can't drop or substantially modify any one of these four components without affecting also the others. This means for example that we cannot drop causality, as Heisenberg tried initially, but still work with the linear-sequential notion of time and the rest of the F apparatus as if nothing had happened.

As will be shown below, many of the enigmas of quantum physics are due to prematurely using, and then, based on respective evidence, trying to give up or substantially modify one of the four cornerstones of the F apparatus in isolation.

If we need to substantially change, or abandon, any of the four constituents of a categorial apparatus, we need to change also the other three components of our categorial apparatus. This may seem to be an inconvenience at first glance, but it actually isn't. This insight is the crucial lever for overcoming the "facticity imprisonment" of modern science.

At the latest since the advent of quantum physics and—independent of that—since Gödel's ingenious proof of 1931 [2], we know that the above-described F apparatus doesn't allow for a comprehensive picture of reality. There are aspects of reality that we can't capture in this framework. The advent of quantum physics was so decisive because we had previously always been able to fool ourselves with an epistemological trick. Confronted with phenomena that we could not grasp adequately by means of the F apparatus we could always postulate the existence of an underlying mechanism—which we just had not yet discovered or understood. This trick allowed us to stabilize our structurally deprived worldview, and it worked until the advent of quantum physics.

In quantum physics we came for the first time so close to the very fabric of reality that hypostasizing underlying causal mechanisms no longer worked. Especially with the violation of Bell's inequalities, and Aspect's respective experiments [3], the "F bluff" was finally called.

The founding fathers of quantum physics, Max Planck, Werner Heisenberg, Niels Bohr et al., were deeply moved by what they had discovered. Later generations of physicists learned to just live with and even ignore the underlying "skandalon"—as happened so often in the history of modern science. Gödel's proof of 1931 is just another point in case. Albert Einstein definitively belongs into the list of authentically moved founding fathers. With his fantastic intuition he saw that the emergent theory, in deep contradiction to his own findings, couldn't be the final answer. Unfortunately, physics subsequently turned to a large extent away from trying to understand and resolve the fundamental skandalon of quantum physics. Instead, one started to focus on increasingly sophisticated mathematical subtleties in trying to subjugate one theory below the other. Seen from the here developed perspective all these efforts are obviously doomed to fail: Quantum physics and general relativity can't and mustn't be "united" by subjugating one below the other. In the TAU the gap between them is bridged by recognizing that they address different, but complementary chrono-ontological portraits of time and reality. But in order to recognize the autogenetic nature of our universe we must recognize the structural limitations of the F apparatus and complement it by a second categorial apparatus—that allows us to handle autogenetic unfolding and the related structure of strong self-referentiality.

In the following such a second, complementary categorial apparatus is introduced. All four components may seem rather strange at first glance. But this is due to the fact that we automatically tend to project them into the rest of the F apparatus—where each of them would immediately cause havoc. But, as mentioned above, this is not how they are meant to be used. The crucial question is whether they *together* form a consistent second categorial framework that enables us to address those phenomena that cannot be grasped in the F framework.

In order to have a name for it, we call the second categorial framework the "E apparatus," which should loosely refer to the actual taking place of reality in the sense of event, emergence, or even epiphany.

In the E apparatus, the four slots are analogously filled with four fully interdependent and mutually supporting components. The E apparatus corresponds to an aspect of reality for which the notion of *statu nascendi* is introduced. It concerns "reality in the making," i.e., as long it has not yet achieved the chrono-ontological format of facticity. Its four constituents are shown in Fig. 3.

Each of the four components requires some further explanation that will be provided in the following. However, it should be stressed again that none of the four components of the E apparatus makes any sense if we project it onto the residual F apparatus—as we automatically tend to do.

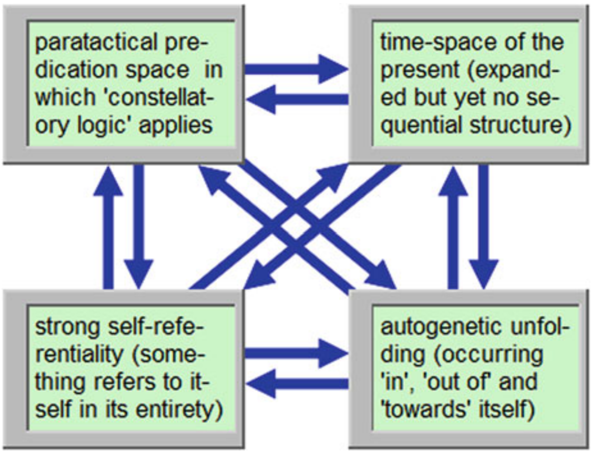


Fig. 3 The E apparatus

As in the F apparatus, all four components of the E apparatus only make sense together. This means that they describe an inherently consistent but fundamentally different and—as demonstrated below—complementary portrait of time and reality. Since there is no specific order between the four components, it may be best to start with the notion of autogenesis since it also lends its name to the autogenetic universe described here.

2 The Notion of Autogenesis

Autogenesis means that something unfolds out of itself, within itself, and toward itself. *Out of itself* means that there is no external causal driver. *Within itself* means that this process is not embedded within an external framework, e.g., local spacetime. *Toward itself* means that increasingly richer structures come into being which may eventually even become aware of themselves. As the respective agents are part and parcel of their self-unfolding universe, the latter starts to become aware of itself in them.

A crucial point in the notion of autogenesis is that—if the process of reality is essentially one of constellatory self-unfolding—all phenomena like perception, awareness, qualia, self-consciousness, mind, etc. must no longer be considered as a total *aliud*, i.e., something of a completely different nature. Instead, they can be seen as the result of a continuation, iteration, and intensification of what has happened throughout and since the very beginning: self-unfolding.

Only understanding reality as essentially a process of self-unfolding allows us to overcome the inelegant and unsatisfying dualism between mind and matter, or *res extensa* and *res cogitans*, etc. This new stance should, however, not be confused with any form of panpsychism that attributes some form of self-awareness to everything, already from the very beginning. The notion of an autogenetic universe simply claims that ongoing, constellatory self-unfolding is the deepest nature of reality and that the phenomenon of self-awareness can, therefore, come into being as an integral part and by repeated iteration of this very same basic process.

At this point, a brief remark on the notion of "constellation" respectively "constellatory unfolding" may be useful. I define a constellation as a set of components that gain their full meaning only mutually, i.e., in their mutual presence. In constellatory unfolding actually two things occur. The components mutually unfold their full meaning—and in doing so the whole of which they are parts unfolds itself as well, i.e., it becomes more what it is, it becomes more itself.

The notion that probably most closely approaches autogenesis is autopoiesis. However, there is also a fundamental difference between the two. Autopoiesis means that something makes itself in the sense of configuring itself. But in this making, autopoietic processes draw on already existing components that are arranged in a somehow cyclical and therefore self-reproducing manner. The notion of autogenesis is even more radical: It specifies that not even the constituents are external givens, but that they also constitute themselves only within this very process.

Many physicists today imagine the origin of our universe as an initial singularity. This assumption seems to be consistent with many empirical observations in the following way: If we extrapolate many of the presently observed cosmological trends backward in time, this leads to assuming such an "initial event" in which everything starts to unfold from a maximally condensed (or completely void) "*point* of departure."

All of our physical laws obviously lose their "grip on reality" at that *point* because they require a somehow unfolded reality at the least. No unfoldedness and no grip on reality mean no theory. We cannot calculate all the way into—or even beyond—this initial singularity. However, all of what we observe points to such an initial Big Bang. There are some attempts to somehow bypass this incalculable state and speculate what might lie "before" it. But seen from the conceptual framework proposed here, it does not make much sense to assume any kind of "causal continuity" despite an "interim absence" of the local spacetime fabric. Even the very notion of an *interim* absence of sequential time is quite hard to accept in philosophical terms. By definition, anything that is not a part of sequential time has neither a beginning nor an end. It is characterized by radical temporal non-locality, i.e., it is a co-existing, complementary aspect of anything that ever happens in sequential time.

Two points should be stressed here. It is a categorial mistake of epic proportion to assume that such a temporally non-local origin should ever "disappear" in local time, and that it should not profoundly affect or even characterize everything that happens thereafter. The view proposed here holds that all of reality emerging from such an origin is still profoundly and irreducibly characterized by this origin. As reality unfolds, it increasingly gains facticity—and this is why our normal physical laws start to gain traction. But in everything that occurs or—more precisely—*in the phenomenon that it actually occurs*, a strong and irreducible aspect of autogenetic unfolding remains as a cross-cutting feature in all of reality. Loosely speaking, we could call this a "reverberation" of the origin. Formulated in a more precise way, it is not actually a *re*-verberation since the initial event is not over. Due to its temporal non-locality, it can't be part of the past. It (co-)exists always, as an irreducible but *factually* not provable aspect—as a kind of essential, but equally unseizable and unerasable "flavor" of all that it is.

The second point is closely related to the above-mentioned phenomenon. As reality unfolds, it gains facticity and can therefore be described by physical laws. But as has just been elaborated, this irreducible and—due to its temporal non-locality—also unreduced aspect of a causal self-unfolding or self-constitution still remains. The notion of autogenesis is introduced in order to address this aspect. Autogenesis is a cross-cutting aspect of all of reality. The degree to which it comes to the fore can differ. As we will see, it becomes more intense in the degree to which the respective phenomenon is characterized by strong self-referentiality. This means that it can play a very remote role, e.g., in simple mechanical phenomena. But its relevance increases with the expanding role of strong self-referentiality—from quantum physical reduction, through the phenomenon of life, to the emergence of consciousness and eventually even mind.

Autogenesis occurs via constellatory unfolding and is characterized by genuine, i.e., not anticipatable novelty. Autogenesis in not part of local spacetime; it occurs "still" in the non-local TSP. And it is not a "process" whose description could be chopped up into infinitesimally small steps. Neither is a comprehensive causal account possible. It "just so happens," and does so "in one go," i.e., under an aspect of time, in which time is expanded but not yet sequentially structured. The taking place of the genuine novel can only occur in the present. But this is neither point-like nor expanded in a sequential manner. It has the character of a time–space. This time offers the "onto-phanetic" platform or stage, on which reality can occur and "take place"—and only this becomes part of local spacetime.

Autogenetic unfolding is neither simply random nor erratic, nor does it allow for a comprehensive causal account. Whatever occurs, unfolds in and out of the *constellation* of what is *present* and it follows its own, emergent logic—which only constitutes itself as part of this very unfolding. Like in Jazz improvisation, the specific "rules and principles" that shape the process emerge only during this actual "taking place."

An indicative notion for this constellatory self-unfolding of reality and the ongoing autogenesis of our universe that I sometimes use is "concertus mundi." The concertus mundi plays, and can be experienced, only in the primordial form of time: in the TSP.

3 The Time–Space of the Present

The main claim is that modern science is based on a too narrow notion of time. The linear-sequential aspect of time does exist and it is very important and powerful. But it is a specific, derivative aspect of time that applies only under certain conditions. It becomes applicable only once reality has "taken place," i.e., when it acquired the chrono-ontological format of a fact. As long as this is not the case, time is very different. What we call "time–space of the present" (TSP) is this primordial, not yet sequentially structured form of time.

Time is here the (temporal) "platform" or "stage" on which all occurs in the first place. Only as—on that temporal platform—reality "takes place," the sequential structure of time emerges—as a derivative feature of time, related to the factual aspect of reality.

The TSP is an aspect of (physical) time in its own right, and it is even the primordial form of time: Time "before" it gains its linear-sequential structure in conjunction with the factization of reality.

In quantum physical terms, the TSP is shape of time before reduction.

But what about predictions, e.g., the ability to predict future facts? Doesn't this imply that the linear-sequential aspect of time expands into the future as well? Yes, it does. But the point is that we are talking about future *facts*, i.e., we again refer to the factual aspect of reality—just mirrored into the future. The Past, a point-like, i.e., virtual present, and the future—all three together are the constituents of the linear-sequential aspect of time. The TSP is an entirely different, complementary aspect of time in its own right.

This is the reason why trials to reconstruct quantum physics from the difference between past and future—but both thought within the linear-sequential notion of time—eventually failed. The philosophically most thought-through trials in the direction were undertaken by von Weizssäcker [4]. But even they eventually failed because time was still seen as essentially linear-sequential.

The crucial step is to overcome this "sequentiality imprisonment" of our notion of time and the related "facticity imprisonment" of our notion of reality. Under the TSP aspect reality is pre-factual. The TSP is the aspect of time that belongs to the actual occurring of reality. The sequential structure of time applies only once reality has "taken place." *Only facts are part of local spacetime, and local spacetime is applicable only to the factual aspect of reality*.

But in which shape is reality before factization, what can we say about its actual occurrence? It can neither be conceptualized as a (sequential) process, nor as a-temporal in the sense of a complete "stasis" or stalemate. The basic mode of reality in the TSP can be described as "constellatory self-unfolding." Reality unfolds itself, i.e., within, out of and toward itself, in and as the constellation of all that is present. Self-unfolding means there are no (external) causal drivers. The whole notion of causality becomes available only together with the emergence of the linear-sequential structure of time, i.e., once reality has acquired the chrono-ontological format of facticity.

The fact that we hitherto didn't recognize the existence of this primordial form of time—as an aspect of (physical) time in its own right—is the reason why we have so much trouble with understanding quantum physics and its relation to classical and relativistic physics.

Without the notion of the TSP we had to project everything already on sequentially structured time. We were unable to appreciate reality in statu nascendi, i.e., the phenomenon and the way it actually occurs—in the TSP.

The premature application of the sequential aspect of time is the reason for all the "enigmas" of quantum physics. Non-locality concerns necessarily always space and time alike. But, without the TSP, we do not have any meaningful notion of non-local time at our disposal. Hence, one automatically projects everything on the inappropriate notion of an already linear-sequentially structured time. This leads into all the elegant but eventually inconsistent or consistent but infinitely ugly and implausible interpretations of quantum physics. (By the latter I refer to the so-called many world interpretations which invoke a completely insane inflation of eventually absolutely meaningless universes. These universes are completely meaningless in a philosophical perspective, because whatever human beings would decide, there will always be parallel universes in which they "drive drunken," "kill their mother in law," "betray their brothers and sisters.")

We will later come back to the by now 360° spectrum of failing trails to interpret quantum physics within the F paradigm. For now it is just important to see that and how the TSP describes a complementary aspect of (physical) time—that becomes consistently thinkable only within the categorial framework to which it belongs.

I would like to close this little elaboration of a complementary aspect of time by a quote:

> Wenn ich recht für mich bin und guter Dinge, etwa auf Reisen im Wagen, oder nach guter Mahlzeit beym Spatzieren, und in der Nacht, wenn ich nicht schlafen kann, da kommen mir die Gedanken stromweis und am besten. Woher und wie, das weiß ich nicht, kann auch nichts dazu. Die mir nun gefallen, die behalte ich im Kopf und summe sie wol auch vor mich hin, wie mir Andere wenigstens gesagt haben. Halt' ich das nun fest, so kömmt mir bald Eins nach dem Andern bey, wozu so ein Brocken zu brauchen wäre, um eine Pastete daraus zu machen, nach Contrapunkt, nach Klang der verschiedenen Instrumente etc. etc. etc. Das erhitzt mir nun die Seele, wenn ich nämlich nicht gestört werde; da wird es immer größer; und ich breite es immer weiter und heller aus; und das Ding wird im Kopf wahrlich fast fertig, wenn es auch lang ist, so daß ich's hernach mit Einem Blick, gleichsam wie ein schönes Bild oder einen hübschen Menschen, im Geist übersehe, und es auch gar nicht nacheinander wie es hernach kommen muß, in der Einbildung höre, sondern wie gleich alles zusammen. Das ist nun ein Schmauß! Alles das Finden und Machen geht in mir nur wie in einem schönstarken Traume vor: aber das ueberhören, so alles zusammen, ist doch das Beste [5].

The author of this wonderful vignette is W.A. Mozart. In my humble English I would try to translate is roughly as follows:

> When being by myself and in good humor, e.g. when travelling or walking after a good meal, or in the middle of the night when I can't sleep – then thoughts stream towards me, and in the best way. Where they come from and how, I don't know – it is not induced by myself. Those that I like, I keep in my mind and I seem to hum them, too – that is at least what others tell me. When holding them in my mind, one comes to the other – like making a paté – including counterpoint, and all according to the sound of the individual musical instruments etc., etc., etc.. All this heats up my soul, if I am not disturbed, and it becomes bigger and bigger, and I unfold it, always richer and lighter. The whole thing becomes almost entirely ready in my head - even if it is very long - so that I can see it in my mind: all together, at one glance, like a beautiful picture or a beautiful person – and it doesn't come sequentially, as it will have to later on, but in my imagination I hear it all together, in one instance. What a delight! All the finding and making occurs in me like in an equally beautiful and impressive dream - but overhearing it, so all together, that is by far the best.

I think it difficult not to be moved by these incredibly vivid and deep sentences. Almost like in his music, Mozart ingeniously combines lightness and serenity with depth and richness. But does that have anything to do with science?

One can take the position that is doesn't. Then these words just describe the hugely distorted subjective perception of physical time by an outstanding artist. Nobody can *force* anyone to give up this stance. The institution of coercive proof exists only in the factual chrono-ontological portrait.

Seen from the here developed TAU, what Mozart describes is an eminently intensive form of experiencing the present—and the constellatory self-unfolding of reality that occurs within it.

In the TAU perspective our experience of the present is not a subjective confabulation. It is the most advanced adaptation of human cognition to the actual taking place of reality. The unfolding of the ability to experience the present can even be seen as the Ariadne thread through the evolution of cognition: What becomes richer and richer—from bacteria through early animals up to mammals and finally human beings—is the ability to appreciate the present and, thus, the actual occurrence of reality.

Seen from this perspective, it becomes clear why the increasing facticity imprisonment of our thinking, culminating in modern science, is so detrimental: It deprives our cognitive access to reality in a most fundamental way: We become increasingly unable to appreciate that actual taking place of reality.

For science this means that all phenomena which are essentially characterized by constellatory self-unfolding become conceptually inaccessible. This holds true from quantum physical reduction to the phenomena of life and mind. For our everyday life this means that all what makes life actually worthwhile—from the experience of beauty, joy or love to the self-constitution of meaning or the phenomenon of free will—becomes at least unexplainable and questionable, if not straightforward denied.

All these phenomena belong irreducibly to the statu nascendi aspect of reality, i.e., constellatory self-unfolding is essential for them. Without a conceptual framework that allows to appreciate also this aspect of reality, we cannot even grasp what these phenomena are all about.

The statu nascendi aspect of reality can never be proven to exist in a factual manner. But it can't be disproven either. Assuming it, i.e., expanding and enriching the categorial framework of addressing reality, remains an issue of deliberate acceptance—which brings us to the next component of the E apparatus, paratactic predication and the related logic of constellations.

4 A Paratactic Predication Space and the Logic of Constellations

Addressing the phenomenon of autogenesis respectively constellatory self-unfolding requires a very specific type of predication. In a classical, Boolean-type of predication, the assumption of autogenetic unfolding causes havoc immediately. A paratactic predication space is one in which propositions are not concatenated in a Boolean or any other formalizable manner. They are just juxtaposed and the overall meaning constitutes itself in and as the configuration of all of them. Since no formal conclusions are possible, even contradicting propositions can coexist—just like a poem in which the sentences "I love her" and "I don't love her" together form a new, semantically richer meaning than the sum of the two statements taken independently. In paratactic predication, the various propositions unfold their full meaning only mutually, i.e., in their authentic presence. In paratactic predication a specific type of logic applies that can be characterized as the "logic of constellations" or "constellatory logic." Its main constituents are the three sub-dynamics that can be described as follows:

- Firstly, there occurs a mutual interpretation or "mutual semantic unfolding" between the individual components of the respective constellation.

- Based on this, there emerges a first overarching meaning of the entire constellation.
- Finally, there is a further sharpening or reinterpretation of the meaning of the initial constituents in the light of the emergent, overarching meaning.

Via its last step, this whole process is looped back into itself and can be iterated, now starting from an already richer meaning of the initial constituents. This constitutes a kind of "semantic powerhouse" that can continue to generate richer and deeper meaning.

The logic of constellations (LoC) describes the constellatory self-unfolding of meaning. In LoC there are no formal truth criteria. The authentic experience of consistence and "Stimmigkeit" (translatable, possibly as a hybrid of beauty, order and harmony) is all that remains for judging whether a configuration of propositions is adequate or not. All great art is based on LoC. Experiencing it means that we are drawn into this dynamics. Not by chance are both, the experience and even more so the production of art often accompanied by "forgetting about time." Great art draws into experiencing the TSP—because it lives there. The underlying dynamics of art is the constellatory self-unfolding of meaning—and this occurs in the TSP, and only there.

The constellatory self-unfolding of meaning corresponds to the statu nascendi aspect of reality. The logic of constellations is the "cognitive counterpart" to the autogenetic unfolding of reality that occurs in the TSP.[2]

The unfolding of meaning that occurs in these three steps is by no means random or arbitrary. It follows its own, emergent logic, i.e., the specific rules that it follows emerge during the process itself. A good example for this improvisation is Jazz. In a good jazz improvisation, no one can anticipate what will occur in the next few minutes. But the process is by no means random either. It just follows its own logic—which emerges during the process itself.

Due to their extreme density, Haikus are good, almost "laboratory examples" for the constellatory self-unfolding of meaning, respectively the logic of constellation according to which it occurs. The following Haiku has been composed by the famous Japanese poet Basho in 1684. The entire constellation of a Haiku is so small and dense that the dynamics of all constellatory semantic unfolding can be observed in great detail and almost in slow motion.

[2]Often the notion of "logic" is restricted to formal rules of drawing conclusions. But this is a much stronger restriction than it seems at first glance. It excludes large parts of the way that we use natural language, in which the processes of semantic unfolding play a crucial role. Therefore, the notion of "logic" is used here in the older, Heraclitean sense. For Heraclites, *logos* still meant both: the most fundamental principles underlying the taking place of reality and the basic rule of thought. As we will see, the autogenetic unfolding of reality cannot be appropriately addressed without semantic unfolding. Semantically static, and therefore just formalizable, concepts are insufficient. The secret of natural language is its extremely sophisticated balance between the principle of semantic constancy—which is necessary for understanding each other and addressing the factual aspect of reality—and the principle of semantic unfolding required for addressing and re-presenting the actual unfolding of reality.

The entire poem consists of the following few words:

quietness at noon
cicadas cry
rock penetrating

How does the logic of constellations materialize in Basho's wonderful haiku? "Cicadas cry" is characterized by abrupt interruptions. Instantly, there is total silence. Equally abruptly, the distributed noise sets in again.

This abrupt start and ending of the noise makes the quietness all the more "hearable" and impressive. Conversely, the cry of the cicadas is even more intensive when interrupted by total silence.

The intensity is so great that even rocks are penetrated. But is it only the cry of the cicadas that penetrates the rocks or also—and possibly even more so—the quietness?

Each of the three components unfolds its own specific meaning in the presence of the others. Out of these "bilateral unfoldings" an overarching scene emerges: A hot, somewhat breathtaking noon between the rocks, penetrated only by the abrupt interplay of noise and silence. Finally, this emergent, overarching scene once again sharpens the meaning of its own constituents: the staggering interchange of silence and noise. Due to this further sharpening of the initial constituents' meaning, the process of constellatory semantic unfolding can go into a next round and can go on and on as a result.

For some people it may seem to converge into particular constellations, while in others it oscillates, and in yet others it may continue to unfold novel horizons of meaning ... As a second example, which lives even more from a radical form of self-referentiality I would like to draw on a famous poem by Gertrude Stein[3]:

A rose is a rose is a rose is a rose

With every iteration, the rose becomes more of what it is: It seemingly unfolds out of itself, within itself, and toward itself. What makes the poem so fascinating is that its iterative semantic self-unfolding ingeniously mimics the characteristic, petal-by-petal self-unfolding structure of the real flower. But how are these poems, and the semantic unfolding that occurs within them, related to science? The crucial point with regard to the overall argument is this:

> Constellatory unfolding is not just a crucial feature of poems; in a way it is the essence of art and what distinguishes art from today's science. But art and science were not always seen as separate, almost contradicting worlds.

[3] In its original version of 1913, the poem reads: "Rose is a rose is a rose is a rose." Later Stein played with several modifications, e.g., by adding the initial "A" or dropping one of the repetitions. She speculated about carving the repetitions on a tree "until it went all the way around." In *Lectures in America*, she commented on her own poem as follows: "When I said, A rose is a rose is a rose is a rose, and then later made that into a ring I made poetry and what did I do I caressed completely caressed and addressed a noun." [6, p. 231].

The split between art and science emerged as science increasingly maneuvered itself into its present “facticity imprisonment.” Self-unfolding is actually a quintessential aspect of the taking place of reality. Art only highlights this aspect of the taking place of reality.
Or in more blunt terms: The realm of art is the enclave, the “game reserve,” into which this aspect of reality was increasingly confined as the ratiomorphic bias took hold of modern thinking. Art was split from science only as the “facticity imprisonment” took hold of our thinking.

Therefore, it makes sense to go back to art and consider what was lost in science. But it is also true that we can only articulate what actually makes art so wonderful if we have the thought pattern of autogenetic unfolding at our disposal. Otherwise, we can only experience the wonderfulness but are incapable of addressing and understanding what actually happens. Art cannot and doesn’t need to be “explained.” Each piece of art is a unique, wonderful instance where meaning unfolds in, out of and toward itself—in a way that can’t be anticipated because it follows its own principles that emerge only as part of the unfolding itself. But we can understand *that* this is the case and *how* this wonderful self-unfolding actually occurs. This understanding is essential for overcoming today’s structural deprivation of science, its deeply rooted incapacity to grasp the *statu nascendi* aspect of reality.

However, coming to grips with autogenetic unfolding not only requires a specific type of predication. It corresponds most closely to a specific form of time, the above described TSP. In the history of philosophy there is a long tradition of trying to think of time in a different way, i.e., not just as a linear-sequential order of events. There was the notion of the *nunc stans*. It refers to an all-encompassing now that is characterized by the absence of sequentiality. But it is also the opposite of a standstill. It is the deepest form of richness, depth, and dynamics—all in one (expanded) moment. To address this aspect of reality requires a specific form of predication and a related type of logic. This is what paratactic predication and the logic of constellations are all about.

5 The Structure of Strong Self-Referentiality

As the fourth and last component of the E apparatus, strong self-referentiality will now be discussed. Self-referentiality means that something refers to itself. Weak self-referentiality exists if something refers to parts of itself. Strong self-referentiality can be defined as *the phenomenon that something refers to itself in its entirety*. The structure of strong self-referentiality and the phenomenon of autogenetic unfolding are twins. Strong self-referentiality is the structural portrait of autogenesis. In turn, autogenesis is the dynamic portrait of the structure of strong self-referentiality. Both twins—autogenesis and strong self-referentiality—cannot be coped within the F framework. But, they are inevitable and irreducible features of reality, as shown by the neither erratic nor causally reconstructable quantum physical reduction for physics or Gödel’s fascinating proof of 1931 for mathematics.

The attempt to classify strong self-referentiality results in three basic types: The first one is the trivial principle of formal identity, which can be described as A = A. A geometric representation of this could be a band that is looped into itself. The next, much more interesting version of self-referentiality contains a "twist" or "torsion," which is usually in the form of a negation, i.e., a "non" in the definition of a set. Just think of B. Russell's set of all sets that do not contain themselves or the barber who shaves all men in his village who *do not* shave themselves. This second type of strong self-referentiality can be geometrically represented by a Mobius band, i.e., a band that is again looped back into itself—but this time the band is being twisted once by 180°. Put in crude terms, this means that "outside turns inside" and "inside turns outside." Coming back to the point of departure requires in this setting not a 360° but a 720° turn. If an idealized, two-dimensional ant moves completely around this band one time, it will find itself on exactly the opposite side—even though it always walked straight forward, i.e., without ever switching sides. Only after a second full circle the ant comes back to its point of departure. As the ant continues its walk, it finds that inside is outside is inside is outside . . .

In a third, most fascinating type of strong self-referentiality, some genuine new feature comes into play or emerges but the underlying *subjectum* is still considered to be the same. The best example for this is obviously personal identity.

People have new experiences all the time, they develop new answers, and gain new features and habits, but considering themselves to be the same is still constitutive for them. An attempt was made to resolve this problem by postulating a "core self" that remains unchanged, to which a bag of possibly changing experiences and qualities is subsequently attached. But this does not really work. In order to be meaningful, the gained experiences and qualities cannot be completely detached from the self. They must be seen as "mine," which means that they should form an integral of the self in question. The result is the original paradox of something—the self in question—remaining "the same—despite change."

The only way to deal with this richest form of strong self-referentiality—characterized by emergent new properties and self-unfolding—is paratactic predication. We must be capable of simultaneously saying that "x is the same" and "x is not the same" without getting into the *ex-falso quodlibet catastrophe* of formal conclusions, but with the possibility that this juxtaposition of countervailing propositions acquires a new, specific meaning of its own. The above-described "logic of constellations" has precisely the purpose of allowing for this kind of richer, not formalizable predications that are characterized by a mutual semantic unfolding of the constituent propositions.

With this we now discussed all four "cornerstones" of the E apparatus, as well as their strong interrelatedness and interdependency. The question that remains open is how these two categorial frameworks, F and E, can coexist, i.e., how they can interact without harming or unraveling each other.

The answer to this question can be approached with a metaphor: The F apparatus provides the lines and shapes while the E apparatus is the color in experiencing reality. The two do not contradict each other because they do not enter into each other's domain. Formally precise and conclusive F statements are never contradicted by E statements. But without the latter, the prior have no "color"—no real meaning or sense—whatsoever. E statements do not allow for conclusion and therefore cannot get in the way of F-type conclusions. Meaning unfolds autogenetically in an E statement: out of itself, within itself, and toward itself.

In practical terms, the relative weight of F and E statements in addressing an issue depends on the degree of self-referentiality of the respective issue. When analyzing the statics of a bridge, there will (hopefully) only be a marginal role for E statements. But when talking about meditation, art, or religion, E-type statements will usually predominate. By having a richer categorial framework at our disposal, composed of both F and E, gives us the conceptual means to address all of reality—from classical mechanics and Boolean predication, through the quantum physical self-constitution of matter and the phylo- and ontogenetic emergence of life, to the phenomena of mind, and art—in a coherent way, i.e., without requiring us to shift the conceptual reference frame. For science, the essential takeaway is that adding the thought pattern of autogenetic unfolding, and all that comes with it, to our conceptual arsenal, we significantly expand what we can perceive, describe, and understand. In paragraph 9 it is described what this may mean for modern physics.

6 Internal Coherence of the E Apparatus and Its Relation to the F Apparatus

All four constituents of the E apparatus are interdependent and constitute each other. The sameness or identity of something that can change and develop requires paratactic predication. As explained above, it must simultaneously be possible to address it as "the same" and as "not the same." But this is not all. What has just been stated also requires and constitutes a trans-successive aspect of time. If something is the same "before" and "after" an event occurred, this implies a temporal platform, a non-local unity of time that overarches what in a purely sequential portrait of time would just be two different points in time. Being the very same also implies existing "at the same time." The TSP provides exactly this complementary, non-local temporal platform on which different states of something that unfolds itself can be seen to exist "at the same time," i.e., in their mutual presence. If this change were to be ultimately caused by an external cause, i.e., via a classical causation, this would imply a rigid sequential time structure. External causation requires clear-cut separability of cause and effect, and this implies that the first clearly precedes the second. This requirement of sequentiality can only be avoided through the phenomenon of self-unfolding and/or autogenesis.

In a similar vein, it can be shown that all other components of the E apparatus depend upon and imply the respective others in an analogous way. This gives us a second, inherently consistent categorial apparatus that allows us to address the *statu nascendi* portrait of time and reality.

All of these descriptions sound rather unwieldy, at least at the beginning. The reason for this is that the modern mind is deeply entrenched in the F mode of thinking, which makes it very difficult to smoothly switch to the E mode and back. A look at the history of Western languages reveals indications that this was not always the case. Most Westerns languages had a "middle voice," a frequently employed grammatical construct that indicates something acts upon—and therefore unfolds—itself. Very prominent in ancient Greek, for example, it still plays an important role in German, English, French, Spanish, and Italian. However, the role of the middle voice was dramatically marginalized during the establishment of modern, techno-scientific civilization and when the "facticity imprisonment" fully took hold of Western thinking. Most technical instruments are deterministic and binary ("on" or "off") in their internal and external manner of functioning. Consequently, the technization of our living world has gone hand in hand with a "binarization" of our thinking—which is diametrically opposed to the phenomenon that something (gradually) "matures" and becomes "more of what it is," i.e., that it unfolds out of itself, within itself, and toward itself.

We now have two complementary categorial frameworks, the F and the E apparatus, at our disposal. Both are internally consistent and coherent, which means that their individual components cannot be exchanged, dropped, or substantially modified without harming the entire apparatus. But what about the two different types of portraits that result from applying these two apparatuses? How do these portraits relate to each other?

The short answer is that all conceivable phenomena imply both aspects in some way. But the relative emphasis can shift in a rather dramatic way—from "almost all F" to "almost all E" as the most adequate way of describing a phenomenon in question. But the countervailing portrait is never reduced to zero. It always remains as a residual, although less prominent feature. For this reason, a comprehensive characterization requires always to combine both portraits.

7 From Two Categorial Apparatus to Three Portraits of Time and Reality

But the transition to a richer categorial framework, i.e., the development of a multiple chrono-ontological set-up, doesn't end here. The assumption of a factual and a *statu nascendi* portrait automatically leads to the questions of "where does the emergent come from," "where is what unfolds before," or "what is the source of the autogenesis"?

These questions lead to the tentative assumption of a third portrait: One that concerns an aspect of comprehensive inseparability or inherently indistinguishable unity. In order to have a name for this tentative third aspect of time and reality, it is called the *apeiron* portrait (with reference to the great pre-Socratic philosopher Anaximander).

The ancient Greek word *peiros* means border, structure, or separation. The *apeiron* is that which has no discernible structure or any distinguishability whatsoever. This implies that it also has no external borders, which is the reason why the apeiron is often interpreted as "infinite" or "endless." But this is just one derivative aspect of the *apeiron*. More important is its inherent indistinguishability. In modern terms, it may be possible to characterize this aspect of time and reality as a superposition of all possibilities.

There are two reasons for assuming the existence of such a third portrait. One is the resulting elegance of the theory, as demonstrated below. The other reason is that physics also seems to include some indications for such a state of indistinguishable unity or "all-in-all" superposedness.

Having mentioned this, the apeiron state may be seen as an absolutely prior state and as the origin of everything. Although this is not wrong, it is incomplete. If we think and speak in this manner, it is easy to once again hypostasize the linear-sequential time structure as the correct overall framework. In order to avoid this and, thus, to make sense, the *apeiron* has to be seen as a pervasively co-existing, all-accompanying aspect of time and reality—just like the *statu nascendi* aspect.

As already mentioned above, the existence and respectively validity of the *statu nascendi* and the *apeiron* aspect can never be proven by pointing to facts, i.e., in a coercive proof. Assuming the validity of these portraits is and remains a matter of deliberate acceptance.

If we accept, in addition to the factual portrait, the existence of these two complementary portraits of time and reality, the result is a three-pronged chrono-ontological framework. The transition to such a multiple chrono-ontology resolves some very old problems in philosophy, e.g., the unresolvable dichotomy between unity and diversity, respectively a monistic or a dualistic "matter/mind" set-up. As long as we have only two countervailing principles, each solution is necessarily biased, as e.g., Hegel famous notion of an "unity of unity and diversity." In a three-pronged framework the third component can always serve as a bridge between the other two, avoiding that they fall apart in a pure dichotomy.

As to the mind/matter relation, the TAU offers also a quite interesting solution. The emergence of live and mind shows the inherent, but not yet unfolded potential of matter. This avoids two unsatisfying alternatives: Having to conclude that mind is a complete aliud to matter, i.e., resorting to an irresolvable dualism, or having to claim that matter already possesses some form of consciousness, i.e., resorting to a form panpsychism. The thought pattern of constellatory self-unfolding allows for the emergence of genuinely novel qualities without giving up the identity of that what unfolds itself. Formulated with regard to time it is the TSP that allows that "something becomes what it is."

The new framework also transcends the antagonistic distinction between realism and constructivism. The three portraits are neither just subjective construals nor are they a sheer representation of what is "out there"—as a reality that is not affected by our way of perceiving it. The emergence of cognition is an integral part of the self-unfolding of our universe. The self-constitution of a subjective perspective enriches what exists objectively. The emergence of self-awareness as a genuinely novel quality is an integral aspect of an autogenetic universe, i.e., one that unfolds out of, within *and toward itself*.

Co-evolutionary development of our cognitive access to time and reality on one hand and their own unfolding on the other hand is no longer strange. Instead, it is now quite natural.

As will be discussed, all three chrono-ontological portraits are interrelated in a very specific way that can be compared to the topology of Borromean rings. But before discussing this, it may be useful to quickly review and somewhat elaborate upon the specificity of these three complementary portraits of time and reality.

7.1 The Factual Portrait of Time and Reality

The factual portrait—painted by the categorial tools of the F apparatus, and onto the co-emergent canvas of local space–time—converges into the picture of a comprehensively determined block universe. All that exists in this portrait has the chrono-ontological format of a fact in local space–time. Even if we have the subjective impression of a progression of time, all that is already "coexists." It is only our strange subjective point of view that gives us the impression that reality develops over time—or in Einstein's words, time is just "a sticky illusion" (Fig. 4).

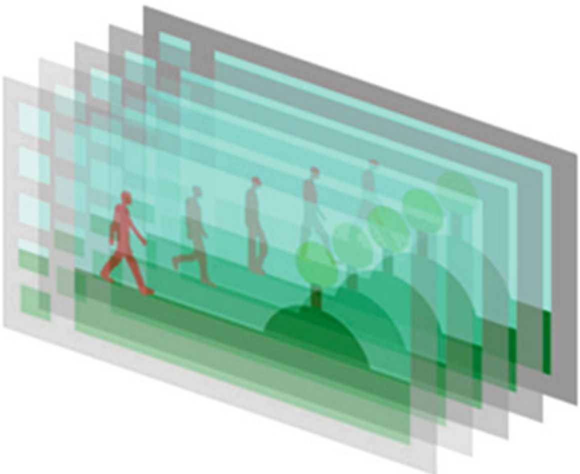


Fig. 4 The factual portrait of time and reality: the block universe. Pickup, Tim. *Block Universe*. http://timpickup.files.wordpress.com/2009/05/block5.jpg (09.12.14). *Classical and relativistic physics imply a block universe in which nothing genuinely new can ever occur*

Fig. 5 The statu nascendi portrait: ongoing unfolding. Becky Jenkins, "Rose Quartz – The Stone of Love," http://www.rainbowremotehealing.com/rose-quartz-stone-love/ (08.11.14)

7.2 The Statu Nascendi Portrait

The E apparatus provides us with a profoundly different picture of time and reality. All that is unfolds, and it even exists only by way of its ongoing, constellatory self-unfolding. The self-unfolding of a flower may serve as a pictorial metaphor for this second, very different chrono-ontological format of reality (Fig. 5).

This portrait contains neither external causality nor a clear-cut separation of past and prior, neither an absolute contradiction between "a" and "non a," nor a full dichotomy between observer and observandum respectively subject and object. It portrays an implicit unity of all in its emergence. What is constantly unfolds, thus becoming more of what it is. Core quantum features such as superposition, entanglement, objective indeterminacy, uncertainty, and quantization are the factually measurable footprints of the inherent pre-locality of the *statu nascendi*.

7.3 The Apeiron Portrait

In this portrait, all that is appears under the aspect of an inseparable unity. Due to this, it is also inherently impredicable. The only way that we can approach this aspect is indirectly or *ex negativo*. However, we must be aware that no finite predication—even if it is negative or indirect—can ever do full justice to what it is addressing.

The inherent impredicability of the *apeiron* aspect is also the reason why it does not require and why it can't have a full categorial apparatus of its own. Categories always imply a certain degree of distinguishability. The strongest form of this is the asymptotic well-definedness of the factual portrait of time and reality. The *statu nascendi* portrait already displays diminished distinguishability. Under the *apeiron* aspect, there remains no distinguishability, whatsoever. Therefore, no third

Fig. 6 The Apeiron portrait: circle/ring. *The circle, in its closure and symmetry around an empty but all-defining middle, is a powerful symbol for the coincidence of the omnipresence, omnipotentiality and omnidentity that constitutes the apeiron portrait*

categorial apparatus is needed or possible. Despite the very strange and inherently "transcendent" character of the *apeiron* portrait, it seems to play a major role in all branches of human cultural evolution. Frequently it is represented—in a still fallible, insufficient way—as a circle or ring (Fig. 6).

An interesting secondary aspect of introducing the *apeiron* portrait of time and reality in this way, i.e., as an integral part of a three-pronged chrono-ontology, is its implication for the phenomenon of religion. On one hand, it allows us to encounter and appreciate it with the fullest respect and even admiration for this courageous, irresolvable attempt at addressing and venerating this deepest aspect of our universe. On the other hand, it also becomes very clear that it is never legitimate, tolerable, or excusable to force own beliefs upon others or coerce them into any given belief.[4]

We now have three complementary portraits in which we can think about and address time and reality or appreciate our autogenetic universe. A major advantage of this new, three-pronged chrono-ontological framework is the avoidance of both pitfalls, a monolithic or a dualistic ontology:

A monolithic architecture always has an irresolvable problem when attempting to derive richness in a natural way. Where is the difference if "all" is essentially just one?

On the other hand, dualistic set-up always requires a more or less artificial and unconvincing construal in order to overcome the initial rift. Where is the unity if the initial set-up is essentially dualist?

[4]Already in the *statu nascendi* portrait, the institution of coercive proof is no longer available. Its only way of being convincing is to make a convincing offer on how to think and appreciate something. Opting for this offer remains an issue of free, deliberate acceptance by those who are addressed. Precisely this holds true, in an even more radical fashion, for all forms of religious beliefs. Anything that we can formulate in a positive way or even insinuate implicitly, remains—per definition—far behind whatever it attempts to address and appreciate. Consequently, as soon as people start to impose or coerce, religion loses all of its rights. Instead, it just serves as a fig leave for manipulative efforts that are usually motivated by such mundane driving forces as power, possession, or control. Religious believe systems are completely legitimate ways of addressing the apeiron aspect of reality—but they must never confound themselves with statements about the factual aspect of reality.

A good example for the latter is Leibniz's "pre-established harmony." It is the huge merit of his lucid thinking that the aporetic character of this solution also becomes so clear. The claim is that only a three-pronged conceptual framework allows us to avoid the unresolvable problems of both, a monolithic or a dualist set-up. And even a three-pronged approach only solves the problem when combined with a specific topology in arranging its three components.

8 The Triality Account and Its Borromean Chrono-Ontology

The three chrono-ontological portraits exist in their own right, are interrelated in a way that will be more closely defined below, and are complementary in the sense that they only offer an adequate picture of time and reality when seen as a whole. The claim is that any categorial framework that excludes one of these three would imply a significantly deprived appreciation of the autogenetic universe in which we live and of which we are an integral part. The name of "Triality Account" is introduced to characterize this three-pronged categorial conceptual framework for conceptualizing and addressing time and reality.

All categorial frameworks imply an ontology. The Triality Account implies a multiple ontology that is constituted by three complementary portraits. The relationships between these three portraits can be compared to the topology of Borromean rings, which means that the three portraits are independent but simultaneously united in a very specific way. In its simplest form, a Borromean topology consists of three rings that are interrelated in such a manner that taking away any one of them causes the other two to simply fall apart in complete separation. In other words, each of the three rings connects the other two—or is itself connected to any other ring just via a third ring. This specific combination of separation and relatedness is fully symmetric, i.e., it holds true for all three rings or all the relationships between them (Fig. 7).

Fig. 7 The three ECHOs of the triality account. *The three portraits of the Triality Account are conceptualized as both, separated and interrelated, like Borromean rings*. O.A. *Borromean Rings*. http://upload.wikimedia.org/wikipedia/commons/thumb/0/07/Borromean-rings_minimal-overlap.svg/640px-Borromean-rings_minimal-overlap.svg.png (09.12.14)

The topology of Borromean rings[5] allows us to separate and simultaneously interrelate the three portraits of the Triality Account in the following way:

- The difference and relatedness between the *factual portrait* (with separability as its cross-cutting feature) and the *apeiron* aspect (in its all-encompassing unity) is provided by the *statu nascendi* portrait's authentic occurring. The actual taking place of reality is the bridge between the resulting facts and the unity from which they emerge.
- The difference and relatedness between the *statu nascendi* and *facticity* is provided by the aspect of an all-encompassing unity or omnidentity. Without this primordial unity, an irredeemable rift would arise between self-contained, fully separable facts and the irreducible flow character and permanent self-transcendence of authentical occurring.
- The difference and relatedness between the *statu nascendi* and the *apeiron aspect* is provided by *facticity*. Without the possibility of factual separatedness, the *statu nascendi* aspect would—in its unfinished separation—"collapse backward" into the inseparable unity of the *apeiron* aspect. Only the emergence of hard, clearly separable facts establishes and ensures the expandedness of our universe in time and space.

Obviously, the statu nascendi is most closely related to the phenomenon of life in general, and even more so to the phenomena of consciousness and eventually autobiographic self-awareness. This means that seen from our perspective as self-conscious human beings, the statu nascendi "feels" like the center, with facticity and the apeiron aspect as somewhat more remote, or even derivative perspectives.

Given the fact that in the TAU our self-awareness marks also a new quality that the entire universe achieves (its actual unfolding *toward* itself, i.e., toward self-perception respectively self-awareness), this elevated and hauled out position of the statu nascendi is completely legitimate. But this doesn't undermine the underlying Borromean interrelatedness of the three chrono-ontological portraits.

It is important, however, not to implicitly position one of these three perspectives as the epistemologically superior meta-perspective. Especially in the attempt to be precise, it is easy to slip into this trap and use the factual aspect as an ultimate reference frame in our thinking. But this is misleading. *The co-existence of all three perspectives also occurs on all of the conceivable meta-levels.* Respecting it is crucial for the multi-faceted Borromean chrono-ontology outlined—and for the sake of maintaining its inherent openness.

This inherent openness is precisely what is missing in the conceptual framework that might be seen as most closely approximating the Borromean framework proposed here. Hegel's philosophy is a grandiose and unique attempt to think through all there is in one stroke—from the self-unfolding of the "absolute idea"

[5]Hans Primas brought this observation to my attention when he kindly commented extensively on an earlier version of this paper in a 2010 workshop on the conceptual foundations of physics that took place at the Parmenides Foundation.

in his *Logic* through the self-emanation of the absolute idea into material reality and the resurgence of the resulting material reality, until it finally understands its origin and genesis. The problem inevitably encountered by such closed systems thinking is that history must seem to come to an end in this very philosophy. And this inherent "closedness" is what finally perverts this entire, admirably sophisticated endeavor into a totalitarian Procrustean bed for thinking about and appreciating our universe.

It appears that Hegel tried to avoid this trap—at least most of the time (with some weaker moments in his philosophy of law and history, see [7, 8])—but the core of his theory, Hegel's *Logic* [9], does not provide a reliable, transferable remedy for this problem. This unresolved structural problem in the very architecture of his philosophy facilitated the totalitarian misuse of his ideas thoughts in the aftermath. On the other hand, the popular Hegel-bashing in most of the modern philosophy isn't even capable to appreciate his unique and grandiose, although eventually failing effort.

The specific way in which a third component is always considered to "alleviate" the initially existing antagonism in Hegel's dialectic is closely related to this danger of "praecox finalism." In contrast to how it is often criticized, this is obviously not conceived as a simplistically repetitive, "wooden" mechanism. The manner of attaining an alleviating synthesis continues to develop throughout the entire process. However, all of this remains conceptualized as a continuous forward spiral. In turn, this explicitly or implicitly hypostasizes the sequential portrait of time as the ultimately correct one. Hegel was right, and his efforts mark a pinnacle in the history of science by attempting to develop a coherent framework in which one can think about all there is. But in the attempt to provide a closed system, Hegel can't allow for an authentic occurrence whose outcome cannot be anticipated. Hegel's philosophy missed the notion of a strong, meaningful present.

Striving for a coherent conceptual framework is, and inevitably remains, the *regulative idea* of science (in the wonderfully deep, Kantian sense of striving for something, although we know that we will never fully attain it). The fundamental challenge is, however, not to confound coherence with closedness, i.e., to develop an inherently open philosophical system that enables us to combine an undiminished striving for a coherent overall framework with the modesty of knowing—and explicitly declaring—that no finite formulation or thought will ever be able to do full justice to this underlying ambition. In the framework developed here, the latter is not a Sisyphus-type condemnation. To the contrary: It ensures a coherent conceptual framework that allows for addressing matter, life, and mind in a "modest ToE" (a so-called theory of everything)—but one that explicitly confesses its own incompleteness, and thus maintains its own openness.

In summary, the Borromean chrono-ontology proposed here tries to offer a coherent but inherently open conceptual framework for appreciating our autogenetic universe. It consists of three different, but interrelated and mutually complementing ways to portray time and reality (Fig. 8).

The TAU can also be seen as a radicalized version of evolution theory. Traditional evolution theory essentially describes autopoietic processes in which existing components and solutions are recombined into novel, usually more sophisticated

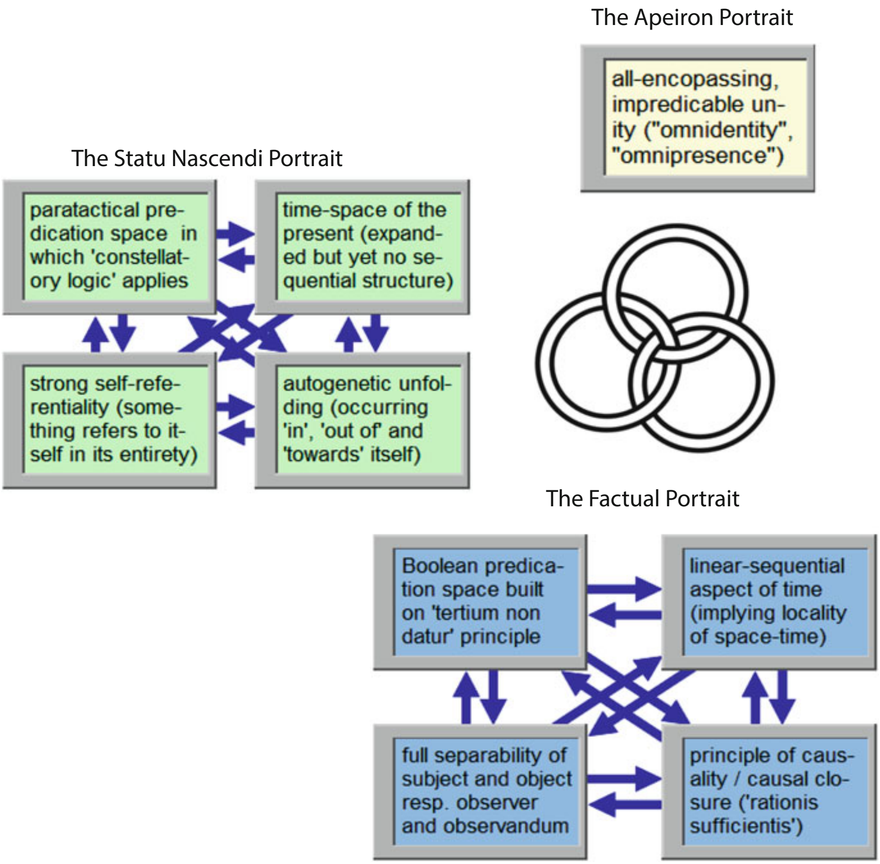


Fig. 8 The basic architecture of a three-pronged Borromean chrono-ontology. O.A. *Borromean Rings*. http://upload.wikimedia.org/wikipedia/commons/thumb/0/07/Borromean-rings_minimal-overlap.svg/640px-Borromean-rings_minimal-overlap.svg.png (09.12.14)

entities. But, traditional evolution theory operates entirely within the factual portrait of time and reality. In the conceptual framework of TAU, these limitations can be overcome. Once constellatory self-unfolding becomes the most fundamental feature, evolutionary dynamics turn out to be a special case of it. Matter, life, and mind can be seen as iteratively higher orders of constellatory self-unfolding—with evolutionary selection being a powerful dynamic feature within all three domains. At the same time it becomes possible to appreciate life and, even more so, mind as significantly richer than matter, although not a complete "aliud"—because they emerge as higher orders of the same underlying feature, constellatory self-unfolding.

If we project this new chrono-ontology back onto our initial pictorial metaphor of an autogenetically unfolding universe we get the following mapping (Fig. 9).

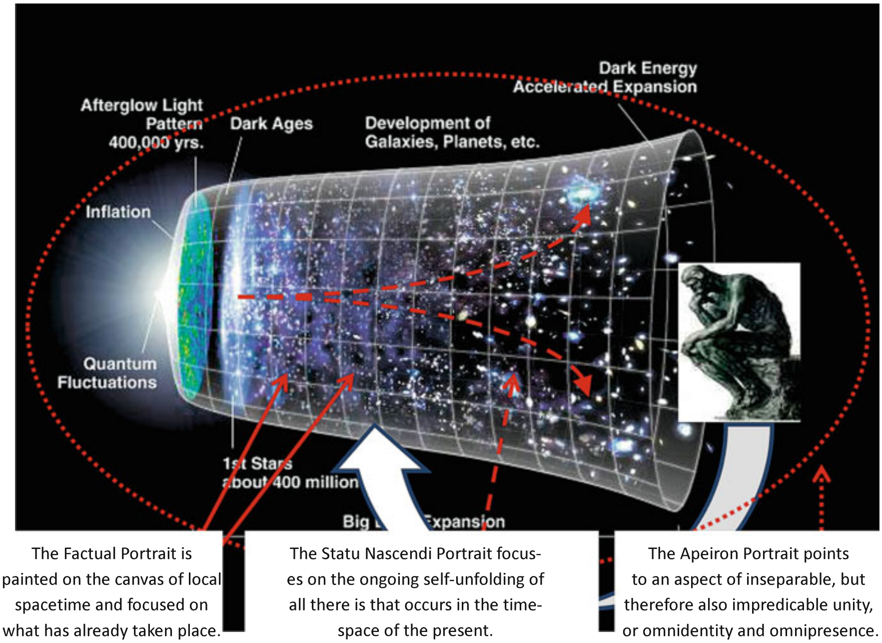


Fig. 9 The theory of an autogenetic universe in a nutshell

Three points of practical relevance should still be stressed in this brief introduction to TAU:

- Our human self-awareness is an emergent aspect of our universe. In us it starts to become aware of itself—and this new quality of the entire universe is the ultimate reason for the infinite and unfathomable dignity of every single human being.
- In an autogenetic universe explainability and wonderfulness are no longer at the expense of each other, they deepen mutually.
- The most appropriate basic tenor in a self-unfolding universe is thankful attentiveness—and in this tenor theoretical and practical reasons (finally) start to converge.

The ideal of human dignity is probably the most advanced achievement of the cultural evolution of mankind so far. At the same time, it will probably be also the most endangered achievement in the decades to come. Western societies tend to show an increasing negligence toward it—from Guantanamo, through NSA to the habitual, pan-OECD ruthlessness with regard to extreme human misery. Many of the new "global players," both, in the political and the commercial sphere, appear to not understand the essence of what human dignity means. Intended and unintended side effects of technological progress constitute another threat dimension—up to the wired ideas of surpassing human beings by "transhuman" computer intelligence or “uploading” preconfigured cognitive dispositions into the brains of real human beings.

Seen from the here developed framework most, if not all of these problem threats are rooted in the “facticity imprisonment” of our thinking, i.e., in reducing reality to its factual and time to its linear-sequential aspect—due to relaying exclusively on the F apparatus in our cognitive approach to time and reality.

The over-fixation on power, possession, and control is probably the most characteristic feature of modern civilization. It is a counter-reaction to perceiving time predominately as “the dent of time,” i.e., as an omnidevouring maelstrom that permanently destroys all there is. But this notion of time is profoundly deprived. Time is primarily that what allows all to happen in the first place. It provides the primordial time–space in which reality can occur. The sequential aspect of time, and with it the phenomenon of decay, is only a derivative aspect of the “primordial present.”

The title of this paper, “The Forgotten Present,” has a dual meaning. On one hand it points to a blind spot in our modern appreciation of time. On the other hand, it points to the oblivion of a present in the sense of a gift. The ongoing self-unfolding of our universe, including our very existence as individual human beings, is the ultimate and unsurpassable gift. Helping us to (re-)discover the ongoing, infinitely wonderful self-donation of all there is, is the deepest practical concern of the TAU—and it converges with its main theoretical concern to overcome the hitherto unsurmountable hindrances of modern science to understand the subsequent self-constitution of matter, life, and mind in a coherent approach.

The remaining four paragraphs of this paper should elucidate, at least in a very preliminary and insufficient way, these wider implications and possibilities that starting to appreciate the autogenetic nature of our universe brings with it.

9 A New Approach to Quantum Physics and General Relativity

The theory of the autogenetic universe is basically a philosophical theory. However, re-thinking the way in which we conceptualize time and reality has necessarily far-reaching implications also for physics. In a more general sense, the availability and manageability of the twin thought patterns of autogenetic unfolding and strong self-referentiality lead to profound changes and new perspectives for every field in which these phenomena play a crucial role. This includes the emergence of the local space–time portrait of physical reality.

The rift between quantum physics and GRT is the oldest and deepest wound in the fabric of modern science. And it is precisely there that TAU offers a radical new perspective—in a very natural, almost effortless way. By separating the three chrono-ontological portraits, it becomes possible to see that all genuine quantum

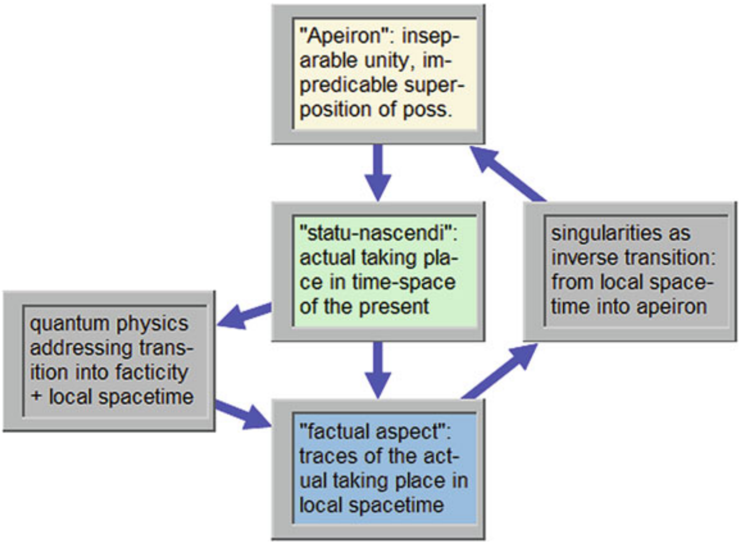


Fig. 10 Quantum reduction and the singularities of GRT as inverse transitions

phenomena are related to a still non-local respectively pre-local constitution of time and reality, i.e., reality that is still in *statu nascendi*.

Seen in the TAU framework, quantum physical reduction addresses the transition of reality from the statu nascendi format into the factual format. Instead, classical and relativistic physics focus on describing the latter, factual format—i.e., the local spacetime portrait of reality. Within this interpretation, the phenomenon of singularities, often marginalized in the past, moves to the center of attention in GRT. In singularities the canvas of local spacetime unravels again—driven by the strong self-referentiality (!) of gravity. Consequentially, quantum physical reduction and the singularities of GRT mark, so to speak, *inverse transitions of time and reality*: into respectively out of the chrono-ontological format of facticity or spacetime locality (Fig. 10).

This also makes it clear that trying to subjugate GRT to QP—or vice versa—is profoundly misleading. Both of them are superb theories as they stand. They are just focused on different portraits of time and reality, which is why they are naturally and quite appropriately incompatible—unless their relationship is considered within the here proposed richer categorial framework that allows to separate the different chrono-ontological format of time and reality.

The novel framework allows also to look at the previously unresolved quarrels between the different interpretations of quantum physics in another light. As long as one can draw only on the categorial apparatus that belongs to the factual/local spacetime portrait, all instances in which the actual taking place of reality, respectively the TSP in which it occurs, come to the fore must appear enigmatic and conceptually irresolvable. Thousands of the best brains have attempted to interpret quantum

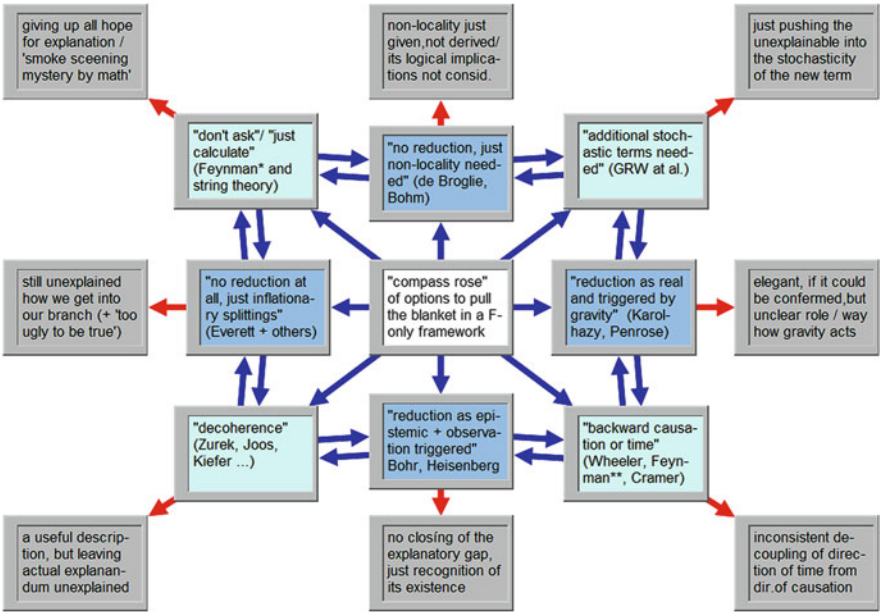


Fig. 11 The "compass rose" of failing trials to understand quantum physics within the limits of the F apparatus

physics, or resolve its relation to GRT, within the limited F apparatus—without success. As almost every possibility has been tried by now, the TAU perspective allows to even map the failing trial as a 360° compass rose. The situation has some similarity with trying to position a blanket that is inherently too short. We can cover every aspect—feet or head, left or right—but never all of them at the same time. Each position covers some parts well, but it also pulls the coverage away from others (Fig. 11).

The problem cannot be resolved by seeking more or smarter ways to position the blanket: It is simply too small. We need to enlarge the blanket, i.e., overcome the categorial constraints of facticity imprisonment. We need to relativize the local spacetime background. This can, however, not be done in isolation but only as part of coherently expanding the categorial underpinnings of both, QP and GRT. By differentiating the three chrono-ontological formats, a "third step" becomes possible and even necessary in both theories. Both of these "third steps" relativize the local spacetime background, and thus the factual portrait, which now becomes an emergent property. As already mentioned above: *The autogenetic universe does not unfold within local spacetime*, *but the emergence of the latter is part of its unfolding*.

In quantum physics, this relativization of the local spacetime background could in a way be interpreted as a "third quantization" or—more precisely (since all quantizations eventually involve finding a fundamental complementary)—as the discovery of a third complementarity. This would be the complementarity between

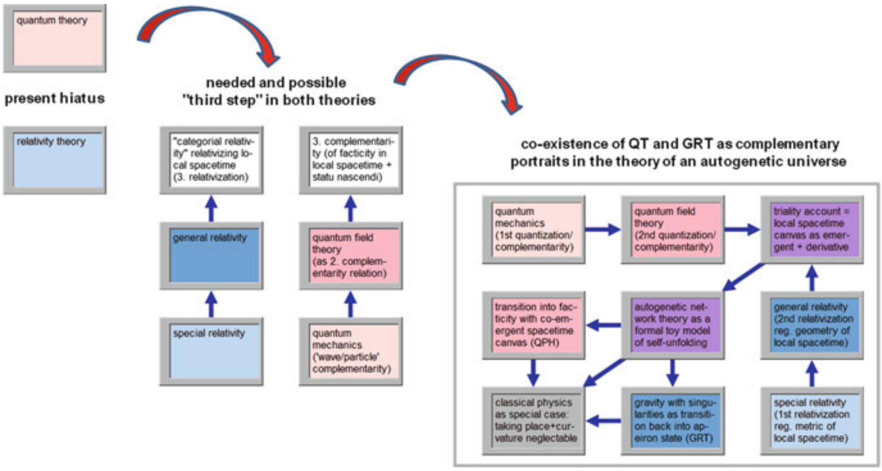


Fig. 12 "Third Quantization" and "Third Relativization"

the local spacetime portrait and the other two chrono-ontological portraits of time and reality. In terms of relativity, the "third step" could be seen as a "third relativization." This would mean that the special and general relativity are followed by a third, and probably ultimate relativization that leads to "categorial relativity." In a precise analogy to the "third step" in QP, this would relativize the very availability of a fully separable local spacetime background. By means of these two "third steps," QP and GRT start to meet and match. Instead of subjugating one to the other, this has the effect of further unfolding them (Fig. 12).

Obviously, this is a new conceptual approach to the problem and not yet a physical theory. (For the current state of trying to formalize the conceptual argument presented here, see the subsequent paper on "Autogenetic Network Theory" by Andrej Nikonov and the author.)

But there may even be inherent limits to formalization. Until now, natural language seems to be the only symbolic system that enables us to address all three chrono-ontological portraits in one symbolic framework. There could have two interesting implications: On one hand, conceptual thinking in natural language—characterized by its most sophisticated balance between the principles of semantic constancy and semantic unfolding—might play a much more important role in physics, once again. On the other hand, TAU may even open up some interesting new perspectives for our understanding of the foundations of mathematics.

A few years ago, Ian Stewart postulated the need for a new branch of mathematics and coined the interesting name of "morphomatics" for it: the science of structure *formation*. Not much has followed in this vein, but the relevance of this demand is undiminished. (For a very preliminary effort in this regard, see the next paragraph.) From a TAU perspective, constellatory self-unfolding is the most fundamental feature of all of reality, including mathematics. Its holy grail of prime numbers may be the right place for also starting to re-think mathematics in the light of autogenetic self-unfolding. Could the sequence of primes itself be strongly self-referential, i.e., could the "atoms" of the number space unfold in a strongly self-referential manner? Could this imply that all algorithms that would allow short-cutting the position of future primes need to contain themselves as a building block? Which would prove that the challenge is in principle unresolvable? Could this, in turn, be more of an advantage than a deficit? For example, as the reason why mathematics fits so well with the physical reality of our autogenetic universe? Looking at the foundations of mathematics from a TAU perspective opens a cornucopia of fascinating, entirely open questions—but they surely cannot and shouldn't be addressed in this already overloaded essay.

In summary, these are three essential implications of TAU for the foundations of physics:

- QP and GRT address different but complementary portraits of time and reality. However, in order to recognize this, we must overcome the historically grown "facticity imprisonment" and advance to a three-pronged, Borromean chrono-ontology.
- Neither QP nor GRT needs to be subjugated to the other. They remain as they stand and are just completed by "third steps" that fully relativize local spacetime.
- Via these two "third steps," one can understand the relation of QP and GRT and build, at least, a conceptual bridge between them.
- Eventually it will be possible to see QP and GRT as describing inverse transitions: into and out of the local spacetime portrait of time and reality.

10 Toward a Modest ToE and a Unified Theory of Structure Formation

Directly building on the new perspectives for the foundations of physics, it becomes possible to generalize the claim of an autogenetic nature of our universe. This leads to something that could be called a "modest ToE." The much sought-after "Theory of Everything" is often imagined as an all-encompassing theoretical framework that would make it possible to explain everything within one coherent conceptual set-up.

Gaining insight into the autogenetic nature of our universe simultaneously encourages and discourages this ambition. It encourages it in the sense of seeking a coherent conceptual framework for describing our universe—from matter to life to the mind. It discourages it in terms of showing that a Laplacian demon respectively the comprehensive availability of coercive proofs are a pure illusion, and not even an attractive one.

Simultaneously considering both, the encouragement and the constructive disillusion, leads to the idea of a *modest* ToE. This maintains and even strengthens the striving for a single, coherent, and possibly even self-evolving explanatory and interpretational framework. However, it explicitly abandons the ambition for an all-encompassing anticipability and provability. A modest ToE strives for comprehensiveness but relinquishes the figment of ever achieving completeness. In other words, a modest ToE is the epistemological counterpart to the idea of an "open systems philosophy."

The cornerstone of a modest ToE for an autogenetic universe is the iterative applicability of the thought pattern of autogenesis—i.e., the possibility to interpret matter, life, consciousness, and finally (autobiographic) self-awareness respectively the phenomenon of mind as iteratively higher orders of self-constitution that build upon each other. (The difference between a three- or four-layer architecture reflects the possibility to either "jump" directly from the self-constitution of life to the self-constitution of mind or to foresee the emergence of consciousness as an interim layer in its own right. The recognition of pre-human consciousness in certain animal species and the wish to underline the specific value and dignity of explicit, autobiographic self-awareness make a four-layer classification of ontic domains the more attractive option.)

It is characteristic for these iterative layers of self-unfolding that each higher layer sheds novel light on the implicit potential of what has been there before. Only the emergence of mind allows us to fully appreciate the inherent potential of all the lower levels. In an autogenetic universe, everything that occurs continues to unfold the meaning of all that has been there before (Fig. 13).

Across all orders of autogenetic self-unfolding, as well as across all of the self-unfolding that occurs within them, structure formation is probably the most cross-cutting phenomenon. Or, in even more radical terms: The taking place of reality in

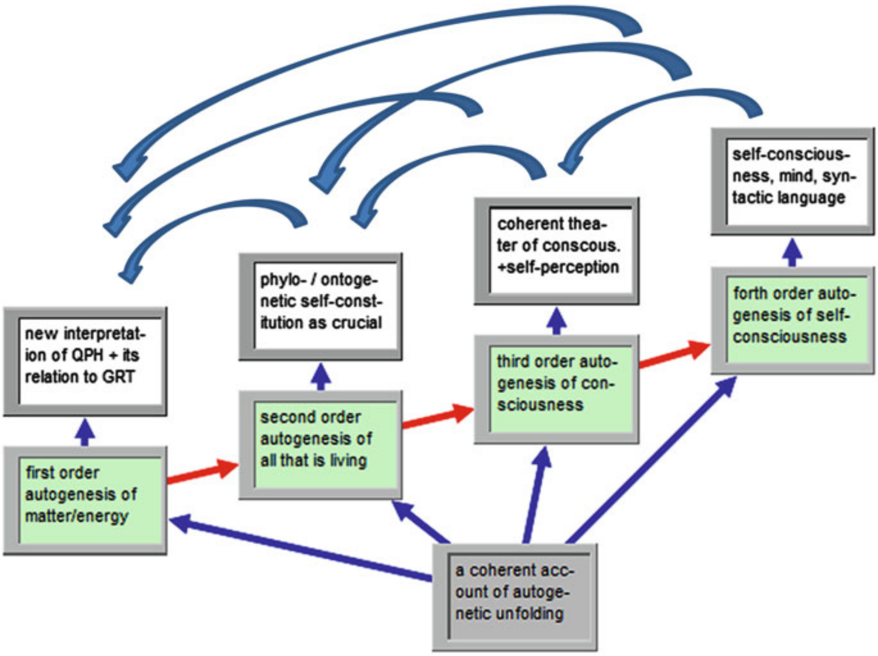


Fig. 13 Third quantization

an autogenetic universe can be described as a permanent process of generating new depth, richness, and meaning—and this process is inevitably accompanied by the emergence of new structures on the physical or semantic level.

Consequently, it would be highly desirable to come to a better and more coherent understanding of the phenomenon of structure formation and its underlying dynamics. Approaches to structure formation have only been local and more or less ad-hoc up to now. A good example for this is Prigogine's elegant theory of "dissipative structures" [10]. What has been missing is a unified conceptual framework to describe and understand structure formation as such, including its different underlying dynamics and mechanisms and how they interact in all of the empirically observable processes of structure formation.

From a TAU perspective and with the thought pattern of constellatory unfolding at our disposal, such a unified conceptual framework becomes feasible. It is a theory that describes how existing entities can come together, unite into new forms, and therefore allow (a) genuinely new qualities and entities to emerge and (b) to continue to further unleash and unfold their own meaning and potential by the very same token.

In such a unified theory, only four basic types of structure formation processes exist and result from just three underlying mechanisms. All three of these mechanisms play a role in every empirically observable process of structure formation, but their relative weights can differ greatly. The prevalence of one of the three underlying mechanisms characterizes three of the four basic types of structure formation processes; the fourth and most sophisticated one is characterized by a delicately balanced combination of all three mechanisms.

All types of structure formation have specific advantages and disadvantages. It is fascinating to see how evolution constantly utilizes and recombines them across all of the temporal and spatial scales—fully exploiting the theoretically available option space as a result. It is equally fascinating to see how these four basic types of structure formation, as well as the three generative mechanisms from which they result, can be found not only in material processes but—as an analogy—also in the domain of semantic and conceptual entities.

In an initial approximation, the four basic types of structure formation and their respective strengths and weaknesses can be described as follows (Fig. 14).

The underlying mechanisms that result in these four basic types of structure formation can even be reduced to three: the more or less fixed *aggregation* of components (A), the morphogenetic influence of a *background* field (B), and the mutual *coordination* of dynamic trajectories (C) (Fig. 15).

The claim is threefold:

- All empirically observable instances of structure formation can—at least in a quite useful approximation—be described and explained by a weighted "ABC" formula (for *A*ggregation, *B*ackground, and *C*oordination).
- SF1 can be reconstructed as dominated by mechanism A, SF2 by mechanism B, SF3 by mechanism C, and SF4 by a combination of all three mechanisms.

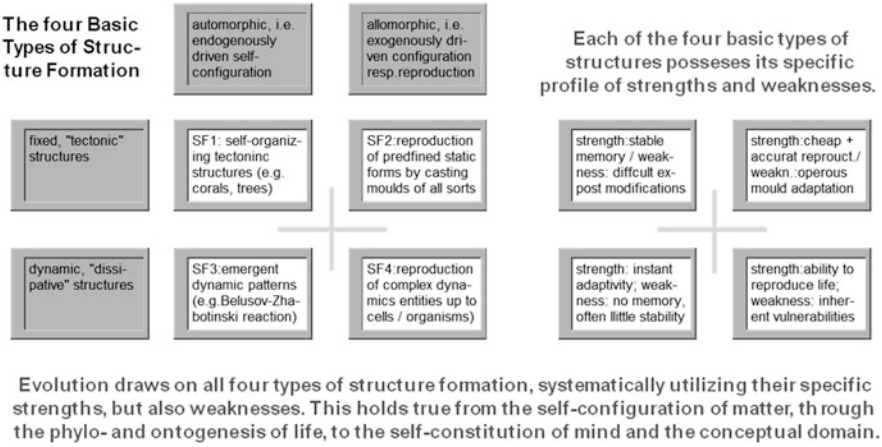


Fig. 14 Four basic types of structure formation with respective strengths and weaknesses

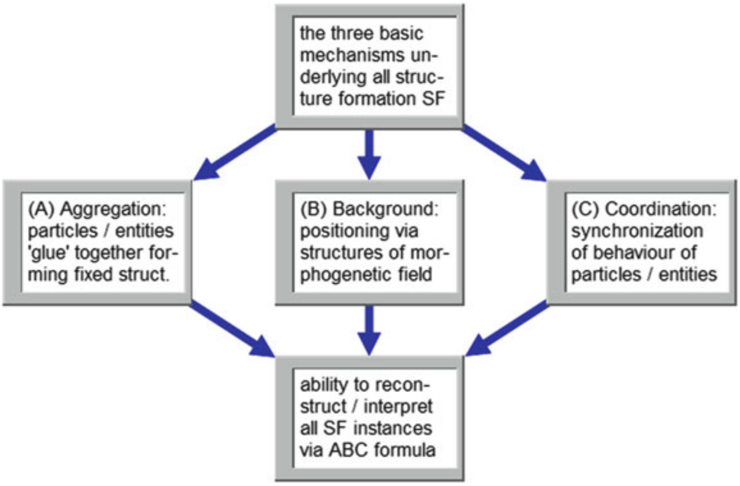


Fig. 15 The three basic mechanisms underlying all structure formation (SF)

- All four types of structure formation, and the gradual transitions between them, can be generated by a simple model that only allows for changing the relative weights of the mechanisms A, B, and C.

Good examples for SF1 are the many diffusion-limited aggregation or tree-like pattern formation processes that are characterized by the self-organizing emergence of static structures. Good examples for SF2 are iron filings that position according to the structures of a magnetic field. Good examples for SF3 are the Rayleigh-Bénard convections, or the so-called Bénard cells in which mutual coordination of the trajectories minimizes friction, therefore optimizing the energy transfer in a hot liquid [11]. Lastly, good examples for SF4 are all of the instances in which biological, social, or conceptual entities (like organisms, social institutions, or memes) reproduce themselves.

The structures and processes that make up the human brain (which is probably the most complex, highly integrated dynamic system that we know) can be described and analyzed by this new framework as a most sophisticated interplay between all four types of structure formation. From the underlying molecular processes through the formation of neurons and their linkages to the formation of perceptions, concepts, and finally even theories we can recognize all four types of structure formation dynamics—utilized according to their specific strengths and weaknesses. And in all these structure formation processes we can identify the three underlying mechanisms A, B, and C, from which they result. These processes occur on an ongoing basis and across all temporal and spatial scales, ranging from ABC dynamics on the molecular scale and in milliseconds to, e.g., the maturation of the human brain or the development of theories that may last for several years and even decades.

Evolution itself can ultimately be interpreted as higher order structure formation. But, in the TAU framework, evolution can't be reduced to "natural selection." At least equally important is that and how the genuinely novel comes into being. Selection alone doesn't do the trick. It is one aspect of the more general phenomenon of constellatory self-unfolding. On the basis of the latter the here proposed "modest ToE" is erected that allows us to address our entire, autogenetic universe—from the initial singularity to us thinking this very thought in this very moment—in one coherent conceptual framework.

Summing up: In an autogenetic universe constellatory self-unfolding is the deepest and most general feature. But, constellatory self-unfolding materializes necessarily as structure formation. Therefore, a unified theory of structure formation—applicable form the self-constitution of matter, through the emergence of life to the epiphany of mind—is an integral part of the here proposed "modest ToE" of an autogenetic universe.

11 A Radically New Approach to Human Cognition: The CPTF Compound

Starting to understand the autogenetic nature of our universe allows for a radically novel appreciation of human cognition, in which the hitherto marginalized or repudiated phenomena of *c*onsciousness, *p*resence (authentic, conceptual), *t*hinking, and *f*ree will move to the center. All four characteristics of human cognition are closely interrelated, and this is the reason why we talk about a "CPTF compound." All four components belong together, i.e., one can't consistently think any of them without assuming also the others, and all together they constitute the prerequisites, the "cognitive infrastructure," for the emergent self-awareness of the autogenetic universe in us.

It is impossible to imagine agents who are aware of themselves without also experiencing that they exist in a present. And the same holds true the other way round. Self-awareness and presence are two sides of the same coin, but this coin is, so to speak, a higher dimensional one, i.e., it has even more sides to it. Only syntactic language and the ability to form explicit concepts—in short, conceptual thinking—enables us to say "I" and turn the light cone of attention toward ourselves. No syntax means no self. Authentic selfhood, in turn, requires the—at least implicit—acceptance of free will. If we were just robots that only think that they "live," "feel," and "think," but actually follow in all that a predetermined script, we would not authentically live, feel, and think in the first place. The full meaning of these notions is only fulfilled, if we assume that we are authentically present and can react to what occurs.

The existence of free will can never be proved in a factual manner. The quest of free will remain constitutively undecidable. This undecidability is constitutive because being able to prove that someone is free is a fact based on a coercive proof, which would undermine this person's freedom. Free will is and necessarily remains an issue of deliberate acceptance—both in the theoretical sense of accepting a hypothesis and in the practical sense of accepting a gift.

The entire CPFT compound is and necessarily remains an issue of deliberate acceptance. None of the four components can be proven to exist by means of a factual, coercive proof. They never exist in the chrono-ontological format of a fact because they only "exist" if they actually occur, i.e., in their authentic taking place. All four components are irreducibly part of the *statu nascendi* aspect of time and reality. Consequently, we cannot really understand what they are—at least in the emphatic sense outlined here—if we do not have the *statu nascendi* portrait at our disposal.

It should be stressed that the here outlined new approach to human cognition, with the CPTF compound as its defining center, is in no way an argument against the most committed and scrutinizing research into the empirical, "factual" correlates of human cognition. The factual portrait of time and reality is an important and valuable part of the overall picture. But we definitely should avoid limiting our understanding, thinking, and appreciation to what we can see with these specific "cognitive spectacles." Otherwise, we will continue to miss the best aspects of both, the autogenetic universe around us and its self-awareness within us.

Much of today's cognitive science is still besotted by the computer metaphor for human thinking, which in turn is a direct consequence of the facticity imprisonment of our thinking. Obviously there are factual traces of human thinking. But its constitutive features—"that, what makes us human"—can't be appreciated in this chrono-ontological portrait. The reason for this is that strong self-referentiality and constellatory self-unfolding are constitutive for them. Ongoing self-constitution does leave factual traces, but these are derivative aspects, not its essence. Drawing again on the already used metaphor: As long as we look only at the traces we never get to see the wanderer that leaves them behind.

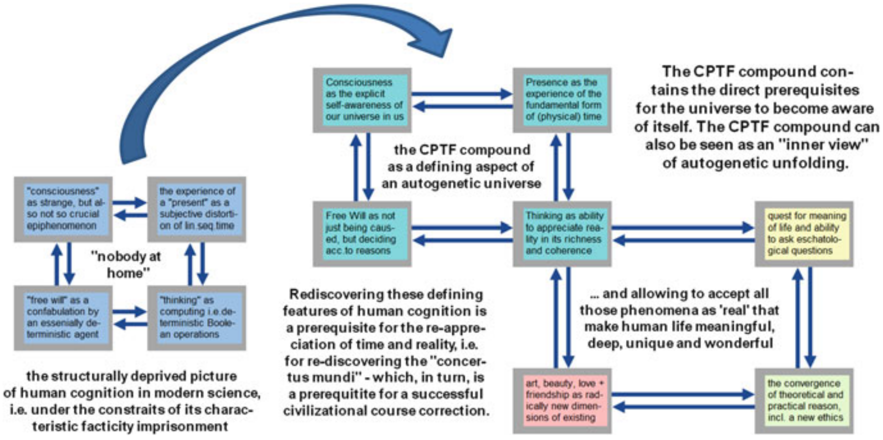


Fig. 16 Outlines of the radically novel take on human cognition that becomes possible when appreciating the autogenetic nature of our universe. Re-appreciating human cognition

In today's scientific efforts to understand human cognition it is recognized to be extremely complex. But that what actually makes us human—free will, most intimately related to the ability to think and select reasons for one's own decisions and believes, or explicit, autobiographic self-awareness, most intimately related to the experience of an authentic presence—is often avoided, by some lesser pundits even proactively repudiated, claiming that their nonexistence could be factually (!) proven (Fig. 16). (The overall approach is somewhat reminiscent of a distant family acquaintance in my childhood. She always praised her Steinway piano because of the many champagne glasses that could be placed on it.)

In an autogenetic universe human cognition plays a pivotal role in two regards. On one hand, it is, as discussed, an instance where the entire, self-unfolding universe starts to become aware of itself. On the other hand, it can be seen as the "vanguard" in the ongoing self-unfolding of our universe. Or, formulated even more radically: Our autobiographic self-awareness is a "live inside view" from the forefront of the ongoing self-unfolding of our autogenetic universe.

Only by becoming aware of itself, our universe becomes fully *auto*genetic—in the sense of not only being self-constituting but also of constituting a self, i.e., an emergent selfhood. This new approach does not deny the evolutionary development of human cognition. Quite to the contrary: It once again makes us aware of the incredible richness, sophistication, and fabulous qualities that have emerged and continue to emerge in the constellatory self-unfolding of our autogenetic universe.

Starting to see human cognition from this angle—as an inflection point at which the entire universe gains a new quality in every single human being—is also crucial for (re)gaining the basic tenor of thankful attentiveness vis-à-vis and as an integral part of our autogenetic universe. As will be explained below, this richer appreciation of our world, including ourselves, is a prerequisite for overcoming the present downward spiral of our modern civilization—which is not so much a result of bad intentions, but of a categorically deprived appreciation of time and reality.

In summary: In TAU, the CPFT compound moves from the periphery to the center of understanding human cognition since only its four interrelated components make our universe truly *auto*genetic in the full sense. The prevailing bottom-up study of human cognition should therefore be complemented by an appreciation of this infinitely wonderful phenomenon in its entirety. This is analogous to a great Buddhist saying (that could also be the leitmotif for the TAU):

> If you want to climb a mountain, begin at the top.

12 Practical Implications for Living and Acting in an Autogenetic Universe

Modern civilization is caught in an increasingly self-threatening downward spiral. Despite a breathtaking expansion of our instrumental skills via modern science and technology, we fail dramatically in using these advances in a reasonable way to make our world a better and safer place, for present and future generations. Both hard and soft factors contribute to the present predicament. On one hand, the exploitation of our natural environment has reached—if not already transgressed—the planetary boundaries. On the other hand, we are tolerating incredible acts of disrespect for human dignity, both within societies and between them. Exploding and increasingly unmanageable complexity, political leaders who are permanently chased by mass media, short-sighted electorates, and massive "first mover disadvantages" when trying to shift to more reasonable behavior: All of these are crucial soft factors that constitute a web of baleful feedback loops together with the hard factors. But seen from a TAU perspective, all of these factors are still the epiphenomena of an underlying problem. And, paradoxically, this is our greatest opportunity.

If we reduce reality to its factual aspect and time to its sequential structure, the world that we live in becomes inherently narrow, poor, and shallow. Time appears mainly as the "dent of time"—that, what constantly pulls away and erodes everything there is. As a natural counter-reaction to this, people try to seize and hang on to everything that they can grasp.

And this leads us to the following conjecture:

> The deepest movens of modern civilization is the over-fixation on power, possession, and control (abbreviated as PPC in the following). But this basic tone is not mainly an issue of "egoism" or just "materialistic values." It is rooted in a categorially deprived appreciation of time and reality. Only understanding and overcoming this fundamental epistemological constriction will enable us to overcome the fundamental predicament of modern civilization.

As soon as we understand the autogenetic nature of our universe, i.e., its ongoing, constellatory self-unfolding, the most meaningful and sensible thing that we can do is to foster its further unfolding, to the best of our abilities. This means that a modified "categorical imperative" emerges in which theoretical and practical reason

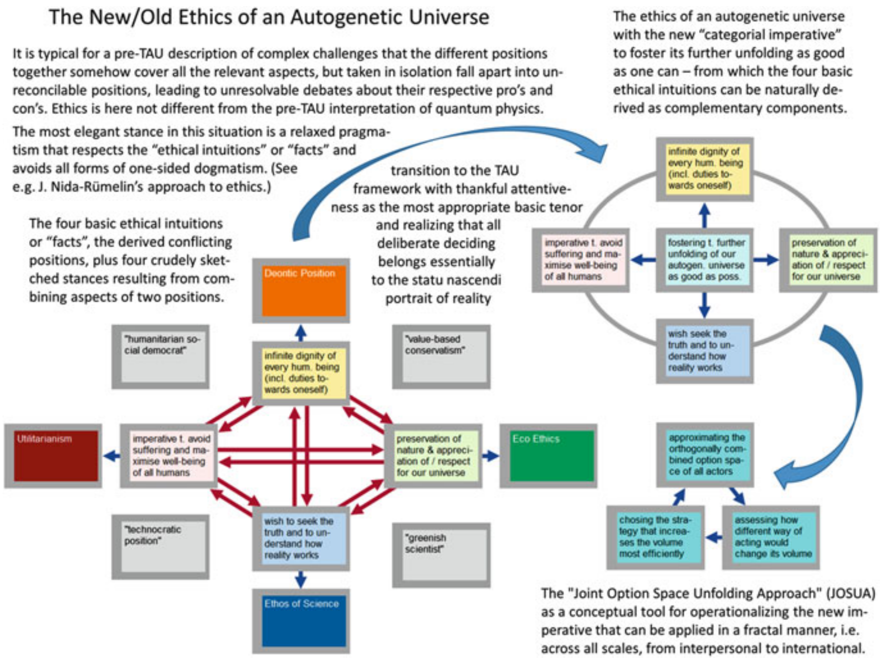


Fig. 17 Ethics of an autogenetic universe [13]

converge: *Try to foster the further unfolding of our universe as good as you can*, *in all that you do*.

Hand in hand with this new leitmotiv of acting, there emerges also a novel basic tone: Appreciating the ongoing unfolding of reality, the "concertus mundi," in *thankful attentiveness* becomes the most appropriate tenor—in relation to the world, other human beings, and ourselves.

Both the modified categorical imperative and the new basic tenor transcend traditional morals or ethics. The resulting behavior maximizes what is good for others and for ourselves at the same time. Fostering the further self-unfolding of our universe is the most ethical behavior *and* the best we can do for leading a rich and deep, meaningful and serene life. In terms of our actions, it implies a new, fractally applicable paradigm that could be characterized as a "joint option space unfolding approach" (JOSUA). According to this principle, the best manner of action is one in which the *joint* option space—the combined option space of the other(s) and ourselves—unfolds the most (Fig. 17).

Education is a good example for JOSUA. It is widely understood, actually in all non-fanatic branches of the cultural evolution of mankind, that a good education is the best we can give to our children. But exactly this is also the best thing for the parents—since it optimizes their chances of having interesting, happy, successful, and responsible children. In no other way can parents unfold the future option space of their children, of themselves, and of the world in general better than by providing them with a good education.

The here just very briefly outlined "meta-ethics" is a direct implication of TAU.[6] As long as we limit ourselves to the factual portrait of reality, ethics remain limited (and deprived) to moral(istic) principles. Within the TAU framework, instead, i.e., when constellatory self-unfolding is recognized as the most fundamental feature of reality, fostering the further self-unfolding of our universe is no longer any kind of "aliud," but just the most obvious and natural expression of being part and parcel of this ongoing self-unfolding of our universe.

My hunch is that in order to overcome today's increasingly self-threatening character, the accelerating downward spiral of modern civilization moral appeals, and even an infinite amount of good will, would not be sufficient. What we need is a most fundamental re-thinking, based on discovering the autogenetic nature of our universe. Only this will unleash the required momentum for the necessary attitudinal and behavioral changes. Only recognizing, that

– developing the tenor of thankful attentiveness,
– striving to foster the further self-unfolding of our universe,
– and acting according to JOSUA

is also the very best we can do to ourselves, will trigger the necessary, self-enhancing, and virally spreading attitudinal and behavioral changes.

But, there is additional reason for hope. The internet and social media provide, for the first time ever in the history of mankind, the infrastructure that allows for a fast global re-thinking.

Or, as Friedrich Hölderlin put it:

> Wo aber Gefahr ist, wächst das Rettende auch.
> (But, where danger is, that what can save us grows too.)

Closing Remarks

Obviously much more could and should be said on the conceptual framework of TAU, and its broad spectrum of theoretical and practical implications. The task of this paper was just to give an initial, essayistic but, hopefully, still synoptic overview on *what it takes*, *what it means*, *and what it leads to when we appreciate the autogenetic nature of our universe*.

The overall architecture of the here outlined re-thinking requires to

- unearth the categorial underpinnings of our theories, show their pitfalls, and indicate how they can be overcome by expanding the categorial foundations of our thinking,
- apply the new framework to the natural sciences, thereby helping overcome the structural bottlenecks in our present understanding of matter, life, and mind,

[6] Usually the notion "meta-ethics" refers to epistemological considerations about ethics. Here it is meant to indicate both, such a reflective perspective on ethics and the phenomenon that ethics starts to transcend itself by becoming convergent with enlightened self-interest.

Fig. 18 The architecture of the here offered comprehensive re-thinking

- show how this leads to re-conceptualizing human cognition and our role in the world,
- reconstruct the strengths and weaknesses of modern civilization, including the reasons for its present downward spiral,
- indicate that and how—based on all of the above—we may manage to overcome the present predicaments (Fig. 18).

As mentioned already, this approach massively violates the well-motivated practices of today's highly compartmentalized sciences. However, reason is the ability to appreciate an issue in its entirety. This means that a fundamental re-thinking, as proposed here, should be done in one go.

For really foundational theories there exist no longer external truth criteria. They are self-contained conceptual entities, which can gain their convincingness only out of the "elegant richness" as well as the "warmth and welcomingness" of the account of reality that they offer.

The novel account of time and reality offered here tries to combine synoptic coherence with inherent openness and the explicit awareness of its own incompleteness.

The most essential step in this trial is to re-think our notion of time. To rediscover "The Forgotten Present"—both in the sense of the primordial feature of time and in the sense of a gift, the ongoing self-donation of reality—is the entry point to this endeavor.

Albert Einstein not only saw the lack of a meaningful role of the present as a crucial deficit of modern physics. He also described what a fundamental re-thinking is all about:

> Concepts that have proven useful in ordering things easily achieve such authority over us that we forget their earthly origins and accept them as unalterable givens. Thus they come to be stamped as "necessities of thought," "a priori givens," etc. The path of scientific progress is often made impassable for a long time by such errors. Therefore it is by no means an idle game if we become practiced in analysing long-held commonplace concepts and showing the circumstances on which their justification and usefulness depend, and how they have grown up, individually, out of the givens of experience. Thus their excessive authority will be broken. They will be removed if they cannot be properly legitimated, corrected if their correlation with given things be far too superfluous, or replaced if a new system can be established that we prefer for whatever reason. [12]

We live in an autogenetic universe that has started to appreciate itself in us. Autogenesis respectively constellatory self-unfolding are not just features of its very beginning. They are the most fundamental and cross-cutting characteristics of all of reality, including ourselves (Fig. 19).

The here offered conceptual framework should allow us to appreciate the autogenetic nature of our universe. The hope is that this will also enable us to better live within it.

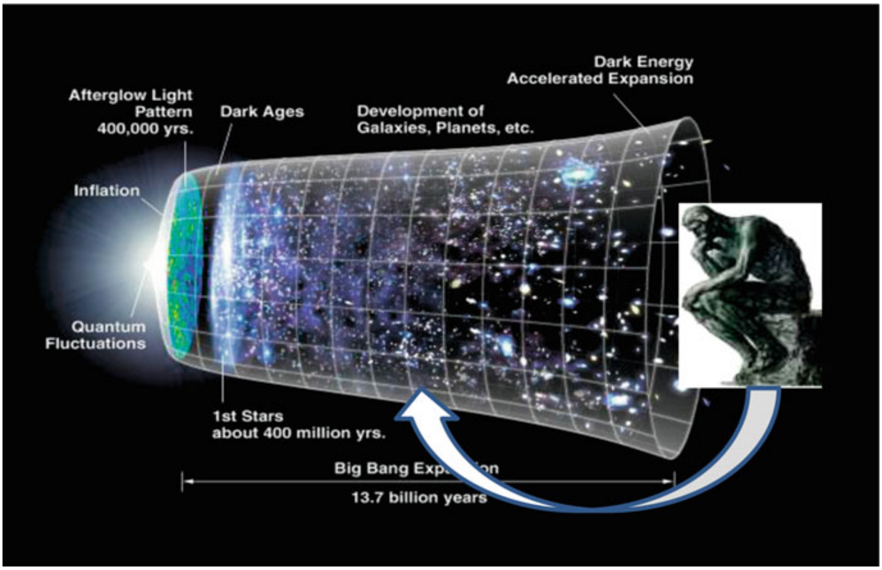


Fig. 19 Autogenetic Universe. NASA/WMAP Science Team. *Timeline of the universe*. Ca. 2006. [Modified by Ryan Kaldari 2010]. http://en.wikipedia.org/wiki/File:CMB_Timeline300_no_WMAP.jpg (09.12.14). Rodin, Auguste. *Le Penseur*. Bronze. H. 180 cm, B. 98 cm, D. 145 cm. Museé Rodin. Paris 1903. [Erstentwurf 1880]. http://www.musee-rodin.fr/fr/collections/sculptures/le-penseur (09.12.14)

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Autogenetic Network Theory

A. Nikonov and A. von Müller

Abstract Autogenetic network theory is a minimalistic toy model for a physical world built up by elements and relations. There is no fundamental background spacetime merely representing a stage for the dynamics of matter. Instead constellations of simple objects generate spacetime in an emergent fashion. Since there are no intrinsic weights of the elements or relations, the primary goal of the theory is to explore if a single class of parameterless links can account for a richer variety of physical characteristics of spacetime, forces and matter. In this introduction the basic building blocks of the theory are characterised and their correspondence to the typical spacetime background based representation of physics is motivated. Furthermore it is demonstrated how the network description could possibly solve some inconsistencies of standard physics as the analysis of the black hole entropy. In addition, as the factual perspective alone may not be sufficient for the complete understanding of the physical world, a possible integration of the triality account philosophy and the network representation of physics is proposed.

1 Introduction

> But you have correctly grasped the drawback that the continuum brings. If the molecular view of matter is the correct (appropriate) one, i.e., if a part of the universe is to be represented by a finite number of moving points, then the continuum of the present theory contains too great a manifold of possibilities. I also believe that this too great is responsible for the fact that our present means of description miscarry with the quantum theory. The problem seems to me how one can formulate statements about a discontinuum without calling upon a continuum (space–time) as an aid; the latter should be banned from the

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> theory as a supplementary construction not justified by the essence of the problem, which corresponds to nothing "real". But we still lack the mathematical structure unfortunately. How much have I already plagued myself in this way!
> (A. Einstein in a letter to Walter Dällenbach, November 1916, translated by John Stachel.)

Autogenetic Network Theory (ANT) is strongly inspired by causal set theory [5] that assumes a partially ordered set as representation of discretized spacetime. This partial order reflects the Minkowski spacetime structure that induces the time-like order between all events. The idea of causal sets is to take this partial order relation as starting point instead and derive the smooth spacetime structure as emergent [12] phenomenon. In CST matter is usually thought to arise from fields living on the set, whereas the aim of autogenetic networks is to describe matter by the set itself. This minimizes the number of physical objects that need to be considered for the description of reality and maximizes the potential to describe objects and phenomena in an emergent manner.

The idea that time and space might not be fundamental at all has intrigued physicists and philosophers for centuries. In particular, the idea that spacetime is relational, rather than absolute, was introduced by Leibniz [11] and later elaborated by Ernst Mach. The so-called Machian [2] principle is said to have guided Einstein in pursuit of its general theory of relativity. However, it is strictly speaking, not consequently realized in the theory he final presented. In some of the possible space–times described by the Einstein equations, the Machian principle is satisfied—in others, it is not. This is mainly due to the distinguished role still attributed to the notion of empty spacetime, described by the structure of flat Minkowksi space. If one takes the ideas of Mach and Leibniz seriously in a more radical way, one is guided towards a theoretical description, in which only matter is seen as fundamental. Space and time arise as merely derived concepts. The hope is that from minimal structures, expressed mathematically by sets and relations, one arrives at a theory of quantum gravity, in which four dimensional spacetime is neither theater, nor actor in the totality of physical events. They are rather emergent at some stage of complexity of the underlying model.

2 Factual Representation

The basic constituents of ANT are two countable sets $\mathrm{E} = e_i$ of elements and $R = r_i$ with $r_i \subset E\ \forall i$ of relations between those elements. A relation is a subset of elements which could carry weights. However, let us analyze how much structure emerges without link parameters. Furthermore also the elements are "maximally simple" building blocks. Standing alone they cannot be characterized by further intrinsic parameters, thus, only with respect to their external links they may differ. The aim of the theory is to express physical parameters like masses and charges as extrinsic properties emerging from specific relational configurations.

Representing the physical world as such a network, the background potentially used for either imagining or drawing the representation is physically irrelevant.

Fig. 1 Representation of ANT with elements as *points*

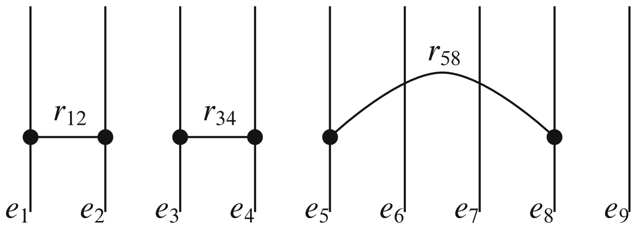

Fig. 2 Representation of ANT with elements as *lines*

Consequently it must not enter any fundamental law. Graphically ordering the elements as points on a circle does not represent anything different from lining them up. Secondly the elements may be graphically represented by any other object if its components do not enter any physical law and it is always taken as a whole. For example, taking lines as objects both their smooth structure and size must be ignored. As a single point it merely represents an attribute-less element of the autogenetic network.

Including the relations in the representation where the elements are denoted as points and integrating the three relations: r_{12}, r_{34}, and r_{58} reality is graphically represented as Fig. 1.

Representing the elements as lines, a physical reality equivalent to Fig. 1 is portrayed by Fig. 2.

2.1 Partial Order

As final constituent of physical ontology and in the tradition of causal set theory [5] a partial order between the relations is introduced. It satisfies the following properties:

- reflexive: For all $r \in R$, we have $r \leq r$
- antisymmetric: For all $r_1, r_2 \in R$, we have $r_1 \leq r_2 \leq r_1 \Rightarrow r_1 = r_2$
- transitive: For all $r_1, r_2, r_3 \in R$, we have $r_1 \leq r_2 \leq r_3 \Rightarrow r_1 \leq r_3$
- locally finite: For all $r_1, r_2 \in R$, we have $\text{card}(\{r_3 \in C \,|\, r_1 \leq r_3 \leq r_2\}) < \infty$

$Card(A)$ is the cardinality of the set A and C the set of all spacetime events. The fact that the number of events in between any pair of events is finite can be regarded as spacetime discreteness.

The CST program [5] has motivated a partial order between point like events as being equivalent to a Lorentzian structure of any potentially underlying manifold.

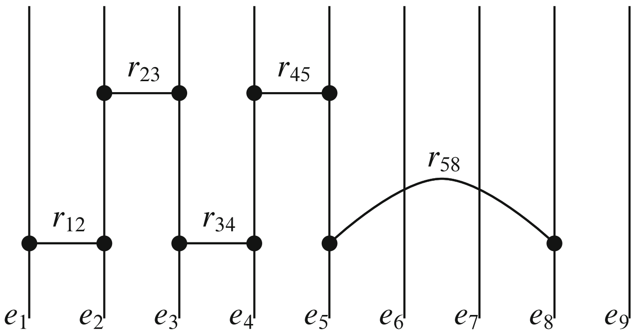

Fig. 3 Partial order between the relations

The slogan "order + number = geometry" sums up that two spacetimes can be bijectively mapped to each other if one knows both their causal structure and their local volume elements. In CST the local volume of a spacetime region is just proportional to its number of points which is countable by definition. The correspondence to special relativity works because two events being ordered are equivalently time-like related with respect to the spacetime embedding, whereas two elements that are not ordered are space-like related respectively.

In ANT the order refers to the relations whereas in causal sets it refers to point like events. However, what we do know considering relativistic physics is indeed the fact that two interacting points are light-like related which is neither time- nor space like. Regarding links as interactions it is natural to regard their endpoints as light-like related. As a consequence ANT reflects naturally the relativistic spacetime structure.

Integrating the partial order between the relations we obtain a graphical representations as in Fig. 3. Shown is an autogenetic network with five partially ordered relations. The representation is equivalent to the following partial order: $r_{12} \leq r_{23}$, $r_{34} \leq r_{23}$, $r_{34} \leq r_{45}$, $r_{58} \leq r_{45}$.

The necessity of adding the partial order as independent ingredient to the theory remains to be discussed further. Alternatively there are proposals of obtaining it automatically from the underlining degree distribution of the graph [10]. The advantage would be an even further reduced set of ontological assumptions with only the two sets of elements and relations remaining. But for the introduction of the model we assume the partial order relation as independently given instead.

2.2 Space—Matter—Point Dualism

In causal sets the discrete structure represents the structure of empty spacetime [5], whereas autogenetic networks express material ingredients as well. Each point is reflecting both a position in spacetime and a point of matter. Motivated by Einstein's theory of general relativity we do not treat spacetime and matter as separate objects that interact, but rather as two different perspectives upon the same autogenetic network. Consequently we will only assume the physical reality of the connected network instead of spacetime, matter, wave functions, fields, etc. as independent ontological components.

Autogenetic elements E are reflecting both positions in spacetime and points of matter. In comparison with the wave-particle dualism the elements play always both roles simultaneously. Due to the wave collapse the wavy character of an object gets promoted to being more localized. There is no equivalent process in the dualism portrait here. Every element can be regarded as spatial or materialistic instead. The two characterizations are fully compatible whereas in the wave-particle dualism they are not. The difference between spacetime and matter is of no mathematical or physical nature, but more related to physicists being used to dividing the representation of the world into such categories. Indeed it may be convenient to focus on either the spatial or material character of the elements in order to generate applicable physical models. However, this convenience does not justify the assumption of a fundamental independence.

We want to demonstrate how to regard one subset of elements as space and another as matter. For example, consider only the description of one particular material element/test particle. We can ignore that the other elements also represent matter and instead treat them as devices that merely generate spacetime. With an eye on convenient bookkeeping the test particle can be denoted by P and the space-generating elements by x_i. The x_i elements relate to each other in such a way that one effective spatial dimension emerges. This is shown in Fig. 4. In this illustration nine spatial elements $x_i : i \in 1, É, 9$ generate the x-axis reaching from the left to the right. The material element P connects to x_1 reflecting the element being positioned at the furthest point to the left this axis.

A related representation where the information about the discreteness of the x-axis is smoothed out appears like the graph below. As above P is positioned at the left of the x-axis. Only the true nature of this x-axis being made up by multiple discrete elements and relations is ignored (Fig. 5).

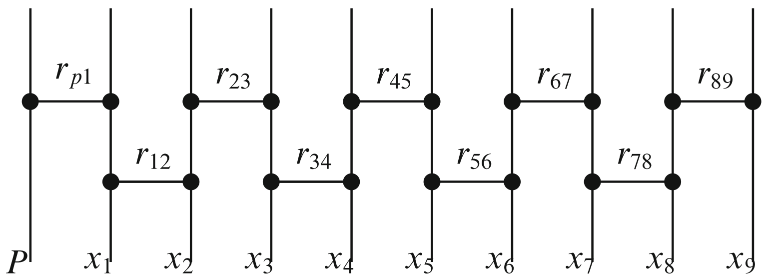

Fig. 4 Particle position in terms of ANT

Fig. 5 Particle position in a smooth one dimensional space

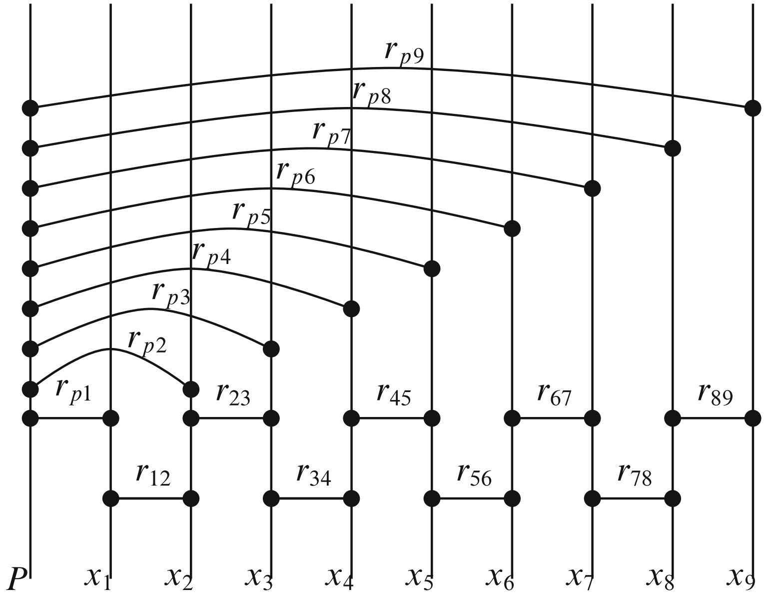

Fig. 6 Particle dynamics in terms of ANT

2.3 Motion

After having represented how material elements obtain effective positions in an emergent spacetime we now include dynamics in the system. As before the set of elements is divided into one material element P and the rest being spacetime generators X. Being connected to spacetime generators the test particle P obtains a corresponding position. But for the representation of motion multiple relations from the element P to different spatial elements are included. Since the relations between P and the spacetime generators X are partially ordered, they correspond to the change of positions with discrete timesteps. Consequently a trajectory of P through spacetime made up by X is represented. In Fig. 6 one can identify the vertical dimension as eigentime of element P in the sense that it orders the relations of P. Speaking in terms of this time that corresponds to the partial order relation, P relates first to spacetime element x_1, then to x_2, and so on. Consequently one can characterize this dynamical process as movement from the left to the right of the x-axis.

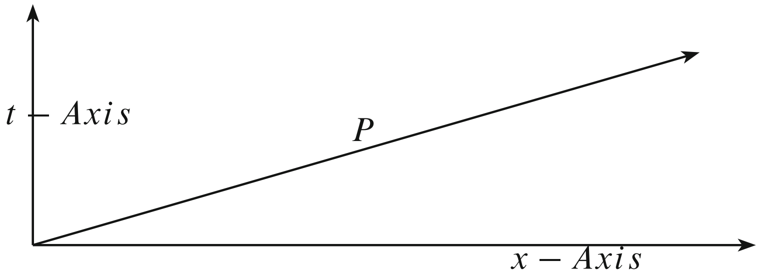

Fig. 7 Particle trajectory with respect to background spacetime

As done in the previous section the graphical representation shown in Fig. 6 can be approximated by Fig. 7. Again both the x-axis constructed by the x-elements and the P trajectory are smoothed out.

3 Triality Account

"My propositions serve as elucidations in the following way: anyone who understands me eventually recognizes them as nonsensical, when he has used them—as steps—to climb up beyond them. (He must, so to speak, throw away the ladder after he has climbed up it.) He must transcend these propositions, and then he will see the world aright" [15, 6.54].

The relational perspective we motivated in the previous chapter corresponds clearly to the ideas that Wittgenstein had in mind when writing tractatus. However, as final remark proposition 6.54 points at the incompleteness of logical language. When doing physics one also searches for such a complete language representing physical ontology. Hence, it is necessary to reflect about Wittgenstein's remark and finally realize the breakdown of the factual description.

Science tends to focus on representing the world by laws which seem to control the observed order in the universe. "Thus people today stop at the laws of nature, treating them as something inviolable, just as God and Fate were treated in past ages" [15, 6.372]. Also physics restricts on describing of what exists, but not why it exists. However, rewriting physical reality in terms of laws, but without giving sufficient reasons for these laws does actually not elucidate anything. Instead of only expressing laws suppressing facts, one might consider asking a question instead: Why is the world not different? As scientists let us optimistically assume that "If a question can be framed at all, it is also possible to answer it" [15, 6.5].

Therefore firstly the notion of possible alternatives for the actual world must be introduced, otherwise the meaning of "different" would not be qualified. Secondly the notion of consistency needs to be defined, so that the actual world becomes more consistent than its alternatives or the most consistent one. In order to discuss the consistency of "possible" worlds with only one corresponding to factual reality a richer notion of reality is necessary. As consistent architecture of such a notion Albrecht von Müller has proposed the triality account which we now couple to a network ontology.

If we do not assume any god-given laws and initial conditions a world of facts can only be described, but not been motivated as anything typical, natural, or consistent. Consequently the triality account embeds the factual within a more complete set of perspectives. The three triality perspectives upon reality form an appropriate set for describing the transitions from possibilities to facts. Possibilities and transitions correspond to the so-called apeiron and the statu nascendi aspect respectively. As the third triality component the factual state relates to the static network representation introduced in the previous chapter. Linking the different levels of reality consistently among each other and to the world, it might finally make a bit more sense. And for the beginning a "bit" might even be enough [13].

Triality account:

- **Factual state:**
  The factual description corresponds to the set of physical facts that in the ANT framework contain the elementary set E and relations R including their order. The complete factual state encodes the spacetime embedding and well-defined relational trajectories of all elements. The four static dimensions are necessary and sufficient for an embedding of the complete partially ordered network. There is no space for time although the partial order is often considered as "factual time." The partial order is an artifact of the necessary separation between all factual elements.
- **Apeiron state:**
  The apeiron is the basis for the creation of relations. It contains the elementary set E and all possible relations $\bar{R}$ that may be created. These are superposed meaning that none of the relations has any definite truth value regarding its factual existence, however, as possibilities they all exist. Possibilities exist without the order acting upon facts. A consistent subset of possible relations corresponds to the factual description. The apeiron state is neither less nor more complete than the factual one, it is just a different aspect. In a sense the apeiron is as superposed with the factual state as possibilities within the apeiron.
- **Statu Nascendi:**
  The statu nascendi is related to the selection process. It contains a selection procedure for the evaluation of all possible relations contained in the apeiron state. The transition state relies on the factual boundary R of the apeiron region $\bar{R}$. Facts that have already emerged influence the apeiron selecting further facts.

In the following illustration we contain three factual relations $r_{12}, r_{34}, r_{67} \in R$ and a subset of superposed relations being a part of the apeiron state. The set $\bar{r}_{52}, \bar{r}_{53}, \bar{r}_{54}, \bar{r}_{56}, \bar{r}_{57}, \bar{r}_{58} \in \bar{R}|e_5$ contains the possible relations (dashed) of the apeiron state that are restricted to the element e_5 (Fig. 8).

Looking at a set from the apeiron perspective where all relations are equally possible, the "statu nascendi" level gives rise to an evaluation procedure finally selecting one out of the many possibilities to be promoted into the factual level.

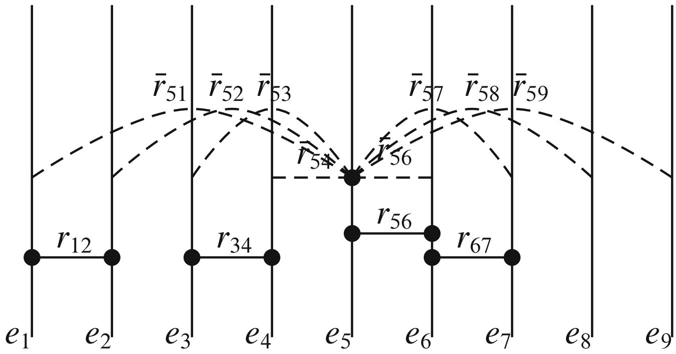

Fig. 8 Mixed state

The **Consistency:** $\bar{R} \to \mathbb{R}$ is defined as map from the set of possible relations $\bar{R}$ to the real numbers. It measures the consistency of something possible becoming a fact which can mathematically be related to a probability measure. In a sense before being existent the fact must have been consistent as a possibility. Let us assume that, given a set $\bar{R}$ of possible relations, the one with the highest consistency is chosen to become a fact $\in R$. On the other hand, high consistencies could set up models with the consistency only being related to a high probability of a relation becoming a fact.

In the figure above the relation r_{57} is consistent since element e_5 has already an indirect relation through e_6 to e_7. Not given any path from the one to the other element is equivalent to the elements existing in parallel worlds. There is no absolute background connecting the elements, only a network based path of factual relations could do so. e_5 could, for example, not relate to e_4 or e_3 since they are factually not connected to the subset that e_5 is a part of. Secondly, for the framework elaborated so far one could even say e_5 cannot relate to e_6 since they are already related. As a consequence the potential r_{57} relation appears to be the unique candidate for becoming a fact. Hitherto, this shall just remain an example supposed to represent autogenesis neither completely nor consistently.

3.1 Apeiron Black Holes

One application of ANT is the evaluation of the black hole entropy. This might seem a little far reaching regarding that we have not yet discussed how simpler parts of physics emerge. Nevertheless, black holes can be represented by an almost pure apeiron state that can be analyzed mathematically. Considering the complete and symmetric apeiron option space is less complicate than the proper mathematical treatment of a mixed state of facts and possibilities.

The introduction of a black hole entropy has shown to be necessary in order not to violate thermodynamics. As a consequence a variety of thermodynamical laws for black holes analogous to the standard thermodynamical laws emerge. From the general relativistic perspective Hawking and Bekenstein [3, 7] have proved that the black hole entropy must be proportional to the area of the event horizon. Nevertheless from the microscopical perspective it still remains unclear how the black hole can store such a high entropy and where exactly its degrees of freedom are located. Here we want to give an explanation of the black hole entropy within the framework of ANT that leads to both the area dependence and the approximate match of the numerical prefactors with the value obtained by Bekenstein [3].

From the perspective of the triality account the black hole reflects a complete apeiron state where order and local spacetime have not yet emerged. We cannot look into the interior of the black hole and thus assume the coexistence of all possible structures made of its N elements. Disregarding this rather epistemic argument, one could equivalently argue that due to the breakdown of factual spacetime, on the apeiron state remains for governing the black hole interior. This can be characterized as counting all possible microstates corresponding to the few macro variables of a specific black hole. As mentioned in the introduction of apeiron state its lack of any partial order leads to the possible configurations being expressed in terms of the sets E and R denoting elements and relations respectively.

We assume the black hole being made up by N elements that lead to $P = \binom{N}{2} \approx N^2/2$ possible relations between those elements. Since each possible relation has two different factual states regarding its existence the partition function Z of the complete ANT is the power set of all possible relations. As usual the entropy can be expressed as $S = k \ln Z$ with k denoting the Boltzmann constant. Consequently the partition function Z and entropy S become:

$$Z = 2^{N^2/2} \tag{1}$$

$$S = k \ln Z = \ln 2 \cdot N^2/2 \tag{2}$$

Adding some set of intuitive physical assumptions that are supposed to be justified rigorously in subsequent papers, we set the number of elements N as proportional to the total mass M of the black hole $M = m_0 N$. We rely on general relativity to the extent that the mass of a simple black hole is related to its Schwarzschild radius $R = 2GM/c^2$. However, we will also present a derivation of general relativity starting from the basic principles of ANT in a subsequent paper. Here we mostly want to motivate why the possibility of embedding general relativity in the ANT framework would possibly have interesting consequences for the analysis of the black hole entropy. From this one immediately obtains:

$$S = k \ln 2 \cdot N^2/2 = k \ln 2 \frac{M^2}{2m_0^2} = kR^2 \frac{c^4 \ln 2}{8G^2 m_0^2} = kA \frac{c^4 \ln 2}{32\pi G^2 m_0^2} \tag{3}$$

Now this formula coincides precisely with the original formula of Bekenstein:

$$S = A\frac{\ln 2 \cdot kc^3}{8\pi \cdot G \cdot \hbar} \tag{4}$$

if the mass m_0 given in the formula above is identified with half the Planck mass:

$$m_0 = \frac{1}{2}\sqrt{\frac{\hbar c}{G}} \tag{5}$$

With regard to the black hole radiation, Hawking obtained an entropy that differed from the formula above by the factor $\frac{1}{2}ln2$ [7]. The entropy obtained is in the right regime, but the prefactor of the m_0 to Planck mass relation has to be discussed further. For now let us just mention the elementary mass matching approximately with the Planck mass. However, instead of taking this as coincidence we are going to justify the Planck mass as naturally emergent in the upcoming paper.

3.2 Statu Nascendi

In the following section we first introduce the notion of nascendi time which differs strongly from the relativistic notion of time. Secondly we motivate two related approaches to derive particular forms of dynamical laws. The triality account is not merely about representing the world by mathematical devices but also attempts to propose how the laws themselves emerge from the apeiron level. The statu nascendi is the crucial part of reality that governs the emergence of structure and thus in a weak sense corresponds to laws. Knowing specific laws from observation, we just attempt to relate these to the statu nascendi. However, the difference between classical laws is that they tend to be functions of factual time whereas statu nascendi processes happen in nascendi time. Classical laws merely describe how reality is, whereas statu nascendi laws describe how reality becomes.

3.2.1 Nascendi Time

We have already described how to represent motion in the given framework. Aside from that, we have not specified which particular laws do govern the dynamics of the system. In order to describe local dynamics, we want to describe processes happening in "non-spatialized" time which we name "nascendi time." This notion of time is supposed to allow the creation of new facts that are chosen from the apeiron state. Let us point out that this is not the time known as fourth spatial dimension that extends space to spacetime. One carefully needs to differ between these two notions of time. From the viewpoint of the triality account general relativistic dynamics is merely spacetime statics.

Given the notion of "nascendi time" we explain how particular relations are chosen from the possible apeiron future and created in the present. In the factual perspective there is no past or future. There is only the global and completely present spacetime. Thus, we need the statu nascendi that uses time for closing a hole of possibilities within the factual perspective through the creation of further facts. Then the past consists of all given facts and the future of the apeiron hole. To sum up we have to analyze how to select which apeiron possibilities become part of the factual state through the statu nascendi process.

4 Triality Dynamics

Summing up this approach in the language of the triality account we derive dynamical equations as part of the statu nascendi. Let us start proposing two approaches to a more quantitative analysis of dynamics consistent with the combination of the triality account and relational networks. The first is based on the notion of resistance distance which then gives rise to an action principle with respect to this discretized spacetime. The second is based on maximum entropy dynamics which has shown to be intertwined with basic quantum principles under further assumptions that are consistent with the relational approach.

4.1 Resistance Action Approach

The accurate and global analysis of consistencies has to integrate the dependencies of multiple relations including their consistencies. As a natural starting point global consistency of the complete network can be defined as the sum of all local consistencies of single relations. An interesting option for a specific choice of the consistency is the inverse resistance [6, 9] between the two points in a network. Keeping intrinsic and absolute weights out of the theory, each relation must have an equivalent unit resistance. Then with the usual rules for electrical resistances being assumed for the resistances between two elements, arbitrary resistances can be obtained effectively. Based on the idea that the consistent world did simply not "resist itself from coming into being," the consistency of a possibility is defined as inverse of the resistance $C = \frac{1}{R}$. Then this resistance can be related to the consistency of a possible link between the two points. Based on the resistances of single links one can formulate the complete network action S as the sum of all link resistances $S = \sum_{l_i} R_i$. Minimizing this action with respect to given boundary conditions we can determine specific configurations of subnetworks.

Alternatively putting this argument into more probabilistic terms the resistance of a possible relation between a and b measures the probability to perform a random walk from a to b within the network. In a sense one could imagine new links being generated by such random walks. Then the probability is related to the flux of walks from a to b and thus inverse proportional to their resistance. Maximizing the consistency or probability of the links would then correspond to minimizing the sum of resistances. Let us point out that although the random walks seem to be of probabilistic nature, the resistance action aims for the minimization of randomness. Finally autogenetic networks allow for a governing principle that relates neither to an exact determinism nor to complete randomness which is the motivation for the notion of autogenesis. However, mathematically ANT allows for the recovery of probabilistic and deterministic tools. Taking these toolsets not as fundamental but only as practical and effective might then allow for an improved and more objective analysis.

4.2 Maximum Entropy Approach

Embedding the apeiron region in a factual boundary gives rise to specific constraints. Taking into account factual constraints the entropy describes the richness of apeiron islands. Maximizing the entropy then gives rise of the probability distribution representing the selection of facts from apeiron possibilities.

In many physical situations the nascendi future looks like the standard spacetime future being everything at the future side of a specific Cauchy surface. One tries to predict the local generation of the facts growing into the future from the boundary to the past. As a consequence, the resistance distance and the strict localization of elements breaks down and one cannot calculate the global action anymore. Thus the open factual can only be handled through the statistics of anticipation. As mathematical tool to obtain the correct statistics we propose the maximum entropy principle.

One characterization of the Shannon entropy is its identification with the expectation value of the information contained in the given outcome. Let us assume that the outcome A has the probability $p(A)$. The information I_A of outcome A measures the number of bits required to represent this outcome and the entropy S is the expectation value of this information over all possible outcomes.

$$I_A = \ln p_A$$
$$\Rightarrow S = -\sum_X p(X) \ln p(X)$$

Consequently, maximizing the entropy value is equivalent to maximizing the expected information gain. As a Bayesian would say, that one simply minimizes irrational assumptions about the likelihood of specific outcomes. However, with a focus on modeling reality instead of merely motivating rational ways of reasoning, one has to argue differently.

Since information represents facts, the increase of information corresponds to the increase of facts. Logical procedures that generate dynamics correspond to deterministic laws. These produce tautological representations from the factual initial condition instead of new facts. Thus, given a more deterministic selection procedure, one obtains a stronger restriction of future possibilities. Furthermore, less options can be encoded by a smaller amount of information, for instance, two options by only one bit, four options by two bits, and so on. Consequently the maximization of necessary bits reflects the minimization of deterministic restrictions upon possible options. This is consistent with assuming the future being an option space as rich as possible—the apeiron.

Given the multitude of options contained in the apeiron, one can describe the probabilities of them becoming a fact. Starting from a pure apeiron perspective no deterministic selection principle can be imposed. Such a principle would restrict the diversity of possible results and thus contradict the assumption of equal possibilities in the apeiron state. Another way of formulating our motivation is—since we want to explain the deterministic causes—starting without determinism by maximizing the entropy. Then each deterministic cause can be added as a constraint on the possible options of the future development. This allows us to clearly identify the apparently random and the deterministic part of the theory. Randomness in our case might just be due to the local ignorance of the apeiron state with its indeterminacy not fitting into our boolean logical treatment. From the complete factual perspective facts appear deterministic instead. Thus there is no need to assume randomness at a fundamental level.

4.3 Epistemic, Realistic, Triality

The maximum entropy principle is usually strongly related to Baysianism. Also applying the resistance action approach forces a partial ignorance of the complete and correlated network relations. In a sense the consideration of what we know and what we do not know gives rise to specific dynamical laws. In particular Jaynes [8] and others have successfully demonstrated Baysianism being a powerful mathematical tool, that allows to handle information and leads to the emergence of dynamical laws. It can be regarded as appropriate tool to derive laws for the autogenetic network. Taking this epistemic view one might attempt to declare the apeiron as lack of knowledge. There is yet a crucial difference of the epistemic perspective to the real nature of the apeiron. It does represent an independent realistic state that is both completely independent from any observer and the factual state.

Let us also point out how the two viewpoints differ with respect to the nature of time. Dynamical behavior that happens in nascendi time has almost nothing to do with the addition of knowledge about preexisting facts. Only in specific physical cases and through the projection upon a mathematical level the epistemic view matches the realistic one. However, portraying the world as static spacetime block with epistemic time forbids the question about the origin of this block or any of its local parts. Consequently, only the promotion of an unknown to a truly undetermined state gives rise to nascendi time. This opens space for the creation and explanation of facts.

After all it becomes clear that the triality account simply means a complete acceptance of reality. Given the apeiron level of reality, the possibilities are regarded as physically existing entities as well whereas usually only the facts are taken as such. And typically the acceptance of reality gives rise to a more complete understanding. Finally, the physical, mathematical, and philosophical consistency of the triality architecture is guaranteed through the well-defined connection of its three levels.

5 Towards Quantum Gravity

Physics involves different perspectives upon reality. Quantum mechanics seems to rely in particular on the statu nascendi that promotes the apeiron into facts, whereas general relativity is based on a complete state of facts leading to a well-defined global spacetime. Only from understating the relation and transition between these states of reality, we can start improving our approach to make quantum mechanics and gravity consistent. As described within the relational theory we have found suitable units for the factual state. These are the smallest units that we can use to stepwise promote from apeiron into the factual state. For particular considerations, the transition between the states of the triality account is necessary in order to build up a model for the full spectrum of physical reality. For some evaluations a completely factual or purely apeiron perspective becomes necessary instead. Specific mathematical devices like the metric are based on the complete factual state. On the other hand, the notion of the quantum mechanical time is based on the statu nascendi. The notion of entropy arises from coupling the apeiron to the factual state.

Sometimes mathematical devices relate to different states of reality occur in a single physical law. Then, this mathematical expression is about different aspects of reality like possibilities and facts simultaneously. One has to verify the consistency of the link between those aspects that the formula provides. Therefore the state transitions need to be carried out while preserving the specific context of the fixed mathematical expression. Only then theoretical statements based on different states can be used simultaneously.

5.1 GrANT

The approach to gravitation starts with introducing an approximation of the resistance distance being valid on a large scale. On this scale we start with a coarse graining of reality in terms of big clusters supposed to represent large objects that interact through gravitation. The “cluster distance” is related to the number of links between the objects. Then we look at the dynamics of the complete network and how the distance between the objects or their number of links respectively changes. From this we step by step motivate the general and well-known expression of gravitation in terms of Newtons law. Similar to an approach by John Baez [1] but keeping the consistency with the set ontology we extend this to the more general Einstein equation of general relativity.

Further topics related to GrANT are the de Sitter topology of the universe that is related to its expansion and the cosmological constant. Taking ANT as starting point, we are going to motivate why this topology indeed seems to be rather the natural solution of the Einstein equation than pure coincidence. In an approach by Krioukov et al. [10] it is shown that a causal network can be indeed naturally embedded in de Sitter space which is the physical spacetime we do observe. Since this approach was originally carried out for causal sets, we are going to discuss how it might relate to ANT as well.

Usually general relativity is based on the factual and complete perspective upon global spacetime. Potentially the discretization and relational treatment of both quantum mechanics and gravity lead to a new approach to unify the two. Although the relevance of the apeiron occurs especially in the quantum regime, even general relativity has to deal with large apeiron holes in specific regimes. We have already motivated the analysis of black holes as complete apeiron state. As one knows, black holes are interesting candidates for testing the connection of quantum mechanics and general relativity. A theory of quantum gravity would have to be able to somehow deal with that.

5.2 QuANT

Within the ANT framework quantum mechanics is supposed to be based on exactly the same basic ontology as gravity is. Both quantum and gravitational dynamics shall not only be consistent but also be derived from the same basic principles. However, quantum effects are rather related to the micro-perspective on single elements whereas gravity relates to the statistics of many elements.

Motivated by an approach proposed by Ariel Caticha [4] we attempt to handle quantum phenomena with the maximum entropy principle. This method allows the derivation of probability distributions for single particle positions. However, purely epistemic viewpoints as the one presented by Ariel Caticha do not explicitly motivate the inclusion of hidden variables necessary for obtaining stochastic quantum behavior. Furthermore some aspects regarding the energy functional and specific stochastic independencies remain to be discussed. And although it seems a fresh and natural approach to the correct understanding of quantum mechanics, further steps towards a relativistic version are necessary.

Considering ANT the inconsistency of single particle dynamics on a fixed background is obvious, one has to regard the background dynamics simultaneously. Here we can only mean an effective background corresponding to the non-locally coupled elements. Corresponding to the approach by Ariel Caticha and from the local particle of the test particle these background elements seems like “hidden variables,” with respect to ANT “hidden” can be translated into “related.” Only due to the fact that the Schrödinger equation attempts the description of a single particle, the rest of the network becomes in a sense “hidden,” objectively it is not. We will show that this perspective seems to be the origin of quantum mechanics. Quantum behavior simply relates to the expected back reaction of the dynamical network on the particle trajectory. Considering both the degrees of freedom of the local particle and the global network restores the symmetry between the elementary constituents of ANT.

To sum up, based on QuANT we motivate a stochastic interpretation of the wave function, its link to the particle dynamics and the extension to a relativistic formulation. Regard that ANT is inherently relativistic, as a consequence also a consistent quantum version would have to be. Furthermore not only implicitly the link between QuANT and GrANT will become obvious, but we will also start the discussion of quantum gravitational topics from the ANT perspective.

6 Conclusions and Outlook

We have presented both the triality account as representation of the possible perspectives upon reality and the relational set framework as representation of spacetime and matter. Combining the two theories, we have demonstrated how the nature of change in time can be expressed. Many theories choose to represent reality by a well-defined ontological set and extra dynamical laws for their generation, by contrast, our motivation is to start with different perspectives upon the physical elements instead and then derive the laws for their supposed generation. One might postulate a set of dynamical laws, we on the other hand motivate how dynamics can arise rather emergently. As method to obtain specific probability distributions for possible results we mentioned the maximum entropy principle. Ariel Caticha has applied this principle to derive quantum mechanics [4]. We explain how this derivation might be related to both the relational viewpoint and the apeiron framework.

A different topic mentioned is the application of the theoretical framework to resolve quantum gravitational problems like the derivation of the black hole entropy. We have shown how the original Bekenstein entropy [3] formula can be derived with the correct prefactor. From the triality perspective we explain how the black hole can be regarded as complete apeiron state where the factual perspective upon the relations is promoted into superposed possibilities. As a consequence of the superposition the factual order relation breaks down and the number of configurations can be counted in order to derive the entropy.

In the subsequent papers we are going to elaborate and apply the resistance distance. Furthermore we are going to explain in greater detail the approach to quantum mechanics related to the ideas by Ariel Caticha [4] and its correspondence with the framework presented here. Besides we are going to present an approach to general relativity which relies on the non-tensorial reformulation of the Einstein equation by John Baez [1]. Related to this we are going to discuss the geometrical embeddings of autogenetic networks into four dimensional de Sitter spacetime [10]. Although specific issues related to a proper embedding remain to be discussed, a proper correspondence of the network with the typical spacetime representation of dynamics is one of the current research priorities. This must be regarded as crucial step towards the correct matching with the successful physical theories developed within the last centuries. However, ANT provides the causal structure necessary for special relativity, the clear identification of spacetime and matter and further aspects typical for the observed spacetime. Thus, at least in principle it features the crucial structure for a rigorous embedding.

Another topic is electrodynamics that was originally developed as relational theory by Weber, then rephrased in terms of fields by Maxwell, and later expressed again as relational and relativistic theory that can be expressed without any fields by Wheeler and Feynman [14]. Formulating or even deriving the Wheeler–Feynman version of electrodynamics in terms of ANT is the more natural approach, yet the effective emergence of field like quantities can also be examined. As convenient mathematical tool fields can be obtained in the same sense as coordinates arise from the embedding of the set in an underlying spacetime.

Due to the simplicity of the theory and the minimal set of physical entities and properties assumed by ANT, the potential connection to the physical world might seem unclear. Fortunately, although one has to start almost from scratch, one knows a lot about physical phenomena to be recovered. Furthermore no argument against complex autogenetic networks potentially representing physical realities has been found yet. Thus it would be interesting to explore to what extent an actual mapping can be developed. It would clearly take effort to reproduce accurately any physical results that have already been expressed in other frameworks. However, regarding possible improvements on expressing things mathematically rigorous and philosophically clear, it might be worth a try.

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Relational Events and the Conflict Between Relativity and the Collapse

Thomas Filk

Abstract It is shown that some of the conundrums of quantum theory, which are related to the locality structure of space–time, appear less astounding when space–time is considered as relational, and the localization of an event is defined by the relations this event has to other events. In particular, a relational space (or a relational space–time) might indicate how the dilemma of Bell's theorem—either quantum theory has no "elements of reality" or it is non-local—can be avoided. Furthermore, it is argued that quantum theory may be more amiable to the implementation of a "present" as compared to classical physics. This present should be considered not as the point-like separation between a future and a past but rather as a temporally extended process related to decoherence. Two models of how a notion of the present can be combined with a relational theory of space–time are presented.

1 Introduction

The debate about whether space is "absolute"—in the sense of an empty background which serves like a "stage" (an expression used by Einstein [17]) for physical processes—or "relational"—i.e., only the relations between physical objects matter and our perception of space is the result of an abstraction—has a long history. In the next section I will briefly review some of the highlights related to Descartes, Newton, and Leibniz with respect to this discussion. When extended to space–time, not the relations between physical objects but the relations between physical events matter.

The conceptual and even more the mathematical simplicity of the dynamical laws of physics with respect to a background space may have been one of the main reasons why Newton's ideas of an absolute space and time still dominate, at least for the time being, today's formulation of physics. The replacement of Newton's

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space and time by a Minkowski space–time in special relativity does not change the absolute character of this concept, and even in general relativity the metric field $g_{\mu\nu}(x)$ is defined with respect to a background manifold (the elements of which are denoted by "x") which has an absolute character. (Even though it is often claimed that relativity is a theory of "space-time," it is, strictly speaking, only a theory of the geometric properties of space–time; and even more rigorously formulated, it is a theory of the geometric properties of space–time as measured by physical instruments.) Quantum mechanics and quantum field theory almost exclusively refer to Newton's absolute space or Minkowski's absolute space–time. Even the paradigm of a relational formulation of classical mechanics—the model of Barbour and Bertotti [5]—assumes a distance function between objects as given without explaining where this distance function comes from. Furthermore, according to this theory there can be empty space between the physical objects which—at least in my definition of a relational theory—does not exist.

In this article, I will argue in favor of a relational interpretation of space (and/or space–time). The reasons are not so much of a philosophical nature, but I will show that some of the conundrums of quantum mechanics look much less "weird" when interpreted in the context of a relational space(-time) as compared to an absolute space–time in the sense of Newton. Similarly, the formulas used for calculating the outcomes of scattering processes in quantum field theory may be reinterpreted in terms of a generalized relational formalism for events.

The main reason why a relational concept of space (and space–time) mitigates some of the strange aspects of quantum theory is related to the notion of "locality," which is different in this framework. As we shall see, in a relational space a single entity can be at several locations simultaneously, where "can be" has to be interpreted in the sense of "is related to." Furthermore, two objects can be far apart from each other (with respect to a natural concept of distance which will be explained in Sect. 4.2) even though they are "next" to each other in the relational sense.

In addition, a relational concept of space, space–time, and matter may indicate how "the forgotten present" can enter the physical stage again. One of the strangest and least well-understood concepts of quantum theory is related to the "reduction process" or "collapse" of the quantum state. While the notion of a present is difficult to implement into the formalism of classical (non-quantum) physics, the reduction process in quantum theory may be considered as a marker of a present. This subject will be touched upon in Sect. 6.

In view of the difficulties of combining the theory of relativity with the fundamental postulates of quantum theory, many attempts have been made which are based on relational concepts of space or space–time. The most prominent of these approaches is due to Sorkin et al. ([11], for a more recent review see [45]), a different ansatz is due to Barbour and Smolin [6]. Matrix theory (see, e.g., [4]) may be considered as a relational theory of (at least) 1+1-dimensional quantum gravity), also spin networks (e.g., [42]), loop quantum gravity (e.g. [47]), and "causal dynamical triangulations" [2] are going in this direction (the literature on these subjects is vast, and the given references are just examples). A more recent approach in this direction can be found in [38]. Other approaches are due to Regge (see, e.g., [34]) or Gambini [28].

A final warning: A "relational interpretation of quantum theory" is due to Rovelli [41]. This relational interpretation is not directly related to the relational concepts of space or space–time and the re-interpretation of quantum theory which I will describe here. Therefore, I will largely avoid the expression of "relational quantum theory." If necessary, I will refer to the interpretation advocated in this article as being "micro-relational."

The article is structured as follows: The next Sect. 2 gives a brief historical account of the debate about relational and absolute space, in particular as far as Newton was involved. The purpose of this section is to point at some historical documents which show that the decision for or against an "absolute" space was not a natural consequence of scientific results or philosophically convincing arguments. In Sect. 3, I will introduce a particularly simple model of a relational space and also elaborate on the conceptual problems of "identification" and "location" of objects. The following section (Sect. 4) generalizes the concept of a relational space and relational locations and applies this generalization to quantum theory. Essentially, some of the "weird" expressions of quantum theory will be re-interpreted in terms of a relational picture. Section 5 extends the relational concepts for space, objects, and locations to a relational space–time, i.e. to events. Before I will comment on a possible implementation of "the present" into a theory of relations in Sect. 7, I will make some remarks on the notion of time in quantum theory (Sect. 6). In particular, I will argue in favor of an "extended present" and a corresponding "temporal extension" operator in quantum theory. The article ends with a brief summary and outlook.

2 Newton Versus Descartes and Leibniz

Between 1715 and 1716, a famous exchange of letters took place between Gottfried Wilhelm Leibniz (1646–1716) and the Anglican clergyman Samuel Clarke (1675–1729) [13]. Actually, both participants addressed their letters to Caroline of Brandenburg-Ansbach who then put forward the letters to the other partner. The exchange was initiated by Caroline who asked Leibniz whether he had any objections against Samuel Clark as a translator of Leibniz's book "Theodicee," and she remarked in her letter that Samuel Clark is acquainted with Sir Isaac Newton. Leibniz still had not forgotten that the president of the Royal Society (which at that time was Newton) had decided in favor of himself regarding the priority claims in the development of calculus and, therefore, in his reply to Caroline, Leibniz could not help but making an offensive side-remark against Newton by claiming that for Newton "space is just the Sensorium Dei." Caroline knew Leibniz from her younger years in Hannover, but now, with the prospect of becoming the future queen consort of England (she was the wife of King George II of Britain) she had to react to this insult of England's scientific hero and so she passed on Leibniz's letter to Samual Clark asking for a reply in this matter.

This initiated an exchange of a total of ten letters of increasing length, the last one dated from November 1716 from Clark to Leibniz which Leibniz could not answer because of his death on November 14th of 1716. Today, historians of science largely agree that the replies of Samuel Clark to Leibniz were at least approved by Newton if not formulated by him in person. For scientists, these letters constitute one of the most interesting scientific discussions on fundamental concepts of physics. The authors addressed subjects like the relational versus absolute character of space and time, the existence or non-existence of a vacuum, the concept of inertia forces, the free will of God and many other matters.

One of the central differences in the opinions of the two opponents refers to the conception of space and time. In reference to the work of Newton, Samuel Clarke argues in favor of an absolute concept of space and time. Space is conceived as an unbounded "stage" for the physical presence of matter, while "time" refers to an eternal and ubiquitous universal "clock" indicating a constant flow. Neither time nor space requires for their existence the presence of matter, even though by us the location and the moment of an event can only be perceived "relative" to material objects whose locations and motions serve as reference measures for space and time.

On the other hand, Leibniz defends a relational concept of space and time. For him, neither space nor time exists in an objective ontological sense but both are mental constructs abstracted from our perception of the relations between material objects and the changes of these relations. Only the material objects and the relations among them exist, and "space" is only an abstraction derived from our perception of these material relations. Similarly, time is just the order of a succession. In particular, for Leibniz there is no meaning in statements like "God could have created the Universe at a different time or at a different location in space" (arguments of this type occur frequently in this exchange of letters). Several times he makes use of his "principle of the identity of the indistinguishable"—if two situations are in principle indistinguishable, they have to be identified—as well as his "principle of sufficient reason" [35]: for everything which happens there has to be a reason why it happened in this particular way and not in a different way.

For Newton it was not the first time that he argued against a relational concept of space in order to defend his idea of an absolute space (which was largely based on religious believes [21]). A widely unknown manuscript of Newton which was never published and today is referred to by its first lines "De Gravitatione" [37] was obviously written as an attack against Descartes notion of a relational space. The exact time of origin of this manuscript is not known. Some scholars consider it as a preliminary draft of the "Principia" and date it to the time shortly after 1680, while others consider it as an early work of Newton, maybe even as early as the mid-1660s.

His main attack is directed against the notion of a relational space as described by René Descartes in his "Principia Philosophiae" [15] from 1644. For Descartes, empty space does not exist. The relations between objects define immediate neighborhoods and "motion" is the change of relations among these objects. Newton counters that Descartes is not even able to define the notion of a velocity, because this needs the distance between an initial position of an object and a final position, but as the relations have changed in the course of time the initial position may no longer be defined when the final position is reached. His critique is correct but it also applies to his absolute space if the positions in this space cannot be observed directly. Newton needs reference points which define a reference system, and this construction is also necessary in the relational concept of space as advocated by Descartes.

Philosophically, his arguments are not very convincing, but the concepts of an absolute space and an absolute time make it much easier to argue in favor of a "relativity principle" (the laws of nature are the same in all inertial reference systems) and his first law of mechanical motion. The mathematical simplicity of the concept of an absolute space and an absolute time, in particular with respect to the definitions of velocity, position, forces, and acceleration, may have been the main reason why not only Newtonian dynamics but also his ideas of an absolute space and an absolute time became the foundations of physics for the next three hundred years.

Presumably, Newton's main attack against Descartes was related to the third planetary law of Kepler, which states that the ratio of the third power of the orbital periods to the second power of the principle axis of the ellipse is constant. Newton was able to derive this relation from his theory of gravity, while in Descartes's model of rotating vortices this relation does not hold. However, "De Gravitatione" never comes to this point and so Newton postponed the details of this critique to his "Principia."

3 Relational Objects: Relational Space

In order to describe the concept of a relational space, let us first consider a set $V = \{a, b, c, d, e, f\}$ with six elements and represent this set by six points (see Fig. 1).

The actual identification of the elements turns out to be an ill-defined problem. While in mathematics it is tacitly assumed that each element of a set can be identified, the practical realization in physics may turn out to be problematic if the elements cannot be distinguished. Unless the elements carry a unique feature (color, shape, number, marker,...) they cannot be identified (see Fig. 2).

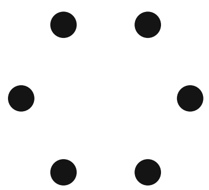

Fig. 1 Six *dots* representing a set V of six elements $\{a, b, c, d, e, f\}$. But "Which is which?," and what does it mean to say "Element b *is* somewhere"?

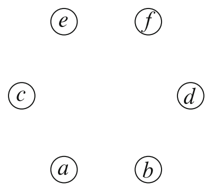

Fig. 2 Only if the elements carry a unique feature—in this case a *letter*—they can be distinguished and identified

In principle, a unique feature of a physical object may also be its position or location in space. But the elements in a set do not have a particular location unless we define a mapping, e.g. $V \to \mathbb{R}^2$, which associates with each element of V a point p in the Euclidean plane.

This is (more or less) the Newtonian picture. Space (usually described by $\mathbb{R}^3$ but for simplicity in this case $\mathbb{R}^2$—actually, we should even use $\mathbb{E}^3$, the affine Euclidean space for which an origin is not yet specified) is given to us by God who may also have access to the coordinates of this space or to other methods of identifying the points in this space. We have to define reference points (the center of a coordinate system as well as the coordinate axes) in order to identify points in space. (Even then we have to define a length scale for our coordinate axes and we have to assume that this length scale can be "transported" such that we are able to measure the distances between any two elements of space—this additional structure becomes relevant for general relativity and shall not be addressed here.) Other methods of identification like "upper left" or "lower right" also make use to this absolute space and predefined reference directions.

Indeed, strictly speaking, the situation in physics is reversed: We use physical objects to identify points in space. Instead of saying "object a is at point p" we should rather say "point p is the point where object a is." If sufficiently many reference points are defined by physical objects (and the possibility to measure distances is given) we can identify other objects "relative to" the reference points. This concept of "relative location" has already been emphasized by Newton who was well aware that we cannot identify the points of space in an absolute manner, even though he seemed to be convinced that in principle this is possible (and for him God is able to do so). In the following I will refer to "absolute space" in Newton's sense of a "God's eye perspective," and I will refer to "relative space" in the sense mentioned above: Certain material reference objects (which are still embedded in an absolute space) serve as reference objects in order to be able to define relative locations.

The concept of a "relational space" is completely different from the two notions of space defined above. In a relational space, the objects are not embedded into a given space, but there are relations defined between the objects, and under suitable conditions (see below) these relations allow for a unique identification of an object. For Descartes these relations were immediate neighborhoods. According to his notion of space, objects do not have a distance with nothing in-between but they touch each other. There is no empty space.

In Fig. 3, the set V of six elements has the additional structure of a relation, expressed by the lines between certain pairs of elements. In mathematics, given a set V, a relation E is a subset of $V \times V$, the Cartesian product of V with itself (i.e., of the set of ordered pairs of elements of V). Thus, $E \subset V \times V$. For the moment I will restrict myself to symmetric relations, i.e., if $(x, y) \in E$ then also $(y, x) \in E$ and I will not distinguish between (x, y) and (y, x). (However, for a causal structure, which I will use in Sect. 5 as the relational structure of events, I will distinguish between (x, y) and (y, x).) For instance, for our set V such a set of relations can be

$$E = \{(a, c), (c, e), (e, f), (f, d), (c, d), (e, d), (d, b)\}.$$

This set of relations is illustrated in Fig. 3, where I represented a relation between two elements by a line which joints the two elements in the graphical representation of the set V. Very often I will refer to the relation between two elements as "being neighbors" and the set V together with the relations E will be called a *graph*. For symmetric relations the graph is called *undirected*, otherwise it is directed. We can represent a graph by its adjacency matrix:

$$A_{xy} = \begin{cases} 1 \text{ if } (y, x) \in E \\ 0 \text{ otherwise} \end{cases} \quad (1)$$

I should emphasize, however, that the representation of this graph by nodes in a plane with lines connecting these nodes serves merely as an illustration of the relations and has no intrinsic meaning whatsoever.

In Fig. 3, each element in V can be uniquely characterized by the type of relations it has to other elements. A helpful concept for such a characterization is the *degree* of an element, i.e. the number of elements to which it is related. This allows to identify all six elements of V, e.g., by the following characterizations:

a : has degree 1 and its neighbor has degree 3.
b : has degree 1 and its neighbor has degree 4.
c : has degree 3 and one of its neighbors has degree 1.
d : has degree 4.
e : has degree 3 and one of its neighbors has degree 2.
f : has degree 2.

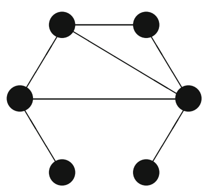

Fig. 3 If, in addition to the elements in Fig. 1, also relations between the elements are defined, we can (under suitable conditions) identify the elements by their relations among each other

In this particular case, given the degree and, in addition, the degree of the neighbors allows the unique identification of elements.

This property of being uniquely identifiable by the relational structure is far from trivial. Actually, the graph of Fig. 3 with six elements is the simplest graph with this structure. One can easily show that no graph with less vertices has this property (if we exclude the trivial case of one single vertex). In Fig. 4 we deleted one of the relations (between e and d). From the graphical representation it is obvious that a reflection of the structure at a hypothetical vertical line through the center does not change the structure. Therefore, we cannot distinguish this structure from the reflected one, or, in other words and not referring to the graphical representation, the element a cannot be distinguished from b, c cannot be distinguished from d, and e cannot be distinguished from f. This graph has a symmetry in the following sense: There exists a non-trivial permutation of the elements of V such that the relations in E remain unchanged. Whenever such a symmetry exists, the points of the graph cannot be uniquely identified by the relational structure.

We can also mix the two possibilities of identification. Figure 5a shows a cyclic graph where none of the elements can be uniquely identified. However, if we "mark" two of the elements (e.g., e and d), all the other elements can be characterized by their relations to these two elements.

The question of identifiability becomes also relevant in the context of general relativity. The fundamental degree of freedom in general relativity is the metric field

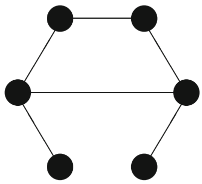

Fig. 4 Very often, in particular for small graphs, the relations do not allow a unique identification of the elements. In this example the graph has a symmetry ($a \leftrightarrow b, c \leftrightarrow d, e \leftrightarrow f$), i.e., there is no way to distinguish a from b, c from d, and e from f

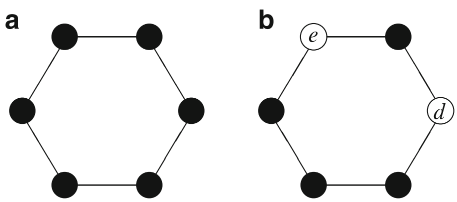

Fig. 5 (**a**) Even though a relation between the elements is defined, this relation is not sufficient to characterize even one of the elements. (**b**) If, in addition, two elements are marked by some property (here e and d), we can identify all elements of the set by their relations with respect to these two elements

$g_{\mu\nu}(x)$. But what does x refer to? From a mathematical point of view, x is a point of a manifold, and at this point the components of the metric field assume the values $\{g_{\mu\nu}(x)\}$. Again, strictly speaking, we have to reverse the statement: The point x is the point where the metric field assumes the value $g_{\mu\nu}$. And as the components of the metric field itself cannot be observed directly due to the diffeomorphism invariance of the theory (i.e., the invariance of freely choosing a coordinate system) but only components of the curvature tensor can be observed, we have to mark and refer to space(-time) points by referring to the curvature: Point x is the point where the curvature assumes a certain value.

Up to now I have discussed mainly the identifiability of elements, but now I want to address the question of "Where *is* an object?"

Given the framework of an absolute space, we define the location of an object by referring to the point where an object *is*. This is done by specifying the mapping $p : p \mapsto x(p)$ which associates with the object p its point $x(p)$ in space.

When we conceive space as defined by a relational structure, we still have two possibilities to give meaning to the question "Where *is* an object?" We can associate one of the nodes of our spatial structure as the location of an object, i.e., we define the location of an object by a mapping $\text{pos} : p \rightarrow V$ and say, e.g.: "The object p is at the spatial point x, i.e., $\text{pos}(p) = x(p)$, which is the only point of degree 4."

But why only going half the way? We can also define the location of an object by its relations to the spatial points. In the following I will often distinguish physical objects like particles from those entities which make up space, and I will refer to the latter ones as spatial points (even though the usage of "point" is misleading because the geometric shape of something is also determined by its relations to other objects; a better expression might be "spatial units"). In this framework, there is no independent meaning of location without reference to the neighbors. In this case the position of an object p is defined by its spatial neighbors, e.g., $\text{pos}(p) = \{e, f, b\}$ (Fig. 6).

In this case it is also possible that a particle can be "in two different spatial regions simultaneously." For example, see, e.g., Fig. 7.

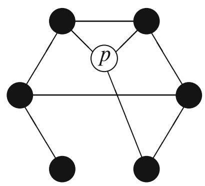

Fig. 6 The location of an object p in a relational structure is defined by the spatial points to which it is related. With the identification of the spatial points as before, the position of p is given by the subset $\text{pos}(p) = \{e, f, b\}$

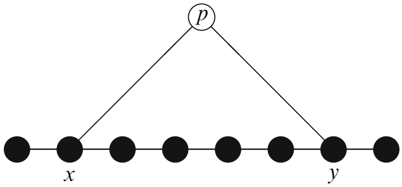

Fig. 7 In a relational setting for the concept of "position," a particle can be "at two locations simultaneously." In the given example, the spatial structure is the "relational line" and the object p "is" at the points x and y simultaneously

4 Relational Space and Quantum Mechanics

This section serves mainly a pedagogical purpose in the sense that in the end I want to suggest not a relational theory of objects but a relational theory of events. The structure of such a framework will be outlined in the next section. It is assumed, although I cannot present a proof at the moment, that in a non-relativistic limit the relational theory of events can be reformulated as a relational theory of objects. As a relational theory of objects is easier to visualize than a relational theory of events, this section outlines such a theory.

4.1 The Generalized Relational Structure of "Location"

As we have seen in the previous section, in a relational framework the position of an object is defined by the spatial points to which it is related. Therefore, in a relational picture, an object can be "at several spatial points simultaneously." Exactly this feature is one of the conundrums in the standard formulation of quantum mechanics. The wavefunction $\psi(x)$ of a particle does not mark a particular point of space as the position of that particle, but it defines a whole region of space where, if measured, the particle can be found. This uncertainty in the position of a particle is not due to a lack of knowledge—such a situation would also be familiar in a classical description of physical systems—but intrinsic. According to the standard interpretation of quantum mechanics, one should not even think of the object (e.g., the electron) as of a particle in such a situation. The electron "becomes" a particle, when a proper measurement is performed.

Of course, this standard interpretation of quantum mechanics is not mandatory. There exist formulations of quantum mechanics—Bohmian mechanics [10, 14] being the most prominent one—where an electron can actually be considered as a particle with a well-defined location, and in this case the uncertainty is indeed due to a lack of knowledge. However, this knowledge cannot be increased by proper measurements, but the measurement process in these types of theories is such that a knowledge about particles beyond what is allowed by the quantum mechanical uncertainty relations is *in principle* not possible. I will not discuss such models in this article for two reasons: First, Bohmian mechanics (and indeed, under very general conditions, any deterministic description of quantum theory) is non-local. It is formulated as a theory of fields and particles which exist in an absolute background space and, therefore, it is similar to a Newtonian model. I will argue later that the problem of non-locality may find a solution in a relational framework. This does not exclude the possibility that Bohmian mechanics, reformulated in a relational framework, may also avoid the problem of non-locality. The second reason is related to an incorporation of the concept of a "present." Again, for me Bohmian mechanics is too close to Newtonian models and, therefore, makes the incorporation of a present difficult, while in a relational model this is possible, as will be shown later.

There are many ways to combine a relational model of space(-time) with quantum mechanics (a by far not complete selection of approaches can be found in [12, 46]). In the following I will describe just one possible model (more details can be found in [24, 25]). In this model, the connection from wave mechanics to a relational model is made by generalizing the concept of a relation. For simplicity, I still assume the relational structure of spatial points as defined by a graph, i.e., for two spatial points x and y either a connection is present or absent. However, the relations of an object to the spatial points will be generalized from a subset of V (i.e., a characteristic function $\chi : V \to \{0, 1\}$) to a complex function $\psi : V \to \mathbb{C}$.

Networks of computer servers are often used as an example of graphs. In this case, two servers are said to be "related," if there exists a link from one of the servers to the other. For a particular server p inside such a network, we can define a generalized relational structure in the sense given above by a real valued ψ_p-function which associates with each other server q, say, the average number of bits sent from p to q during a particular time interval. This is a time-dependent function which means that we can also formulate a dynamics for this generalized type of relation.

The relational description of a single particle simply consists in a different interpretation of the wave function and not in a different mathematical structure. The absolute value of $\psi(x)$, i.e. $p(x) = |\psi(x)|^2$, still gives the probability(density) for finding a particle in a particular location. The changes with respect to the standard interpretation of quantum mechanics are minor: Instead of speaking of a "probability amplitude" $\psi(x)$, we refer to this function as a (complex-valued) relation (for which we do not specify any ontology, even though it would not be difficult to find appropriate ontologies). When this relation is probed by a measurement, it changes in the way of a "winner takes it all" manner (such types of reactions are well known in the theory of neural networks, see, e.g., [31]) such that finally only one relation of amplitude 1 remains, indicating the measured position of an object. The "winner" follows from probability: Point x wins the competition with probability $|\psi(x)|^2$.

For such a relational structure, a natural dynamics might be defined as follows. First of all, as we have discretized space, we also discretize time and formulate the dynamics in terms of an iterative mapping which defines $\psi_{n+1}(x)$ (the ψ-function at

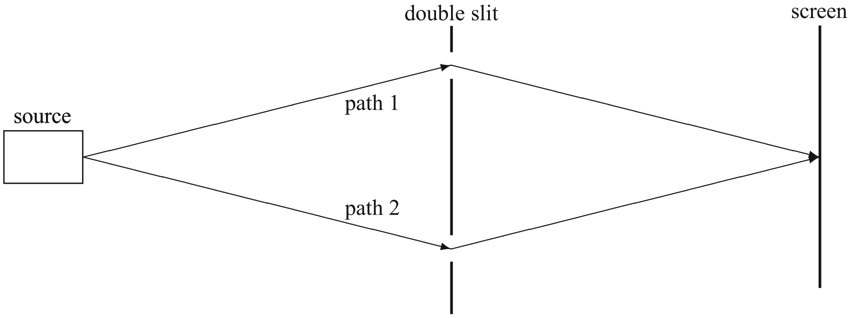

Fig. 8 In the famous double slit experiment the total amplitude is obtained by assuming that *a particle* propagates along path 1 AND path 2. In the micro-relational interpretation *the relations* of a particle propagate along path 1 and path 2

time-step $n + 1$) as a linear function of ψ_n. A natural candidate for such a dynamics is the following equation:

$$\psi_{n+1}(x) = \psi_n(x) + \epsilon \left(\alpha \sum_y A_{xy} \psi_n(y) + \beta V(x) \psi_n(x) \right) . \tag{2}$$

The second term on the right-hand side (proportional to a constant ϵ) corresponds to the change of the generalized relation $\psi(x)$. There are two contributions: The first one describes the propagation of the relation from one spatial point to a neighbored one (expressed by the adjacency matrix A_{xy}), the second one describes an additional change of the relation due to a local potential. This second term may also depend on the valency of point x (i.e., the number of points it is related to). It is not hard to see that under very general conditions such an equation becomes a Schrödinger-type equation in a continuum limit.

In a (temporal) continuum limit, we can again derive the summation-over-paths representation of the quantum propagator like in the standard case. However, the interpretation of this formula now reads: "One relation of the particle follows path 1, a second relation follows path 2, etc." instead of the usual "the particle propagates along path 1 AND path 2 AND path 3 etc." (see Fig. 8). In this case the relational interpretation of the wave function seems to be much less "weird" as the standard interpretation.

4.2 Spatial Distance and Many-Particle Systems

For this relational reinterpretation of the wave function to become a model, several problems have to be addressed. One of the main problems is the distance between two spatial points. Many different definitions of a distance between two points on a graph are in use. From a mathematical point of view, the most natural definition is the "minimal path"-distance: A path from a point x to a point y on a graph can be defined as a sequence of points $(x = x_0, x_1, x_2, \ldots, x_N = y)$ such that for any two neighbored points x_k and x_{k+1} the pair (x_k, x_{k+1}) is an element of E. N is called the *length* of the path. The "minimal path" distance between two points x and y is then defined as the length of the shortest path from x to y.

This definition of distance has the disadvantage that it is not related to the way how we actually measure distances in physics. A more natural definition is related to a (possibly length-weighted) average of all paths connecting two points and can be interpreted in terms of the propagation of the relations of an object on this spatial structure. This definition has sometimes been called "massive propagator distance" [9, 22]. In [23] I have discussed a suitable measure of distance for graphs with an additional causal structure, which is relevant for the relational structure of events.

A second problem of this relational interpretation refers to the treatment of several objects (particles). For a single particle p, we can represent the (generalized) relations by a mapping $\psi_1 : \{p\} \times V \to \mathbb{C}$ which is (as $\{p\}$ consists of only a single element) the same as $\psi : V \to \mathbb{C}$. For two particles p_1 and p_2, a natural extension would be the representation of the generalized relational structure by a mapping on all possible relations of the two objects to V, i.e. $\psi_2 : \{p_1, p_2\} \times V \to \mathbb{C}$. Obviously, this is *not* the same as $\tilde{\psi}_2 : V \times V \to \mathbb{C}$ which is the proper space of 2-particle wave functions in quantum mechanics. The function ψ_2 as defined above can be represented as a product: $\psi_2 \simeq \psi_1(x)\phi_1(x)$. This corresponds to the case of product states, i.e., 2-particle systems which are not entangled. Therefore, the association of a complex "weight" with each relation of the single particles excludes the possibility of entanglement.

In order to include also entangled states, we have to assign a "weight" to *pairs of relations*, each relation from one of the particles. In this way we arrive at a relational structure which can be represented by functions on the configuration space, like in quantum mechanics. Indeed, the fact that the relations of one particle are not independent of the relations of the other particle is the essential new feature of quantum systems.

These constructions are easily generalized: For n (distinguishable) particles the relations are now subsets of $\{p_1, \ldots, p_n\} \times V$. Considering "weights" only on this set of relations restricts the construction to product states. In order to include entangled states, we have to associate "weights" with all possible sequences $(x_1, \ldots, x_n)$ such that (p_i, x_i) is an allowed binary relation.

Before I conclude this section, I would like to give a (maybe somewhat unusual) example for the relational structure of particles from an everyday situation. Consider a boarding card. Until we approach an E-ticket counter in the hall of an airport and present our identity card to the reading device, our boarding card exists only as a "virtual" program or print-out recipe. Only when we make the "measurement" (presenting our identity card or E-ticket) at the E-ticket counter does the boarding card become reality at this particular E-counter. In principle, it could have "come into existence" at all the possible E-ticket counters in the airport. In this sense, the boarding pass existed virtually at all the counters simultaneously, but forced to

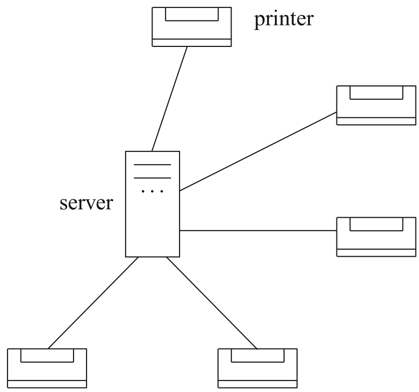

Fig. 9 A server connected to a periphery of counters with printers is a model for a relational concept. An e-ticket exists only virtually as a program instruction in the server. Only when the e-ticket number (or some identity card) is presented at a counter—a measurement—the e-ticket becomes reality at the printer of this counter

become reality by our "measurement" it only comes into existence at one particular counter and can never (at least in the ideal case) come into existence again at one of the other counters (Fig. 9).

A slight change of the model brings it even closer to the quantum situation: Suppose that upon presenting the identity card (or E-ticket) to the counter, the boarding pass only comes into existence at this counter with a certain probability. Only if one would present a copy of the E-ticket at all counters in the airport simultaneously, the boarding pass will come into existence with probability one at one of the counters.

4.3 The Collapse in a (Micro)relational Interpretation

The quantum collapse, or the reduction of the quantum state, is often considered as proof that quantum theory is non-local (see, e.g., [1]). The argument is as follows: If a quantum state is non-local, i.e. the wave-function is non-zero in an extended region or—in case of EPR-states [18]—in two regions which are spatially separated by a large distance, and a measurement is performed which leads to a reduction of the state to a new state in agreement with the results of the measurement, then this reduction violates the micro-causality condition of relativity. The collapse has to be instantaneous, and even though there is no transfer of energy or signaling, there seems to be a "spooky action at a distance."

In order to be explicit, let us consider an EPR-state for two (distinguishable) spin-$\frac{1}{2}$ particles:

$$|\Psi\rangle_{\text{EPR}} = \frac{1}{\sqrt{2}}(|\text{up}\rangle_1|\text{down}\rangle_2 - |\text{down}\rangle_1|\text{up}\rangle_2). \tag{3}$$

The two particles (referred to 1 and 2) may be billions of light years apart. If a spin measurement is performed on one of the electrons (e.g., 1) and the result is, say, "up," the total state is reduced to the separable state

$$|\Psi\rangle_{\text{red}} = |\text{up}\rangle_1 |\text{down}\rangle_2. \tag{4}$$

According to the standard interpretation of quantum mechanics (and also the interpretation of, say, Bohmian mechanics), the single particles do not have a definite spin before the first measurement has been performed, but on the other hand one always observes an anti-correlation of spins, even if the two measurements are performed within the causal complement of each other. Bohmian mechanics is a non-local theory, because the "guidance wave" indeed collapses globally and instantaneously upon the first measurement on one of the particles. In a similar sense, all other interpretations of quantum theory, where the collapse has an ontological counterpart (and is not merely a "change of knowledge" on the side of the observer), describe this state reduction as a non-local process. Even the Many-Worlds interpretation [16, 20] for which there is no collapse is non-local in the sense that the splitting of the carrier of the wave function from a single to a double universe is non-local.

The relation between the quantum state reduction and non-locality is still a matter of highly emotional debates. The results of John Bell [8] are taken as proof that quantum theory is non-local. (Proponents of a purely subjective interpretations of quantum theory attribute the wave function to our knowledge about the world and the reduction of the wave function as a change of this knowledge as a result of a measurement. But does this knowledge refer to an objective entity? If yes, then this entity behaves non-local, if not, what does this knowledge refer to?) The main problem for any non-local interpretation of the quantum state reduction is the question "with respect to which reference system does the collapse occur?"

Before I will discuss the reduction problem in the framework of the micro-relational interpretation, let me state that independently of whether the relational interpretation is true or not, as long as we do not have a satisfactory incorporation of gravity into the quantum formalism, I do not consider the concept of "locality" as well defined. It could well be that what we call "non-locality" is just a remnant of quantum effects on small scales. In the same way as worm-holes are not considered as "non-localities" even though we might travel in seconds into a completely different part of our (or another) universe. The "space-time" foam of Wheeler or other quantum gravity effects might very well induce apparently non-local effects even though, when considered in detail, locality may be preserved.

As a second remark I would like to emphasize that a preferred reference system does not contradict relativity. In our universe we even do have a preferred reference system which is the system with respect to which the background radiation is isotropic. By chance, this happens to be the same system with respect to which the observable mass in our universe seems to be at rest. The existence of an either is not excluded by relativity, only the formulas have to be reinterpreted in the sense of Poincaré: A Lorentz-invariant theory (like Maxwell's theory, the standard model

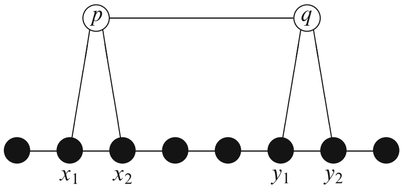

Fig. 10 In a relational setting for the concept of "position," two particles can be "lightyears apart" and still be nearest neighbors in the relational sense. In this picture one object is at a position x_1, x_2 and the other at y_1, y_2 and, nevertheless, both objects are nearest neighbors

of particle physics and, in fact, almost all field theoretical models) predicts the Lorentz-contraction and a time dilation for any physical system which is motion relative to this either. The difference between an either model and relativity is not the mathematical formalism or the observed phenomena, but just the interpretation. Indeed, the fact that an ontological interpretation of the state reduction seems to require a distinguished notion of simultaneity is one of the reasons why I believe that quantum theory is more amiable to the notion of a "present" than classical physics or the theory of relativity. I will come back to this point in Sect. 6.

How does the problem of non-locality appear in the micro-relational interpretation? As Fig. 10 indicates, two particles, which seem to be "billions of lightyears" apart with respect to our classical notion of space, may be nearest neighbors in a relational sense. And as indicated in [24, 25], entanglement may be the large-scale phenomenon of a micro-relation between these two particles. Furthermore, also the state reduction for a single particle with extensive relations to various spatial points is a local effect as all these relations emanate from this single particle. Therefore, the reduction of an extended quantum state to a local quantum state, even if it happens instantaneously, is not necessarily an action at a distance.

4.4 Superluminal Propagation?

One of the more speculative consequences of a relational space (or space–time), and, in particular, relational locations of objects in such a space, is the possibility of superluminal propagation, however, without any violation of causality principles. The fact that the structure of space is given by the relations among the spatial points, and that these relations are not constrained by continuity requirements, leads to the possibility that certain spatial points might have a direct relation to other spatial points which otherwise appear to be far apart. Such relations would have a similar effect on a microscopic scale as wormholes on a macroscopic scale.

Whether or not such short-cuts among the spatial relations exist or become relevant depends on the details of the dynamics for those spatial relations, which I haven't specified. A very speculative possibility arises, however, if also other relations, apart from the ones between spatial points, can be used for the propagation of relations. In other words, if the adjacency matrix in Eq. (2) also contains the relations from other particles to spatial points or even the relations among other particles, then non-local states may give rise to superluminal propagation.

To discuss a simple example, consider the double slit experiment again. While the particle passes the slits it has relations to the spatial points in both slits simultaneously and we encounter a situation which is similar to the one depicted in Fig. 7. If a second particle comes propagating from the left-hand side (which means, that it has relations to the points on the left-hand side and that these relations change according to some local algorithm) and some of its relations reach the spatial point x, the question is, whether its relations can also propagate directly from x to p and then further to y. In this case it could be detected with a finite probability at point y after changing its relations in just two steps from the point x.

While in real situations the distance between x and y might be large (e.g., of the order of 10^{30} Planck units or even more) the short-cut $x \to p \to y$ consists of only two steps and would appear superluminal.

The main question remains, whether relations like (x, p) and (p, y) or also relations like (q, p) in Fig. 10 can be used for propagations. The type of interactions between particles may also restrict the type of relations which a particle can utilize for propagation.

5 Relational Space–Time: Relational Events

The previous two sections mainly dealt with a relational spatial structure, the relational concept of location of an object in such a spatial structure, and its generalization to spatial relations which are "weighted" by a complex factor. In this section, I will describe a relational structure of space–time.

When dealing with space–time, the relevant "objects" (the elements of space–time) are events. For an absolute concept of space–time (e.g., Minkowski space–time), the events mark particular space–time points. Or, in other words, an event is located at a particular space–time point.

In a relational picture, the locations of space–time points are defined by their relations to other space–time points (Fig. 11). Therefore, in order to define the location of a particular event—like the emission of a photon by an electron—we have to specify these relations. Again, in quantum theory, and this time I refer to the formalism of quantum field theory, I generalize the concept of a relation from a yes-or-no concept to a complex valued relation.

Without going into details, I just consider the simple event of Coulomb scattering of two electrons in the lowest approximation. Two elementary events—the emission of a photon of one electron and the absorption of the photon by the other electron—constitute this scattering (Fig. 12). Usually, the asymptotic states are characterized by their momenta, but for simplicity I consider the process as determined by four external events x_1, x_2, x_3, x_4 which correspond to two initial states of the electrons and two final states of the electrons, respectively. Suppressing all indices referring

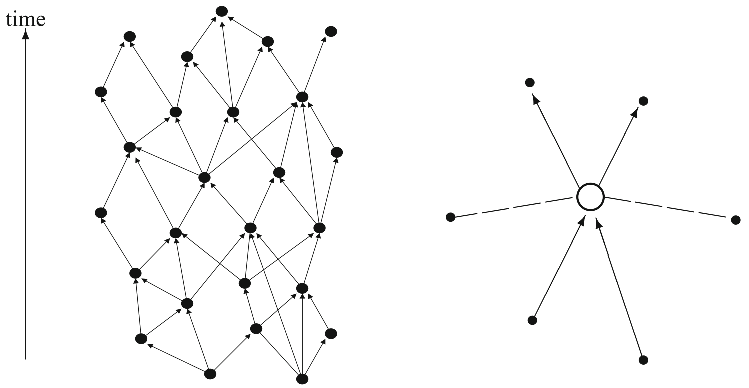

Fig. 11 (*Left*) The events of "space-time" are endowed with a causal structure. (*Right*) A physical event can be related to the events of "space-time" in three different ways: it can be causally influenced by events in its past, it can influence events in its future and there maybe "space-like" relations to events which are in the causal complement. The distinction between "space-like" events and time-like or light-like events maybe related to the real and imaginary part of the causal Green's functions

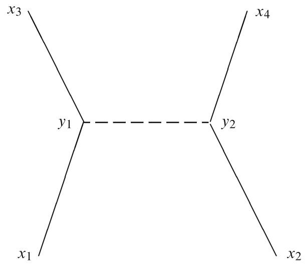

Fig. 12 The lowest order approximation of a Coulomb scattering of two electrons by an exchange of a (virtual) photon. The points x_i are kept fixed while one has to integrate over all possible positions of the intermediate events at y_1 and y_2

to the spin of the electrons and the polarization of the photons as well as factors of π and other normalization factors etc., the amplitude for this process can formally be expressed as

$$A(x_1, x_2, x_3, x_4) \propto \int \mathrm{d}y_1^4 \int \mathrm{d}y_2^4 \; S(x_1, y_1) S(x_2, y_2) G(y_1, y_2) S(y_1, x_3) S(y_2, x_4). \tag{5}$$

Here, $S(x, y)$ denotes the electron propagator (from space–time point x to space–time point y) and $G(y_1, y_2)$ the propagator of the exchanged photon. In general, the contributions from these propagators are complex functions. Each propagator defines a generalized relation between the event (say y_1) and other events (in this case y_2, x_1, and x_3). The fact that we have to integrate over the "location" y_1 of this event indicates that this event does not happen at a particular point but, in principle, everywhere in space–time. This, at least, is the usual interpretation of this integration: we have to sum over all histories, i.e. all positions for this event. In the micro-relational picture, this integration is interpreted as a "sum" over all relations for the event, say "emission of a photon," to all the other events of the space-time canvas. (Actually, as the exchange propagator for the photon between event y_1 and y_2 does not have to be on mass-shell, emission of a photon and absorption of a photon cannot be distinguished and should rather be interpreted as "interaction with a photon.")

So, in the micro-relational interpretation, events do not have a particular location, they also do not happen "simultaneously everywhere," as the situation is often phrased in quantum theory, but they have relations to all other events. The amplitude for a particular process in quantum field theory is just the remainder of the sum over all these relations. (For more details, see [24, 25].)

6 A Temporal Extension Operator

In this section I will briefly comment about some aspects of the role of time in quantum theory. In particular, I will argue why quantum theory supports the notion of a present (in contrast to classical physics) and why such a present should not be described as a "point-like" or singular structure but rather as something which has an extension.

Two ingredients of quantum theory seem to make the inclusion of a distinguished present more appropriate than within a classical formalism. First, quantum theory is non-local in the sense that the changes of non-local quantum states as the result of (local) measurements (or interactions with an environment) are considered to be instantaneous. Examples of non-local quantum states are Bell states of entangled particles or (approximate) momentum eigenstates of single particles. A local measurement, e.g. a spin measurement of one of the particles in a Bell state, leads to an instantaneous change of the quantum state. The corresponding quantum correlations have been experimentally confirmed even within the causal complement of the events associated with the measurements [3].

If this change of a quantum state is correlated with an ontology, the natural question arises, with respect to which reference system the change of the quantum state occurs? A natural explanation (and, in my opinion, the only explanation in sight) is that there exists a preferred reference system in nature and that the collapse of the quantum state (or the splitting of this state in a many-worlds-interpretation) occurs with respect to this system. The existence of such a preferred reference system is not forbidden by Relativity. As has been noted already by Poincaré, any Lorentz invariant theory allows for two different interpretations of the spatial and temporal coordinates, which, however, lead to exactly the same experimentally observable phenomena: (1) Einstein's interpretation, according to which "time" and "length scales" are defined by physical clocks and "rulers";

there is no preferred reference system. (2) Poincaré's interpretation, according to which there is a preferred reference system (the "either"-system); however, every physical object which is subject to a Lorenz-invariant dynamics is contracted and slowed down in its dynamics when moving with respect to the either. This Lorentz contraction and time dilation make it impossible to detect the either system with physical objects as measuring devices.

The conclusion seems to be that classical relativistic physics *allows* a preferred reference system which, however, is not detectable, while quantum theory seems to *require* a preferred reference system which, however, cannot be detected because of the no-signaling theorem in quantum theory (according to which the instantaneous change of a quantum state as the result of a measurement cannot be used for super-luminous signal transfer). If a distinguished present exists, this implies a distinguished reference system. While such a system does not exist in non-quantum Lorentz-invariant theories, it seems to exist in quantum theory.

The second reason why quantum theory is more amiable to the notion of a present is also related to the reduction of a quantum state. In [27] we have argued that the reduction of a quantum state marks the transition from "virtual possibilities" to actual facts. If anything at all, this transition can be considered as the marker of a "present." More generally, we can consider the decoherence process, which leads from a superposition (with respect to classical states) to decoherent classical states, as the material correlate of the present. According to this view it becomes obvious why we consider the present not as something momentary or instantaneous but something extended: The decoherence process is not instantaneous but temporally extended. This temporal extension is measured against an idealized classical clock. Typical decoherence processes in quantum mechanics which lead to macroscopic classical states happen in time scales between nano seconds (e.g., the registration of a photon on a photographic plate) up to seconds or even longer (for well-prepared isolated quantum systems).

This view allows to define a measure for the present. The following considerations are not meant to be an exact mathematical treatment but rather an outline of such a procedure. First of all, we associate a present with a process, i.e., each process has its own present. For this reason, some processes can have an extended present of more than seconds while other processes have a present which is of the order of nano-seconds. (Note that this process-dependent extension of the present does not contradict the previous remarks concerning a distinguished reference system.) Next we define a "degree of facticity": This will typically be a decoherence measure for a classical fact, or, with a slightly different emphasis, a degree of effort or costs in order to "undo" an event. In a third step we define a "measure for the present" by taking the derivative of the facticity defined above (Fig. 13).

The degree of facticity depends, e.g., on the number of particles (degrees of freedom) which have participated in a certain fact. When a single electron or atom passes through a double slit and finally hits a screen and it did not have any interaction with the environment, which encoded the slit through which the photon passed, there is no facticity associated with the statements "the electron passed through slit 1" or "the electron passed through slit 2." If, however, the electron or

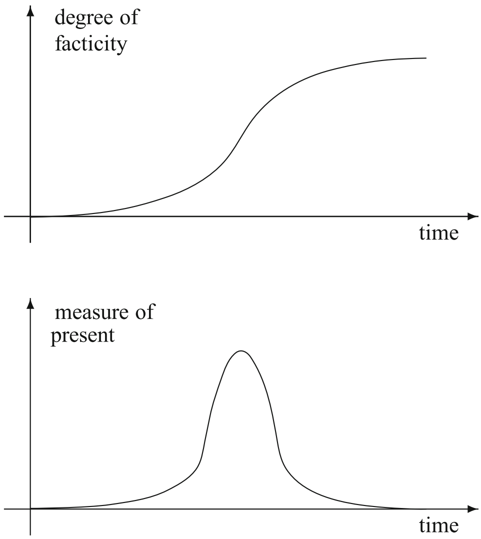

Fig. 13 (*Left*) Degree of facticity as defined by some decoherence measure. (*Right*) The derivative of the degree of facticity may be considered as a measure for the present

atom has interacted with, say, a photon, such that a measurement of this photon can reveal the slit through which the electron passed, then the passing through one slit or the other has already acquired a certain degree of facticity. If now, this photon is measured by a macroscopic measuring device, the passing through one of the slits becomes an almost irreversible fact. If, on the other hand, the photon is reflected by some kind of mirror and reabsorbed by the electron or atom (such a reabsorption of a photon happens, e.g., in interactions of the electron or atom with the vacuum or in the quantum eraser [43, 44]), the facticity is erased. Interaction with a single photon defines a low degree of facticity.

The fact that in quantum theory any event has a temporal extension is already inherent in the energy-time uncertainty relations: $\Delta E \cdot \Delta t \geq \hbar/2$. The temporal extension of an event in which an energy with an uncertainty of ΔE is exchanged is at least $\Delta t = \hbar/\Delta E$. Other instances of temporal extensions are, e.g., related to arrival times (see, e.g., [36] and the references therein). Further details concerning temporal extension and temporal non-locality can be found in [26].

It is well known that in quantum theory there exists no time operator (an operator which corresponds to the measurement of the instant of time when an event takes place). The standard argument against such a time operator is due to Pauli [40] and is based on the observation, that such a time operator has to satisfy canonical commutation relations with the energy operator. On the other hand, if there existed an operator having canonical commutation relations with the energy operator, then this operator can generate eigenstates of the energy operator with arbitrarily low energy. But the energy of physical states has to be bounded from below (the state with minimal energy is the vacuum state, and if there existed no state with lowest energy there could be no stable physical system).

Even though a time operator seems to be forbidden in quantum theory (there are many attempts to define a time operator, see, e.g., [7, 30, 39, 48], but in all cases fundamental physical requirements are sacrificed), the considerations above may lead to an alternative: Instead of a time-operator, one can define a "temporal extension operator." (One approach in this direction has been described by Basil Hiley [32, 33].) The spectrum of a temporal extension operator ΔT should be positive (because no physical event has a temporal extension which is less than zero). Furthermore, such an operator is not associated with a physical system, but with a physical event. Therefore, the commutation relations with H need not be defined, but the commutation relations with a variant of the energy operator which corresponds to the energy transfer in an event, ΔH, maybe related to the canonical commutation relations.

7 Two Relational Models of the Present

In this section, I want to combine the idea of a relational space–time (in the sense of relational events) with the concept of a present. In a simple setting one might consider the building-up of relations as the process related to the present. In this case the primordial situation consists of a set of unrelated events and during the course of time more and more relations are inserted and become the threads of the canvas of space–time. This picture resembles the process ideology of Whitehead [49] and is sketched in Fig. 14. (A similar model is also advocated in [19].) Note that in this pictorial representation there are two "time-directions": (1) The direction from bottom to top which corresponds to a physical time experienced by an internal observer, and (2) the direction indicated by the four pictures (from left to right) which corresponds to a time from a "God's-eye"-perspective.

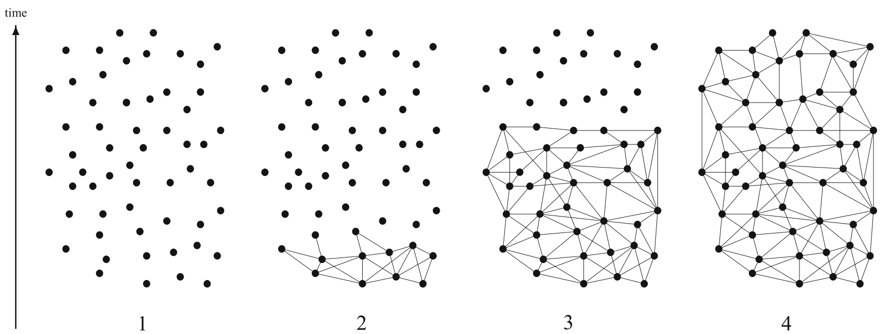

Fig. 14 The "flow of the present" as a building-up of relations. The primordial state (**1**) consists of only unrelated events. The "becoming of facts" is expressed in an increasing number of relations between these events (**2**–**4**). Note that there are two time directions (*see text*)

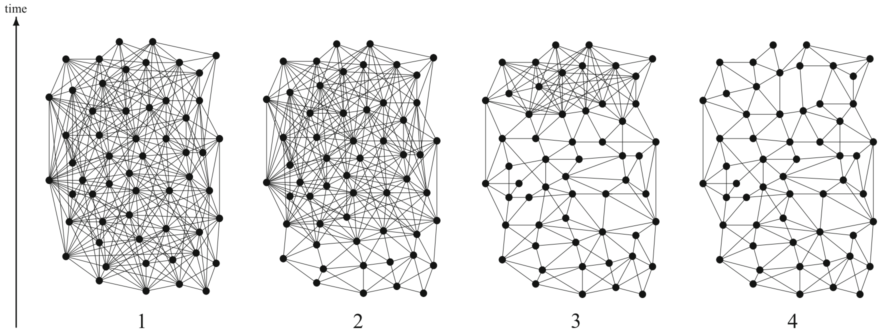

Fig. 15 The "flow of the present" as a pruning of relations. In the primordial state (**1**), almost all possible relations exist. The "becoming of facts" consists in a reduction of relations (steps **2** and **3**) until only factual relationships remain (**4**)

A simple model for such a "two-time" situation is provided by a computer simulation: When a computer simulates, say, the Game of Life of John Conway [29], there is the external time in which the computer operates, which is also my time as the programmer. The second time direction corresponds to the algorithmic steps which lead to an update of the configurations of the cellular automaton. We can halt the computer program for as long as we want (measured with respect to our time), an intrinsic "being" in the Game of Life would never feel such a halt. It only counts the number of algorithmic steps.

As already mentioned in the last section, I consider the transition from virtual possibilities to facts as the marker of the present. With a glance at quantum theory where the reduction of a quantum state, i.e. the reduction from many possibilities, expressed by a superposition of these possibilities, to a single fact marks the present, we can reverse the previous picture (Fig. 15): The initial state is represented by an almost complete graph for which the relations between events indicate the almost unlimited number of virtual possibilities (Fig. 15, 1). During the process of "becoming a fact" most of these relations are cut such that only a few "factual" relations survive (Fig. 15, 2–3). These make up our reality. The final state of this model-universe (4) consists of a structured set of compatible relations which now constitute a factual space–time history. Again, this model comprises two "times," a physical time experienced by internal observers and an extrinsic time.

8 Summary and Conclusion

I have argued that the concept of "locality" receives a completely different meaning when the positions or locations of entities (or events) are defined in a relational sense as compared to an absolute space or space–time. In particular, many counter intuitive aspects of quantum theory appear less strange from this perspective. A relational space or space–time as well as a relational structure between particles might also be a way to circumvent the constraints given by Bell-type inequalities: the "elements of reality" and the requirement of locality are no-longer mutually exclusive.

Finally, I have argued that quantum mechanics as compared to classical mechanics is more amiable to the notion of a "present" for two reasons: (1) an ontological state reduction seems to require a distinguished reference system which is also a prerequisite for the notion of a present, and (2) the collapse or reduction of a quantum state indicates the pruning of possibilities and the transition to facts, which can serve as a marker for the present.

Acknowledgements I acknowledge many stimulating discussions during the Parmenides-Workshop "The Forgotten Present" (Munich-Pullach, April 29th – May 2nd, 2010) with all participants, in particular with Julian Barbour, Avshalom Elitzur, Domenico Giulini, Basil Hiley, and Hans Primas.

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Relativistic Interactions and the Structure of Time

Dustin Lazarovici

Abstract While our physical description of the world does not contain an objective present, it still adheres to the notion of "instants" or "instantaneous states" that appear through the formulation of physical laws as initial value problems. That these ideas survived even the revolutionary transition from classical Newtonian space–time to relativistic space–time is mostly due to the concept of fields as mediators of relativistic interactions. But the duality of fields and particles is problematic, leading to singularities caused by self-interactions. In this article, it is thus argued that the conception of physical reality as a succession of instantaneous states may be a fundamental fallacy underlying some of the very concrete technical problems that we encounter in modern physics. By the example of the Wheeler–Feynman theory we demonstrate the chances and challenges associated with a conceptual revision that takes relativistic space–time more seriously. Finally, we discuss the possible implications for our philosophical understanding of the structure of time.

1 Introduction

Although the present *per se*, the moving point of *Now*, does not appear in the objective, physical descriptions of the world, the concepts of *instants* or *instantaneous states* do appear, at least implicitly, through the formulation of dynamical laws as differential equations requiring the specification of *initial conditions*. Given the state of a physical system (e.g., the universe) at a time t, the physical laws determine the complete history of that system, i.e. its state at any other time t prior to or later than t'.[1] Mathematically, we call this an *initial value problem*.

[1] In principle, dynamical laws can be stochastic rather than deterministic, meaning that future and/or past states may not be uniquely determined by the initial state. But this distinction won't be essential to our discussion.

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As we conceive of time as a continuum and as our model of the continuum is the field of reals, t and t' denote real numbers representing point-like moments in time, "point-like" meaning "without temporal extension." Note that the time t for which the "initial state" is specified is, in principle, arbitrary and not ontologically distinguished in any way, meaning that none of this touches on the deeper issue of the present, the way in which the *Now* is—or seems to be—ontologically different from past and future. But this particular form into which our dynamical laws are usually cast is highly significant because it reflects a certain conception about the nature of time that is deeply rooted in our current understanding of the world. That is the idea of time as a (continuous) sequence of point-like *instants* and, accordingly, the conception of physical reality as a *succession of instantaneous states*.

A.N. Whitehead describes this conception as follows:

> [The] fact that the material is indifferent to the division of time leads to the conclusion that the lapse of time is an accident, rather than of the essence, of the material. The material is fully itself, in any sub-period however short. Thus the transition of time has nothing to do with the character of the material. The material is equally itself at an instant of time. Here an instance of time is conceived as in itself without transition, since the temporal transition is the succession of instants.
>
> The answer, therefore, which the seventeenth century gave to the ancient question of the Ionian thinkers, 'What is the world made of?' was that the world is a succession of instantaneous configurations of matter—or of material, if you wish to include stuff more subtle than ordinary matter, the ether for example. We cannot wonder that science rested content with this assumption as to the fundamental elements of nature. The great forces of nature, such as gravitation, were entirely determined by the configurations of masses. Thus the configurations determined their own changes, so that the circle of scientific thought was completely closed. This is the famous mechanistic theory of nature, which has reigned supreme ever since the seventeenth century. It is the orthodox creed of physical science. [1, p. 51]

This "orthodox creed" survived even the eruptions caused by Einstein's theory of relativity, although it is certainly challenged by its denial of absolute simultaneity. Newtonian space–time consists of identical copies of three-dimensional Euclidean space, parametrized by an absolute, external time. In a relativistic setting, there is no preferred foliation of four-dimensional space–time into three-dimensional hypersurfaces of simultaneity (Fig. 1). This, however, does not necessarily imply that it is impossible to slice up the history of the universe into snapshots of instantaneous states. It rather tells that (if achievable) there will be infinitely many, equally valid ways to do so. Usually, it is still possible to formulate relativistic laws as initial value problems—as equations, that is, whose solutions are determined by initial data on a space-like "Cauchy-surface"—and physicists often find it convenient to do so, even if it means to bring the equations into a form that is no longer manifestly Lorentz invariant (or diffeomorphism invariant in the general relativistic case). In other words, even in a relativistic setting we commonly expect that the history of a physical system can be told as a succession of instantaneous states such that an instantaneous configuration of the physical variables determines its own evolution—albeit with the caveat that the same story may be told differently with respect to different frames of reference (but see [2]!).

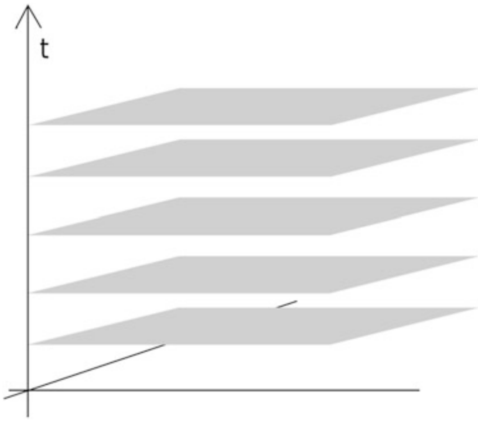

Fig. 1 Scheme of Newtonian space–time. The planes represent copies of three-dimensional Euclidean space at different times

But this expectation, I will argue, may turn out to be a prejudice rather than a sound demand, a premature application of the anthropocentric perspective and classical intuitions about time and space. Whitehead called it an instance of the "Fallacy of Misplaced Concreteness" [1, p. 52]. I believe that those preconceptions may not only obscure our philosophical understanding of time but may actually lie at the bottom of very concrete and very persistent difficulties that physics is still facing today. The signs are subtle, yet I believe they can be found in physics itself, for instance, as I will try to demonstrate, upon reflection on relativistic interactions, field theory and the problem of self-interactions.

2 Why Fields?

Newtonian Mechanics is certainly the paradigm of a classical theory, by which I mean a theory based on a classical model of time and space. The instantaneous state of a Newtonian system is given by the spatial configuration of its constituent particles together with their velocities or momenta. The particles have a well-defined position and a well-defined velocity at each moment in time and move according to a law of motion that respects the symmetries of three-dimensional Euclidean space.

Newton, well aware of the fact that his *laws of universal gravitation* constituted an unprecedented breakthrough in the human understanding of nature, was nonetheless convinced that they didn't tell the complete story about the causal connections involved in the attraction of bodies. To him and most of his contemporaries, the idea of an "action at a distance" through empty space, without mediation by some "agent," was unacceptable.

> It is inconceivable that inanimate Matter should, without the Mediation of something else, which is not material, operate upon, and affect other matter without mutual Contact. [...] That Gravity should be innate, inherent and essential to Matter, so that one body may act upon another at a distance thro' a Vacuum, without the Mediation of any thing else, by and

> through which their Action and Force may be conveyed from one to another, is to me so great an Absurdity that I believe no Man who has in philosophical Matters a competent Faculty of thinking can ever fall into it. Gravity must be caused by an Agent acting constantly according to certain laws; but whether this Agent be material or immaterial, I have left to the Consideration of my readers. (Newton in a letter to Bentley, dated Feb. 25, 1692/3, quoted in [3])

As I don't dare putting my word against Newton's, here is Erwin Schrödinger on a similar issue:

> Hidden residues of *animism* could be found in physics even in modern times. From the common understanding of nature, they haven't even disappeared to this day. As *Mach* rightfully observes, some residues of animism are attached to the abstract idea, that we designate by the conceptual pair of cause and effect. [...] In physics, the force has established itself as the "cause of motion". This understanding is clearly derived from the act of will of muscle innervation and the feeling of pressure that accompanies this act whenever a limb of our body sets a solid body into motion or brakes its motion. We may insist, at least, that we have removed from the physical notion of force the attribute of *intention* that is so inseparably linked to its psychophysiological example; it remains dubious whether we succeeded, as long as we are setting the cause-effect-relation in its place, the causa efficiens for the causa finalis. It still *causes* the result, even though unconsciously, without intention. It *is* someone or something. For a nobody or nothing cannot cause at all. Thus *Kirchhoff* argued that the force in mechanics must be understood *solely* as the product of mass and acceleration. In this way, the *Newton*ian law of motion, claiming this equality, becomes neither tautological, nor trivial. To the contrary, freed from the slag, its true content just comes to light more clearly: the bodies determine each other's acceleration—not the velocities or anything else. [4, pp. 32–33] (translation by D.L.)

Obviously there is a lot to discuss here, about the status of physical laws, about the nature of scientific explanations and the question if and how *causal* notions should figure into them. But all of this is far beyond the scope of this paper. So let it suffice to state my believe that, although Newton's points are still of relevance today, Schrödinger expresses the more sophisticated and more modern understanding of physical laws. There is no need for *causal relations* over and above the *functional relations* that fundamental laws of motion posit between the variables describing the elementary physical entities. And indeed, if we free ourselves from an overly mechanistic (or "animistic") picture of physical interactions and think in terms of laws, rather than causal "agents," we see that an action at a distance, as Newtonian gravity seems to describe, is neither absurd nor accidental. It rather reflects the intimate connection between the form of the dynamical laws and the underlying model of time and space. In a classical setting, the simplest and most natural way to define the interaction of particles is on the three-dimensional surfaces of simultaneity—i.e., throughout each copy of three-dimensional space at equal times—because this is precisely the structure of Newtonian space–time. From there on, we have a pretty compelling case for Newton's law of gravity as the simplest, non-trivial equation compatible with the symmetries of Euclidean space.

Let's now turn our attention to the second pillar of classical physics, the Maxwell–Lorentz theory of electromagnetism. At first glance, there are two crucial differences between electromagnetic interactions and gravitational interactions. 1. Electromagnetic interactions are not instantaneous, but "propagate" with the speed of light. 2. Electromagnetic interactions are not direct particle interactions, but mediated by electromagnetic fields. Strictly speaking, in Maxwell theory particles do not act on particles. Particles create (or excite) the electromagnetic field. The electromagnetic field acts on particles. The instantaneous state of a system in Maxwell–Lorentz theory thus includes the spatial configuration of the particles and their momenta *and* the configuration of the electromagnetic field.

Hence, if you were worried about action at a distance in Newton's theory—be it about gravity acting *instantaneously* or gravity acting *unmediated*—the field concept seems to be a solution to both. In fact, it may even seem like the finite propagation speed of electromagnetic effects provides compelling empirical evidence for the existence of a "mediator" of electromagnetic forces. After all, as we are looking up to the firmament, our visual receptors may be excited by stars that have long ceased to exist and radio transmissions from Mars rover take several minutes to reach mission control on earth. Doesn't this mean that there must be *something* actually moving or evolving in time and space? Something that has propagated, in some sense, from a distant source all the way to our current location?

Electromagnetism is a relativistic theory. Indeed, it had been relativistic—or Lorentz invariant, let's say—long before we even understood what it meant to be Lorentz invariant, long, that is, before Einstein and Minkowski understood that this somewhat peculiar symmetry of Maxwell's equations was not accidental but expressive of a radically new structure of time and space. Minkowski space–time, in contrast to Newtonian space–time, doesn't come with a preferred space-like foliation, its geometric structure is not one of ordered slices representing "objective" (= Lorentz invariant) hyperplanes of absolute simultaneity. But Minkowski space–time does have an objective (geometric) structure of *light-cones*, with one double-light-cone originating in every point. And so, applying the same reasoning I suggested for Newtonian Mechanics, the simplest and most natural way to define a particle interaction in Minkowski space–time is to have the particles interact *directly*, not along equal-time hyperplanes but along light-cones, for this is the geometric structure at our disposal (Fig. 2). In other words, if $z_i(\tau_i)$ and $z_j(\tau_j)$ denote the trajectories of two charges particles, it wouldn't make sense to say that the particles interact at "equal times" as it is in Newtonian theory. It would however make perfectly sense to say that the particles interact whenever

$$(z_i^\mu - z_j^\mu)(z_{i,\mu} - z_{j,\mu}) = (z_i - z_j)^2 = 0. \tag{1}$$

For an observer finding himself in a universe guided by such laws it might then seem like the effects of particle interactions were propagating through space with the speed of light. And this observer may thus insist that there must be *something* in addition to the particles, something moving or evolving in space–time and mediating interactions between charged particles. And all this would be a completely legitimate way of speaking, only that it would reflect more about how things *appear* from a local perspective in a particular frame of reference than about what is truly and objectively going on in the physical world. From "Gods perspective" there are no fields (or photons, or anything of that kind)—only particles in space–time interacting with each other.

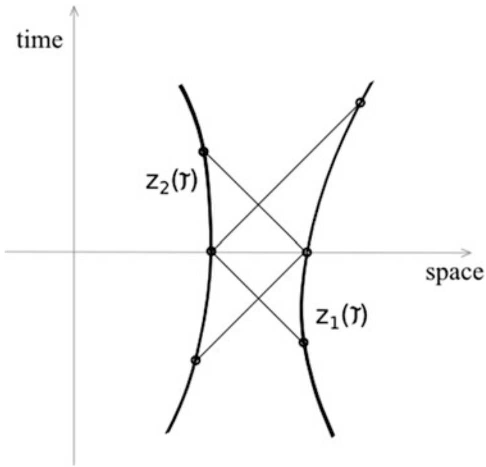

Fig. 2 Interaction of two particles along light-cones

3 Electromagnetism Without Fields

The scenario described above may be hypothetical, but it is not entirely fictitious, for such a formulation of electrodynamics actually exists and is able to reproduce the empirical predictions of the Maxwell–Lorentz theory. That theory is known today as *Wheeler–Feynman electrodynamics* or, for reasons we are going to explore later, as the *Wheeler–Feynman Absorber theory* [5, 6]. It can be defined by a principle of least action for what is arguably the simplest relativistic action for describing interacting particles:

$$S = \sum_i \Big[-m_i \int \sqrt{\dot{z}_i^\mu \dot{z}_{i,\mu}}\, \mathrm{d}\lambda_i - \frac{1}{2} \sum_{i \neq j} e_i e_j \int \int \delta\big((z_i - z_j)^2\big) \dot{z}_i^\mu \dot{z}_{j,\mu}\, \mathrm{d}\lambda_i\, \mathrm{d}\lambda_j \Big]. \tag{2}$$

Since Wheeler–Feynman electrodynamics and Maxwell–Lorentz electrodynamics are (under certain assumption, see below) for all practical purposes empirically equivalent, it may seem that the choice between the two candidate theories is merely one of convenience and philosophical preference. But this is not really the case since the sad truth is that the field theory, despite its phenomenal success in practical applications and the crucial role it played in the development of modern physics, is inconsistent.

The reason is quite simple. The Maxwell–Lorentz theory for a system of N charged particles is defined, as it should be, by a set of mathematical equations. The equation of motion for the particles is given by the Lorentz force law

$$m_i \ddot{z}_i^\mu = e_i\, F^{\mu\nu}(z_i)\, \dot{z}_{i,\nu} \tag{3}$$

describing the acceleration of a charged particle in an electromagnetic field. The electromagnetic field, represented by the field-tensor $F^{\mu\nu}$, is described by Maxwell's equations. The homogenous Maxwell equations tell us that the antisymmetric tensor $F^{\mu\nu}$ (a 2-form) can be written as the exterior derivative of a potential (a 1-form) $A^\mu(x)$, i.e. as

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu. \tag{4}$$

Finally, the inhomogeneous Maxwell equations couple the field degrees of freedom to matter, that is, they tell us how the charges determine the configuration of the electromagnetic field. Fixing the gauge-freedom contained in (4) by demanding $\partial_\mu A^\mu(x) = 0$ (Lorentz gauge), the remaining Maxwell equations take the particularly simple form:

$$\Box A^\mu = -4\pi j^\mu, \tag{5}$$

with $\Box = \partial_\mu \partial^\mu$ the d'Alembert operator and j^μ the 4-current density, which for N point charges is:

$$j^\mu(x) = \sum_{i=1}^{N} j_i^\mu = \sum_{i=1}^{N} e_i \int \delta^4(x - z_i(\tau_i))\dot{z}_i^\mu(\tau_i)\,\mathrm{d}\tau_i. \tag{6}$$

Now, *given* the trajectories $z_i(\tau_i)$ $i = 1, \ldots, N$ of the particles, the solutions of (5) are well known. By linearity of Eq. (5) we can sum the contribution from each particle. A special solution is given by the ("advanced" and "retarded") *Liénard–Wiechert* potentials:

$$A^\mu_{i,\pm}(x) = e_i \frac{\dot{z}_i^\mu(\tau_i^\pm)}{\left(x^\nu - z_i^\nu(\tau_i^\pm)\right)\dot{z}_{i,\nu}(\tau_i^\pm)}, \tag{7}$$

where $\tau_i^+(x)$ and $\tau_i^-(x)$ are the solutions of

$$\left(x - z_i(\tau)\right)^2 = 0. \tag{8}$$

To this we can add any solution of the free wave equation

$$\Box A^\mu = 0. \tag{9}$$

Note that the light-cone structure of relativistic space–time is naturally reflected in these solutions of the Lorentz-invariant equation (5). The Liénard–Wiechert field at space–time point x depends on the trajectories of the particles at the points of intersection with the (past and future) light-cones originating in x.

But this is not the end of the story, since the theory actually requires us to solve (5) and (3) *together*. And this set of coupled differential equations is ill-defined. The Liénard–Wiechert field (the solution of (5)) is singular precisely at the points where it is needed in (3), namely on the world-lines of the particles! This is the notorious problem of the *electron self-interaction*: a charged particle generates a field, the field acts back on the particle, the field-strength becomes infinite at the point of the particle—and the interaction terms blow up. Hence, the simple truth is that the field concept for managing interactions between point-particles doesn't work—unless one relies on formal manipulations like renormalization [7] or modifies Maxwell's laws on small scales [8].

The good news, however, is that—as we have seen—we didn't need the fields in the first place! Taking the idea of a relativistic interaction theory seriously, we can "cut the middle man" and let the particles interact directly. John Wheeler and Richard Feynman thought that way, so did Fokker [9], Schwarzschild [10], and many others, all the way back to Gauss in the nineteenth century [11].

The equations of motion derived from the Fokker–Wheeler–Feynman action (2) correspond to the Liénard–Wiechert solutions of the Maxwell–Lorentz equations *without self-interaction*.[2] That is, we can write

$$m_i \ddot{z}_i^{\mu} = e_i \sum_{j \neq i} F_j^{\mu\nu}(z_i)\, \dot{z}_{i,\nu}, \tag{10}$$

where $F_j^{\mu\nu}$ is given by (4) and the *time-symmetric* Liénard–Wiechert solution

$$A_j^{\mu} = \frac{1}{2}\Big[A_{j,+}^{\mu} + A_{j,-}^{\mu}\Big]. \tag{11}$$

Note that the total "force-tensor" ${}^{(i)}F^{\mu\nu} = \sum_{j \neq i} F_j^{\mu\nu}(z_i)$ for the i-th particle is thus a functional of all the other trajectories, i.e.

$${}^{(i)}F^{\mu\nu} = {}^{(i)}F^{\mu\nu}\big[z_1(\tau_1), \ldots, \cancel{z_i(\tau_i)}, \ldots, z_N(\tau_N)\big].$$

The status of the Maxwell equation's (5) in Wheeler–Feynman theory is now somewhat analogous to the status of Laplace's equation in Newtonian gravity. We can get the simplest (or arguably so) Gallilean invariant theory by writing the force as the gradient of a potential and having that potential satisfy the simplest nontrivial Galilean invariant equation, which is the Laplace equation:

$$\Delta V(x,t) = \sum_i \delta(x - x_i(t)). \tag{12}$$

[2]But note that the Liénard–Wiechert fields are just one of infinitely many possible solutions of the Maxwell equations.

Similarly, we can get the (arguably) simplest Lorentz invariant theory by writing the force as the exterior derivative of a potential and having that potential satisfy the (arguably) simplest nontrivial Lorentz invariant equation, which is (5). As concerns the equation of motion for the particles, the form of (10) is pretty much the natural choice for of a relativistic pair interaction. If the trajectories are parametrized by proper time, the Minkowski norm of the 4-velocity is a constant of motion. We thus have

$$\frac{d}{d\tau}\left(\dot{z}_i^{\mu}\dot{z}_{i\,\mu}\right) = 2\ddot{z}_i^{\mu}\dot{z}_{i\,\mu} = 0 \tag{13}$$

for any physical trajectory, which is immediately satisfied if the acceleration is proportional to $F^{\mu\nu}\dot{z}_\nu$ for some anti-symmetric 2-tensor $F^{\mu\nu}$. This is all to show that the fundamental equations are at least as natural and transparent if we understand them as part of a direct-interaction theory rather than a field theory.

In Newtonian gravity, we can make sense of the gravitational potential at any point in space by conceiving its effect on a hypothetical *test particle*, feeling the gravitational force without gravitating itself. However, nothing in the theory suggests that we should take the potential seriously in that way and conceive of it as a physical field. Indeed, the gravitational potential is really a function on configuration space rather than a function on physical space, and it is really a useful mathematical tool rather than corresponding to *physical* degrees of freedom. From the point of view of a direct interaction theory, an analogous reasoning would apply in the relativistic context. It may seem (and historically it has certainly been the usual understanding) that (5), in contrast to (12), is a *dynamical* equation, describing the temporal evolution of *something*. However, from a relativistic perspective, this conclusion seems unjustified. Taking four-dimensional space–time seriously, the formal analogy between (5) and (12) is pretty much complete.

3.1 A Remark on Time-Symmetry

In the philosophy of time Wheeler–Feynman theory is often discussed in connection with the *arrow of time* and the *radiative asymmetry*. The radiative asymmetry is the fact that we observe charged particles to emit radiation “into the future” but not “into the past.” That is, we observe electromagnetic waves *spreading outwards* from moving sources, not *converging on them* as they accelerate.[3] The puzzle is that electromagnetic radiation is described by Eq. (5), which is *time-symmetric*. However, it seems that in order to account for the radiation phenomena we observe, we have to consistently choose time-asymmetric solutions, namely *retarded* fields rather than advanced fields or a linear combination of both.

[3] This is just the usual way of speaking, of course. We do not observe electromagnetic waves *directly* and in the Wheeler–Feynman description there are none.

One motivation of Wheeler's and Feynman's to study the direct-interaction theory was to explain this temporal chauvinism by a similar reasoning as we use to explain thermodynamic irreversibility in statistical mechanics. Note that the Wheeler–Feynman interaction is manifestly time-symmetric, particles interact along their past *and* future light-cones (see Eqs. (11) and (10)). For their explanation, Wheeler and Feynman assume a large number of homogeneously distributed particles, forming a medium they call the *absorber*. Then they study the behavior of a charged particle *outside* of the observer, that is, in a space–time region with sufficient spatial distant from all the particles of the absorber [5]. They go on to argue that whenever a certain thermodynamic condition holds for the absorber medium (the so-called *absorber assumption*), interactions with the single particle are effectively described by the full retarded Liénard–Wiechert field, including the radiative "back-reaction" that Dirac derived in his renormalized theory [7] (see also [12]). Some authors (e.g., [13]) have criticized the reasoning of Feynman and Wheeler, to me they seem very concise and convincing. Be that as it may, the issue we want to discuss here is not what the Wheeler–Feynman theory can teach us about the *arrow of time* but rather what it can teach us about the *structure of time* and the types of physical laws that we can and should consider. So let's proceed along those lines.

4 The Case for an Extended Instant

If you take mathematical soundness to be a necessary criterion, there is a compelling case for the Wheeler–Feynman as the most serious candidate for a classical theory of electromagnetic interactions. Nevertheless, it is certainly not the theory presented in standard textbooks. The reasons for this are partly historical, partly sociological, but mostly the fact that the equations of motion are simply not of the familiar type and thus—at least by currently available means—notoriously difficult to handle. As we can see from the action functional (2) (or, alternatively, from (10) together with (11) and (7)), the force acting on particle at some space–time point x depends on the trajectory of the other particles at their points of intersection with the past and future light-cone originating in x—it is not determined, as we have come to expect, by the "present state" of the system, where "present state" means the configuration of the physical system on a suitable space-like hypersurface including x. This is to say that the Wheeler–Feynman equations of motion are not naturally posed as *initial value problems* and it is yet unclear if they can be formally reduced to such.

This circumstance, that we don't yet have a fully developed mathematical solution theory for this type of equations, is however not only testimony of their intricacy but also of the fact that only few physicists and mathematicians have acknowledged their relevance. Anyways, as things stand today, we frankly don't know what kind of *initial data* (and/or boundary conditions) have to be specified in order to ensure existence and uniqueness of solutions (see [14, 15] for the current status of the solution theory). And what this implies, in other words, is that we don't

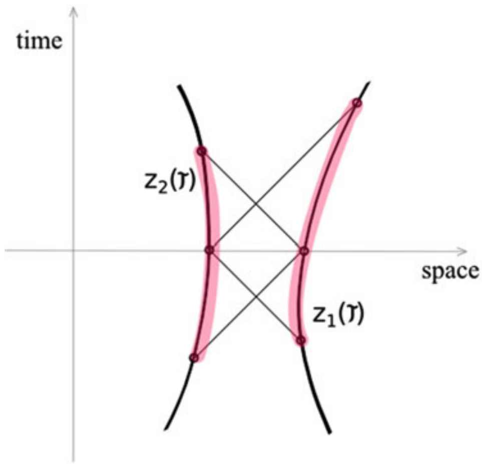

Fig. 3 Interaction of 2 particles in space–time; are the marked trajectory-segments sufficient and/or necessary to determine a unique solution?

know what a *state* is in a Wheeler–Feynman-type theory, if a "state" is supposed to contain the physical data necessary and sufficient for determining the complete evolution of the system.

A hint to the right direction may be provided by the *energy functional* that can be defined as a conserved quantity for the Wheeler–Feynman dynamics [6]. This energy is not assigned to *instantaneous configurations* of the physical variables, but determined by entire *segments of space–time trajectories*, enclosed by intersections with the other particles' light-cones (see Fig. 3). Thus, Wheeler and Feynman claim, it would seem natural to specify such trajectory strips as initial data, at least for the two-particle problem (see their remark below Figure 3 in [6]; for a rigorous discussion of a simplified model, see [16]). What we might end up calling the dynamical *state* of a system would thus consist of temporally extended parts of its particles' world-lines, corresponding to something like an *extended* or *epochal instant*, spanning more than just a space-like cross-section of space–time.

In situations where the space-like distance between the particles (or interacting bodies) is very small (compared to the distances covered by light-like trajectories on relevant time-scales), the extension of the pertinent world-line segments is very short, so that the particles share a common "present" of small temporal extension that may appear almost point-like. This is similar to the phenomenon that we have the impression of seeing the world surrounding us as it is "right now," although we know that the light emitted from objects, no matter how close, requires a finite time to reach us.[4]

It may also turn out, if the solution theory is as nice as one can hope for, that the equations of motion have a unique solution for any Newtonian Cauchy data, that is, any specification of the particles' positions and momenta on a space-like

[4]Of course, that time might be negligible compared to the processing time of our brain and visual receptors, but that is not the issue here.

Cauchy surface (see [14] for optimistic results along those lines). In this case, the equations of motion could be formally reduced to an initial value problem and we could take the 6N-dimensional phase-space of classical mechanics as the "state"-space of Wheeler–Feynman theory. But this "state"-space would mainly serve to parametrize solutions—it is not the space of physical variables on which the laws can naturally be defined. An instantaneous configuration of position and momenta would fix the entire history of the system, but it is clear nonetheless that the *content* of this history must be more than a succession of instantaneous configurations. Narrating the history of a Wheeler–Feynman universe as a succession of states, we would leave out the most interesting parts, namely the way in which the things in the world interact, in which the "present" change in the system's state of motion is determined by other events in space–time.

In fact, being more consequent, it seems most appropriate not to describe such laws in terms of "forces" and "interactions" at all (i.e., by (10)), but to adopt a more holistic perspective, which is to take the Lagrangian paradigm seriously and read a principle of the least action applied to (2) as a law designating the permissible *histories* of the system, rather than a law about the temporal evolution of enduring systems through different *states*. In other words, it could be argued that equations of the Wheeler–Feynman-type are more aptly described as laws about world-lines (or even about entire *worlds*), than as laws about interacting particles.

To me, such a "holistic" description of nature would seem quite beautiful. Suppose, however, that for some reason we were committed to physical laws as initial value problems and the conception of physical reality as a succession of instantaneous states. Then here is what we could do: By supplementing our theory with additional variables we can obtain a description on some state-space that is sufficiently rich, so that instantaneous configurations of variables contain the complete dynamical information required to determine their evolution. This procedure, even if successful, would seem quite unreasonable and contrived were it not for the fact that, in our particular case, we can be fairly parsimonious in adding extra structure. All we need to do is to introduce a ($\mathbb{R}^4$-valued) field on space–time as additional physical degrees of freedom to turn our direct-interaction theory into a Cauchy-data theory about particle positions and momenta *plus* the field configuration. But this solution, as we have seen, carries the seed of its own destruction. The equations become singular as they describe the field acting back on the particles producing it. The dualism of particles and fields has led to mathematical inconsistencies, telling us that something has gone fundamentally wrong.

5 Conclusion

For decades physicists had hoped that quantum mechanics will solve the problem of the electron self-interaction and thus get rid of the divergencies that plague the classical field-theory of electromagnetic interactions. Indeed, as the predominant intuition about quantum mechanics was that nature is somehow fuzzy and unsharp on small length scales, people had a rather unsophisticated intuition about quantum mechanics, but you can see how one would think that the fuzziness of the electron's position might work to our benefit by "smearing out" the singularities. Anyways, that didn't happen. The problem of electron self-interaction was not solved, but inherited by the supposedly more fundamental theory of Quantum Electrodynamics, where it went on to make quite a prominent career under the name of "ultraviolet divergences." The best modern quantum field theory can do is to apply formal renormalization schemes and cut off the range of the divergent integrals on small length scales (high energy scales), i.e. precisely where the dynamics of the matter-fields at a point x depend on the properties of the electromagnetic field arbitrarily close to x. If electromagnetism is in many ways the cradle of modern physics (of relativity and field theory), the electron self-interaction is in many ways its original sin.

I am clueless about how to fix quantum field theory. But if we go back to the drawing board and reflect on electromagnetism in the classical regime, it seems clear to me that the issue lies in the dualism of point-particles as the primitive ontology and the field as a mediator of their interactions. This is not to say that there is something wrong with a particle ontology or the field-concept *per se*. For, as I have tried to argue, if we dig deeper, still, we find hints that the source of all evil may actually be a *temporal fallacy*. This temporal fallacy is the persistence of the "mechanistic" world-view (in the sense of Whitehead) and a failure to take relativistic space–time seriously enough and it is reflected, notably, in our preference for initial value problems as the standard form of dynamical laws.

In other words, I believe that this preference for initial value problems is not just a matter of mathematical convenience; We should be aware—and somewhat suspicious—of the fact that this particular form of a physical description is also appealing because it reflects—and probably enforces—many of our preconceptions about time and the world as we perceive it. Although most physicists seem willing to abandon *presentism* in the light of relativity and deterministic laws, the very concept of *initial states* is clearly a residue of presentist thinking. At least it reflects an understanding of time as a sequence of point-like instants. And since solutions of initial value problems are usually trajectories in some state space, they are bound to represent the physical world as a succession of instantaneous configurations of whatever physical variables the theory poses.

Initial value problems also reflect a certain fantasy about human intervention in this physical world, our deep rooted intuition that we can influence the "future" by manipulating the "present" (based, of course, in our even deeper rooted intuition that we always act in the "now"). The Wheeler–Feynman theory of electromagnetic interactions challenges those preconceptions. Of course, if laws of the Wheeler–Feynman type are to describe the world that we live in, they have to account for (or at least be compatible with) the human experience of the world. That is to say, they must be able to describe a world in which we can "prepare" the kind of physical systems that we usually handle and predict their evolution into the future and in which the effects of "backward-causation" are not apparent on macroscopic scales. But, as Wheeler and Feynman argued, all of this may arise in *effective* or, let's say, *thermodynamic* descriptions of macroscopic systems, rather than being manifest on the most fundamental level of the physical theory. Most physicists today are willing to accept that the *arrow of time* is a phenomenon of this kind, something to be explained on the basis of time-symmetric microscopic laws, rather than something to be found in the fundamental laws or nature of time itself.

Our intuitions about the *structure* of time as a succession of instants seem to be more tenacious—and, I believe, more problematic. At least we have to recognize that even Whitehead, who saw the metaphysical problems resulting from those believes so clearly, was much too generous as he granted that "*[t]he great forces of nature, such as gravitation, were entirely determined by the configurations of masses. Thus the configurations determined their own changes, so that the circle of scientific thought was completely closed.*" (See the quotation in Sect. 1.) This "circle of scientific thought," that Whitehead describes, was—and still is—nothing short of closed as concerns the "great forces of nature." In its established form, it is not even logically (or mathematically) consistent. The holes may be as tiny as single points at which singularities appear in the mathematical formulation, but they are devastating nonetheless if our aim is to understand nature on a fundamental level. And they show even more blatantly in modern physics, in particular in quantum field theories, than in the classical theories we have focused on here.

If I read the signs correctly, the coming revolution in fundamental physics may be a progression away from mechanistic theories, towards a more holistic understanding of physical reality. This may require a fundamental revision of our conception of time, just as Einstein's revolution at the beginning of the last century. The first step, however, may just be to take the latter to its logical conclusion.

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Instants in Physics: Point Mechanics and General Relativity

Domenico Giulini

Abstract Theories in physics usually do not address "the present" or "the now". However, they usually have a precise notion of an "instant" (or state). I review how this notion appears in relational point mechanics and how it suffices to determine durations—a fact that is often ignored in modern presentations of analytical dynamics. An analogous discussion is attempted for General Relativity. Finally we critically remark on the difference between relationalism in point mechanics and field theory and the problematic foundational dependencies between fields and spacetime.

1 Introduction

All known fundamental physical laws are of *dynamical* type. Without exception, they are all required to provide answers for *initial-value problems*. This means the following: If we specify the state of a physical system the laws allow us to deduce further states that are usually interpreted as lying to the future, or past, or both, of the initially given one. Except for General Relativity, this is formally achieved by labelling the states by an external parameter t that—without further justification—is interpreted as "time" (whatever this means). In this contribution I wish to point out that this parameter may be eliminated and that measures of duration can be read off the sequence of states obtained from the dynamical laws.

In the traditional formulation, an initial-value problem is said to be *well posed* if and only if the determination of the future (and possibly past) states is unique, and continuously dependent on the initial state. The last condition means that if we sufficiently restrict the variation of the initial state we can let the evolution vary less than any given bound. These conditions are not only satisfied in Newtonian mechanics, which serves as a paradigmatic example in this respect, but also in the mathematically and conceptually and most complicated theories, like Einstein's theory of General Relativity. Albert Einstein, as well as David Hilbert, wrote down

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the field equation of General Relativity in November 1915. But only in the late 1950s did mathematicians succeed to prove that it indeed allowed for *well-posed initial-value problems*. Had this turned out to be false this would have possibly led physicists to abandon General Relativity, despite all its other convincing features. To allow for a well-posed initial-value problem is presumably the single most important sanity check for any candidate fundamental dynamical law in physics.

This is not restricted to classical laws and classical determinism. The fundamental dynamical law in Quantum Mechanics, Schrödinger's equation, also allows for well-posed initial-value problems. The quantum-mechanical state evolves according to this equation just as deterministically and continuously as the state in Newtonian mechanics does according to Newton's or Hamilton's equations. The typical quantum-mechanical indeterminacy that distinguishes it so drastically from classical mechanics does not concern the evolution of states, it concerns the relation of states to observable features of the system under consideration. But this shall not be the issue we address here. Therefore we will restrict attention to classical (i.e. non-quantum) laws. Our concern is the problem of how to characterise, in a physically meaningful way, data that suffice to determine the evolution and how to find a measure of duration merely from that data.

2 Newtonian Mechanics

Newton's famous third law is written in standard modern textbook language as

$$m\ddot{\vec{x}}(t) = \vec{F}\big(t, \vec{x}(t), \dot{\vec{x}}(t)\big)\,. \tag{1}$$

In this form it is meant to apply to an idealised *mass point*, which should be thought of as an extensionless object ("point") of position $\vec{x}$ and mass value m. A single overdot denotes the derivative with respect to the parameter ("time") t (i.e. the rate of change of the dotted quantity) and a double overdot the second "time" derivative. Finally, the right-hand side denotes the force, $\vec{F}$, which in the case of just one particle is supposed to be externally specified and possibly dependent on t, the instantaneous position $\vec{x}(t)$ of the particle and its instantaneous velocity $\dot{\vec{x}}(t)$. Given the function $\vec{F}$, Newton's equation has a unique solution once the initial position and initial velocity of the particle are specified. The solution is the function $t \to \vec{x}(t)$ that assigns a unique position $\vec{x}$ in space to each value t. That is the standard textbook presentation, except that t is from the start always referred to as time (Newtonian time).

Equation (1) tells us that an initial datum that suffices to predict the future is the position and velocity *at the initial reading of time*. The initial reading of time is a particular value of the parameter t that *represents* time, namely that value that represents the initial moment. This is achieved via a *clock*. A clock is another physical system that also obeys an equation of the form (1) for the pointer variable

p as function of parameter t. Whereas t is not directly observable, p is. Given $p(t)$ we may invert this relation and express t as function of p. This is possible if p is strictly monotonous in t. Systems for which this is not the case would not count as clocks. We then eliminate t in $\vec{x}(t)$ in favour of p and obtain a function $\vec{x}(p)$. This function expresses a relation between the clock's pointer position p and the particle's position $\vec{x}$. That relation is observable because p *as well as* $\vec{x}$ are observable. This is in contrast to $\vec{x}(t)$ where t is not observable. The elusive "initial time" is then that reading of p at which we release the particle. This, in essence, is the idea of *ephemeris time* [7].

But what happens if there is no obvious way to single out a system as "clock". For example, imagine we are given $n+1$ (we say $n+1$ rather than n for later notational convenience) mass points moving about under the action of their own pairwise gravitational attraction. No "clocks" or background reference systems against which the motions of the particles could be measured are given to us. The only thing we can measure are the $\frac{1}{2}n(n+1)$ instantaneous relative distances between pairs of points. Could we still ascertain the validity of Newton's laws of mechanics? This is a relevant question since the situation depicted is basically just that astronomers have to face. And yet it took almost 200 years from the writing of Newton's Principia until physicists and mathematicians first answered this question (of which Newton was fully aware) with sufficient clarity.

The basic question that needs to be answered is how we can construct Newton's absolute space and time from observations of relational quantities alone, for it is only with respect to special spatial reference frames and special measures of time that Newton's equations are valid. These special spatial reference frames are called *inertial systems* and the special measures of time *inertial timescales*. This nomenclature was introduced in 1885 by Ludwig Lange (1863–1936) [9]. One year earlier James Thomson (1822–1892), the elder brother of William Thomson (1824–1907), better known as Lord Kelvin, wrote the following [19]:

> The point of space that was occupied by the centre of the ball at any specified past moment is utterly lost to us as soon as that moment is past, or as soon as the centre has moved out of that point, having left no trace recognisable by us of its past place in the universe of space. There is then an essential difficulty as to our forming a distinct conception either of rest or of rectilinear motion through unmarked space. [...] We have besides no preliminary knowledge of any principle of chronometry, and for this additional reason we are under an essential preliminary difficulty as to attaching any clear meaning to the words *uniform rectilinear motion* as commonly employed, the uniformity being that of equality of spaces passed over in equal times.

This was rephrased into a mathematical problem by Peter Guthrie Tait (1831–1901) [18]:

> A set of points move, Galilei wise, with reference to a system of co-ordinate axes; which may, itself, have any motion whatever. From observation of the *relative* positions of the points, merely, to find such co-ordinate axes.

This is precisely the problem we set above in the simpler case of *free* point particles. So suppose we are given some number of point particles that move about freely, i.e. there is no mutual attraction or repulsion due to any force, and suppose this motion does obey Newton's law with reference to some unknown inertial reference system and inertial timescale. How can we reconstruct these by merely observing the relative distances of the points? How many points and how many snapshots do we need to accomplish that?

2.1 Reconstructing Absolute Space and Time

Tait's answer to the above question, given in the same paper [18], is as follows: We wish to reconstruct the inertial system and timescale from an unordered *finite* number of snapshots ("instances") of instantaneous relative spatial configurations. For this we consider $n+1$ mass-points P_i $(0 \leq i \leq n)$ moving inertially, i.e. without internal and external forces, in flat space. Their trajectories are represented by $n+1$ functions $t \mapsto \vec{x}_i(t)$ with respect to some, yet unspecified, spatial reference frame and timescale. The only directly measurable quantities at this point are the $n(n+1)/2$ instantaneous mutual separations of the particles. We now proceed in the following nine elementary steps:

1. The instantaneous mutual separations are given by $n(n+1)/2$ positive real numbers per label t. This is equivalent to giving their squares:

$$R_{ij} := \|\vec{x}_i - \vec{x}_j\|^2 \qquad \text{for} \quad 0 \leq i < j \leq n\,. \tag{2}$$

2. The knowledge of the $n(n+1)/2$ squared distances, R_{ij}, is, in turn, equivalent to the $n(n+1)/2$ inner products

$$Q_{ij} := (\vec{x}_i - \vec{x}_0) \cdot (\vec{x}_j - \vec{x}_0) \qquad \text{for} \quad 1 \leq i \leq j \leq n\,, \tag{3}$$

as one sees by expressing one set in terms of the other by the simple linear relations (no summation over repeated indices here):

$$R_{ij} = Q_{ii} + Q_{jj} - 2Q_{ij} \qquad \text{for} \quad 1 \leq i < j \leq n\,, \tag{4a}$$

$$R_{i0} = Q_{ii} \qquad \text{for} \quad 1 \leq i \leq n\,, \tag{4b}$$

$$Q_{ij} = \tfrac{1}{2}\big(R_{i0} + R_{j0} - R_{ij}\big) \qquad \text{for} \quad 1 \leq i \leq j \leq n\,. \tag{4c}$$

3. We now seek an inertial system and an inertial timescale, with respect to which all particles move uniformly on straight lines. Correspondingly, we assume

$$\vec{x}_i(t) = \vec{a}_i + \vec{v}_i t \qquad \text{for} \quad 0 \leq i \leq n \tag{5}$$

hold for some *time-independent* vectors $\vec{a}_i$ and $\vec{v}_i$.

4. The 11-parameter redundancy by which such inertial systems and timescales are defined is given by

   (a) spatial translations: $\vec{x} \mapsto \vec{x} + \vec{a}$, $\vec{a} \in \mathbb{R}^3$, accounting for three parameters,
   (b) spatial boosts: $\vec{x} \mapsto \vec{x} + \vec{v}t$, $\vec{v} \in \mathbb{R}^3$, accounting for three parameters,
   (c) spatial rotations: $\vec{x} \mapsto \mathbf{R} \cdot \vec{x}$, $\mathbf{R} \in O(3)$ (group of spatial rotations, including reflections), accounting for three parameters,
   (d) time translations: $t \mapsto t + b$, $b \in \mathbb{R}$, accounting for one parameter, and
   (e) time dilations: $t \mapsto at$, $a \in \mathbb{R} - \{0\}$, accounting for one parameter.

   The redundancies (a) and (b) are now eliminated by assuming P_0 to rest at the origin of our spatial reference frame. We then have, assuming (5),

$$Q_{ij}(t) = \vec{x}_i(t) \cdot \vec{x}_j(t) = \vec{a}_i \cdot \vec{a}_j + t\,(\vec{a}_i \cdot \vec{v}_j + \vec{a}_j \cdot \vec{v}_i) + t^2\, \vec{v}_i \cdot \vec{v}_j\,. \quad (6)$$

5. Measuring the mutual distances, i.e. the Q_{ij}, at k different values t_a $(1 \leq a \leq k)$ of t we obtain the $k\,n(n+1)/2$ numbers $Q_{ij}(t_q)$. From these we wish to determine the following unknowns, which we order in four groups:

   (1) the k times t_a,
   (2) the $n(n+1)/2$ products $\vec{a}_i \cdot \vec{a}_j$,
   (3) the $n(n+1)/2$ products $\vec{v}_i \cdot \vec{v}_j$, and
   (4) the $n(n+1)/2$ symmetric products $\vec{a}_i \cdot \vec{v}_j + \vec{a}_j \cdot \vec{v}_i$.

6. The arbitrariness in choosing the origin and scale of the time parameter t, which correspond to the points (d) and (e) above, can, e.g., be eliminated by choosing $t_1 = 0$ and $t_2 = 1$. Hence the first group has left the $k - 2$ unknowns $t_3, \ldots, t_k$. The last remaining redundancy, corresponding to the spatial rotations in point (c), is *almost* eliminated by choosing P_1 on the z axis and P_2 in the xz plane. This suffices as long as P_0, P_1, P_2 are not collinear. Otherwise we choose three other mass points for which this is true. Here we exclude the exceptional case where all mass points are co-linear. We said that this "almost" eliminates the remaining redundancy, since a spatial reflection at the origin is still possible.
7. Tait's strategy is now as follows: for each instant in time t_a consider the $n(n+1)/2$ Eq. (6). There are $k-2$ unknowns from the first and $n(n+1)/2$ unknowns each from groups (2), (3) and (4). This gives a total of $k\,n(n+1)/2$ equations for the $k - 2 + 3n(n+1)/2$ unknowns. The number of equations minus the number of unknowns is

$$(k-3)n(n+1) + 2 - k\,. \quad (7)$$

   This is positive if and only if $n \geq 2$ and $k \geq 4$. Hence the minimal procedure is to take four snapshots ($k = 4$) of three particles ($n = 2$), which results in 12 equations for 11 unknowns.
8. Recall that we assumed the validity of Newtonian dynamics and that the given trajectories correspond to force-free particles. This implies the existence of inertial systems and hence also the existence of solutions to the equations above.

For positive (7) the equations determine the $3n(n+1)/2$ unknowns in groups (2)–(4) which, in turn, determine the $6n-3$ free components of $\vec{a}_i$ and $\vec{v}_i$ up to an overall sign, since $3n(n+1)/2 \geq 6n-3$ if and only if $n \geq 2$. Note that we have $6n-3$ rather than $6n$ free components for $\vec{a}_i$ and $\vec{v}_i$, since we already agreed to put P_1 on the z axis, which fixes two components of $\vec{a}_1$ and $\vec{v}_1$ each, and P_2 in the xz plane, which fixes one component of $\vec{a}_2$ and $\vec{v}_2$ each. Note also that we cannot do better than determining the $\vec{a}_i$ and $\vec{v}_i$ up to sign, since the Q_{ij} are homogeneous functions of *second* degree in these variables.

9. Once the $2n$ vectors $\vec{a}_i$ and $\vec{v}_i$ are obtained, so is clearly the inertial system (up to orientation) and the inertial timescale. This is as far as Tait's solution to Thomson's problem goes.

One remarkable thing about Tait's solution is that the spatial inertial system and the inertial timescale are determined together. This is not really surprising: The mathematical problem of calculating the k labels t_a representing "instants" cannot be separated from the characterisation of the instants themselves. In this sense it might be said—following Julian Barbour [2]—that instants are not to be located in time, but that time is rather to be found in instants. Thus it seems that the philosophical discussion concerning the reality of time (see e.g. [10] for an up-to-date account) is then really a discussion concerning the reality of instants. But in point mechanics, instants are relational configurations, the reality of which cannot be doubted without mocking the theory.

2.2 *Mechanics Without Parameter-Time*

If time can be read off instances, as claimed above, we should, at least in principle, be able to altogether eliminate the parameter t from the laws. How does the t-less version of Newtonian mechanics look like? One answer has been well known for a long time, albeit in a somewhat hybrid form in which the absolute positions in space still feature. It goes under the name of Jacobi's principle, after Carl Gustav Jacobi (1804–1851). It takes the form of a geodesic principle in configuration space. That means, it determines the physically realised paths in configuration space between any pair $(\mathbf{q}_i, \mathbf{q}_f)$ of given points to be that of shortest length. Here "length" is measured in some appropriate metric that encodes the essential dynamical information.

Note that the parameter t plays no rôle: its value at the initial and final point need not be specified. Rather, the measure of inertial time elapsed between the initial and final configuration can be calculated *after* the dynamical trajectory has been determined through the geodesic principle. Let there be n mass points whose positions are $(\vec{q}_1, \ldots, \vec{q}_n) =: \mathbf{q}$, moving under the influence of a potential $V(\mathbf{q})$. The configuration space is $\mathbb{R}^{3n}$ and its Riemannian metric, with respect to which the physically realised trajectories of constant Energy E are geodesics, is given by $g = (E - V)T$, where T in the positive-definite bi-linear form that appears in the expression for the kinetic energy ("kinetic-energy metric"). The inertial time that has elapsed along the length-minimising trajectory between $\mathbf{q}_i$ and $\mathbf{q}_f$ is then given by

$$\Delta t(\mathbf{q}_i, \mathbf{q}_f) = \int_{\mathbf{q}_i}^{\mathbf{q}_f} \sqrt{\frac{T\big(d\mathbf{q}/d\lambda, d\mathbf{q}/d\lambda\big)}{E - V(\mathbf{q})}}\, d\lambda\,. \tag{8}$$

This may be understood as saying that time has to be chosen in such a fashion so as to lead to the standard form of energy conservation. Indeed, from (8) we get

$$E = T\big(d\mathbf{q}/dt,\, d\mathbf{q}/dt\big) + V(\mathbf{q})\,. \tag{9}$$

Note that (8) only depends on the pair $(\mathbf{q}_i, \mathbf{q}_f)$ and not on the way we parametrise the path. Hence the choice of the parameter λ is arbitrary. Therefore we have a well-defined map

$$\Delta t : \mathbb{R}^{3n} \times \mathbb{R}^{3n} \rightarrow \mathbb{R}_+ \tag{10}$$

which, for given energy E, assigns to each pair of points in the configuration space the inertial-time duration of the physical journey connecting them. As we will discuss next, there is a certain analog to Jacobi's Principle in General Relativity, with some additional issues arising due to the fact that the fundamental mathematical entities being fields rather than point-particles. Finally we point out that there is a generalisation to Jacobi's principle in models of point mechanics without absolute space. In these models only the instantaneous relative distances enter the laws and the time lapse can again be calculated from the dynamical trajectories. First attempts were Reissner's (1874–1967) [16] and Schrödinger's [17], with the full "relativisation" of time being achieved only much later in [3]. See also [4] for more on the modern context and translations of the papers by Reissner, Schrödinger, etc.

3 General Relativity

Einstein's equations are equations for entire spacetimes, that is, pairs (M, g) where M is a four-dimensional differentiable manifold endowed with a certain geometric structure called *Lorentzian metric*, which is here represented by g. Given such a pair (M, g) and a specification of certain aspects of physical matter, it makes unambiguous sense to say that (M, g) does, or does not, satisfy Einstein's equations. No external notion of time enters the picture at this stage. This, clearly, is for good reasons: Spacetimes do not evolve (in "time" external to them); they simply are! In addition, no conditions concerning structures internal to (M, g) need to be imposed, such as sequential ordering of substructures (to be interpreted as "instants"), absence of closed timelike curves (i.e. journeys into ones own past) or

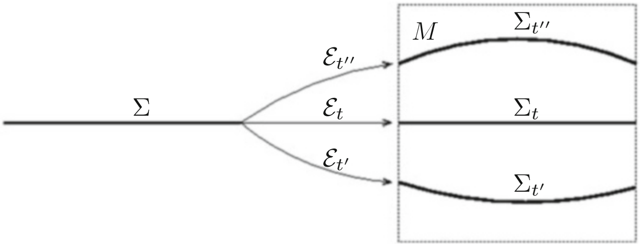

Fig. 1 Spacetime, M, is foliated by a one-parameter family of embeddings $\mathcal{E}_t$ of the three-manifold Σ into M. Here t is a formal label without direct physical significance. Σ_t is the image in M of Σ under $\mathcal{E}_t$. Each such Σ_t is an *instant*

causal evolution of geometry. On the other hand, Einstein's equations are *compatible* with the *additional imposition* of such structures. It required the hard work of mathematicians of many years to show that a reasonable set of such additional conditions exist which ensure that Einstein's equations allow for well-posed initial value problems in the sense explained above.

In particular, these conditions ensure that the spacetime can be thought of as the history of space. In a loose mathematical sense this means that spacetime is a staking of spaces, each one being an instant. More precisely, spacetime is foliated by a one-parameter family of embeddings of space into spacetime. This is schematically represented in Fig. 1. For that to make mathematical sense we must be sure that a single space, Σ, suffices to foliate spacetime. Its geometry may change from leaf to leaf, but not its essential properties as differentiable manifold, for otherwise we could not speak of *its* evolution. In particular this means that its topological properties are preserved during evolution, like its connectedness and its higher topological invariants; see Fig. 2.

One of the fundamental difficulties with the notion of spacetime as history of space is its inherent redundancy: There are many ways to describe one and the same spacetime as the evolution of space. This is explained in Fig. 3. This means that if we cast Einstein's equations into the form of evolution equations for "space", we cannot expect unique solutions, contrary to what is usually required for well-posed initial-value problems. The point here is that the non-uniqueness is not arbitrary. It is precisely of the amount that accounts for the different ways to move space through a *fixed* spacetime, no more and no less. This is closely related to the infamous "Hole Argument" [15]. That relaxation of the uniqueness requirement is familiar from the so-called gauge-theories and does not imply any renunciation from determinism of fundamental laws, at least as long as the degree of arbitrariness in the analytical expression of the evolution is under complete mathematical control. Physical configurations are then taken to be the equivalence classes under the relation that identifies any two apparently different evolutions that give rise to the same spacetimes (more precisely: diffeomorphism-class of spacetimes).

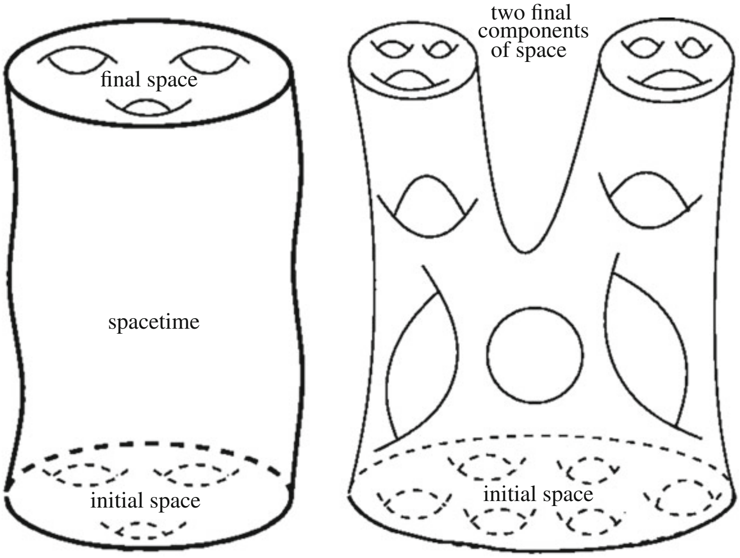

Fig. 2 Schematic rendering of spacetimes. The one on the *left* may be viewed as time evolution of space. Time runs upwards and space corresponds to the horizontal sections, here depicted by a 3-holed surface. In the spacetime on the *right* an initial connected space at the *bottom*, represented by a single 6-holed surface, evolves into two 3-holed pieces. This spacetime cannot be viewed as time evolution of a single space and shall be excluded from the discussion

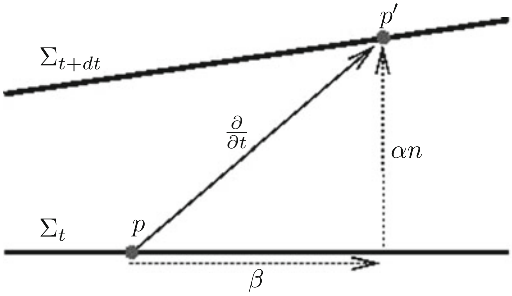

Fig. 3 There is a large ambiguity in moving from an initial space-slice Σ_t "forward in time". For $q \in \Sigma$ the image points $p = \mathcal{E}_t(q)$ and $p' = \mathcal{E}_{t+dt}(q)$ are connected by the vector $\partial/\partial t|_p$ whose components tangential and normal to Σ_t are β (three functions) and αn (one function), respectively. Hence there is a four-function worth of ambiguity to move Σ_t in a given ambient spacetime

3.1 The Chronos Principle

Modulo the difficulties just mentioned, we may ask whether we can extract a notion of time merely from the information of instants. An instant here is a spatial configuration, that is a pair (Σ, h), where Σ is a three-dimensional manifold and h is a Riemannian (i.e. positive definite) metric. One obvious question concerning Einstein's equations is this: given two instants (Σ_1, h_1) and (Σ_2, h_2), can we associate a measure of time by which they are apart if we assume that both 3-geometries occur in a spacetime that satisfies Einstein's equations. This is known as the "sandwich conjecture" in General Relativity and known to fail in many known examples which are, however, of special symmetry that renders the problem singular. For example, it is obvious that specifying any two flat 3-slices in Minkowski space does not give us any information on their separation. Similarly, it has been shown that in the spherically symmetric case a similar underdetermination prevails [13]. On the other hand, it has been an old hope that a suitable analog of Jacobi's principle, and in particular formula (8), is also valid in General relativity. This has been first proposed in the classic and well-known paper [1] of 1962. An apparently less well-known contribution appeared 12 years later, in which the following "Chronos Principle" in General Relativity was proposed, according to which time is a measure for the distance of instantaneous configurations (instants) [6]. Moreover, it was asked in [6] whether such measures existed such that one would not have to know the entire spatial configuration in order to determine the time span.

> This postulate contains the statement that it is not necessary to look at the change in configuration of the entire universe to measure time. It is sufficient to measure the change in configuration of only a localized region of the universe, and one is assured that the local time thus obtained will be equal to that of any other region, and indeed equal to the global time. ([6], p. 76)

It is this localisation property that renders this reading of time from instants physically viable. Let us therefore see how it can be satisfied. The answer, quite surprisingly, leads more or less directly to General Relativity. We shall give the argument in a slightly simplified form.

As already stated, Einstein's equations can be cast into evolutionary form. In that form one may identify a kinetic-energy metric, just like in point mechanics. It reads:

$$ds^2 = \int_\Sigma d^3x \; G^{ab\,nm}[h(x)] dh_{ab}(x) dh_{nm}(x), \tag{11}$$

where $G^{ab\,nm}[h(x)]$ is a certain expression that depends on the metric tensor h of space but not on its derivatives (ultralocal dependence). It is sometimes called the Wheeler-DeWitt metric. The measure of time will be obtained by a rescaling of the kinetic-energy metric, just like in (8). Hence one writes

$$d\tau^2 = \frac{ds^2}{\int_\Sigma d^3x \; R(x)} \,. \tag{12}$$

Here R must be a scalar function of the spatial metric h. The simplest non-constant such function is the scalar curvature, which depends on h and its derivatives up to order 2. The condition that the measure of time be compatible with arbitrarily fine localisations $\Sigma \to U \subset \Sigma$ requires the integrands in the numerator and denominator of (12) to be proportional. Without loss of generality we can take this constant of proportionality (which cannot be zero) to be 1 (this just fixes the overall scale of physical time) and obtain

$$G^{ab\,nm}[h(x)]\frac{dh_{ab}(x)}{d\tau}\frac{dh_{nm}(x)}{d\tau} - R[h](x) = 0\,. \tag{13}$$

This is a well-known formula (the so-called Hamiltonian constraint) in General Relativity. Hence Relativity just satisfies the localisation property with the simplest conceivable local rescaling function R. Finally, physical time is now given in terms of three-dimensional geometric quantities by a Jacobi-like formula, which is just the analog of (8) in the case $E = 0$:

$$\Delta\tau(\mathbf{g}_i, \mathbf{g}_f) = \int_{\mathbf{g}_i}^{\mathbf{g}_f} \sqrt{\frac{G\big(d\mathbf{g}/d\lambda, d\mathbf{g}/d\lambda\big)}{-R\big[\mathbf{g}(\lambda)\big]}}\, d\lambda\,. \tag{14}$$

4 Conclusions and Open Issues

Following [2] we tried to argue that the notion of "time from instances" is inherent in classical point mechanics as well as in General Relativity. We also saw that in General Relativity that notion of time is not as hopelessly global as one might have feared. In fact, one can argue that General Relativity just realises the simplest *localisable* notion of that sort of time.

But there are also points that remain open (to me):

1. Solutions to dynamical equations of motion in the form of (generalised) geodesic principles are subsets of (dynamically realised) configurations in the space of (kinematically possible) ones. These subsets are delivered to us in the form of unparametrised curves. So, even though the parameter does not matter, the structure of a one-dimensional sub-continuum remains. In particular one (or two) preferred orderings are selected. What is the significance of that? What makes us experience this solution configurations according to this order?
2. Can we, on the space of 3-geometries, characterise a function that structures it according to some definition of geometric entropy? How would its gradient flow be related to the dynamics of General Relativity?
3. Suppose the spacetime we live in did not allow for any symmetries and were sufficiently generic, so as to not allow for two *different* isometric embeddings of any of its possible 3-geometries. (Such spacetimes exist and are, intuitively speaking, the generic case, though their degree of generality or naturalness is not easy to characterise mathematically.) This means that each instant would have its unique place in spacetime. Would this count as a perfect representation of the "Now" in a physical theory (here General Relativity), or could/should we ask for more?

Finally I wish to comment on the transition of point mechanics to field theory. In point mechanics, the requirement to only employ purely relational quantities is met by eliminating all explicit reference to absolute space and time. This has been gradually achieved in the papers of Reissner, Schrödinger, and Barbour & Bertotti. But what is the precise analog of that requirement in field theory? A standard answer to this is that the theory should be *background independent*. The intended meaning of that phrase is that the theory should not employ structures which are not dynamically active. Closer inspection shows that it is quite hard to translate this intended meaning into a clear mathematical condition [8]. The problem is that whatever the mathematical formulation is, it seems quite easy to turn it into an equivalent one by some formal rewriting that renders it (formally) background independent. It is often taken for granted that the requirement of *diffeomorphism invariance* (also known as "general covariance") is sufficient, because that would deprive spacetime points of their independent individuality. This is true to some extend, but it seems not to go as far as one might have hoped for. Modern (quantum-)field theory does not get rid of space and time.

Markus Fierz was deeply concerned about the problematic relation between spacetime and fields. In a remarkable letter of October 9–10th 1951 to Wolfgang Pauli[1] he wrote ([11], Vol. IV, Part I, Doc. 1287, p. 379)

> There exist [in classical physics–DG] solutions [to field equations–DG] with empty domains, that is, emptied from all fields. Hence one needs a theory of space which is independent of what fills space. There is the geometry of space and the laws of things in space. [...]
> Space is still absolute in Relativity Theory insofar as one may characterise it without referring to its 'content', and because it may even exist without any content. [...]
> In a [hypothetical–DG] full Theory of Quantum Fields, in which the act of observation and the possibility to localise are described correctly, it should not be necessary to introduce space separately. Opposite to what Einstein hoped, the laws of space should follow from the laws of Nature (not the laws of Nature from geometry). But this can only be hoped for if there is no such thing as empty space, that is, if you cannot clear [ausräumen] space. Fields are not in space, they span space. Space is not a geometric idea [Gedankending], it is a certain aspect of the world."
> In this sense, space in Relativity Theory is absolute and this is why Einstein suggested to call it aether. In a proper field theory the theory of localisation should deliver a theory of space. Space should somehow be 'created' by test bodies and hence be a function of the observer in a much deeper sense than in Relativity Theory.

Pauli replied on October 13 in a way that would also be typical for many modern relativists ([11], Vol. IV, Part I, Doc. 1289, pp. 385–386):

> Your wording does not do justice to Relativity Theory, which is just an attempt to connect geometry and laws of nature concerning things [Dinge] in the spacetime world. [...] All people happily proclaim just the opposite to what you said in your letter: namely from now on only the connection of spacetime and things is absolute. [...]

[1] There exist two versions of this letter, one from October 9th and one from October 10th. Here we quote from the fist only.

> I am quite indignant about this part of your letter, since it shows to me that the, compared to me, slightly younger generation of physicists (not to speak of the still younger ones!) have completely repressed [verdrängen] General Relativity - and because I know how important Einstein considered this point to be. [...]
> After this urgent correction (diagnosis: 'repression' [Verdrängung]!) one can ask whether the dependence of space (i.e. spacetime) from the things [Dingen] according to General Relativity is sufficient. To pose the question already means to negate it. [...]
> I agree that the impossibility to accommodate Einstein's postulate (i.e. Mach's original point of view) within General Relativity is a deep and significant sign for the inadequacy of classical field physics.

So we see that after his usual grumble Pauli finally agrees at least on the existence of a fundamental difficulty, which was, after all, well addressed by Fierz' original complaint. Even today all candidate theories of quantum gravity make use of non-dynamical structures that represent some sort of space or spacetime (of various dimensions). Hence I believe Fierz' complaint is as relevant today as it was 60 years ago.

Everyone knows the opening words of Hermann Minkowski's (1864–1909) famous address "Raum und Zeit", delivered in Cologne on September 21st, 1908 [12]:

> Gentlemen! The views of space and time which I wish to lay before you have sprung from the soil of experimental physics. Therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.

But it seems not to be so well known that Minkowski felt the enormous abstraction and possible physical over-idealisation of the concept of spacetime *as such*, as he clearly indicated in his introduction, before going into the description of what we now call "Minkowski space" (space meaning spacetime). He wanted his readers to understand the points of spacetime as individuated entities:

> In order to not leave a yawning void [gähnende Leere], we wish to imagine that at every place and at every time something perceivable exists. In order to avoid saying 'matter' or 'electricity' for that something, I will use the word 'substance' for it. We focus attention on the substantive [substantiellen] world point at x, y, z, t and imagine to be capable to recognise this substantive point at any other time.

That substantivalist's view of Minkowski spacetime is still inherent in its mathematical representation in modern field theory. One sign of this is the interpretation of its automorphism group (the Poincaré group) as proper physical symmetries rather than gauge transformations. Recall that a proper physical symmetry transforms solutions to dynamical equations into solutions, but the transformed solution is considered physically different (distinguishable) from the original one. In contrast, gauge transformations just connect redundant descriptions of the same physical situation.

Individuating spacetime points is natural if we think of spacetime to be a geometrically structured *set*. A set, by Cantor's definition, consists first of all of a set which may then carry certain geometric structures of various complexities. But recall what according to Cantor's definition it already takes to be a set [5]:

> By a *set* we understand any gathering together M of determined well-distinguished objects m of our intuition or of our thinking (which are called elements of M) into a whole.

Minkowski's "substance" may serve to distinguish events. But is that substance not eventually just another physical system obeying its own dynamical laws? If so, what kind of "dynamical law" can that be if there is no non-dynamical substance left with respect to which we can define change. Surprisingly—or perhaps not—this is just the same difficulty that stood at the very beginning of modern theories of dynamics. In "de gravitatione", written well before the Principia, presumably between 1664 and 1673 (the dating is still controversial), Newton said [14]:

> It is accordingly necessary that the determination of places and thus of local motions is represented in some unmoved being of which sort space or extension alone is that which is seen as distinct from bodies. [...]
> About extension, then, it is probably expected that it is being defined either as substance or accidents or nothing at all. But by no means nothing, surely, therefore it has some mode of existence proper to itself, by of which it fits neither to substance nor to accident.

"Das noch Ältere ist immer das Neue"
Wolfgang Pauli

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Irreversibility and Collapse Models

Mohammad Bahrami, Angelo Bassi, Sandro Donadi, Luca Ferialdi, and Gabriel León

Abstract Irreversible phenomena are of fundamental importance because they characterize a direction of time. Irreversibility has been observed in three different physical situations, namely, in thermodynamics (monotonic increase of entropy), quantum theory (measurement process), and cosmology (black holes and their entropy). There is no consensus on how these three kinds of irreversibility are connected, and whether there is any common ground that can explain them consistently, or if one of them is more fundamental than the others. A solution to the above questions is to work with a physical theory that picks a preferred direction of time. Collapse models, as quantum non-linear and stochastic theories, may provide us with such a solution. After discussing the features of collapse models in detail, we review the phenomenological implications of these models, with particular attention to the aforementioned issues.

1 Introduction

In our daily experience, we have a common-sense about the direction of time and how to distinguish the past from the present. However, one of the most controversial and unsolved problems of modern physics is the so-called arrow of time. This problem is strictly related to the concept of irreversibility and of irreversible

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processes. Irreversible processes are of fundamental significance, because they make the two directions of time to be physically inequivalent [1].

Historically, the notions of irreversibility and the direction of time are connected with the second law of thermodynamics and the concept of entropy. Although there is no clear consensus on how irreversibility and entropy are connected, the forward direction of time is usually identified as the direction in which the entropy increases [2]. Statistical mechanics attempts to derive the phenomenological laws of thermodynamics from the underlying time-symmetric microscopic dynamics, aiming at providing an explanation for the question of the monotonic increase of entropy [3]. However, according to the Loschmidt paradox [4], it should not be possible to derive macroscopic irreversibility if one only starts from the reversible microscopic laws. Up to now, the answers to Loschmidt paradox have been sought in two different ways: either by introducing special boundary conditions while keeping the time-symmetric dynamical laws or by using different fundamental laws of nature which are time-asymmetric [2, 5]. If we follow for the second answer, then a stochastic nonlinear dynamical law of nature can describe irreversible behaviors. Among fundamental stochastic nonlinear dynamics are *collapse models*, which are described by modified Schrödinger equations with nonlinear and stochastic terms [6–36]. In [37], David Albert has argued how the thermodynamic arrow of time can be derived from a specific collapse model.

In quantum theory, the very nature of the measurement process involves the concept of irreversibility. This irreversibility, which cannot be derived from the Schrödinger equation, is usually called as the "measurement problem." The linear dynamics of quantum mechanics allows for superpositions of any object, e.g. in two different positions in space. However, at the macroscopic level we never observe such spatial superpositions. Then, there should be a mechanism which suppresses superpositions at the macroscopic level, while allowing them at microscopic level. Such a mechanism creates a fundamental irreversibility, and introduces an arrow of time. *Collapse models* have been originally proposed to solve this problem of standard quantum theory.

The unification of quantum theory and General Relativity provides a framework in which the concept of entropy is connected with the quantum measurement problem. It is known that up to this date, we do not have a fully working theory of Quantum Gravity. However, we have learnt a lot. In particular, one of the most important challenges that a complete theory of quantum gravity must address is the calculation, from first principles, of the Bekenstein–Hawking entropy for black holes [38, 39]. Several explanations have been given to this problem by using the two most popular models of quantum gravity: String Theory and Loop Quantum Gravity. One cannot deny the progress made by both theories and their successes [40]. However, there are particular situations, like Schrödinger's Black Hole [41]—where strong quantum effects are combined with strong gravitational effects—which point at a difficulty with present quantum theories of gravity. It is a well-known result that the entropy of a black hole is given by $S \propto A/4$, where A corresponds to the area of the event horizon. In the Schrödinger Black Hole case, a quantum measurement-like process makes A to be indeterminate at some specific times, and thus we are not able to assign the corresponding entropy to the black hole. In this particular case, String Theory and Loop Quantum Gravity cannot provide a clear answer to the problem simply because they only modify gravity and work with standard quantum theory, measurement problem included. In this regard, some modifications in standard quantum theory should be introduced in order to address the aforementioned problem. This means that one should modify both quantum theory and General Relativity in order to construct a successful unified theory.

Moreover, if one tries to follow a canonical quantization procedure for gravity (such as the old Wheeler-de Witt approach [42], or in its modern formulation, e.g. in the form of Loop Quantum Gravity [43]), the resulting theory would be an atemporal theory. A notion of time, or its general relativistic counterpart, is no longer present in the theory, simply because the Hamiltonian vanishes when acting on the physical states allowed by the theory. This is known as *the problem of time in quantum gravity* [44]. A way out to this problem is to define an effective wave function according to a specific procedure, whose evolution is described by a modified Schrödinger equation [45–47]. Following this procedure (that we will discuss in more detail in Section III.B), one is able to specify the space–time and its slicing using the standard lapse and shift functions. The particular realization of this construction depends on the situation and the specific theory of matter fields which one is considering. However, as noted in [45–47], the standard Schrödinger equation emerges, only as an effective description, with a confined range of validity. Therefore, it is likely to obtain modifications that could lead to departures from a unitary evolution, like those proposed by collapse models.

In summary, as we described before, the concept of irreversibility is of fundamental significance in order to introduce the idea of the direction of time. We briefly reviewed three irreversible phenomena in thermodynamics, quantum theory, and cosmology. We argued that the dynamics of collapse models, which is given a modified Schrödinger equation, can provide a possible explanation to the aforementioned irreversibilities. In this paper, we will elaborate collapse models and their relevant implications in more detail. This paper is organized as follows. In Sect. 2, we give an introduction to collapse models, describing the most important models (GRW, CSL, QMUPL). We also briefly review the relativistic generalization of the CSL model. In Sect. 3, we first examine the experimental implications of collapse models, and then we briefly describe the possible application of these models to inflationary models in cosmology.

2 Collapse Models

Although standard quantum mechanics has obtained many successes since its formulation, it has also generated puzzles that persist to this day. These puzzles, which are all connected to the measurement problem, originate from the linear character of the quantum dynamics, whose range of validity is not clearly defined. Coping with these puzzles, some physicists believe that one has to modify quantum mechanics by introducing some nonlinear stochastic corrections. These corrections should be stochastic in order to avoid superluminal signaling [48–51]. The new nonlinear and stochastic theories are usually called as "*collapse models*" [6–36].

According to collapse models, a noise field couples non-linearly with the system (usually with the spatial degree of the freedom of the system), inducing a spontaneous random localization of the wave function in a sufficiently small region of the space. A suitably chosen collapse parameters make sure that micro-systems evolve practically (but not exactly) with the linear Schrödinger dynamics, while for large macro-systems, the non-linear effects are so strong that the wave function is always perfectly localized in space. We review the following three collapse models:

– GRW: The first collapse model; it describes the evolution of any system of distinguishable particles, whose localizations are driven by discrete jumps in time (Poisson process).
– CSL: This is a generalization of the GRW model, where discrete jumps are replaced by a continuous diffusion process, and the behavior of identical particles is described as well.
– QMUPL: This model has the great advantage of being physically realistic, and, at the same time, mathematically simple enough to allow for a thorough analysis.

2.1 The GRW Model

Quantum Mechanics with Spontaneous Localizations (QMSL) model, also known GRW model, is the first collapse model, proposed by Ghirardi et al. [7]. This model is based on the following assumptions:

1. Each particle of a system of n-distinguishable particles is subject to a spontaneous, sudden, and random localization, which is described by a Poisson's process in time with the mean rate λ^i_{GRW} (for the i-th particle).
2. The evolution between two successive collapses is governed by the Schrödinger equation.
3. The localization process is described as follows:

$$|\psi\rangle \rightarrow \frac{\hat{L}^i_{\mathbf{a}}|\psi\rangle}{\|\hat{L}^i_{\mathbf{a}}|\psi\rangle\|}, \tag{1}$$

   where $\hat{L}^i_{\mathbf{a}}$ is the self-adjoint localization operator of the i-th particle around the center $\mathbf{a}$.
4. The localization operator is chosen to be

$$\hat{L}^i_{\mathbf{a}} = \left(\frac{\alpha}{\pi}\right)^{3/4} e^{-(\alpha/2)(\hat{\mathbf{q}}_i-\mathbf{a})^2}, \tag{2}$$

where $\mathbf{q}_i$ is the position operator of the i-th particle, and $r_C = 1/\sqrt{\alpha}$ is the correlation distance of the localization function, which is chosen to be $r_C = 10^{-7}$ m.

5. The probability density to have a localization around $\mathbf{a}$ is given by

$$P_i(\mathbf{a}) = \|\hat{L}^i_{\mathbf{a}}|\psi\rangle\|^2. \tag{3}$$

As we see, a wave function evolves according to the Schrödinger equation and, when a spontaneous collapse occurs, it is localized according to Eqs. (1) and (2). Moreover, Eq. (3) tells us that the localization is more probable in the region of space where the probability to find the particle (according to the rules of standard quantum mechanics) is higher. In order to understand how the collapse mechanism works, let us consider the following simple example. Consider a superposition of two Gaussian wave functions, centered at $x = \pm a$ with widths equal to $1/\sqrt{\gamma}$:

$$\psi(x) = \frac{1}{\mathcal{N}}\left[e^{-(\gamma/2)(x+a)^2} + e^{-(\gamma/2)(x-a)^2}\right], \tag{4}$$

where $\mathcal{N}$ is a normalization constant. For $a \gg r_C \gg \frac{1}{\sqrt{\gamma}}$ one can easily show that the probability that the collapse occurs at positions $x = \pm a$ is about $1/2$, while this probability is about zero at positions far from centers of two Gaussian wavepackets [6]. Therefore, the collapse process localizes the particle in accordance with the Born probability rule.

In the coordinate representation, the GRW master equation for the density matrix of a single particle is given as follows:

$$\frac{\partial}{\partial t}\langle \mathbf{x}|\hat{\rho}(t)|\mathbf{x}'\rangle = -\frac{i}{\hbar}\langle \mathbf{x}|[\hat{H}, \hat{\rho}(t)]|\mathbf{x}'\rangle - \lambda_{\mathrm{GRW}}\left(1 - e^{-(\alpha/4)(\mathbf{x}-\mathbf{x}')^2}\right)\langle \mathbf{x}|\hat{\rho}(t)|\mathbf{x}'\rangle. \tag{5}$$

The parameter λ_{GRW} is chosen equal to $\lambda_{\mathrm{GRW}} = 10^{-16}\ \mathrm{s}^{-1}$. This value makes the model consistent with quantum mechanics at microscopic level: the collapse effect for micro-objects is practically negligible. The contribution of the collapse to the evolution of the density matrix is:

$$\langle \mathbf{x}|\hat{\rho}(t)|\mathbf{x}'\rangle \simeq e^{-\Gamma(|\mathbf{x}-\mathbf{x}'|)t}\langle \mathbf{x}|\hat{\rho}(0)|\mathbf{x}'\rangle, \tag{6}$$

with the reduction rate $\Gamma(|\mathbf{x}-\mathbf{x}'|)$ given by:

$$\Gamma(|\mathbf{x}-\mathbf{x}'|) = \lambda_{\mathrm{GRW}}\left(1 - e^{-(\alpha/4)(\mathbf{x}-\mathbf{x}')^2}\right). \tag{7}$$

One of the most important features of collapse models is the so-called *amplification mechanism*. If we assume that the reduction rates for the N constituents of a macroscopic object are equal ($\lambda^i_{\mathrm{GRW}} = \lambda_{\mathrm{GRW}}$), one can prove that the reduction rate for the center of mass of an N-particle system is *amplified* by a factor N with respect to that of a single constituent [7]. In other words, $\lambda_{\text{macro}} = N\lambda_{\text{GRW}}$. For a macroscopic object with an Avogadro's number ($\mathcal{N}_A$) of constituents, the collapse occurs fast enough compared to the human perception time ($\sim 10^{-3}$s). In fact, according to the amplification mechanism

$$\lambda_{\text{macro}} = \mathcal{N}_A \, \lambda_{\text{GRW}} = 10^7 \text{ s}^{-1}, \tag{8}$$

which means that the wave function of a macroscopic object collapses almost instantaneously and superpositions are immediately suppressed.

Another important feature of the GRW model is that the collapse mechanism pumps energy into the system. This property can be used to set bounds on the value of λ_{GRW} coming from cosmological data analysis [11]. This energy increase is very small, e.g., for a particle with the mass $m = 10^{-23}$ g, one has:

$$\frac{\delta E}{t} \simeq 10^{-25}\text{eV} \cdot \text{s}^{-1}\,; \tag{9}$$

which means that it takes 10^{10} years (the age of Universe) to have an increase of 10^{-8} eV. As one can see, according to the GRW model the kinetic energy of a free particle increases monotonically. It is as if the noise field had an infinite temperature. In Sect. 2.3, we will discuss how this behavior can be modified in such way that the energy approaches a finite asymptotic value, by introducing dissipative effects to the dynamics.

2.2 The CSL Model

The continuous spontaneous localization (CSL) model, proposed in 1990 by Ghirardi et al. [9], generalizes the GRW model in two ways: it deals with continuous collapse processes in time, and it allows to describe the dynamics of identical particles (second quantization formalism). The CSL dynamics is given by the following stochastic differential equation in the Itô form [6]:

$$\begin{aligned} d\psi_t = \Bigg[-\frac{i}{\hbar}\hat{H}\,dt + \sqrt{\gamma}\int d\mathbf{x}\,\Big(\hat{\mathcal{N}}(\mathbf{x}) - \langle\hat{\mathcal{N}}(\mathbf{x})\rangle_t\Big)\,d\xi(\mathbf{x},t) \\ -\frac{\gamma}{2}\int d\mathbf{x}\,\Big(\hat{\mathcal{N}}(\mathbf{x}) - \langle\hat{\mathcal{N}}(\mathbf{x})\rangle_t\Big)^2 dt\Bigg]\psi_t\,, \end{aligned} \tag{10}$$

where $\gamma > 0$ describes the strength of the collapse mechanism, $\langle\hat{\mathcal{N}}(\mathbf{x})\rangle_t = \langle\psi_t|\hat{\mathcal{N}}(\mathbf{x})|\psi_t\rangle$ is the standard quantum average of the operator $\hat{\mathcal{N}}(\mathbf{x})$, which is defined in terms of the creation $a^{\dagger}(\mathbf{x}, s)$ and annihilation $a(\mathbf{x}, s)$ operators of a particle at point $\mathbf{x}$ with spin s as follows:

$$\hat{\mathcal{N}}(\mathbf{x}) = \sum_s \left(\frac{\alpha}{2\pi}\right)^{3/2} \int d^3y\, e^{-(\alpha/2)(\mathbf{x}-\mathbf{y})^2} \hat{a}^{\dagger}(\mathbf{y}, s)\hat{a}(\mathbf{y}, s). \tag{11}$$

with $r_c = 1/\sqrt{\alpha}$ where $r_C = 10^{-7}$ m, as in GRW model. Finally, $\xi(\mathbf{x}, t)$ is a family of independent standard Wiener processes with:

$$\mathbb{E}[d\xi(\mathbf{x}, t)] = 0, \qquad \mathbb{E}[d\xi(\mathbf{x}, t)d\xi(\mathbf{y}, t)] = \delta^{(3)}(\mathbf{x} - \mathbf{y})dt. \tag{12}$$

One can also consider a mass-proportional model, where the strength of the collapse is given by $(m/m_0)^2\gamma$, and $m_0 = 1$ amu. After averaging over the noise, the CSL master equation for the density matrix is given by:

$$\begin{aligned}\frac{\partial}{\partial t}\langle \mathbf{x}, s'|\hat{\rho}(t)|\mathbf{x}', s''\rangle = &-\frac{i}{\hbar}\langle \mathbf{x}, s'|[\hat{H}, \hat{\rho}(t)]|\mathbf{x}', s''\rangle \\ &+\frac{\gamma}{2}\sum_{i,j}\left[2D(\mathbf{x}_i - \mathbf{x}'_j) - D(\mathbf{x}_i - \mathbf{x}_j) - D(\mathbf{x}'_i - \mathbf{x}'_j)\right] \\ &\times\langle \mathbf{x}, s'|\hat{\rho}(t)|\mathbf{x}', s''\rangle,\end{aligned} \tag{13}$$

with

$$D(\mathbf{x} - \mathbf{y}) = \left(\frac{\alpha}{4\pi}\right)^{\frac{3}{2}} e^{-(\alpha/4)(\mathbf{x}-\mathbf{y})^2}, \tag{14}$$

and the vectors $|\mathbf{x}, s\rangle$ are defined as follows:

$$|\mathbf{x}, s\rangle = \hat{a}^{\dagger}(\mathbf{x}_1, s_1)\ldots\hat{a}^{\dagger}(\mathbf{x}_n, s_n)|0\rangle. \tag{15}$$

For one particle, the CSL master equation becomes:

$$\frac{\partial}{\partial t}\langle \mathbf{x}|\hat{\rho}(t)|\mathbf{x}'\rangle = -\frac{i}{\hbar}\langle \mathbf{x}|[\hat{H}, \hat{\rho}(t)]|\mathbf{x}'\rangle - \gamma\left(\frac{\alpha}{4\pi}\right)^{\frac{3}{2}}\left[1 - e^{-(\alpha/4)(\mathbf{x}-\mathbf{x}')^2}\right]\langle \mathbf{x}|\hat{\rho}(t)|\mathbf{x}'\rangle. \tag{16}$$

Comparing the above equation with the GRW master equation, one finds:

$$\lambda_{\text{GRW}} = \gamma\left(\frac{\alpha}{4\pi}\right)^{\frac{3}{2}}. \tag{17}$$

It is worth mentioning that also in this model the mean energy is not conserved in the course of time [9]. The other important feature of the CSL model, as in the GRW model, is the *amplification mechanism*, which is responsible for non-observation of macro-superpositions. A handy formula for the collapse rate in the CSL model is given by Adler [11]:

$$\Gamma = n^2 N \lambda_{\mathrm{GRW}}, \tag{18}$$

with n the number of particles within the distance r_{C}, and N the number of such clusters.

2.3 The QMUPL Model

Among collapse models, the so-called QMUPL (Quantum Mechanics with Universal Position Localization) model is particularly interesting, being an excellent compromise between mathematical simplicity and physical adequacy. This model was first introduced by Diosi [12, 13] and subsequently studied in [14–22], both from the mathematical and the physical point of view. It is particularly relevant because it is the simplest model describing the evolution of the wave function of a system of N distinguishable particles, subject to a spontaneous collapse in space. The QMUPL wave function dynamics is defined as follows:

$$d\psi_t = \left[-\frac{i}{\hbar}\hat{H}\,dt + \sum_{n=1}^{N} \sqrt{\lambda_n}\, (\hat{\mathbf{q}}_n - \langle \hat{\mathbf{q}}_n \rangle_t) \cdot d\mathbf{W}_n(t) - \frac{1}{2} \sum_{n=1}^{N} \lambda_n\, (\hat{\mathbf{q}}_n - \langle \hat{\mathbf{q}}_n \rangle_t)^2\, dt \right] \psi_t, \tag{19}$$

where $\hat{H}$ is the standard quantum Hamiltonian, $\mathbf{q}_n$ is the position of the n-th particle, $\mathbf{W}_n(t)$ are N independent three-dimensional Wiener processes, and the parameters λ_n are N positive coupling constants, given by [20]:

$$\lambda_n = \frac{m_n}{m_0}\lambda_0, \tag{20}$$

where m_n is the mass of the n-th particle, and λ_0 determines the strength of the collapse mechanism. Note that in QMUPL model, the noise field couples to the position of the system, one for each constituent of the system. This choice is simpler than the ones made in the GRW and CSL models, where the noise field couples to more complex functions.

After averaging over the noise, the dynamics of the density operator becomes:

$$\frac{d}{dt}\langle \mathbf{Q}|\hat{\rho}(t)|\mathbf{Q}'\rangle = -\frac{i}{\hbar}\langle \mathbf{Q}| \left[\hat{H}, \hat{\rho}(t)\right] |\mathbf{Q}'\rangle - \frac{1}{2}\sum_n \lambda_n (\mathbf{q}_n - \mathbf{q}'_n)^2 \langle \mathbf{Q}|\hat{\rho}(t)|\mathbf{Q}'\rangle. \tag{21}$$

with $\mathbf{Q} \equiv \{\mathbf{q}_n\}$. For a one-particle system, for the collapse rate one gets:

$$\Gamma(\mathbf{x}) = \lambda_0\, \mathbf{x}^2/2. \tag{22}$$

Comparing this expression with Eq. (7) for the GRW and CSL model (we remind that the two models predict the same evolution for the statistical operator, in the case of a single particle), we see that Eq. (22) represents the small-distance ($|\mathbf{x}| \ll 1/\sqrt{\alpha}$) Taylor expansion of Eq. (7). Therefore we can identify:

$$\lambda_0 = \frac{\alpha \lambda_{\text{GRW}}}{2} = \frac{\alpha^{5/2}\gamma}{16\pi^{3/2}}, \tag{23}$$

where we have set $m = m_0$. Accordingly, we can state that the QMUPL model is an approximation, at the statistical level, of the GRW and CSL models for small superposition distances, since the collapse rate for the QMUPL model and the ones for the GRW and CSL models coincide for $|\mathbf{x}| \ll r_C$. In fact, recently it has been proven in mathematical detail that, by taking the limits

$$\lambda_{\text{GRW}} \to \infty, \qquad \alpha \to 0, \qquad \lambda_{\text{GRW}} \cdot \alpha = \text{const.} \tag{24}$$

in the GRW model, one recovers the QMUPL model [23].

Once again, a common feature that the QMUPL model shares with the other collapse models is the energy increase. Here, it is appropriate to mention that the energy increase is not an unavoidable consequence of the collapse process. A possible resolution is to include dissipative terms. This has been done with the QMUPL model in [25]. In this dissipative QMUPL model, the energy of the system approaches asymptotically a finite value, related to the temperature of the noise. If the initial energy of the system is smaller, then it increases in time; if initially larger, then it decreases. In addition, another important motivation to build such a dissipative model is that when the noise has a finite temperature, one can think of identifying it with a physical field of Nature.

We should also mention that we have only considered the case of models, where the collapse is driven by a Markovian noise. A Markovian noise, whose correlation function is given by a Dirac delta cannot truly describe a physical random field. In order to obtain more realistic models, the available models should be extended to include non-Markovian noises. This issue has been investigated in [26, 27]. Moreover, due to mathematical simplicity of QMUPL model, such extensions are easier for this model [28, 29].

2.4 Relativistic Collapse Models

The collapse models analyzed so far are non-relativistic models. The main problem for a relativistic extension is that in the non-relativistic models the collapse of the wave function is *instantaneous* (GRW model) or anyhow it occurs in a nonlocal way (in the CSL and QMUPL model). This nonlocality is necessary, in order for the models to violate Bell inequalities (a violation, which has been confirmed experimentally). In particular, if one assumes that the collapse occurs instantaneously at time $t = 0$ in a reference frame, in any other frame the space-like surface $t = 0$ is not the same as the one identified by $\tilde{t} = 0$ (where $\tilde{t}$ is the time in the other reference frame). This implies that the picture of the collapse is obviously not covariant, therefore the collapse cannot be instantaneous in any reference frame.

In order to construct a consistent relativistic collapse model, one has to satisfy other constraints, which can be briefly summarized as follows:

- The model has to be non-linear and stochastic, in order to solve the measurement problem of quantum mechanics.
- The model must be non-local, in order to reproduce quantum correlations (EPR-like). This implies that the collapse of the wave function has to be either instantaneous or superluminal.
- The model must not allow for faster-than-light signaling: non-local features of the model cannot be exploited to send signals at superluminal speed.
- The model has to be stochastically invariant under Lorentz transformation [30].

Several relativistic models have been proposed so far, none of which can be considered completely satisfactory. However, there was some progress in this regard. The first attempt consisted in giving the relativistic generalization of the CSL model. In order to make such a model relativistically invariant, Eq. (10) was replaced with the following Tomonaga–Schwinger type equation (written in the Stratonovich formalism):

$$\frac{\delta\psi(\sigma)}{\delta\sigma(x)} = \left[-\frac{i}{\hbar}\,\mathcal{H}(x) + \sqrt{\gamma}\,(\mathcal{L}(x) - \langle\mathcal{L}(x)\rangle)\,V(x) - \gamma\,(\mathcal{L}(x) - \langle\mathcal{L}(x)\rangle)^2\right]\psi(\sigma). \tag{25}$$

where the 3rd term on the right side is the collapse term. Here σ denotes an arbitrary space-like hyper-surface of space time, on which the wave function is defined, and x is now an event on the hyper-surface. The operators $\mathcal{H}(x)$ and $\mathcal{L}(x)$ are respectively the Hamiltonian density and the local density of the noise fields onto which one decides to localize the wave function, while $V(x)$ denotes a stochastic process on space time. The mean of $V(x)$ is equal to zero, while its correlation function, in order to satisfy the invariance requirement previously stated, must be a Lorentz scalar. This feature is actually the one which raises the main difficulties.

The simplest choice for a Lorentz invariant correlation function is the Dirac delta:

$$\mathbb{E}[V(x)V(y)] = \delta^{(4)}(x - y). \tag{26}$$

However, such a correlation function causes an infinite production of energy (per unit time and unit volume), therefore it is not physically acceptable. The reason is that the fields are locally coupled to the noise, and since this is assumed to be white, too many particles are created out of the vacuum. The natural solution to this problem would be to consider a weaker noise (a non-white noise). However, this is not an easy goal to reach, for the following reason. The third term of Eq. (25) , $\gamma\,(\mathcal{L}(x) - \langle\mathcal{L}(x)\rangle)^2$, guarantees that the correct quantum probabilities are reproduced by the collapse mechanism, and its form is strictly related to the white nature of the noise field. Changing the noise implies that such a term has to be replaced with a non-local function of the fields. In this way one destroys the local construction of the Tomonaga–Schwinger equation, and it is very likely that the model turns out to be inconsistent.

Other attempts have been made to obtain a consistent collapse models, among which we mention the works of P. Pearle involving a tachionic noise [31], and the relativistic GRW model by Tumulka [32], which so far works only for non-interacting particles. An interesting toy model is the one proposed by Ghirardi [52]. They suggested that the collapse process occurs instantaneously along all space-like hypersurfaces crossing the center of the jump process. Such a model reproduces the predictions of quantum mechanics, it is Lorentz invariant, it does not allow for faster-than-light signaling and it does not lead to any apparent contradiction. Unluckily, the dynamics for the collapse process is missing, which is the reason why this cannot be considered a physical model. However, such a model suggests that there is no reason that forbids relativistic collapse models to be a viable program.

3 Implications of Collapse Models

Collapse models lead to many new physical situations, some of which can be exploited to discriminate experimentally between them and the standard quantum theory. This novel physical implications have also been used to resolve some problems in other branches of science, e.g. in cosmology [45, 53] or thermodynamics (e.g., the arrow of time) [37]. In this section, we will discuss in detail the experimental and the cosmological implications of collapse models.

3.1 Experimental Implications of Collapse Models

One of the advantages of collapse models is that, in principle, they can be tested experimentally. The effects of the noise field can be observed in many different ways, e.g. the suppression of spatial superpositions, or the spontaneous emission of radiation from matter. For a review on the phenomenology of collapse models, see [24]. The great challenge is to design an experiment in the suitable range (e.g., mass of the system, delocalization distance, etc.) and in the proper experimental conditions where the effects of the noise field can be distinguished from the effects of the environment, because an external environment can also induce a collapse-type effect [54, 55].

We review the most promising methods proposed so far, as experimental tests of collapse models. Among them, the possible failure of the superposition principle for macroscopic systems, and the process of spontaneous radiation emission from matter have been received particular attention in the literature. In the following, we will elaborate these two effects in more detail.

3.1.1 Radiation Emission

According to quantum mechanics, systems in the stationary ground state (e.g., electrons in their atomic ground state) are stable and do not emit radiation. However, in collapse models the noise field constantly interacts with the system, and thus can force the system to release its energy by the emission of radiation. In general, the rate of photon emission, $\Gamma(p)$, as a function of the photons' outgoing momentum $p = |\mathbf{p}|$ is given by:

$$\Gamma(p) = \sum_s \int d\Omega_{\mathbf{p}} \frac{d}{dt} \left\langle \Psi \left| \hat{a}^{\dagger}_{\mathbf{p},s}(t)\, \hat{a}_{\mathbf{p},s}(t) \right| \Psi \right\rangle, \tag{27}$$

where $|\Psi\rangle \equiv |\psi\rangle \otimes |0\rangle$ is the tensor product of the generic initial state $|\psi\rangle$ of the system and the vacuum state $|0\rangle$ of the electromagnetic field; $\hat{a}^{\dagger}_{\mathbf{p},s}$ is the creation operator of the emitted photon with momentum $\mathbf{p}$ and polarization s, and $\Omega_{\mathbf{p}}$ is the solid angle.

So far, the emission rate $\Gamma(p)$ has been calculated only for three simple systems: a free (charged) particle [56–58], a particle bounded by an harmonic potential [56], and the hydrogen atom [57]. Two different dynamical reduction models have been used for the calculations: the QMUPL model [56] and the CSL model [57]. The main advantage of the QMUPL model over the CSL model is that one can obtain exact analytical expressions for the free particle and the particle bounded by an harmonic potential [56]. On the other hand, the CSL model is physically more realistic, but the relevant computation can only be done with perturbative methods [57, 58].

For example, using the CSL or the QMUPL model, the emission rate of the free particle is:

$$\Gamma(p) = \frac{e^2 \hbar\, \lambda_{\mathrm{GRW}}}{2\pi^2 \varepsilon_0 c^3 m_0^2 r_C^2\, p} \sim \frac{10^{-32}}{pc} \mathrm{Day}^{-1}. \tag{28}$$

This value is too small to be observed. This issue can be in principle solved if one considers the emission from a large number of particles (e.g., an Avogadro number of particles) [24]. However, in this case, the environmental interactions (which may also induce radiation emission) increase, smearing out the effects of the noise field. Thus, one has to design the experiment in such a way that the collapse effects are, at least, comparable with the environmental effects. In this regard, a very good choice for the system is Germanium (Ge) in solid phase, which can be prepared with the least possible impurities. Moreover, the experiment should be performed in a very protected environment—like in underground laboratory—in order to minimize the source of noise. The emission rate from Ge, as predicted by collapse models, has not been computed yet. By treating the four valence electrons of the Ge atom as free particles, a first rough estimation of the emission rate was obtained [58]. However, to design an experiment, more precise estimates are needed.

3.1.2 Neutrinos and Kaons Oscillations

Particles with definite flavors (e.g., with different strangeness) do not have definite masses. As a consequence, flavor eigenstates are given by a superposition of mass eigenstates. Since the mass eigenstates have different time evolutions, a particle with flavor f_1 may be found in flavor f_2 in the course of time. This is the basic idea of the particle oscillation [59]. Probabilities given by standard quantum mechanics and by collapse models to this phenomenon are different. It has been suggested that this difference can be detected in an experiments, e.g., with neutrinos [60].

According to the standard quantum theory, the probability that a particle with flavor f_1 ends up in the flavor f_2 is given by:

$$P_{f_1 \to f_2}(t) \propto 1 - \cos\left(\frac{t}{\hbar}\left[E_1(\mathbf{p}_i) - E_2(\mathbf{p}_i)\right]\right), \tag{29}$$

where $E_j(\mathbf{p}_i) = \sqrt{\mathbf{p}_i^2 c^2 + m_j c^4}$, $\mathbf{p}_i$ is the initial momentum, and $j = 1, 2$ [59]. Accordingly, the probability that a state with flavor f_1 ends up in a state with flavor f_2 shows an oscillatory behavior in time, with a frequency proportional to the difference of energies, as given by Eq. (29).

According to the CSL model, the time evolution of the mass eigenstates is different. One can unfold such an evolution by using perturbation theory in the interaction picture, where the standard part of the dynamics is taken as the unperturbed Hamiltonian, and the noise term as the perturbation. This is a very good approximation because, even for a particle with large masses, the coupling with the noise is very small. Accordingly, the modified expression for the transition probability $P_{f_1 \mapsto f_2}$, as given by the mass proportional CSL model, is given by Bassi et al. [61]:

$$P_{f_1 \mapsto f_2} \propto 1 - e^{-t\zeta} \cos\left(\frac{t}{\hbar}\left[E_1(\mathbf{p}_i) - E_2(\mathbf{p}_i)\right]\right), \tag{30}$$

with $\zeta = \frac{\gamma}{16\pi^{3/2} r_C^3 m_0^2 c^4}\left[\frac{m_1^2 c^4}{E_1(\mathbf{p}_i)} - \frac{m_2^2 c^4}{E_2(\mathbf{p}_i)}\right]^2$. It is clear from the above equation that the noise field suppresses the flavor oscillation. To give an estimate, for neutrinos with initial momentum $|\mathbf{p}_i|c = 1$ MeV, the damping term is about $\zeta \sim 10^{-55}$ s^{-1}, which is too small to be observed with current technology.

3.1.3 Toward Macroscopic Delocalized States

Recently there has been great interest in creating quantum delocalized states of many-particle systems, with the aim of testing the limits of validity of quantum mechanics (i.e., the superposition principle) towards the macroscopic scale [62–70]. This is leading to cutting-edge experiments involving spatial superpositions of *center-of-mass* of larger and larger systems, e.g. nano-scale magnets [62], superconducting rings [63], ensemble of photons [64] or atoms [65], macromolecules [66, 67] and optomechanical systems [69]. In particular, during the past decade, there has been great progress in interference experiments with macromolecules [66, 67], and optomechanical systems [69]. It seems that they will open up the possibility of testing the predictions of collapse models.

Experiments with Macro-Molecules

The basic idea of matter-wave interferometry is to measure the interference between the beams of matter diffracted from a double-slit, a grating or a beam-splitter. The interference can be studied in the far-field (Fraunhofer regime), or the near-field (Fresnel regime). According to collapse models, the noise field destroys the relevant quantum coherence of the beams, so the visibility of the interference pattern will be different from the visibility predicted by the standard quantum theory. At a critical scaling (e.g., mass scale and delocalization scale) this difference becomes observable; in this way, one can test collapse models.

The far-field matter-wave interference has been implemented with electrons, neutrons, atoms, diatomic molecules (e.g., Na_2, K_2, and I_2), cold-helium clusters, and hot buckyball molecules (e.g., C_{60} or C_{70}) [67]. Due to the mass and the delocalization scales of these experiments, the bounds obtained on collapse parameters (λ, r_c) are rather weak, i.e., the collapse effects are not observable here. Because of limitations in operating in the far-field regime (in particular, because of the short de Broglie wavelength and the limited coherence of beam sources [67]), the mass and the delocalization scales cannot be shifted to the proper range where the collapse effects become important.

However, the recently developed near-field matter-wave diffraction techniques can enable the extension of the mass and the delocalization scales to the favorable range [67, 68]. With these novel techniques, it has been possible to observe quantum interference of macromolecules with mass up to 6,910 amu, maximum size of 60 Å, and delocalization distance of 266 nm [66, 67]. Nevertheless, to observe the collapse effects, the mass scale should be extended to the scales of the order of $\sim 10^8$ amu [68]. Recently, a near-field matter-wave interferometry with clusters of the mass in the range 10^6 to 10^8 amu, prepared in the spatial superposition over the distance of 80 nm, has been proposed [68], where it has been argued that the environmental decoherence effects can be manipulated in a way that they can be distinguished from collapse effects. In this experiment, the superposition principle should fail, as prescribed by collapse models. The corresponding reduction of the visibility of the interference pattern is given by:

$$\frac{\mathcal{V}_{\mathrm{CSL}}}{\mathcal{V}_{\mathrm{SQM}}} = \exp\left\{-2\lambda_0 T_0 N \left(\frac{m}{m_0}\right)^3 \left[1 - \frac{\sqrt{\pi} r_C}{Nd}\mathrm{erf}\left(\frac{Nd}{2r_C}\right)\right]\right\}, \tag{31}$$

with $\mathcal{V}_{\mathrm{CSL}}$ the CSL visibility of the interference pattern, $\mathcal{V}_{\mathrm{SQM}}$ the quantum mechanical visibility, d the grating period, $T_0 = m_0 d^2/h$ the Talbot time, $m_0 = 1$ amu, m the mass of the interfering particles, and N the Talbot order of grating [68]. For masses above 10^6 amu, the collapse effect should be observed according to the current ranges of collapse parameters and the available technologies.

Experiments with Opto-Mechanical Systems

There has been seminal progress toward preparing the quantum superposition of nano- and micro-scales opto-mechanical resonators (containing billions of atoms) over the distance of the order of the zero-point motion, i.e., $x_0 = \sqrt{\hbar/2m\omega}$, with m the mass and ω the frequency of harmonic potential (see [70], and the references therein). However, opto-mechanical devices have thus far not reached the quantum regime due to technical difficulties [71]. Meanwhile, the proposed experiments (e.g., see [69]) are in the range where, according to collapse models, the superposition principle is still maintained.

Recently, by merging the techniques of quantum opto-mechanics and insights from matter-wave interferometry, an optomechanical double-slit experiment has been proposed [70]. In this experiment, the nano-sized objects (with the mass $m \sim 10^7$ amu, and the size $D \sim 40$ nm) are prepared in a spatial superposition over the distance of the order of their size ($d \sim D$). It has been argued that in this scaling, collapse models predict the failure of the superposition principle. As usual, one has also to cope with decoherence. The main sources of decoherence are the scattering from blackbody radiation and scattering from environmental particles.

This experiment consists of four steps: (1) preparation of a mechanical resonator in its ground state, i.e., a Gaussian wave of width x_0, (2) the free expansion of the Gaussian wave to the width σ (where $\sigma \gg x_0$), (3) preparation of a superposition of two Gaussian waves of the width σ_d separated by the distance d (where $\sigma^2 \gg 2d\sigma_d$), and (4) observation of the interference by measuring the position after further free evolution. However, there are still some restrictions in this experiment (in particular, the implementation of step (3)) that should be overcome to make this proposal feasible.

3.2 Cosmological Implications

The standard Λ-Cold Dark Matter (ΛCDM) model, i.e. the standard cosmological model including cold dark matter and dark energy, has become in recent years very praised among cosmologists [72] due to the major agreement among its predictions and observations [73]. At the same time, the interpretation of the quantum theory behind the predictions of ΛCDM is often considered as the philosophical speculation with no observable consequences, and thus dismissed. However, as we will argue in the following, this attitude is not completely correct.

One of the cornerstones of the ΛCDM model is the inflationary paradigm [74]. The modern standard inflationary scenario states [75] that all the cosmic structures (namely galaxies, stars, planets and eventually human beings) are originated from quantum fluctuations of the vacuum state of the inflation field. The general setting is that the perturbations evolved in a Friedmann–Robertson–Walker background space–time with a nearly exponential expansion. Once the physical wavelength associated with these fluctuations becomes larger than the Hubble radius, they are assumed to be classical density perturbations. When the universe becomes matter dominated, primeval density inhomogeneities are amplified by gravity and grow into the structure we see today. The photon density perturbations left a particular imprint in the Cosmic Microwave Background (CMB), which is associated with fluctuations in the photons temperature. The signature left by the photons in the CMB is one of the most important predictions of the inflationary paradigm and indeed is confirmed by recent observational data [73].

However, the inflationary model is not able to point out the physical mechanism responsible for generating the inhomogeneity and anisotropy of our universe, if one starts from an exactly symmetric state associated with the early universe [45, 53]. In other words, it is not clear how out of an initial condition which is homogeneous and isotropic both in the background space–time and in the quantum state that describes the "fluctuations," and based on a dynamics that supposedly preserves those symmetries, one ends with a non-homogeneous and non-isotropic state characterizing the late actual universe. This issue has been recently acknowledged by some cosmologists (sometimes this problem is referred to as the quantum-to-classical transition of the primordial fluctuations) [76, 77], though the majority of the researchers in the field sustain that there is no problem at all (e.g., refer to [78]).

In a recent series of works [46, 53, 79, 80], the aforementioned problem has been analyzed in detail. In order to overcome this fundamental issue, the authors conclude that one has to invoke a collapse of the wave function. This collapse breaks the original symmetries of the quantum state of the inflaton field. The inhomogeneities of the field (described by the post-collapse state) are related to the (classical) perturbations in the metric by Einstein's semiclassical equations. The result of the evolution of such perturbations is related to the actual anisotropies and inhomogeneities observed in the CMB. It is evident that the mechanism (that breaks the symmetries) should be a physical process independent of external entities (i.e., "observers," "measurement devices," or "an environment") as by definition the universe contains everything.

All the results, achieved so far by using the collapse hypothesis within the inflationary scenario, have been obtained by characterizing the expectation values of the quantum quantities involved [81–84]. However, no specific collapse model has been used. Consequently, the next step is to consider a physical collapse mechanism; this is the point where collapse models enter into the picture. As we explained before, the reduction of the wave function is due to the interaction of the system with a noise field. In particular, in the inflationary scenario, one can consider that the collapse of the wave function of each mode of the inflationary field is caused by the interaction of such modes with the noise field. Hence, the advantage of using collapse models in the inflationary regime is twofold:

1. One can explain from first principles why we observe the inhomogeneities and anisotropies in the universe.
2. One can use the observational data (e.g., the anisotropies of the temperature of the CMB) as a test for collapse models, and/or to put some bounds on the collapse parameters.

Before discussing in more detail the idea of implementing collapse models in the cosmological scenario, we would like to address an issue related to the nature of the cosmological noise field, which is still debated. Usually, in collapse models, the noise field is considered as a classical stochastic field which fills the whole space–time. This field, as all other physical fields in nature, could have a quantum origin, although not a standard one with an hermitian coupling, otherwise the underlying dynamics would be linear, and there would be no collapse of the wave function. It should also interact with the matter degrees of freedom, and at least at the fundamental level, be related with the gravitational degrees of freedom. To delve into the previous idea, let us consider the following situation. Consider a superposition of a physical object being at two different locations in space. If one takes into account the gravity field produced by the object, then we end up with a superposition of two different space–times, each one associated with a different position of the object. However, as discussed by Penrose [85], the superposition of distinct (space–time) manifolds clashes with the standard superposition principle as given by the Schrödinger equation, and a reduction of the wave function must occur. One can speculate that the noise field interacts with the gravitational degrees of freedom, causing the suppression of superpositions of the geometry. Thus, one could conjecture that the classical noise field arises as a phenomenological modification of the Schrödinger equation. However, the relation between the gravitational degrees of freedom and the noise field should be treated, at least in principle, at the quantum level. This idea is similar to what proposed by Penrose [33, 34] and Diosi [35, 36], independently. They argue that the gravity plays a central role in the collapse mechanism, and thus the unification of quantum mechanics and the theory of gravitation would likely involve modifications in both theories, rather than only the theory of gravity, as is more frequently assumed.

In the absence of a full quantum theory of gravitation, the setting that provides the clearest picture of the cosmological implications of collapse models is that of semiclassical general relativity [86]. Such a description has to be taken just as an effective one and of limited range of applicability. The semiclassical picture of gravitation in interaction with quantum fields allows for a quantum treatment of matter fields and a classical treatment of gravitation. We will assume that the fundamental degrees of freedom for the gravitational interaction are not those characterizing the standard metric variables, but some other variables that are indirectly connected to them. Thus, an approximate description of gravity and quantum matter fields is given by semiclassical Einstein's equation:

$$G_{ab} \equiv R_{ab} - (1/2)g_{ab}R = 8\pi G\langle \hat{T}_{ab}\rangle, \tag{32}$$

where the left-hand side displays the metric description of gravity as a classical quantity, while the right-hand side displays the other matter fields (including the inflaton), in the standard quantum field theory fashion. It is clear that this approximated description would break down when the collapse takes place. This breakdown is due to the fact that divergence of the left-hand side of Eq. (32), i.e $\nabla_a G^{ab}$, is zero, while the divergence of the right-hand side is not. The reason is that $\nabla_a\langle \hat{T}^{ab}\rangle$ has discontinuities associated with the change of the quantum state at the time of collapse. This breakdown can be formally represented by adding the term Q_{ab} in the semiclassical Einstein's equation which is supposed to become nonzero only during the collapse:

$$R_{ab} - (1/2)g_{ab}R + Q_{ab} = 8\pi G\langle \hat{T}_{ab}\rangle. \tag{33}$$

In this setting, we can describe the evolution of the state of the universe. The initial state of universe is given by an homogeneous and isotropic state for the gravitational and matter degrees of freedom. At some point the quantum state of the matter field is subject to a collapse, which we choose to be effectively described by collapse models. After the collapse, the state of the matters fields will not share the symmetries of the initial state, and the gravitational degrees of freedom are assumed to be, once more, accurately described by Einstein's semiclassical equation. However, since $\langle \hat{T}_{ab}\rangle$ for the new state does not need to have the symmetries of the pre-collapse state, we are led to a geometry that generically will be no longer homogeneous and isotropic. The anisotropies and inhomogeneities of the space–time can be related to the anisotropies of the temperature of the CMB. As a consequence, the CMB provides observational data which can be compared with collapse models predictions. Accordingly, one can test collapse models (or may impose new bounds on the collapse parameters) by using cosmological observations.

Finally, we end with a schematic view regarding the *problem of time* in quantum gravity and collapse models, as we described it in the introduction. In the theories that follow the canonical quantization approach, the canonical variables describe the geometry of a 3-spatial hypersurface, and characterize the embedding of this 3-surface in a four-dimensional space–time. Nevertheless, a notion of time, or its general relativistic counterpart, a time function usually specified by the lapse function and shift vector, is no longer present in the theory, simply because the Hamiltonian vanishes when acting on the physical states allowed by the theory (i.e., those satisfying the momentum and Hamiltonian constraints).

The problem is to recover a space–time description of our universe, which clearly is a fundamental element if one expects to successfully connect the predictions of the theory with observations. An approach to address the problem [45–47] is to consider, simultaneously with the geometry, some matter fields canonically described by a set of ordered pairs of variables $\{(\varphi_1, \pi_1), \ldots, (\varphi_n, \pi_n)\}$, and to define an appropriate variable $T(\varphi_i, \pi_i, \mathcal{G}, \Pi)$ that acts as a physical clock in this "matter gravity theory," where we denote the canonical variables by $(\mathcal{G}, \Pi)$. In other words, one starts with a wave function for the configuration variables of the theory $\Phi(\varphi_1, \ldots, \varphi_n, \mathcal{G})$, which satisfy the Hamiltonian and momentum constraints $\hat{H}_\mu \Phi(\varphi_1, \ldots, \varphi_n, \mathcal{G}) = 0$ with $\mu = 0, 1, 2, 3, \ldots$. The next step is to obtain an effective wave function Ψ for the remaining variables by projecting Φ into the subspace where $\hat{T}(\varphi_i, \pi_i, \mathcal{G}, \Pi)$ acquires some values in a specific range. If one denotes with $P_{T,[t,t+\delta t]}$ the projection operator onto the subspace corresponding to the region between t and $t + \delta t$ of the spectrum of the operator $\hat{T}$, then one attempts to recover a certain evolution equation for $\Psi(t)$ by analyzing the dependence of $\Psi(t) \equiv P_{T,[t,t+\delta t]}\Phi$ on the parameter t. This evolution is expected to be described by a Schrödinger-type equation which involves small modifications of the standard Schrödinger equation. After obtaining a wave function related to the spectrum of the operator $\hat{T}$, one can use it to compute the expectation values of the 3-dimensional geometrical operators for the wave function $\Psi(t)$, i.e. to obtain quantities like $\langle \Psi(t)|\hat{\mathcal{G}}|\Psi(t)\rangle$, and $\langle \Psi(t)|\hat{\Pi}|\Psi(t)\rangle$. Such a set of quantities can be seen as describing the "average" space–time in the $3 + 1$ decomposition. In other words, the above procedure constructs a space–time, where the slicing corresponds to the hypersurfaces on which the geometrical quantities are given by the expectations of the projected wave functions $\Psi(t)$. Consequently one is able to specify the space–time and its slicing using the standard lapse and shift functions.

Evidently, the particular realization of this procedure depends on the situation and the specific theory of matter fields which one is considering. However, as noted in [45–47], the point is that the standard Schrödinger equation emerges only as an effective description, with a confined range of validity. Therefore, it is not unexpected to obtain modifications that could lead to departures from a unitary evolution, like those proposed by collapse models. It thus seems natural to conjecture that these can be the grounds where a variation of the evolution given by Schrödinger equation, involving something akin to a reduction of the wave function, might find its ultimately explanation.

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Time and the Algebraic Theory of Moments

B.J. Hiley

Abstract We introduce the notion of an extended moment in time, the duron. This is a region of temporal ambiguity which arises naturally in the nature of process which we take to be basic. We introduce an algebra of process and show how it is related to, but different from, the monoidal category introduced by Abramsky and Coecke. By considering the limit as the duration of the moment approaches the infinitesimal, we obtain a pair of dynamical equations, one expressed in terms of a commutator and the other which is expressed in terms of an anti-commutator. These two coupled real equations are equivalent to the Schrödinger equation and its dual.

We then construct a bi-algebra, which allows us to make contact with the thermal quantum field theory introduced by Umezawa. This allows us to link quantum mechanics with thermodynamics. This approach leads to two types of time, one is Schrödinger time, the other is an irreversible time that can be associated with a movement between inequivalent vacuum states. Finally we discuss the relation between our process algebra and the thermodynamic origin of time.

1 Introduction

In this paper we address the question of time in quantum mechanics. The first and more commonly chosen option is to treat time as an external parameter as one does in the Schrödinger and Heisenberg equations of motion. In the relativistic domain time is treated as the fourth component of a four-vector. In non-relativistic quantum mechanics, the three space components are regarded as operators, why keep time as a parameter? Surely it should be treated as an operator. However the attempt to treat time as an operator is regarded as a failure for the reasons discussed by Pauli [57] in his seminal paper on this topic. As a caveat, we should point out that recently there have been two papers [29, 30] that have challenged this conclusion.

Rather than following this line of argument I want to make a radical departure and consider both space and time as arising from a deeper level in which process is taken

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as fundamental [3]. In earlier work along these lines Bohm et al. [4], and Hiley [37] proposed that this underlying process be referred to as the *holomovement*. In contrast to the present world view which has it roots in Democritus with its "atoms" having a set of preassigned properties, we want to explore a world envisaged by Heraclitus, where all is change, all is flux. This will lead us, in the first approximation, to introduce two types of time, an unfolding (Schrödinger) time, together with a moment or duron, a "now," which allows us to consider the precise time to be ambiguous within some interval $\Delta T = t_2 - t_1$. We will denote this ambiguous moment by $[T_1, T_2]$ where $T_1(T_2)$ are some suitable elements of an algebra that are functions of $t_1(t_2)$ respectively [39].

Using these ideas we will construct an algebra of moments, which we detail in Sect. 2. In such a structure we cannot attach a meaning to an instant or a sharp "point" in time except through some limiting procedure. We will show that in this limit, we can recapture the quantum formalism in algebraic form. By this we mean we recapture the quantum formalism in what is generally known as Heisenberg's matrix mechanics. But in order to follow such a line of reasoning we must first address some very basic questions.

The first of these questions is to ask, "What is a quantum object?" The answer is surely obvious? An electron is a point-like quantum object! Those simple words hide a perplexing riddle that takes us far from the comfort of our classical world. Let us venture into this *quantum world* and illustrate the problem with a simplistic example originally proposed by Weyl. In this "toy" world, let us represent "shape" and "colour" as quantum operators that do not commute. To make this world even simpler suppose there are only two shapes, sphere and cube, that are the "eigenvalues" of the "shape" operator and only two colours that are "eigenvalues" of the "colour" operator red, and blue.

We require to collect together an ensemble of red spheres.[1] In this world we must use one instrument to measure colour (e.g. a pair of spectacles that enables us to distinguish colours) and another incompatible instrument to measure shape. I decide first to collect together spheres and discard all the cubes. I then decide to collect together those spheres that the colour-measuring device classifies as red. I am done. I have an ensemble of red spheres. So what is the problem? Just recheck that the objects in the ensemble are all in fact spheres. We check by using the first pair of glasses again and find that half are now cubes! No permanent *either/or* in this world. No permanent *and/and* either!

Let us look closer and follow Eddington's [21] suggestion that the elements of existence in the process world can be described by idempotents, $E^2 = E$. The eigenvalues, λ_e, of an idempotent is 1 or 0, existence or non-existence. In symbols

$$E^2 = E, \qquad \text{with} \qquad \lambda_e = 1 \text{ or } 0.$$

[1]This example will be appreciated by cricketers everywhere.

If all idempotents commute, existence is well defined. However in quantum theory, idempotents do not always commute.

$$[E_a, E_b] \neq 0.$$

What then of existence?

$$\text{Either } E_a \text{ or } E_b, \qquad \text{never} \qquad E_a \text{ and } E_b.$$

Existence, non-existence and in between? Clearly no world of classical objects!

What now is the position of reductionism? It won't work because we cannot start with some set of basic building blocks with well-defined properties. We cannot separate objects into ensembles with well-defined properties. How can we build stable structures if we cannot do that? And when the cube is blue, can we rely on it still being a cube as we try to build a structure of blue cubes?

No structures at all? How can this be? Quantum mechanics was introduced to explain stable structures. Without quantum mechanics there is no stability of matter! Without quantum mechanics there would be no atom as we know it; no crystalline structures, no DNA, and no classical world. But we see a the classical world. We are the DNA unfolded in this world! We probe quantum phenomena from our classical world, so naturally we insist on reductionism. We strive to find the elementary objects, the atoms, the leptons, the baryons, the quarks, the strings, the loops and the M-branes from which we try to reconstruct the world.

Surely we are starting in the wrong place. Spencer-Brown [63] and Parker-Rhodes [56] certainly thought so, so too did Lou Kauffman [49]! We should start with the *whole* process and then to make a description we must start from "distinction" or "difference." We start with a broad brush with which to make the initial differences. We then find relations between these differences. Within these preliminary differences we make finer distinctions and establish more relationships between these new differences. We then make yet more finer distinctions, establishing further relationships and so on. In this way we build a hierarchy of orders, to describe a structure process.

Kauffman [49] following Spencer-Brown [63] introduces the notion of "crossing" the boundary of a distinction, symbolised by $\urcorner$ with $\urcorner\urcorner = \urcorner$, an idempotent. We will see in Sect. 4.2 that the distinction cross, $\urcorner$, will be replaced by an idempotent in the algebra with which we will describe our structure-process. Thus in symbolic form

$$[T_1, T_2] \to \overline{T_1\rceil}\, T_2 \to \psi_L(t_1) E_a \psi_R(t_2).$$

Here $E_a^2 = E_a$ is some suitably chosen idempotent and $\psi_L E_a (E_a \psi_R)$ is an element of the left (right) ideal constructed using some suitably chosen idempotent, E_a.

Notice we are not God-like looking *in*, but inside looking *out*. Should we think of these distinctions as passive marks or are we going to allow for the fact that we are part of the process of making these distinctions? Are we participators? Wholeness implies that we and our instruments are inside the whole process, yet our current theories start with the assumption that we and our instruments are outside our cosmos and so we struggle to get back in!

At this stage we must pause. The mere thought of "putting ourselves back into it" traps us into thinking that there is something independent and separate to be put back in. We should never have been *out of it* in the first place! Now I hear alarm bells ringing. "He is going to suggest that we must put subjectivity back into our science whereas we know that the whole success of science has been to keep the subject out!" That is true of classical physics, but quantum physics says we must at least put our measuring instruments back into the system. Go further and ask "Is nature basically subjective?"

As Bohr [10] constantly reminds us, there is no separation between the system and its means of observation. He emphasises that this fundamental inseparability arises as a direct consequence of what he called "the indivisibility of the quantum of action." After warning us of the dangers of using phrases like "disturbing the phenomena by observation" and "creating physical attributes to atomic objects by measurements" he gives an even clearer statement of his position. He writes, "I advocate the application of the word phenomenon exclusively to refer to the observations obtained under specified circumstances, *including an account of the whole experimental arrangement*" (my italics) [10]. Because of the meaning Bohr attaches to the word "phenomenon," he insists that analysis into parts is *in principle* excluded.

However Bohr himself as the observer, is still outside. He claims to be a *detached* observer. No pandering to subjectivity here. But the question that fascinates me is "How do we become detached?" Let me spell out the problem. I am assuming that the universe did come into being from some form of quantum fluctuation along the lines that is currently assumed. The exact details as to whether this takes the form of a unique occurrence or in the form of a multiverse, or yet something else is of no significance for my argument here. Any quantum birth must have evolved into our classical world and the question is what are the essential properties of this evolution for the emergence of a classical world to take place.

Bohm and I have already given a description of how this could happen in the context of the Bohm approach [9], but there we already start half-way along the road when we single out the particle and give it a "rock-like" status. However as we have argued earlier, the quantum particle is not "rock-like." Its properties are not behaving as we would expect. Instead we have a quasi-stable process many of whose properties are constantly transforming. All that ultimately remains are the quasi-invariant processes, the distinctions, the idempotents.

Note the word "quasi-invariant" can be worrying. Fine for the so-called elementary particles like the muon, the Λs, the Ω^- and so on, but surely some properties are immutable such as charge, baryon number, lepton number, etc. But even here when electron meets positron charge and lepton vanish, being left with a pair of photons. Thus we are forced to look at nature from a very different perspective. This perspective does not allow us to start with particles or even fields. I do not need the popular story of decoherence to reach the classical world. That is fine if a classical world already exists. Then decoherence plays a vital role. But we are using another approach in which classical ideas are abstracted from the notion of an indivisible unity that was the baby universe.

2 Activity and Process

I want to start from the flow of experiences we encounter from the time we leave our collective intellectual womb. As Kauffman [49] stresses, the primitive perception is *distinction*. We perceive differences, make distinctions and build an order. We do this through relationships. We relate different differences. We perceive similarities in these differences and then look for the differences in these similarities and so on. In this way we construct a hierarchy of order and structure in the manner detailed by Bohm [3] in his long forgotten paper *Space, Time and the Quantum Theory understood in Terms of Discrete structure Process*.

But the differences of what? Just difference! We experience a flux of sensations, which we must order if we are to make sense of our world. We focus on the invariant features in that flux. What is inside? What is outside? What is left? What is right? And so on. More generally what is A, what is not-A. But the distinction A/not-A is not absolute in a world of process. In a different flux of perceptions, B and not-B may become a distinction. In this context it may not be possible to make the distinction between A and not-A. The processes are ontologically and epistemologically incompatible so that even distinction becomes a relative concept. Ultimately we could reach some domain when the distinction becomes absolute in that domain. Thus emerges the classical world with its absolute and stable distinctions. But note that this ordering does not only apply to the material world. It also applies to the world of thought. Here it is quite clear that the observer, the I of my mind, is part and parcel of the overall structure of the same mind. It is here that we have direct experience with the notion of wholeness. It is also here that we have direct experience of flux, activity and process philosophically highlighted by Schelling [62] and Fichte [26].

But even here it is easy to slip back into the categories of objects being the primary, forgetting that these objects take their form from the very activity that is thinking. I cannot capture this point better than Eddington [21] when he wrote,

> Causation bridges the gap in space and time, but the physical event at the seat of sensation (provisionally identified with an electrical disturbance of a neural terminal) is not the *cause* of the sensation; it *is* the sensation. More precisely, the physical event is the structural concept of that which the sensation is the general concept.

Or perhaps we should use the school of continental philosophers like Fichte [26] who wrote,

> For the same reason, no real being, *no subsistence or continuing existence*, pertains to the intellect; for such being is the result of a process of interaction, and nothing yet exists or is assumed to be present with which the intellect could be posited to interact. Idealism considers the intellect to be a kind of doing and absolutely nothing more. One should not even call it an *active subject*, for such an appellation suggests the presence of something that continues to exist and in which an activity inheres.

Idealism? Probably much too far for physicists, but the emphasis on activity *per se* and *not the activity of a thing* is the message to take. Neither idealism nor scientific materialism, but something different.

How can we hope to begin a description of such a general scheme? Start with Grassmann [34]. In the process of thought we can ask the question "Is the new thought distinct from the old thought, or is it one continuous and developing activity?" We find it easier to "hold" onto our description in terms of the "old," T_1, and the "new," T_2. But are they separate? Clearly not! The old thought has the potentiality of the new thought, while the new thought has the trace of the old thought. They are aspects of one continuing process. They take their form from the underlying process that is thought. Each has a complex structure of yet more distinctions, so that each T can be thought of as the tip of an "iceberg" of activity.

In order to symbolise this basic indivisibility, we follow Grassmann [34] and Kauffman [48, 50] and enclose the relationship in a square bracket, $[T_1, T_2]$. Some properties of this bracket have already been discussed above. Relationship is a start but not enough in itself. Our task then is to order these relationships into a multiplex of structure. To do that we need some rules on how to put these relationships together.

In my paper on *The Algebra of Process* [38] I tentatively suggested two rules of combination. Firstly a multiplication rule, (3), that defines a Brandt groupoid. Secondly I introduce a rule for addition, (5). These two binary relations taken together, of course, define an algebra. Our defining relations are

(1)	$[kA, kB] = k[A, B]$	Strength of process.
(2)	$[A, B]^* = -[B, A]$	Process directed.
(3)	$[A, B][B, C] = [A, C]$	Order of succession.
(4)	$[A, [B, C]] = [A, B, C] = [[A, B], C]$	Associativity.
(5)	$[A, B] + [C, D] = [A + C, B + D]$	Order of coexistence.

Notice $[A, B][C, D]$ is NOT defined.

The importance of the groupoid in quantum theory has been pointed out by Connes [15]. He recalled Heisenberg's [36] original suggestion in which Weyl, also draws to our attention, namely, that $x(t)$, the position of the electron in the atom, must be replaced by

$$X_{mn}(t) = \sum_a R_{mn} \exp[i(\nu_m - \nu_n)t].$$

Notice how the "position, $X_{mn}(t)$" becomes a set of two-point objects, a set of transitions between energy eigenstates labelled by m and n. Thus, once again, we are talking about transitions between one state and another, that is between structures defining *what has been to what will be*.

When written in this form the exponent ensures that the Ritz combination rule of atomic spectra can be satisfied, namely

$$\nu_{mj} + \nu_{jn} = \nu_{mn}.$$

This result is needed when we form variables like $X_{nm}(t)^2$ which appear in the discussion of a quantum oscillator. Heisenberg then proposed that the amplitudes combine as

$$R_{mn} = \sum_j R_{mj} R_{jn}.$$

This was originally recognised as the rule for matrix multiplication. But, as Connes has pointed out, it is based on a more primitive structure, namely the groupoid. Indeed it was a study of Heisenberg's original paper [36] that led me originally to propose the relations (1) to (4) above, although at that time I was unaware that I was dealing with a recognised mathematical structure, a groupoid.

I have shown elsewhere [38, 41] how the quaternions and indeed how a general orthogonal Clifford algebra emerges from the groupoid defined by the relations (1) to (4). I don't want to present these ideas here again [44]. Also a later summary of the main results of the emergence of orthogonal Clifford algebras can be found in Hiley and Callaghan [45]. Rather I want to relate the defining relations (1) to (5) above to a structure introduced by Kauffman [48], which he called the iterant algebra.

To explain the ideas lying behind the iterant algebra, let us start with the plane and divide it into two, an "inside" and an "outside." Now introduce the activity of "crossing" the boundary [63], $\urcorner$, and denote the activity of crossing from inside to outside by $[I, O]$, while the crossing from outside to inside is denoted by $[O, I]$. Here I and O are simply symbols denoting "inside" and "outside." This is the primary distinction.

Kauffman then generalises the notation and introduces a product defined by

$$[A, B] \star [C, D] = [AC, BD] \tag{1}$$

and shows that one can also use this relationship to generate the quaternions. Thus we have two structures with two different products producing the same algebra. But are they so different? When $B = C$ we have

$$[A, B] \star [B, D] = [AB, BD] = B[A, D]. \tag{2}$$

In this way the products have been brought closer together. In fact product (1) above is simply an equivalence class of the Kauffman product. But notice product (1) is undefined in our structure when $B \neq C$ thus, in one sense, giving a more general structure.

We have already suggested that we may write $[A, B] \rightarrow \overline{A}\rceil\, B \rightarrow A_L E_a B_R$. An even more suggestive form is to complete the sequence

$$A_L E_a B_R \rightarrow A\rangle\langle B \rightarrow |a\rangle\langle b|. \tag{3}$$

Here A and B are elements of the algebra, while a and b are the eigenvalues of the elements A and B regarded as operators in a Hilbert space. In fact the symbol $\rangle$ was introduced by Dirac [19, 20] who called it the *standard* ket.[2] It was introduced to prevent multiplication from the right, thus forming a left ideal. To prevent multiplication from the left, a dual symbol $\langle$, (*standard* bra), was also introduced, this time forming a right ideal. It should be clear that the joint symbol $\rangle\langle$ is playing a role analogous to E_a, our idempotent.[3] Although Dirac called $\rangle$ a vector in Hilbert space, it has a more natural meaning in terms of an algebra as we see in Sect. 4.2.

In order to stay on familiar territory we will use the last term in Eq. (3). With this identification we can relate our work to that of Zapatrin [69] and of Raptis and Zapatrin [60] who developed an approach through the incident algebra. In this structure the product rule is written in the form

$$|A\rangle\langle B| \cdot |C\rangle\langle D| = |A\rangle\langle B|C\rangle\langle D| = \delta_{BC}|A\rangle\langle C|. \tag{4}$$

Again this multiplication rule is essentially rule (3), the order of succession above. But there is a major difference. When $B \neq C$ the product in Eq. (4) is zero, whereas we leave it undefined at this level.

Finally we also want to draw attention to the work of Abramsky and Coecke [1] who argue that a process approach to quantum phenomena can best be described in terms of a symmetric monoidal category. Our product (3) above is identical to

[2] The label a was suppressed by Dirac leaving it understood provided no ambiguity arose.

[3] Symbolically we write $\rangle\langle \times \rangle\langle = \langle\rangle\ \rangle\langle$ with $\langle\rangle = 1$.

the product used in this category. There is a very close relationship between the algebraic structure we adopt in Sect. 4.2 and the diagrams used by Coecke [14] which form part of a much more general planer algebra [47].

However I will not discuss these relationships further here as I want to return to the basic ideas that are open to us when we look at process in terms of an algebra.

3 The Intersection of the Past with the Future

We are focusing on process or flux, via a notion of *becoming* which we symbolise by $[T_1, T_2]$. There will be many such relationships forming an ordered structure defining what we have called "pre-space" elsewhere. (See Bohm [7] and Hiley [37].) In other words these relationships are not to be thought of as occurring *in* space–time, but rather space–time is to be *abstracted from* this pre-space. This is a radical suggestion so let me try to develop my thinking more slowly.

Conventional physics is always assumed to unfold in space–time, the evolution being from point to point. In other words physics always tries to talk about time development *at an instant*. Any change always involves the limiting process

$$\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}.$$

But before taking the limit, it looks as if we were taking a point in the past (x_1, t_1) and relating it to a point in the future (x_2, t_2), i.e. relating what *was* to what *will be*. But we try to hide the significance of this step by going to the limit $(t_2 - t_1) \to 0$. Then we interpret the change to take place at an instant, t. Yet curiously the instant t is a set of measure zero sandwiched between the infinity of that which has passed and the infinity of that which is not yet. This is fine for evolution of point-like entities but is questionable when the evolution of extended structures is involved.

When we come to quantum mechanics, it is not positions that develop in time but wave functions, which like the Pauli spinor, can be treated as a special element of the algebra, namely, a minimal ideal in the algebra (see Hiley [41]). Ideals are determined by idempotents and, as we have seen above, idempotents can be used as "separators." But they are more than separators, they are the essence of the individual aspects of the process.

To clarify these notions let us recall Feynman's classic paper [24] where he sets out his thinking that led to his "sum over paths" approach. There he starts by dividing space–time into two regions R' and R''. R' consists of a region of space occupied by the wave function before time t', while R'' is the region occupied by the wave function after time t'', with $t' < t''$. Then he suggested that we should regard the wave function in region R' as contain information coming from the "past," while the conjugate wave function in the region R'' representing information coming from the "future."[4] The *possible present* is then the intersection between the two, which is simply represented by the transition probability amplitude $\langle\psi(R'')|\psi(R')\rangle$. From this Feynman derives the Schrödinger equation. But what I want to discuss here is $|\psi(R')\rangle\langle\psi(R'')|$. This is where all the action is!

Before taking up this point further, I would like to call attention to a similar notion introduced by Stuart Kauffman [51] in his discussion of biological evolution. Here it is clear that we are talking about an evolution of *structure*. Kauffman discusses the evolution of biological structures from their present form into the *adjacent possible*. The adjacent possible contains only those forms that can develop from the immediate previous form. Radical re-structuring is limited, small deformations are more likely. This means that only certain sub-class of forms can develop out of the past. Thus not only does the future form contain a trace of the past, but it is also constrained by what is "immediately" possible. So any development is governed by the *tension between the persistence of the past, and an anticipation of the future* [68].

What I would now like to do is to build this notion into a dynamics. Somehow we have to relate the past to the future, not in a completely deterministic way, but in a way that constrains the possible future development. The basic notion we need is thus structures which when represented in space–time cannot be localised. Central to our structure is the "moment," $[T_1, T_2]$. In algebraic terms it is *a-local* but when represented in space-time is non-local; not only non-local in space, but also "non-local" in *time*. It is a kind of "extension in time," a "duron;" a region of ambiguity where re-structuring is possible. This ambiguity fits comfortably with the energy-time uncertainty principle. Thus a process that involves energy changes cannot be described as unfolding at an instant except in some approximation.

3.1 Bi-Local Dynamics

How then are we to discuss the dynamics of process, a dynamics which depends on this notion of a moment? Let us start in the simplest possible way by proposing that the basic dynamical function will involve two external times, giving rise to a *bi-local* model. Thus we will discuss the time development of two-point functions of the form $[A(t_1), B(t_2)]$. We will show that we are led to a pair of Eqs. (9) and (10) which depend on a mean time and a time difference. We will then show that we capture the usual equations of motion in the limit $t_1 \rightarrow t_2$. We will then go on to exploit the bi-local structure.

Fortunately we do not have to start with quantum physics as we can motivate the idea entirely within classical physics. Such functions are implicit in all variational principles that lie at the heart of modern physics. For example, in his classic work on

[4] This is essentially the same idea that led to the notion of the anti-particle "going backwards in time," but here we are not considering "exotic" anti-matter.

optics, Hamilton [35], recognising the importance of Fermat's least-time principle, basically a principle involving two times. He even suggested that both optics and classical mechanics could be united into a common formalism by introducing a two-point characteristic function, $\Omega(x_1, x_2)$. Following on from Hamilton's work, Synge [64], in his unique approach to general relativity, proposed that a two-point function, which he called the "world function," lies at the heart of general relativity.[5] Can we exploit these two-point functions to develop a new way of looking at dynamics?

Let us start by recalling that the use of the variational principle produces the classical Hamilton–Jacobi equation (see Goldstein [31]). Specifically this emerges by considering a variation of the initial point x_1 of the trajectory. Standard theory shows that by varying the initial point x_1, we can obtain the relations

$$\frac{\partial S}{\partial x_1} = p(x_1) \qquad \frac{\partial S}{\partial t_1} + H_1 = 0, \tag{5}$$

where we have written $H_1 = H\big[x_1, \partial S(x_1, x_2)/\partial x_1\big]$ for convenience and we have replaced the world function Ω by the classical action function S.

What is not so well known is that if we vary the final point B, we find another pair of equations

$$\frac{\partial S}{\partial x_2} = -p(x_2) \qquad \frac{\partial S}{\partial t_2} - H_2 = 0. \tag{6}$$

Here the second Hamilton–Jacobi equation formally becomes the same by writing $t_2 = -t_1$.

Similarly for the quantum propagator $K(x_2, x_1, t_2, t_1)$ which we write as $K(2, 1)$ [25], we find not only

$$i\hbar \frac{\partial K(2,1)}{\partial t_1} + K(2,1)H_1 = 0, \tag{7}$$

but also

$$i\hbar \frac{\partial K(2,1)}{\partial t_2} - H_2 K(2,1) = 0. \tag{8}$$

The similarity in form between Eqs. (5) and (6) and the Eqs. (7) and (8) is not coincidental, but arises from the lifting properties from the classical symplectic group to its covering group, the metaplectic group (see de Gosson [32]). Could this similarity be taken to support the idea that we have a wave coming from the "past" and the "future," thus fitting into the general scheme I am developing here?

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[5] In modern parlance these functions are the generating functions of the symplectomorphisms in classical mechanics (see de Gosson [32]).

Leaving that speculation aside, let us see how we can formally exploit the two Hamilton–Jacobi Eqs. (5) and (6). Consider a pair of points with co-ordinates (x_1, t_1) and (x_2, t_2) joined by a geodesic in configuration space. The world function (generalised action) for this pair can be written as $S(x_1, x_2, t_1, t_2)$. (See de Gosson [33] for a formal treatment of the above structure.)

We will find it more convenient to use "sums" and "differences" rather than the co-ordinates themselves. Thus we change to co-ordinates $(X, \Delta x, T, \Delta t)$ where

$$X = \frac{x_1 + x_2}{2}, \quad T = \frac{t_1 + t_2}{2}, \quad \Delta x = x_2 - x_1, \quad \Delta t = t_2 - t_1,$$

so that the generalised action becomes

$$S(x_1, x_2, t_1, t_2) = S(X, \Delta x, T, \Delta t).$$

Then Eqs. (5) and (6) can be replaced by

$$\frac{\partial S}{\partial X} = \Delta p, \quad \frac{\partial S}{\partial T} = \left[H_2 - H_1\right] \tag{9}$$

$$\frac{\partial S}{\partial \Delta x} = P, \quad \frac{\partial S}{\partial \Delta t} = \tfrac{1}{2}\left[H_2 + H_1\right]. \tag{10}$$

In order to see the meaning of the two equations let us make a Legendre transformation

$$K(X, P, T, E) = P\Delta x + E\Delta t - S(X, \Delta x, T, \Delta t), \tag{11}$$

so that

$$\frac{\partial S}{\partial T} = -\frac{\partial K}{\partial T}, \quad \frac{\partial S}{\partial \Delta t} = E.$$

A general background discussion to these ideas can be found in Bohm and Hiley [6].

Equations (9) and (10) will form the basis of a bi-local classical theory. Now we must show that if we go to the limit $\Delta t \to 0$ and $\Delta x \to 0$ we will reproduce the expected equations of motion. Therefore let us go to this limit. We find

$$\lim_{\Delta t \to 0} \frac{\partial S}{\partial T} = -\left[H_2 - H_1\right] \Rightarrow \frac{\partial S}{\partial T} + \frac{\partial H}{\partial P}\Delta p + \frac{\partial H}{\partial P}\Delta x \approx 0. \tag{12}$$

But

$$\Delta p = -\frac{\partial K}{\partial X} \quad \Delta x = \frac{\partial K}{\partial P},$$

so that Eq. (12) becomes

$$\frac{\partial K}{\partial T} + \{K, H\} = 0, \tag{13}$$

where $\{,\}$ is the Poisson bracket so that Eq. (13) becomes the classical equation of motion for the dynamical variable K. Indeed when K is identified with the probability distribution, this is nothing more than the Liouville equation.

The second equation in (9) becomes

$$\frac{\partial S}{\partial \Delta t} = \tfrac{1}{2}\big[H_2 + H_1\big] = E.$$

Since E, the total energy, is a constant for a closed system, we have

$$\lim_{\Delta t \to 0} \frac{\partial S}{\partial \Delta t} = E. \tag{14}$$

Thus we see that in the limit $\Delta t \to 0$, the dynamics is defined by two equations, namely, Eqs. (13) and (14). They are both conservation equations, the first is the conservation of probability and the second is the conservation of energy. We will now show that the analogue of these two conservation equations also emerge in the quantum case as we will now show.

4 Quantum Pasts and Futures

4.1 The Hilbert Space Approach

Now let us examine the quantum domain and consider Feynman's suggestion mentioned earlier in more detail. Introduce a world function defined by

$$\hat{\rho}(t_1, t_2) = |\psi(t_1)\rangle\langle\psi(t_2)|. \tag{15}$$

We use the symbol $\hat{\rho}$ because it will turn out that we are essentially dealing with a generalised density operator. Let us proceed formally by writing

$$\frac{\partial}{\partial T}(|\psi(t_1)\rangle\langle\psi(t_2)| = \left(\frac{\partial}{\partial t_1}|\psi(t_1)\rangle\right)\langle\psi(t_2)| + |\psi(t_1)\rangle\left(\frac{\partial}{\partial t_2}\langle\psi(t_2)|\right). \tag{16}$$

We could use the two Eqs. (7) and (8) in (16) to proceed, but since Feynman has already derived the Schrödinger equation from these considerations, we prefer to substitute these two equations

$$i\frac{\partial}{\partial t_1}|\psi(t_1)\rangle = \hat{H}_1|\psi(t_1)\rangle \quad \text{and} \quad -i\frac{\partial}{\partial t_2}\langle\psi(t_2)| = \hat{H}_2\langle\psi(t_2)|$$

into Eq. (16) § to find

$$i\frac{\partial\hat{\rho}(t_1,t_2)}{\partial T}+\hat{\rho}(t_1,t_2)\hat{H}_2-\hat{\rho}(t_1,t_2)\hat{H}_1=0. \tag{17}$$

If we now take the limit as $\Delta t\to 0$ when $T\to t$, we find

$$i\frac{\partial\hat{\rho}}{\partial t}+[\hat{\rho},\hat{H}]_-=0. \tag{18}$$

Here $\hat{\rho}$ has become the usual density operator for the pure state $|\psi(t)\rangle$. This equation is the quantum version of Eq. (9) and is, of course, the quantum Liouville equation.

Now let us consider

$$2\frac{\partial}{\partial\Delta t}(|\psi(t_1)\rangle\langle\psi(t_2)|)=|\psi(t_1)\rangle\left(\frac{\partial}{\partial t_2}\langle\psi(t_2)|\right)-\left(\frac{\partial}{\partial t_1}|\psi(t_1)\rangle\right)\langle\psi(t_2)|.$$

So that by using the two Schrödinger equations again, we find this time

$$2i\frac{\partial\hat{\rho}(t_1,t_2)}{\partial\Delta t}+\hat{\rho}(t_1,t_2)\hat{H}_2+\hat{\rho}(t_1,t_2)\hat{H}_1=0, \tag{19}$$

which we recognise as the quantum version of Eq. (10). The "derivative" $\partial/\partial\Delta t$ looks rather odd until one recalls field theory,

$$\lim_{t_2\to t_1}\left[|\psi(t_1)\rangle\left(\frac{\partial}{\partial t_2}\langle\psi(t_2)|\right)-\left(\frac{\partial}{\partial t_1}|\psi(t_1)\rangle\right)\langle\psi(t_2)|\right]=|\psi(t_1)\rangle\overleftrightarrow{\partial_t}\langle\psi(t_2)|.$$

With a little work, we can show

$$|\psi(t_1)\rangle\overleftrightarrow{\partial_t}\langle\psi(t_2)|=T^{00}=E. \tag{20}$$

Thus we can finally write Eq. (19) as

$$2E=[\hat{\rho},\hat{H}]_+. \tag{21}$$

This turns out to be an expression of the conservation of energy equation. Collecting together the main results so far we find

$$i\frac{\partial\hat{\rho}}{\partial t}+[\hat{\rho},\hat{H}]_-=0\quad\Leftrightarrow\quad\frac{\partial K}{\partial T}+\{K,H\}=0,$$

and

$$2E=[\hat{\rho},\hat{H}]_+\quad\Leftrightarrow\quad E=\lim_{\Delta t\to 0}\frac{\partial S}{\partial\Delta t}.$$

Again if K is the classical analogue of the density operator then we would have a correspondence between the classical Liouville equation (13) and the quantum Liouville equation (18). In turn the quantum energy Eq. (21) then corresponds to the classical energy Eq. (14). Thus we have a clear correspondence between the classical and the quantum levels.

4.2 *The Algebraic Approach*

So far we have restricted our discussion to the more conventional mathematics, but I want to exploit a more general way of exploring these ideas using an algebraic approach that has already been discussed in Brown and Hiley [11] and further developed by Hiley [44] and Hiley and Callaghan [45].

In the algebraic approach, a ket $|\psi(t_1)\rangle$ is replaced by an element of a minimal left ideal, $\Psi_L(t_1)$, while $\langle\psi(t_2)|$ is replaced by an appropriate element of a right ideal, $\Psi_R(t_2)$.[6]

We then start by defining an algebraic density element

$$\bar{\rho}(t_1, t_2) = \Psi_L(t_1)\Psi_R(t_2),$$

and write these algebraic elements, Ψ, in polar form

$$\Psi_L(t_1) = R(t_1)\exp[iS(t_1)] \quad \text{and} \quad \Psi_R(t_2) = \exp[iS(t_2)]R(t_2).$$

Here we emphasise that Ψ, R, S are *elements of the algebra* and not elements of a Hilbert space. Then

$$2\frac{\partial\bar{\rho}(t_1, t_2)}{\partial\Delta t} = \left[-\frac{\partial R(t_1)}{\partial t_1}R(t_2) + R(t_1)\frac{\partial R(t_2)}{\partial t_2} - iR(t_1)R(t_2)\left[\frac{\partial S(t_1)}{\partial t_1} + \frac{\partial S(t_2)}{\partial t_2}\right]\right] \times \exp\left(-i\left[S(t_2) - S(t_1)\right]\right),$$

where we have assumed that R and S commute. Then when we go to the limit $\Delta t \to 0$ with $T \to t$, we find

$$\lim_{\Delta t\to 0} 2\frac{\partial\bar{\rho}}{\partial t} = -iR^2\frac{\partial S}{\partial t}. \qquad (22)$$

[6] If R is a noncommutative ring, a left ideal is a subset I_L such that if $a \in I_L$ then $ra \in I_L$ for all $r \in R$.

Thus Eq. (19) then become

$$2R^2\frac{\partial S}{\partial t} + [\bar{\rho}, H]_+ = 0. \tag{23}$$

This equation is identical to equation (11) derived in Brown and Hiley [11]. A yet different derivation of this equation will also be found in Hiley [40]. The reason why I have re-derived this equation in different ways is because I have not seen this equation written down in this form in the literature. However it is implicit in Dahl [18]

In Brown and Hiley [11] we showed that there were two important consequences following from this equation. Firstly the Berry phase and the Aharonov–Bohm effect followed immediately from this equation in a very simple way. Secondly we used this quantum equation to see where the quantum potential introduced by Bohm emerges from what is essentially the Heisenberg picture (see also Hiley [40]). We found that this potential only appeared as a result of *projecting* the algebraic elements onto a representation space. This led us to speculate that all the "action" of quantum phenomena takes place in a pre-space, the structure of which is described by the algebra. All we see is its projection onto a space–time manifold. Thus the space–time manifold is not to be taken as "basic." Rather it is something that is derived from the deeper and more basic structure-process.

It is well known that we cannot display quantum processes in a *commutative* phase space because we are using a non-commutative structure. However this does not rule out the possibility of representing quantum phenomena in terms of a *non-commuting* phase space. In fact this has already been achieved through the Moyal algebra [55], sometimes described as the deformed Poisson algebra.

This structure contains a non-commutative $\star$-product which gives rise to a Moyal bracket, which can be used to produce an analogue of Eq. (18). There also exists a symmetric bracket, the Baker bracket [42], which can be used to produce an equation which is the analogue of (21). Thus these equations seem basic to the type of non-commutative structures that we are using to describe quantum phenomena.

What is even more interesting is that the Moyal algebra provides a natural way to approach the classical limit. The Moyal bracket equation reduces the classical Liouville equation which leads to a conservation of probability, while the equation involving the Baker bracket reduces to the classical Hamilton–Jacobi equation. We have shown the details elsewhere, [11], where we also show that when the quantum form of this equation is projected into a space representation, the quantum potential emerges through what we have called the quantum Hamilton–Jacobi equation. We now see why this QHJ approaches the ordinary Hamilton–Jacobi equation in the classical limit. The appearance of the quantum potential is clearly a consequence of the non-commutative structure required by quantum theory.

This leads to an interesting connection with the work of Gel'fand [52] where it can be shown that for any *commutative* C^*-algebra, one can reconstruct the Hausdorff topological space M underlying the commutative algebra. With a non-commutative algebra there is no unique underlying manifold. One has to introduce a set of "shadow" manifolds, which are constructed by sets of projections from the algebra. In each projection, we get a kind of distortion of the type found in maps when using a Mercator's projection. Therefore it is not surprising to find it necessary to introduce inertial forces, like the one derived from the quantum potential, to account for the predicted behaviour in the shadow manifold. This is very similar to how the gravitation force is manifested in general relativity (For a more detailed discussion of these ideas, see Hiley [44].)

5 Bi-Algebras and Super-Algebras

5.1 Motivation

In this next section I want to extend the algebra and construct a bi-algebra. This is motivated by some proposals made by Umezawa [65] in his discussions of thermal quantum field theory. His aim was to find a common formalism in which both quantum and thermal effects can be incorporated. Unlike the work presented here, Umezawa uses Hilbert space and shows that if we "double" the Hilbert space, then the thermal state can also be represented by a single vector in this double space. For example, in more familiar notation, the thermal wave function can be written in the form

$$|\Omega(\beta)\rangle = Z^{-1/2}\sum \exp[-\beta E_n/2]|\psi_n\rangle \otimes |\psi_n\rangle. \tag{24}$$

Here $\beta = 1/kT$ and $|\psi_n\rangle$ are the energy eigenkets. Z is the partition function. The ensemble average of some quantum operator A would then be given by

$$\langle\Omega(\beta)|A|\Omega(\beta)\rangle = Tr(\rho A),$$

where ρ is the thermal density operator, which in its more usual form is written as

$$\rho = \exp[-H\beta].$$

Those familiar with algebraic quantum field theory will recognise that the doubling of Hilbert space is essentially the GNS construction (Emch [22] and Hiley [41]). In terms of the algebra, this doubling of the number of field elements suggests that any algebraic theory would have double the algebra, but the bi-local theory I have introduced above is the first step to developing a bi-algebraic structure.

In the last section, we have been discussing a two-time quantum theory where the time is being treated as a parameter and not as an element of a general non-commutative algebra. Let us now see how we can generalise the structure to make time part of the larger algebra.

In order to anticipate the quantum approach, we return to classical physics and form a Poisson bi-algebra by introducing a generalised Poisson bracket defined by

$$\{\,\} = \frac{\partial}{\partial X}\frac{\partial}{\partial \Delta p} - \frac{\partial}{\partial \Delta p}\frac{\partial}{\partial X} + \frac{\partial}{\partial \Delta x}\frac{\partial}{\partial P} - \frac{\partial}{\partial P}\frac{\partial}{\partial \Delta x},$$

so that we find the following relationships

$$\{X, \Delta p\} = \{\Delta x, P\} = 1$$
$$\{X, P\} = \{\Delta x, \Delta p\} = \{X, \Delta x\} = \{P, \Delta p\} = 0. \tag{25}$$

This suggests we introduce another pair of brackets of the form

$$\{T, (H(t_2) - H(t_1))\} = \{\Delta t, (H(t_2) + H(t_1))\} = 1. \tag{26}$$

If we were to introduce the quantity $L(t_1, t_2) = H(t_2) - H(t_1)$, we have the classical correspondence to the Liouville operator introduced by Prigogine [59]. This connection will be discussed further when these results are generalised to the quantum domain.

5.2 *The Quantum Bi-Algebra*

In moving to quantum theory, we need to regard the position and momentum as *algebraic elements* and base the theory on pairs of *algebraic* elements, $\{\bar{x}_1, \bar{x}_2, \bar{p}_1$ and $\bar{p}_2\}$. Again I have added the “bar” to emphasise that these are elements of the algebra. In other words we are doubling the algebra to form a bi-algebra. Following the analogous procedure to the classical case, we introduce the notation

$$2\bar{X} = \bar{x}_1 \otimes 1 + 1 \otimes \bar{x}_2, \quad \bar{\eta} = \bar{x}_1 \otimes 1 - 1 \otimes \bar{x}_2 \tag{27}$$
$$2\bar{P} = \bar{p}_1 \otimes 1 + 1 \otimes \bar{p}_2, \quad \bar{\pi} = \bar{p}_1 \otimes 1 - 1 \otimes \bar{p}_2. \tag{28}$$

We then find that the following commutator relations hold

$$[\bar{X}, \bar{\pi}] = [\bar{\eta}, \bar{P}] = i,$$

and

$$[\bar{X}, \bar{P}] = [\bar{\eta}, \bar{\pi}] = [\bar{X}, \bar{\eta}] = [\bar{P}, \bar{\pi}] = 0. \tag{29}$$

These relations are the quantum analogues of the generalised Poisson brackets defined in Eq. (25). These results were already reported in Bohm and Hiley [6].

The aim in this section of the paper is to find a time "operator" that may be connected with irreversibility. Prigogine [59] has already pointed out that we need a theory in which irreversibility plays a fundamental role directly in the dynamics itself. Let us see how we can make contact with his approach.

First note that we can write the quantum Liouville equation (18) in terms of the bi-algebra

$$i\frac{\partial \bar{\rho}_V}{\partial t} + \bar{L}\bar{\rho}_V = 0. \tag{30}$$

Here $\bar{\rho}_V$ is a vector equivalent of the density operator and $\bar{L} = \bar{H} \otimes 1 - 1 \otimes \bar{H}$. The appearance of the "super-operator" $\bar{L}$ enables us to introduce a time "operator" $\bar{T}$, defined through the relation

$$[\bar{T}, \bar{L}] = i. \tag{31}$$

This is the quantum version of the classical form presented by the first equation in (26).

Prigogine [59] argues that this time operator, $\bar{T}$ represents the "age" of the system. I don't want to discuss the reasons for this as I have already made some comments on it in Bohm and Hiley [6] and in Hiley and Fernandes, [39]. A more general discussion of Prigogine's point of view will be found in George and Prigogine [27], and in Prigogine [59],

What I want to do now is to go on to the bi-algebraic generalisation of Eq. (15). This requires the introduction of the "super-operator" corresponding to the anti-commutator, which can be written in the form

$$\bar{E}\bar{\rho}_V = (\bar{H} \otimes 1 + 1 \otimes \bar{H})\bar{\rho}_V = E_+\bar{\rho}_V. \tag{32}$$

Such an operator was first introduced by George et al. [28] in their general discussion of dissipative processes. They, like us, regard this as an expression of the total energy of the system. I have only found one other discussion relating the anti-commutator, $[\bar{\rho}, H]_+$, to the energy of the system. This is the work of Dahl [18] who was concerned with energy storage and transfer in chemical systems.

For completeness I should point out that Fairlie and Manogue [23] have discussed an analogous equation based on the cosine Moyal bracket introduced by Baker [2]. However they explore a very different structure.

As well as introducing the "age operator," $\bar{T}$, we have the possibility of introducing a "time difference operator," $\bar{\tau}$, which we will call the duron. This object satisfies the commutator relations

$$[\bar{T}, \bar{\epsilon}] = [\bar{\tau}, \bar{E}] = i,$$

and

$$[\bar{T}, \bar{E}] = [\bar{\tau}, \bar{\epsilon}] = [\bar{T}, \bar{\tau}] = [\bar{E}, \bar{\epsilon}] = 0, \tag{33}$$

where we have written $\bar{\epsilon}$ for $\bar{L}$ to bring out the symmetry. Hiley and Fernandes [39] have already suggested these relationships in the context of finding "operators" for time. In particular they interpreted $\bar{\tau}$ as the mean time spent passing between two energy states. Here we will suggest a different interpretation.

6 Bi-Algebras and the Bogoliubov Transformations

Before discussing the meaning of $\bar{\tau}$ in more detail let me return to my way of thinking about the bi-algebra. I have proposed that the evolution of a quantum process does not proceed at an instant of time at a point in space, but through the ambiguous region of phase space that I have called a "moment." We consider the relation between the two sides of this moment, describing one side as information coming from the past while the other side is to do with the possible developments for the future.

I have spoken at times rather dramatically about this latter feature as "information coming from the future." But such a way of talking is not that outrageous that it has not been suggested before. For example, Cramer [17] in his transactional interpretation of quantum mechanics uses the advanced potentials to carry information from the future. The transaction is a "handshake" between emitter and the absorber participants of a quantum event. This notion, in turn, has a resonance with an earlier proposal of Lewis [53, 54] who has based his thinking on the following idea. In the rest frame of a photon time dilation suggests that there is no time lapse between emission and absorption and because of the length contraction, there is no distance between the emitter and absorber either. The light ray is a primary contact between the two ends of the process. These are both very radical ideas and unfortunately I have never known what to make of them so I have introduced the notion of a "moment" hoping that Δt, when projected into a space–time frame is small, but as these two examples show this may be a too conservative view to adopt!

Recently I was very happy to meet with Giuseppe Vitiello to discuss some of his extremely interesting ideas on dissipative quantum systems. His ideas are, perhaps, even more conservative and therefore probably more reliable, yet they seem to fit into the overall scheme I am discussing here. His work is reported in a series of papers in Vitiello [66], Celeghini et al. [12, 13] and Iorio and Vitiello [46]. I will rely heavily on the mathematics contained in these papers.

They are interested in quantum dissipation, which they explore in terms of a pair of coupled dissipative oscillators, one emitting energy, the other absorbing energy. In terms of our two-sided evolution discussed above, we find one "side" of the process is seen as representing the system while the other "side" is seen as representing the environment, the latter acting as a sink for the dissipated energy.

In this model the degrees of freedom of the system are described by a set of annihilation operators $\{a_k\}$, while the environment is described by the set $\{\tilde{a}_k\}$. Thus there is a doubling of the mathematical structure. The extra field variables describing the "environment" are a mirror image of the variables used to describe the system. Not only is a spatial mirror image but it is also a "*time-reversed* mirror image" as Vitiello [67] puts it. So the "environment sink" appears to be acting as if it were "anticipating the future."

Let us leave the imagery for the moment and move on to see how the ideas work mathematically. For this we will need to introduce some more formalism. So far we have introduced elements of our bi-algebra by effectively defining two sets of co-products which we will now express formally as

$$\Delta_+ \bar{A} = \bar{A} \otimes 1 + 1 \otimes \bar{A} \quad \text{and} \quad \Delta_- \bar{A} = \bar{A} \otimes 1 - 1 \otimes \bar{A}. \tag{34}$$

We have then shown that when we go to the limit $\Delta t \rightarrow 0$, we produce two dynamical equations, namely,

$$i\frac{\partial \bar{\rho}_V}{\partial t} + \bar{L}\bar{\rho}_V = 0 \quad \text{and} \quad \lim_{\Delta t \rightarrow 0}\left(2i\frac{\partial \bar{\rho}_V}{\partial t}\right) + \bar{H}_+\bar{\rho}_V = 0. \tag{35}$$

But what do we make of the general co-products and the commutation relations listed in Eqs. (27)–(29)? To explore these let us first make a Bargmann transformation from the Heisenberg algebra to the boson algebra of annihilation and creation operators. This will enable us to immediately relate our work to that of Vitiello [66] and Celeghini et al. [13]. Thus writing

$$a = \bar{x}_1 + i\bar{p}_1 \qquad \tilde{a} = \bar{x}_2 + i\bar{p}_2$$
$$a^\dagger = \bar{x}_1 - i\bar{p}_1 \qquad \tilde{a}^\dagger = \bar{x}_2 - i\bar{p}_2.$$

We can immediately make contact with Eq. (24) by using the well-known generator of the Bogoliubov transformation

$$G = -i(a^\dagger\tilde{a}^\dagger - a\tilde{a}). \tag{36}$$

Then applying this to the vacuum state $|0,0\rangle$, we find a new vacuum state $|0(\theta)\rangle$ given by

$$|0(\theta)\rangle = \exp(i\theta G)|0,0\rangle = \sum_n c_n(\theta)|n\rangle \otimes |n\rangle. \tag{37}$$

This means that by doubling the algebra we can immediately see the similarity with Eq. (24) and this opens up the possibility of linking thermodynamics and quantum phenomena in a direct way, which is different from the thermal ensemble methods used in Bose–Einstein and Fermi statistics. Doubling the algebra means doubling the degrees of freedom, so that we have a new process in addition to the usual dynamics.

Umezawa [65] gives a detailed discussion of a possible way of understanding this extra degree of freedom. We will not discuss his ideas here, but suggest another way of exploiting these extra degrees of freedom to prove a better understanding of the notion of time. To bring this possibility out let us first go deeper and develop the boson bi-algebra a bit further by defining the following co-products based on Eqs. (27) and (28),

$$\Delta_{+}a = a \otimes 1 + 1 \otimes a = a + \tilde{a}; \qquad \Delta_{-}a = a \otimes 1 - 1 \otimes a = a - \tilde{a}. \qquad (38)$$

$$\Delta_{+}a^{\dagger} = a^{\dagger} \otimes 1 + 1 \otimes a^{\dagger} = a^{\dagger} + \tilde{a}^{\dagger}; \quad \Delta_{-}a^{\dagger} = a^{\dagger} \otimes 1 - 1 \otimes a^{\dagger} = a^{\dagger} - \tilde{a}^{\dagger}. \qquad (39)$$

We see immediately that these co-products are identical to those introduced by Celeghini et al. [13] but we can go further and form

$$A = \tfrac{1}{\sqrt{2}}(a + \tilde{a}) = \sqrt{2}(\bar{X} + i\bar{P}); \quad A^{\dagger} = \tfrac{1}{\sqrt{2}}(a^{\dagger} + \tilde{a}^{\dagger}) = \sqrt{2}(\bar{X} - i\bar{P}), \quad (40)$$

and

$$B = \tfrac{1}{\sqrt{2}}(a - \tilde{a}) = -\sqrt{2}(\bar{\eta} + i\bar{P}); \quad B^{\dagger} = \tfrac{1}{\sqrt{2}}(a^{\dagger} - \tilde{a}^{\dagger}) = -\sqrt{2}(\bar{\eta} - i\bar{\pi}). \quad (41)$$

These operators lie at the heart of their approach. In our approach we see that these operators have a very simple interpretation. They are simply the annihilation and creation operators of the mean position variables and the difference variables respectively. Thus

$$\bar{X} = \tfrac{1}{\sqrt{8}}(A + A^{\dagger}) \quad \text{and} \quad \bar{P} = \tfrac{i}{\sqrt{8}}(A - A^{\dagger})$$

$$\bar{\eta} = \tfrac{1}{\sqrt{2}}(B + B^{\dagger}) \quad \text{and} \quad \bar{\pi} = \tfrac{i}{\sqrt{2}}(B - B^{\dagger}).$$

In other words the operators A and B are the algebraic way of defining the ambiguous moments of in our algebraic phase space. They are the variables that we need to describe the unfolding process that forms the basis of our paper.

Now I want to follow Celeghini et al. [13] further and generalise our approach by deforming the bi-algebra. We do this by defining the co-product

$$\Delta_{+}a_q = a_q \otimes q + q^{-1} \otimes a_q \qquad \Delta_{+}a_q^{\dagger} = a_q^{\dagger} \otimes q + q^{-1} \otimes a_q^{\dagger}, \qquad (42)$$

where we will write $q = e^{\theta}$ where θ is some parameter, the physical meaning of which has yet to be determined. Then

$$A_q = \frac{\Delta a_q}{\sqrt{[2]_q}} = \tfrac{1}{\sqrt{[2]_q}}(e^{\theta}a + e^{-\theta}\tilde{a}); \; B_q = \tfrac{1}{\sqrt{[2]_q}}\frac{\delta}{\delta\theta}\Delta a_q = \tfrac{1}{\sqrt{[2]_q}}(e^{\theta}a + e^{-\theta}\tilde{a}) + h.c. \qquad (43)$$

The A_q and B_q are then the deformed equivalents of Eqs. (40) and (41). Notice also that

$$\Delta_- A_\theta = \frac{\delta}{\delta\theta}\Delta_+ A_\theta \quad \text{and} \quad \Delta_- A = \lim_{\theta\to 0}\frac{\delta}{\delta\theta}\Delta_+ A_\theta, \tag{44}$$

so that the two sets of co-products defined in Eqs. (38) and (39) are not independent. With these definitions it is not difficult to show that we can write

$$A(\theta) = \tfrac{1}{\sqrt{2}}(a(\theta) + \tilde{a}(\theta)) \quad \text{and} \quad B(\theta) = \tfrac{1}{\sqrt{2}}(a(\theta) - \tilde{a}(\theta)). \tag{45}$$

So that

$$a(\theta) = \tfrac{1}{\sqrt{2}}(A(\theta) + B(\theta)) = a\cosh\theta - \tilde{a}^\dagger \sinh\theta, \tag{46}$$

and

$$\tilde{a}(\theta) = \tfrac{1}{\sqrt{2}}(A(\theta) - B(\theta)) = \tilde{a}\cosh\theta - a^\dagger \sinh\theta. \tag{47}$$

This is immediately recognised as the Bogoliubov transformation from the set $\{a, \tilde{a}\}$ of annihilation and creation operators to a new set $\{a(\theta), \tilde{a}(\theta)\}$. This result justifies the use of the Bogoliubov generator given in Eq. (36), which was used to construct the GNS ket given in Eq. (37).

7 Unfolding Through Inequivalent Representations?

Having put a formalism in place, I now want to consider how all this leads to a radically new way of looking at the way quantum processes unfold in time. My ideas go back to the early eighties when David Bohm and I were discussing how we could think about the type of process underlying quantum phenomenon. Most of this work was unpublished essentially because I did not have an adequate understanding of the mathematics needed. However Bohm [7] did publish some of the background relevant to the ideas I am developing here. There perhaps for the first time he makes a clear statement as to what we were thinking. I quote

> All these relationships (of moments of enfoldment) have to be understood primarily as being between the implicate "counterparts" of these explicate moments. That is to say, we no longer suppose that space-time is primarily an arena and that the laws describe necessary relationships in the development of events as they succeed each other in this arena. Rather, each law is a structure that interpenetrates and pervades the totality of the implicate order.

Implicit in this was the idea that space–time itself would emerge at some higher explicit level [37]. All of this early discussion could easily be dismissed as "somewhat vague," but we did try to make it more specific by arguing that the inequivalent representations contained within quantum field theory would play a key role. However we could not see how to make the mathematics work.

In the general context of Bohm's ideas, the vacuum state should not be regarded as absolute and self-contained. Rather each vacuum state provides the basis for what we called an explicate order so that a set of inequivalent vacuum states could be thought of as providing an array of explicate orders, all embedded in the overall implicate order in which all movement is assumed to take place. The movement between inequivalent representations, between inequivalent vacuum states, is then regarded as a movement from one explicate order to another.

This movement, as we have seen, is described by a Bogoliubov transformation and should be distinguished from the unfolding–enfolding transformation that Bohm describes with the metaphor of the jar of glycerine demonstration [5]. Mathematically this is just an automorphism of the kind $e' = MeM^{-1}$ as was discussed in [43]. Within this structure we found the explanation as to why in a single Hilbert space formalism nothing *actually* happens. The inner automorphisms of the algebra of operators are simply a re-description of the *potentialities* of the process so that every unitary transformation becomes merely a re-expression of the order. In this sense everything is a *potentiality*.

But what about the actual occasions? This has been the continuing difficulty of the "measurement problem." Where do the actual events arise in the quantum formalism? First we should notice that in quantum field theory, the vacuum kets $|0(\theta)\rangle$ belong to inequivalent representations of the boson algebra. Our suggestion is that not only is there a movement within each inequivalent representation but there is also another movement involved and this is the movement *between* inequivalent representations and thus between these inequivalent vacuum states. The key question how is this movement described mathematically.

The answer appears to lie in the relationship between the two co-products described by Eq. (45) as Celeghini et al. [13] have already pointed out. It is this feature that allows us to discuss the movement between inequivalent representations. To explain this idea let us define

$$p_\theta = -i\frac{\delta}{\delta\theta}. \tag{48}$$

We can then think of p_θ as a conjugate momentum to the internal degree of freedom θ so that this momentum can be thought of as describing the movement between inequivalent Hilbert spaces. This identification becomes even more compelling once we realise that

$$-i\frac{\delta}{\delta\theta}a(\theta) = [G, a(\theta)] \quad \text{and} \quad -i\frac{\delta}{\delta\theta}\tilde{a}(\theta) = [G, \tilde{a}(\theta)]. \tag{49}$$

Here G is the generator of the Bogoliubov transformation given in Eq. (36). Indeed if we use this generator then for a fixed value of $\bar{\theta}$ we have

$$\exp(i\bar{\theta}p_\theta)a(\theta) = \exp(i\bar{\theta}G)a(\theta)\exp(-i\bar{\theta}G) = a(\theta + \bar{\theta}), \tag{50}$$

which is equivalent to the transformation from $|0(\theta)\rangle \to |0(\theta + \bar{\theta})\rangle$. Furthermore and even more importantly from our point of view the movement is expressed in terms of an inner automorphism of the algebra.[7]

7.1 The Role of Time

Finally I want to return specifically to the question of time. In the bi-algebra we have two elements of time,

$$\bar{T} = \Delta_+ t = t \otimes 1 + 1 \otimes t \quad \text{and} \quad \tau = \Delta_- t = t \otimes 1 - 1 \otimes t. \tag{51}$$

Since $\Delta_+ t$ and $\Delta_- t$ are related, T and τ are not independent. If we regard $\psi_\theta(x,t)$ as the eigenfunction of $\bar{T}$ so that $\bar{T}\psi_\theta(x,t) = t\psi_\theta(x,t)$, then we will represent τ by $-i\partial/\partial t$. In the conjugate representation $\phi_\theta(x,\tau)$ is the eigenfunction of τ, then $\bar{T}$ will take the form $i\partial/\partial t$. Here I am merely exploiting the analogy between the x- and the p-representations where the operators are $(x, -i\partial/\partial x)$ and $(p, i\partial/\partial p)$ respectively.

How are we to understand this structure? When $\bar{T}$ is diagonal, we remain within a single Hilbert spaces parameterised by θ. Its eigenvalue, t, will then be the Schrödinger time. This means the potentialities are changing with time although no irreversible process is taking place. The system remains within this Hilbert space, getting older as it were but not actualising. Bohm [8] calls $\bar{T}$ the implication parameter and regards it as a measure of the age of the system.

Our proposal is that an actual change comes about when a transformation to a different inequivalent vacuum state occurs or, in other words, to a new Hilbert space. Notice that during this transformation, $\bar{T}$ is no longer diagonal implying that Schrödinger time is ambiguous during the transition process. Thus the Schrödinger equation is no longer valid.

A new process unfolds and τ becomes diagonal. This means that the time between inequivalent states is well defined signifying a Bogoliulov transformation is taking place. This would then tie in with the idea of Hiley and Fernandes [39], where they regarded τ as a measure of the time between states, but in this paper it is regarded as a measure of time between inequivalent vacuum states. The fact that θ and its conjugate p_θ do not commute implies that transition between

[7] The inner automorphism is a way of expressing the enfolding and unfolding movement.

inequivalent states is not sharp and requires just the kind of ambiguity we have suggest accompanies the notion of a moment.

This kind of ambiguity is not surprising as quantum theory already tells us that energy and time are complementary variables. So why do we insist on the evolution of a process with a definite energy occurring at a definite instant of time? Mean energy can be conserved but surely to have change, we must have some ambiguity in each moment of time to allow for the creation of a new structure. Here we are exploiting the tension between what has gone with what is to come. We must have a break between the structure that has been and the new structure that is to come. This implies that there are many coexisting instants of time with various weightings in the same moment. In this way not only does quantum theory contain spatial non-locality but that it also contains a "non-locality" in time as proposed by Peres [58].

This discussion suggests a very different view of time evolution. It is not a substitution of one point-like event evolving into another infinitesimal later point-like event. Rather there is an enfolding–unfolding of an extended structure as has been suggested by Bohm [7] when he writes

> Becoming is not merely a relationship of the present to a past that is gone. Rather, it is a relationship of enfoldments that actually are together in the present moment. Becoming is an actuality.

In Umezawa [65] the parameter θ is associated with temperature. Indeed it is tempting to regard θ as the inverse of β, i.e. θ is proportional to the temperature. However I am reluctant to make this a definitive step at this stage because I am very aware of the idea of modular flow introduced by Rovelli [61] and Connes and Rovelli [16] which has some direct relevance to what I am discussing here. These papers have an extensive discussion on the thermodynamic origin of time. They have probed deeper into the mathematical structure implicit in the work I am discussing and have shown how the Tomita–Takesaki theorem provides this connection between time and the thermal evolution of a quantum system. There are clearly connections between this work and the tentative proposals I have outlined in my paper. There is much more to be said but this must be left for another publication.

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The Problem of Time and the Problem of Quantum Measurement

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Dedicated to Malala Yousafzai, for her extraordinary courage and support for the cause of education and knowledge

Abstract Quantum theory depends on an external classical time, and there ought to exist an equivalent reformulation of the theory which does not depend on such a time. The demand for the existence of such a reformulation suggests that quantum theory is an approximation to a stochastic non-linear theory. The stochastic non-linearity provides a dynamical explanation for the collapse of the wave-function during a quantum measurement. Hence the problem of time and the measurement problem are related to each other: the search for a solution for the former problem naturally implies a solution for the latter problem.

1 Why Remove Classical Time from Quantum Theory?

Dynamical evolution in quantum theory is described by the Schrödinger equation. The time parameter which is used for describing this evolution is part of a classical spacetime. By classical spacetime we mean both the underlying spacetime manifold and the gravitational field [equivalently the metric] which resides on it. As we know, the gravitational field is determined by the distribution of classical matter according to the laws of the general theory of relativity. What is perhaps not so well appreciated is that, in accordance with the Einstein hole argument, a physical meaning cannot be attached to the points of the underlying manifold unless a dynamically determined metric tensor field resides on it [1, 2]. Thus one can reasonably assert that classical

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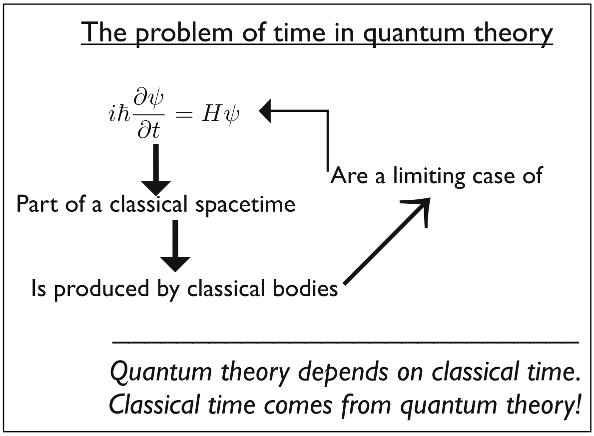

Fig. 1 The circularity of time in quantum theory

spacetime, and hence also the time parameter used to describe evolution in quantum theory, is determined by classical bodies and fields. Now, the dynamics of classical objects is itself a limiting case of quantum dynamics. We see here the circularity of time in quantum theory. Quantum theory depends on classical time. But classical time is well defined only after one considers the classical limit of quantum theory (Fig. 1).

We hence conclude that there should exist an equivalent new formulation of quantum theory which does not depend on classical time. We have argued elsewhere that such a new formulation is a limiting case of a stochastic non-linear theory. The non-linearity, which has to do with gravity, becomes significant in the approach to the Planck mass/energy scale and possibly plays a role in explaining the collapse of the wave-function during a quantum measurement [2, 3].

How should one go about constructing such a reformulation, which we will call Generalized Quantum Dynamics [GQD]? One is foregoing classical time, and along with it, the point structure of a spacetime manifold. A natural possibility is to replace the original spacetime by a non-commutative spacetime. Such a spacetime, and its associated dynamics, called Non-commutative Special Relativity [NSR], was proposed by us in a recent work [4]. In NSR, evolution is described via a "proper time" constructed from taking the Trace over the non-commutative spacetime metric.

As will be described in the next section, a GQD is arrived at by constructing the equilibrium statistical thermodynamics of the underlying NSR [5]. Section 4 then sketches ongoing work on how one possibly recovers classical spacetime and classical matter fields, from considerations of statistical fluctuations around a GQD. This work, when complete, would be central to achieving a fundamental understanding of why superpositions of position states are absent in the macroscopic, classical world (Fig. 2).

One notices that in the transition from a GQD to the classical world, there is no sign of ordinary quantum theory [which depends on classical time]! That recovery

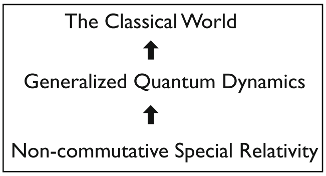

Fig. 2 From a non-commutative spacetime to the classical world, via GQD

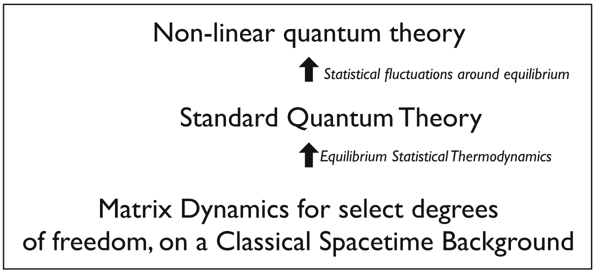

Fig. 3 From trace dynamics to a nonlinear theory, via standard quantum theory

must take place separately, and that is where the connection of the time problem with the measurement problem emerges. In Fig. 2, by classical world is meant a universe which is *dominated* by classical matter fields. Only when such a dominance is given, can one talk of the existence of a classical spacetime; otherwise, the Einstein hole argument will again come into play and forbid the occurrence of the ordinary spacetime manifold. However, not all matter is classical; there is a sprinkling of "quantum" fields, whose dynamics must be derived from first principles, given a classical time.

This is what is achieved by the theory of Trace Dynamics [6–9] which is the classical dynamics of non-commuting matrices on a background classical spacetime. The equilibrium statistical thermodynamics of this matrix dynamics is shown to be the ordinary quantum theory. Statistical fluctuations around equilibrium are shown to lead to non-linear modifications of the quantum theory, and this non-linearity is responsible for collapse of the wave-function during a quantum measurement (Fig. 3). In the limit when the non-linearity becomes strongly dominant, the non-linear theory reduces to classical mechanics.

The connection between the problem of time and the problem of measurement is the following. In our opinion, Trace Dynamics should perhaps not be treated as a stand-alone theory. Because it gives a matrix (equivalently operator) status to matter degrees of freedom, while retaining a point-like structure for spacetime. This will again run into the kind of difficulties implied by the Einstein hole argument: a non-commutative nature for matter degrees is not consistent with a commutative nature for spacetime degrees, unless a dominant classical matter background is available. Thus, a logical starting point for Fig. 3 is to place it at the top of Fig. 2. First one starts from a NSR and derives a GQD, and from there the classical world with a classical time. On this classical world one considers the matrix dynamics for select degrees of freedom (which are sub-dominant and not classical), and this eventually leads to a non-linear quantum theory. The physics which solves the problem of

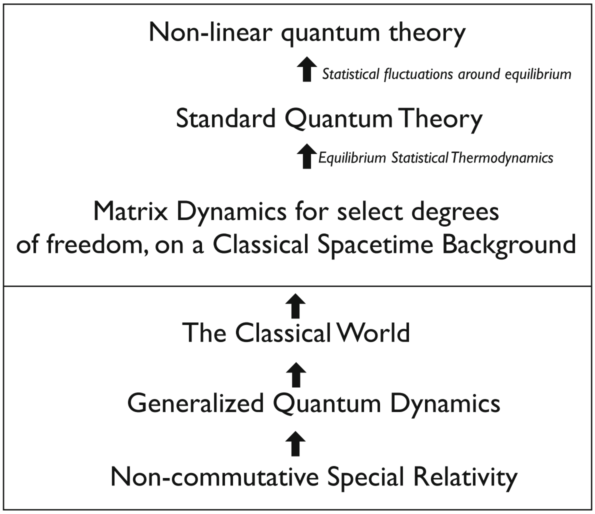

Fig. 4 Solving the problem of time, and the problem of quantum measurement

time in quantum theory is strongly correlated with the physics that solves the measurement problem in quantum theory (Fig. 4).

Figure 4 captures the philosophy of our approach, and the essence of this article. One starts from an NSR and arrives at a GQD. This is described in Sect. 2. The transition from a GQD to the classical world is discussed in Sect. 4 (the logical place would be Sect. 3, but this work is as yet incomplete, and hence its discussion is left till the end). The derivation of ordinary quantum theory and the solution of the quantum measurement problem is discussed in Sect. 3.

2 A Generalized Quantum Dynamics

The mathematical formulation leading up to a GQD [5] is strongly motivated by and based on the theory of Trace Dynamics developed by Stephen Adler and collaborators [6]. The new added element is the assumption of a non-commutative spacetime with operator (equivalently matrix) coordinates $(\hat{t}, \hat{x}, \hat{y}, \hat{z})$, for which a proper time is defined by taking a trace over a line-element:

$$ds^2 = Tr\, d\hat{s}^2 \equiv Tr[d\hat{t}^2 - d\hat{x}^2 - d\hat{y}^2 - d\hat{z}^2]. \tag{1}$$

This line element is invariant under coordinate transformations of the non-commuting coordinates, with their commutation relations being completely arbitrary. Fermionic/Bosonic matter degrees of freedom, described by non-commuting matrices, live on this spacetime, and are respectively characterized by whether they belong to odd/even sector of the graded Grassmann algebra. A classical dynamics of these non-commuting matrix degrees of freedom $\hat{q}^i$ can be constructed to describe evolution with respect to the proper time s: we call this a non-commutative special relativity [NSR]. Thus as in special relativity, a "particle" is assigned a set of four coordinates $(\hat{t}, \hat{x}, \hat{y}, \hat{z})$, a four velocity is defined by taking their derivative with respect to the proper time, and a canonically conjugate four momentum $\hat{p}^i$ is defined by taking the "trace derivative" of the Trace Lagrangian (trace of a polynomial function of coordinates and velocities) with respect to the four velocity. From the Trace Lagrangian, one derives Lagrange equations of motion, a Trace Hamiltonian, and Hamilton's equations, as in ordinary mechanics [4].

The central feature of this matrix classical dynamics, which makes it different from point particle classical dynamics, is that it possesses a novel conserved charge:

$$\hat{Q} = \sum_{r \in B} [\hat{q}_r, \hat{p}_r] - \sum_{r \in F} \{\hat{q}_r, \hat{p}_r\}, \tag{2}$$

where the commutators are for bosonic degrees of freedom, and anticommutators are for fermionic degrees. We note that the commutators/anti-commutators also include pairs such as $[\hat{E}^i, \hat{t}^i]$ and $\{\hat{E}^i, \hat{t}^i\}$, where $\hat{E}^i$ is the energy variable canonically conjugate to $\hat{t}^i$. This conserved charge $\hat{Q}$, which has the dimensions of action, is a consequence of the global unitary invariance of the Lagrangian and the Hamiltonian. It would be trivially zero in the case of point-particle mechanics, but that is not the case here, and its existence is all the more remarkable, because the individual $q - q$, $q - p$, and $p - p$ commutators/anti-commutators are non-zero and completely arbitrary. The existence of this charge plays a central role in the emergence of quantum theory from this underlying level, as we will see shortly.

This matrix dynamics on a non-commutative "flat" space-time is according to us the fundamental dynamics, its symmetries being invariance of the operator spacetime metric under Lorentz transformations, and the global unitary invariance of the Lagrangian.

However this is not the dynamics we observe in our laboratory experiments. Hence one proposes that this dynamics must be coarse-grained over, much the same way that coarse graining over the microscopic degrees of freedom reproduces the statistical thermodynamics of macroscopic systems. Thus we shall develop the statistical thermodynamics of the above classical matrix dynamics, employing entirely conventional methods and techniques of equilibrium statistical mechanics. The classical matrices are analogous to the atoms of a gas, and the coarse-graining is analogous to constructing the thermodynamics of the gas, leading to its approximate macroscopic thermodynamic description. It is remarkable that the thermodynamics of this matrix dynamics will be the sought for GQD, which is a precursor to quantum theory, and in that sense quantum theory is an emergent phenomenon.

One starts by showing that a measure $d\mu$ can be defined in the phase space of the matrix degrees of freedom, and Liouville's theorem holds, demonstrating the conservation of phase space volume. A probability density distribution $\rho(H, T; \hat{Q}, \lambda)$ is defined in the phase space, where the "temperature" T and the matrix λ are respectively the Lagrange multipliers introduced to respect the conservation of the Hamiltonian and the charge $\hat{Q}$. A canonical ensemble is constructed and an equilibrium distribution is arrived at by maximizing the entropy

$$S = -\int d\mu \rho \log \rho \tag{3}$$

subject to the conservation constraints. As anticipated, the equilibrium distribution is given by

$$\rho = Z^{-1} \exp(Tr\lambda \hat{Q} - HT) \tag{4}$$

with Z being the partition function. An important result which can be proved is that the canonical ensemble average of $\hat{Q}$ is of the form

$$\langle \hat{Q} \rangle_{AV} = i_{\text{eff}} \hbar \tag{5}$$

where $\hbar$ is a real positive constant of dimensions of action, and $i_{\text{eff}} = \text{diag}(i, -i, i, -i, \ldots, i, -i)$ such that $Tr\, i_{\text{eff}} = 0$.

Now, the phase space measure, and the canonical average of an observable $\mathcal{O}$ given by

$$\langle \mathcal{O} \rangle_{AV} = \int d\mu \rho \mathcal{O} \tag{6}$$

are invariant under constant shifts of dynamical variables in phase space. This leads to an important Ward identity for a polynomial function $W(z)$ of the dynamical variables z in phase space.

Under the assumptions that T is identified with the Planck scale, and we work much below that scale, and secondly that in the Ward identity the conserved charge $\hat{Q}$ can be replaced by its canonical average $i_{\text{eff}}\hbar$ the Ward identity simplifies greatly, to the following:

$$\langle \mathcal{D} z_{\text{eff}} \rangle_{AV} = 0; \quad \mathcal{D} z_{r\text{eff}} = i_{\text{eff}} [W_{\text{eff}}, z_{r\text{eff}}] - \hbar \sum_s \omega_{rs} \left(\frac{\delta \mathbf{W}}{\delta z_s} \right)_{\text{eff}} . \tag{7}$$

This equation contains the essence of the sought for GQD! Here, z_{eff} is that matrix component of the matrix dynamical variable z which commutes with i_{eff}. Different choices of the polynomial W lead to different important results which contain the mathematical essence of GQD.

If W is chosen to be the operator Hamiltonian H, this Ward identity becomes the Heisenberg equations of motion

$$\langle \mathcal{D} z_{\text{eff}} \rangle_{AV} = 0; \quad \mathcal{D} z_{r\text{eff}} = i_{\text{eff}} [H_{\text{eff}}, z_{r\text{eff}}] - \hbar \dot{z}_{r\text{eff}}. \tag{8}$$

A dot denotes derivative with respect to the proper time s. We recall that the operator time $\hat{t}$ is one of the dynamical variables z.

Next, if we choose $W = \sigma_v z_v$, we get

$$i_{\rm eff}\mathcal{D}z_{r\rm eff} = [z_{r\rm eff}, \sigma_v z_{v\rm eff}] - i_{\rm eff}\hbar\omega_{rv}\sigma_v \tag{9}$$

which gives the emergent canonical commutation rules for the bosonic and fermionic degrees of freedom. Thus we obtain, what we call effective canonical commutators of the canonically averaged matter degrees of freedom. For a bosonic pair

$$[q^\mu, q'^\nu] = 0; \quad [q^\mu, p_\nu] = i_{\rm eff}\hbar\delta^\mu_\nu, \tag{10}$$

while for a fermionic pair

$$\{q^\mu, q'^\nu\} = 0; \quad \{q^\mu, p_\nu\} = i_{\rm eff}\hbar\delta^\mu_\nu. \tag{11}$$

This leads to the desired non-commutativity amongst configuration variables and the corresponding momenta of matter degrees of freedom, at the emergent level. Evidently, there is included now a operator time—energy commutation relation. In anticipation of the standard quantum theory that will eventually emerge from here, we identify the constant $\hbar$ with Planck's constant.

In this sense, a Generalized Quantum Dynamics which does not refer to a classical time emerges from the underlying NSR in the statistical thermodynamic limit [5]. One does have a concept of time-evolution, but this evolution is with respect to the proper time s constructed from the trace of the operator spacetime line-element. In Sect. 4 we will discuss how one possibly proceeds from this GQD to recover classical time.

Furthermore, since at the fundamental matrix level, the theory is Lorentz invariant as shown in [4], if we add another assumption of boundedness of $H_{\rm eff}$ and existence of zero eigenvalue of $\vec{P}_{\rm eff}$ corresponding to a unique eigenstate ψ_0, there exists a proposed correspondence between canonical ensemble average quantities and Lorentz-invariant Wightmann functions in the emergent field theory,

$$\psi_0^\dagger \langle P(z_{\rm eff})\rangle_{\widehat{AV}}\psi_0 = \langle {\rm vac}|P(\mathcal{X}_{\rm eff})|{\rm vac}\rangle.$$

We can also obtain an equivalent Schrödinger picture corresponding to the emergent Heisenberg picture of space-time dynamics. For that, we define

$$U_{\rm eff}(s) = \exp\left(-i_{\rm eff}\hbar^{-1}sH_{\rm eff}\right),$$

such that

$$\frac{d}{ds}U_{\rm eff}(s) = -i_{\rm eff}\hbar^{-1}H_{\rm eff}U_{\rm eff}(s).$$

Then, for a Heisenberg state vector ψ we form Schrödinger picture state vector $\psi_{\rm schr}(s)$, for space-time degrees of freedom

$$\psi_{\rm schr}(s) = U_{\rm eff}(s)\psi,$$
$$i_{\rm eff}\hbar\frac{d}{ds}\psi_{\rm schr}(s) = H_{\rm eff}\psi_{\rm schr}(s).$$

Thus we obtain Schrödinger evolution for the phase-space variables at the canonical ensemble average level.

We note that time and space continue to retain their operator status, although they now commute with each other.

3 Trace Dynamics and the Quantum Measurement Problem

Let us once again have a look at Fig. 4. We have thus far outlined how the lowermost arrow [NSR to GQD] is realized. In the next section we will discuss the next arrow [GQD to the classical world]. For the purpose of the present section, let us assume the classical world as given: matter fields are classical and classical spacetime obeys the laws of general relativity. The universe is dominated by classical matter, which is responsible for the generation of a classical spacetime—in particular there exists a classical time with respect to which evolution can be defined.

In such a classical world, how does one realize quantum theory, so essential to successfully describe the very large number of quantum phenomena observed in the laboratory? The traditional approach of course is to start from a classical dynamics for a system with given configuration variables and their canonical momenta, to replace Poisson brackets by commutation relations, hence introducing Planck's constant, and to replace Hamilton's equations of motion by Heisenberg equations of motion [equivalently the Schrödinger equation].

This approach [and the equivalent path-integral formulation], although extremely successful, ought to be regarded as not completely satisfactory, and "phenomenological" in nature. Because it pre-assumes as given the knowledge of its own limiting case, namely classical dynamics. One should not have to "quantize" a classical theory; rather there should be some guiding symmetry principles for developing a quantum theory, and then deriving classical mechanics from quantum theory as a limiting case. This requirement is in the same spirit whereby one does not arrive at special relativity by "relativizing" Galilean mechanics, or one does not arrive at general relativity by "general relativizing" Newtonian gravitation. The more fundamental theory stands on its own feet, and the limiting case only arises as an approximation—the prior knowledge of the limiting case should not be essential for the construction of the fundamental theory.

An offshoot of arriving at quantum theory by "quantization" is that this leaves us without an understanding of the absence of macroscopic superpositions [the

Schrödinger cat paradox] and of the quantum measurement problem. [Unless of course one accepts the many-worlds interpretation as an explanation, or one believes in Bohmian mechanics as being the correct mathematical formulation of quantum theory].

Trace Dynamics [6] sets out to derive quantum theory from an underlying matrix dynamics where select matter degrees of degrees $\hat{q}^i$ are described by non-commuting matrices [whereas the rest of the matter fields, which dominate the Universe, continue to be treated as classical] and a classical [Minkowski] spacetime is a given. These matrices represent bosonic/fermionic degrees of freedom, depending on whether they belong to the even/odd sector of the graded Grassmann algebra. Like in the previous section, a classical dynamics is constructed for these matrix degrees, with the difference that now time evolution is with respect to a classical time, as opposed to a proper time constructed from the operator spacetime line-element. Given a Trace Lagrangian, one derives Lagrange's equations of motion, a Hamiltonian, and Hamilton's equations of motion. Once again, as a consequence of global unitary invariance there is a conserved charge with dimensions of action, the Adler–Millard charge

$$\tilde{C} = \sum_{r \in B} [\hat{q}_r, \hat{p}_r] - \sum_{r \in F} \{\hat{q}_r, \hat{p}_r\}, \tag{12}$$

where the commutators are for bosonic degrees of freedom, and anticommutators are for fermionic degrees. This time round though, there is no pair such as (E^i, t^i) in the commutators, because time is not an operator. In fact it should be emphasized that the construction in this section proceeds in very much the same fashion as in the previous section, except that a classical spacetime is given. More precisely, the approach adopted in the previous section was developed by us completely following the work of Adler and collaborators as described in this section. This matrix dynamics is Lorentz invariant, under transformation of the ordinary spacetime coordinates.

An equilibrium statistical mechanics for this matrix dynamics is constructed, as before, by maximizing the entropy, and as before it can be shown that the canonical average of $\tilde{C}$ takes the form

$$\langle \tilde{C} \rangle_{AV} = i_{\text{eff}} \hbar. \tag{13}$$

A Ward identity holds, from which one deduces, after replacing the Adler–Millard charge by its canonical average, the standard quantum relations of quantum theory, the Heisenberg equations of motion, and by taking the non-relativistic limit one can write the equivalent description of the dynamics in terms of the Schrödinger equation. The correspondence between canonical ensemble averages and Wightmann functions is proposed as before. In this way one recovers ordinary relativistic quantum field theory, and its non-relativistic limit, from the underlying classical matrix dynamics. This is the step described by the lower arrow in the upper half of Fig. 4.

Something very remarkable is achieved next, by the upper arrow in the top half of Fig. 4. One examines the role played by the statistical fluctuations around equilibrium, for the case of the non-relativistic Schrödinger equation. These are taken into account by revisiting the Ward identity, and instead of replacing $\tilde{C}$ by its canonical average, one replaces $\tilde{C}$ by the canonical average plus correction terms. These correction terms represent the ever-present statistical fluctuations around equilibrium, analogous to the Brownian motion corrections to equilibrium thermodynamics. These fluctuations induce a [linear] modification of the non-relativistic Schrödinger equation, the modifications being caused by the stochastic fluctuations, and if one assumes the fluctuations to be of the white noise type, they can be described by the Itô representation of Brownian motion.

In order to make contact with the quantum measurement problem, one must now make a somewhat ad hoc assumption [which must eventually be justified from a deeper understanding of Trace Dynamics, and perhaps of the possible involvement of gravity]. The point is that the Schrödinger equation, after including fluctuations, turns out not to be norm-preserving. Now one knows from particle number conservation in non-relativistic quantum theory that norm must be preserved during evolution. While norm-preservation must eventually be proved from deeper principles, for now one defines a new wave-function by dividing the original wave-function by its norm, so that the new wave-function preserves norm. This new wave-function obeys a *non-linear* Schrödinger equation while continuing to depend on the statistical fluctuations.

This non-linear Schrödinger equation contains within itself a special class, which coincides with the so-called models of Continuous Spontaneous Localization [CSL] developed by Ghirardi, Rimini, Weber and Pearle [10–13] to explain the absence of macroscopic superpositions and to provide a dynamical explanation for the collapse of the wave-function during a quantum measurement. A prototype of such models is the one particle stochastic non-linear Schrödinger equation [14]

$$d\psi_t = \left[-\frac{i}{\hbar} H\,dt + \sqrt{\lambda}(q - \langle q \rangle_t)dW_t - \frac{\lambda}{2}(q - \langle q \rangle_t)^2 dt \right] \psi_t, \tag{14}$$

where q is the position operator of the particle, $\langle q \rangle_t \equiv \langle \psi_t | q | \psi_t \rangle$ is the quantum expectation, and W_t is a standard Wiener process which encodes the stochastic effect. Evidently, the stochastic term is nonlinear and also nonunitary. The collapse constant λ sets the strength of the collapse mechanics, and it is chosen proportional to the mass m of the particle according to the formula $\lambda = \frac{m}{m_0}\,\lambda_0$, where m_0 is the nucleon's mass and λ_0 measures the collapse strength.

This equation can be used to prove the absence of macroscopic superpositions and solve the quantum measurement problem, and furthermore its predictions for experiments in the mesoscopic regime differ from those of the standard linear Schrödinger equation [9, 13, 15]. This allows the stochastic non-linear quantum dynamics, and hence Trace Dynamics, albeit indirectly, to be confirmed or ruled out by laboratory tests in the foreseeable future. The structure of the equation naturally provides an amplification mechanism—collapse becomes more and more important for larger systems. Furthermore, as can be anticipated by the very nature of its construction [norm-preservation], this non-linear equation dynamically reproduces the Born probability rule for the random outcomes of successive quantum measurements on an observable.

Although more remains to be done [why fluctuations should preserve norm; can the CSL model be uniquely derived from trace dynamics, is the collapse constant λ a new constant of nature, or is it determined by already known fundamental constants via involvement of gravity in collapse], it is unquestionably true that trace dynamics provides a very natural and attractive avenue for understanding the origin of probabilities during quantum measurement, although the Schrödinger dynamics is by itself deterministic. It has to do with the universal presence of statistical fluctuations: if the Schrödinger equation is a thermodynamic approximation to the underlying matrix dynamics, the stochastic non-linear corrections to the Schrödinger equation which are responsible for dynamical collapse, and the origin of probabilities, are a consequence of the unavoidable presence of fluctuations around thermodynamic equilibrium.

It should also be emphasized that the theory of wave-function collapse discussed here [CSL] is a non-relativistic theory, as also is the starting point wherein the connection between trace dynamics and CSL is developed. Despite several attempts, a relativistic theory of wave-function collapse does not yet exist [9]. One clear difficulty is that the norm-preservation condition, which permits the construction of the non-linear stochastic Schrödinger equation, is not necessarily available anymore.

4 From the Generalized Quantum Dynamics to Trace Dynamics

The ideas discussed in this section are a report on work in progress, and hence have not yet taken final shape in terms of a mathematical formulation.

Trace Dynamics takes a classical spacetime as given, and on this given spacetime it considers the matrix dynamics of selected degrees of freedom, for which quantum behaviour is derived. To our understanding, a fully consistent treatment of these select degrees, which is in accordance with the Einstein hole argument, should also associate an operator space-time with these degrees, as discussed in Sect. 2. However, and this is crucial, one makes an *assumption* that this operator spacetime associated with these select degrees of freedom makes a very negligible impact on the classical spacetime produced by the dominant classical matter fields. This assumption is what allows one to proceed with a pre-given classical spacetime while developing trace dynamics. It is possible however, as discussed towards the end of this section, that this assumption may have to be revisited, in order to understand better the fundamental nature of EPR quantum correlations [no signalling, but yet an "action at a distance", as during the collapse of the wave-function].

One must face next the hard problem of understanding the transition from a GQD to a classical world. At a simplistic level, one could take the following approach. One should consider the statistical fluctuations about the equilibrium, at which GQD holds. However, one knows how to do that only in the non-relativistic case. The non-relativistic limit of the GQD cannot be defined by "going to speeds much less than speed of light", since time and space are still operators and there clearly is no classical notion of speed here. However, in the Lorentz transformations which define the invariance of the operator spacetime line-element, the one-parameter invariance along a given direction is defined by the parameter β which in the classical limit is defined as v/c. A non-relativistic limit of GQD can hence be defined by taking the limit $\beta \ll 1$. In this limit one can demand that the fluctuations preserve norm in the Schrödinger equation, in which case the Schrödinger equation is transformed to a non-linear equation, of which the CSL type stochastic equation is a special case. Evolution is described with respect to the proper time s defined from the trace of the operator spacetime element, and the Hamiltonian depends on configuration degrees of freedom which include operator time. As before, one can consider the many-particle macroscopic limit and show that macroscopic superpositions are absent. However, something else extremely significant happens now. The absence of macroscopic superpositions in the matter sector implies the absence of superpositions of different spacetime quantum states corresponding to the operator status of space and time, thereby leading to the *emergence of a classical spacetime*. This is an important lesson, even though yet understood only in the non-relativistic and flat case: the emergence of a classical macroscopic description for matter comes hand in hand with the emergence of classical spacetime—the two are inseparable, and this inseparability is entirely in accord with the Einstein hole argument. If quantum theory is an emergent phenomenon [emerging from trace dynamics], so is classical spacetime an emergent phenomenon [emerging again from the generalized trace dynamics]. The matrix degrees of freedom may well be called the "atoms of spacetime".

A greater challenge is to understand the relativistic case: how is the ordinary spacetime of special relativity to be recovered from GQD, when the norm-preservation condition is not apparently available.

An even greater challenge is to recover classical gravity! When one proceeds from GQD to recover the classical world, not only should the classical spacetime manifold emerge, but there must emerge also classical gravity, which satisfies Einstein equations. Only then can consistency with the Einstein hole argument be ensured. Now GQD by itself has no gravity. Thus it seems we must return again to the lowermost level, and propose that gravity be introduced at the level of matrix dynamics itself, possibly by going from the "flat" operator spacetime element to the "curved" operator spacetime element:

$$ds^2 = Tr\, d\hat{s}^2 \equiv Tr \hat{g}_{\mu\nu} d\hat{x}^{\mu} d\hat{x}^{\nu}. \tag{15}$$

The expectation is that operator Einstein equations can be assumed to hold at the matrix dynamics level, and coarse graining would lead to Einstein equations for the canonically averaged operator metric, self-consistently coupled with the "curved space" GQD which depends on the canonically averaged operator metric. [While of course this idea remains to be developed mathematically, one cannot help noticing the resemblance it bears to the Schrödinger–Newton system studied by Diósi [16] and Penrose [17] and others [9] in the context of studying gravity induced dynamical wavefunction collapse]. From here, one possibly proceeds to study the impact of statistical fluctuations on the equilibrium GQD and canonically averaged Einstein equations. This system is now non-linearly self-coupled, and it could be that one may not have to by hand bring in the assumption of norm-preservation to arrive at a stochastic non-linear CSL type collapse model which obeys the Born rule. In the macroscopic limit, such a non-linear system could be responsible for making both macroscopic objects and the associated spacetime and gravity behave classically. Once such a classical world is recovered, one can implement the construction described in Sect. 3, for arriving at quantum theory starting from trace dynamics for the select degrees of freedom.

Our ideas may provide a useful way out for a better understanding of the apparent "action at a distance" which seems to prevail during the seemingly instantaneous collapse of the wave-function and in EPR-type quantum correlations. Perhaps one must not entirely disregard the implications of the operator space-time metric line-element associated with the [sub-dominant] quantum system, as was done in Sect. 3 while deriving quantum theory on a given classical space-time background. A quantum system always "carries" such a line-element with itself, in the sense that the most fundamental matrix level of description always exists, although we coarse grain it to arrive at what we observe at a higher level. Seen from the viewpoint of this operator line-element, which is non-commutative in nature, there is no point-structure to the spacetime associated with it, no definite light-cone structure, and no pre-given causal order, although it does have operator-level Lorentz invariance. Thus from the point of view of this line-element, "wave-function collapse" can well happen in a unsurprising manner which otherwise appears as "instantaneous action at a distance" from the point of view of the externally given classical spacetime, because the latter possesses a causal structure. But this latter causal structure is not intrinsic to the quantum system under study—its something we choose to employ for our convenience, and then we "cry foul"! Indeed since there is no violation of special relativity in an EPR measurement, the apparent strangeness could simply be a case of trying to describe the process from an inaccurate perspective. Support for our idea also comes from an important recent paper [18], where it has been shown that if one does not assume a predefined global causal order, there are multipartite quantum correlations which cannot be understood in terms of definite causal order and spacetime may emerge from a more fundamental structure in a quantum to classical transition.

In summary, in this work we have addressed the two key fundamental obstacles which still hold us back from getting a better understanding of quantum theory: the problem of time and the problem of quantum measurement. The problem of time suggests that a fundamental description of spacetime which is more compatible with quantum theory than the conventional one is a non-commutative spacetime.

The passage from a non-commutative spacetime to the commutative one that we see around us is through a coarse graining: akin to a passage from microscopic Newtonian mechanics to macroscopic thermodynamics via statistical mechanics. Quantum theory also emerges as the equilibrium description from the underlying level via a coarse graining. Statistical thermodynamics invariably implies Brownian motion fluctuations around equilibrium, and these are what result in quantum theory being an approximation to a stochastic non-linear theory, and dynamically explain the collapse of the wave-function and the emergence of probabilities during a quantum measurement. Thus the problem of time and the problem of quantum measurement are related to each other; their solution possibly springs from the same underlying source. Ongoing laboratory experiments are testing whether quantum theory is indeed an approximation to a non-linear theory, and these experiments also indirectly test the idea that the issues of time and measurement in quantum theory are related to each other.

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A much more detailed bibliography of works relevant to this article can be found in [9].

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Classical and Quantum Probability: The Two Logics of Science

Philippe Blanchard

The true logic of this world is probability theory

J.C. Maxwell

Abstract We review and discuss first briefly the algebraic framework of classical and quantum physics and commutative and noncommutative probability theory. After that we propose a mathematical definition of decoherence sufficiently general to accommodate quantum systems with infinitely many degrees of freedom and give an exhaustive list of possible scenarios that can emerge due to decoherence. We conclude with some messages of quantum science.

1 Introduction

According to Parmenides "Thinking and Being refers to the same" (to gar auto nein estin to kai enai). The main purpose of the Parmenides Foundation is to advance the understanding of complex thinking. Reasoning, creative thinking, making choices are cognition processes. Choice theory [2] explores what it means to act rationally.

Information and Probability are the two main tools making possible a rational decision process. Information can increase in time and sometimes make the probabilistic point of view obsolete. Therefore probability theory has a very immediate and strong connection to our life. The past is gone, the present has measure zero, and the future is random. Classical probability theory helps us to sharpen this guess.

[1]Dedicated with admiration and affection to Rudolf Haag on his ninetieth birthday

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The logical structure of this theory is remarkably simple. To calculate probabilities of events two axioms are used:

– Axiom of additivity for mutually exclusive events, symbolically

$$\text{Prob}(E_1 \text{ or } E_2 \text{ or} \ldots) = \sum_i \text{Prob}(E_i)$$

– Axiom of complementarity

$$\text{Prob}(\text{not } E) = 1 - \text{Prob}(E) .$$

An important exception is encountered in Quantum Mechanics. Let a source S of neutrons be placed behind a screen with two holes A and B. The arrival of the particles at another screen can be measured and the place of arrivals are random; we associate to each region M of the screen the probability $P_A(M)$ that a neutron emitted at S will arrive in M if the hole B is closed. Similarly we consider $P_B(M)$. If both holes are open, one could expect that $P_{A\cup B}(M)$ should be the sum $P_A(M)+P_B(M)$. The experiments show that it is not the case

$$P_{A\cup B}(M) \neq P_A(M) + P_B(M) .$$

This suggests that some sort of multivalued logic should be used and that "False" is not the same as "Not True." Moreover an experiment to determining through which hole the neutron goes interferes so strongly with the neutron that the axiom of additivity is restored [24].

The central thesis of Jaynes's book "Probability: the Logic of Science" (Cambridge UP 2004) occurs in the following passage:

> Our theme is simply: probability as extended logic. The "new" perception amounts to the recognition that the mathematical rules of probability theory are not merely rules for calculating frequencies of "random variables": they are also the only consistent rules for conducting inference (i.e. plausible reasoning) of any kind and we shall apply them in full generality to that end.

For Jaynes the Copenhagen interpretation of Quantum Theory presents problems I will discuss in this paper.

The mathematical formalism and the orthodox interpretation of non-relativistic quantum mechanics are stunningly simple but leave the gate open for alternative interpretations aimed at solving the dilemma lying in the Copenhagen interpretation. Orthodox Quantum Mechanics considers two types of incompatible time evolutions U and R, U denoting the unitary evolution implied by Schrödinger's equation and R the reduction of the quantum state. U is linear, deterministic, local, continuous and time reversal invariant, while R is probabilistic, non-linear, discontinuous, non-local, and irreversible.

We will start in Sect. 2 by motivating the algebraic description of physical systems, starting from operational principles (observables, states, preparation, and measurement), explain how classical and quantum physical systems are described within the algebraic framework, and introduce the basic idea of algebraic probability [4].

In Sect. 3 we will present some basic features of environmental decoherence. The part of the universe that must be considered is the system. The rest of the universe is called the surroundings or the environment. The system and its environment jointly make up the universe.

The algebraic approach allows classical and quantum physics to be treated within one mathematical framework. The classical probability theory has a place in the algebraic framework. This formulation admits a generalization to the so-called Quantum Probability Theory. In this approach Quantum Theory appears as a generalization of classical probability theory, rather than a modification of classical mechanics.

The crucial and only difference between realistic classical and quantum situations is the non-commutativity of the algebra of observables. This unique modification implies the following important consequences [14]:

– Existence of Heisenberg's uncertainty relations.
– Using the GNS-construction to obtain the standard Hilbert space formalism and the superposition principle for pure states leading to quantum interferences.
– Quantum interferences imply that quantum theories can in general not be embedded into classical hidden-variables theories and that the notion of mutually exclusive events becomes blurry.
– Quantum theories are intrinsically non-deterministic, from which it follows that the probabilities of histories (i.e., time ordered sequences of possible events $(P_1, P_2, \ldots P_n)$ do not follow in general $0-1$ laws anymore, even if the state used to predict these probabilities is pure.
– In general no meaningful notion of conditional probability of the event P_j given its past and future exists.
– Open Quantum Systems happen everywhere and all the time and explain the occurrence of decoherence implying the approximate appearance of the classical world from quantum theory seen as a universally valid theory.

Next in Sect. 3.1 open systems in the algebraic framework are discussed and Sect. 3.2 introduces our definition of decoherence. Since it is one of the merits of our approach to decoherence that it permits an exhaustive classification of possible decoherence scenarios, an overview over them will be given in Sect. 3.3.

The concept of decoherence has attracted much attention during recent years, see [9, 12]. Decoherence is a quantum process that dynamically describes the apparent loss of quantum coherence due to the coupling of the system we observe to other degrees of freedom which escape direct observation. Typical examples are given by scattering processes, in which the off-scattered particles and/or radiation are not detected. In such cases quantum correlations between the observed system S and its environment become delocalized in an effectively irreversible manner. These quantum correlations can neither be seen by observations of one or the other system alone, nor interpreted as statistical correlations existing between existing states of local systems. They truly reflect the non-local nature of quantum physics.

Decoherence is of major importance for both theoretical and experimental physics, and has direct implications in chemistry and biology. Moreover, it is linked to fundamental problems such as that of quantum measurement, observation, quantum information theory, and related philosophical issues. Decoherence has been accepted as the mechanism by which classicality emerges in a quantum world. However despite the recent progress and the voluminous literature on decoherence there has been some confusion about its meaning and its full potential in the past.

To provide a rigorous definition a notion of decoherence formulated in the algebraic framework of quantum physics has been proposed in [5]. It captures the essential features of environmental decoherence, generalizes it, and permits a through and rigorous analysis.

2 The Algebraic Framework

Just as Newton invented calculus to describe classical mechanics, von Neumann invented a splendid theory of algebras of operators to describe quantum mechanics. He realized that Hilbert space and the class of linear operators on it provide the correct mathematical framework to formalize the laws of quantum mechanics introduced by Heisenberg, Schrödinger, and Dirac.

2.1 States and Observables

Every physical theory must involve the dual concepts of observables and states. In classical statistical mechanics an observable is a Borel real function $f(q, p)$ on the phase space $\Gamma = \{(q, p) \in \mathbf{R}^{3n} \times \mathbf{R}^{3n}\}$, a state is a probability measure μ on Γ and to each pair (μ, f) consisting of an observable f and a state μ we have associated the probability measure μ_f on $\mathbf{R}$ which takes the Borel set $E \subset \mathbf{R}$ into $\mu[f^{-1}(E)] \equiv P(\mu, f, E)$.This number is the probability that a measurement of f will be in E when the system is in the state μ. Heisenberg proposed the idea that in Quantum Mechanics we should represent an observable by a complex square matrix playing the role of the function f on phase space Γ.

To build up a mathematical theory describing any physical system we must associate mathematical objects to the preparation and measurement processes such that they determine a probability distribution for each pair consisting of a preparation and a measurement. A preparation procedure will be denoted by φ and a measurement by A. The probability distribution predicted by the theory will be denoted by $P(\varphi, A, E)$, E being a Borel set in $\mathbf{R}$.

If $P(\varphi_1, A, \cdot) = P(\varphi_2, A, \cdot)$ for any instrument A, we will define φ_1 and φ_2 as equivalent, in symbols $\varphi_1 \sim \varphi_2$. An equivalence class of the relation is called a state.

Similarly if for two instruments A_1 and A_2, $P(\varphi, A_1, \cdot)$ and $P(\varphi, A_2, \cdot)$ agree for all states $\varphi \in \Sigma$ the instruments are called equivalent. The corresponding equivalence classes of this equivalence relation are called observables; the set of all observables will be denoted by O. For any pair $A \in O$ and $\varphi \in \Sigma$ we write $\varphi(A)$ or $< A, \varphi >$ for the expectation value of the distribution $P(\varphi, A, \cdot)$. See [3].

2.2 C-Algebras*

Now we construct some algebraic structure on O. If we rescale A by a real number λ and thus obtain the observable λA, then we should have $\varphi(\lambda A) = \lambda\varphi(A)$ for any $\varphi \in \Sigma$ since $\varphi(A)$ is an expectation value. Moreover we can square the scale of the apparatus and thus obtain the observable $A^2 \in O$ and more generally A^n for any $n \in \mathbf{N}$. This permits us to calculate the n-th moment $\varphi(A^n)$ of $P(\varphi, A, \cdot)$ from which we can reconstruct $P(\varphi, A, \cdot)$; this is the Hamburger moment problem.

In many situations for $A, B \in O$ (e.g., kinetic and potential energy) we can find a third observable $C \in O$ such that $\varphi(C) = \varphi(A) + \varphi(B)$. In fact we assume that for any pair A, B there is $C \in O$ such that $C = A + B$ (if not O has to be completed). With this structure O becomes a real vector space and each state φ is a linear functional on O. We can introduce a notion a positivity:

$$A \geq 0 \Longleftrightarrow \varphi(A) \geq 0 \qquad \forall \varphi \in \Sigma\,.$$

For the trivial observable $\mathbf{1}$ we require the normalization $\varphi(\mathbf{1}) = 1$. This allows to introduce a norm on O:

$$\|A\| = \sup\{|\varphi(A)| : \varphi \in \Sigma\}\,.$$

Moreover it follows that $|\varphi(A)| \leq \ A\ $ i.e. φ is a continuous linear functional on the normed space O. From $\varphi(\ A\ \mathbf{1} \pm A) \geq 0$ for all $\varphi \in \Sigma$ we conclude that

$$\|A^2\| = \|A\|^2$$

for any $A \in O$.

We assume that O can be embedded in a complex algebra $\mathcal{A}$ where $\mathcal{A}$ has, in general, a noncommutative product $AB \neq BA$ and an antilinear involution $A \to A^*$ satisfying

$$(\lambda A + \mu B)^* = \overline{\lambda} A^* + \overline{\mu} B^*$$
$$(AB)^* = B^* A^*$$

for any $A, B \in \mathcal{A}$ and $\lambda, \mu \in \mathbf{C}$. The natural extension of the positivity condition is

$$\varphi(A^* A) \geq 0 \quad \varphi \in \Sigma \quad A \in \mathcal{A}\,.$$

The norm is defined by $\|A\| = \sup\{|\varphi(A), \varphi \in \sum\}$ and we shall assume the generalization of $\|A^2\| = \|A\|^2$, namely

$$\|A^*A\| = \|A\|^2 \qquad A \in \mathcal{A}$$
$$\|AB\| \leq \|A\|\|B\| \qquad A, B \in \mathcal{A}\,.$$

From $\varphi[(\lambda A + \mathbf{1})^*(\lambda A + \mathbf{1})] \geq 0$ follows

$$\varphi(A^*) = \overline{\varphi(A)}, \qquad \|A^*\| = \|A\|$$

for any $A \in \mathcal{A}$ by a suitable choice of λ. If $\mathcal{A}$ is complete in the norm $\| \ \|$, then $\mathcal{A}$ is called a C^*-algebra.

The observables correspond to the self-adjoint elements, i.e. $A^* = A$ since in this case we have $\varphi(A) \in \mathbf{R} \quad \forall \varphi \in \Sigma$. Σ is therefore defined by

$$\Sigma \simeq \{\varphi \in \mathcal{A}^* | \varphi(A^*A) \geq 0 \quad \forall A, \varphi(\mathbf{1}) = 1\}\,.$$

In this way we have argued in favor of a general mathematical model for states and observables which covers all known physical applications and admits a sufficiently rich structure to facilitate rigorous development. This algebraic framework of physics was proposed by Segal [26] and developed by Haag and Kastler [17]. For an elementary introduction, see [27] and to give full details [13, 16, 25].

There are two important examples of C^*-algebras. Let $\mathcal{H}$ be a Hilbert space and let $\mathcal{B}(\mathcal{H})$ denote the set of all bounded linear operators on $\mathcal{H}$ equipped with the usual operator norm: $\mathcal{B}(\mathcal{H})$ is a C^*-algebra. This example exhausts the class of noncommutative C^*-algebras in the sense that it can be shown that every C^*-algebra is isomorphic to some concrete C^*-algebra on a suitable Hilbert space, which is the content of the GNS construction (see Sect. 2.3). The second important example is provided by the set of all continuous function $C_0(\Omega)$ vanishing at infinity equipped with the sup-norm $\|f\| = \sup\{|f(\omega)| \omega \in \Omega\}$.

2.3 Representations

To recover the traditional Hilbert space framework of quantum physics one uses a representation of $\mathcal{A}$ by bounded operators on a Hilbert space. The Gelfand–Naimark–Segal construction is used to construct representations.

Theorem *Let $\mathcal{A}$ be a C^*-algebra and φ a state on $\mathcal{A}$. Then there exists a Hilbert space $\mathcal{H}_\varphi$ and a representation $\Pi_\varphi : \mathcal{A} \to \mathcal{B}(\mathcal{H}_\varphi)$ where $\mathcal{B}(\mathcal{H}_\varphi)$ is the algebra of all bounded operators on the Hilbert space $\mathcal{H}_\varphi$, such that*

$$\varphi(A) = < \xi_\varphi, \pi_\varphi(A)\xi_\varphi >$$

and such that ξ_φ *is cyclic for* Π_φ*, i.e.* $\overline{\Pi_\varphi(\mathcal{A})\xi_\varphi = \mathcal{H}_\varphi}$*. The representation* Π_φ *on* $\mathcal{H}_\varphi$ *is uniquely determined by* φ *up to unitary equivalence (i.e., physical indistinguishability). For a proof see [11].*

The set of all vector states on $\Pi_\varphi(\mathcal{A})$ *can be interpreted as the set of all states which can be prepared by instruments described by* $\Pi_\varphi(\mathcal{A})$*. Allowing mixtures and thus considering convex combinations of vector spaces it is natural to close the concrete* C^**-algebra* $\Pi_\varphi(\mathcal{A})$ *in a topology where convergence is equivalent to the convergence of expectation values. This topology is the weak operator topology or equivalently the ultraweak topology. Let* $\mathcal{M}$ *be this closure and by the celebrated von Neumanns bicommutant theorem [11] we have*

$$\mathcal{M} = \Pi_\varphi(\mathcal{A})'',$$

with the commutant $\mathcal{M}'$ *defined by*

$$\mathcal{M}' = \{x \in \mathcal{B}(\mathcal{H}) | [x, y] = 0 \quad \forall y \in \mathcal{M}\}$$

for any subset $\mathcal{M} \subset \mathcal{B}(\mathcal{H})$*. Such an algebra is called a von Neumann algebra. The density matrices on* $\mathcal{H}_\varphi$ *correspond to the normal states on* $\mathcal{M}$*.*

If $\mathcal{M} \subset \mathcal{M}'$ $\mathcal{M}$ *is called abelian or commutative and if* $\mathcal{M} = \mathcal{M}'$*,* $\mathcal{M}$ *is called maximally abelian. If the center* $\mathcal{Z}(\mathcal{M})$ *with*

$$\mathcal{Z}(\mathcal{M}) = \mathcal{M} \cap \mathcal{M}'$$

is trivial, i.e. $\mathcal{Z}(\mathcal{M}) = \{\lambda \mathbf{1}\}$*,* $\mathcal{M}$ *is called a factor and* $\mathcal{M}$ *is associated to a pure quantum system.*

Example 1 (Classical System) Let Ω be the phase space which we assume to be locally compact. As C^*-algebra we take $\mathcal{A} = C_0(\Omega)$ where $C_0(\Omega)$ denotes the algebra of all measurable functions vanishing at infinity. As state space Σ we obtain by the Riesz–Markov theorem the set of all probability measures on Ω. The pure states correspond to the Dirac measures $\delta_\omega, \omega \in \Omega$. In these states we know with probability 1 that the system is in ω explaining why classical systems are called realistic or deterministic. For every $\mu \in \Sigma$ we have

$$\mathcal{M}_\mu = \overline{\Pi_\mu[C_0(\Omega)]}^w = L^\infty(\Omega, \mu)$$

acting on the Hilbert space $\mathcal{H}_\mu = L^2(\Omega, \mu)$.

Example 2 (Quantum System with n degrees of freedom) We consider m particles moving on $\mathbf{R}$ and let $Q_1 \dots Q_n$ and $P_1 \dots P_n$ be the canonical self-adjoint position and momentum operators. These operators satisfy the CCR

$$[Q_i, P_j] = i\delta_{ij}\mathbf{1} \qquad [Q_i, Q_j] = [P_i, P_j] = 0 \qquad (**)$$

for all $i, j = 1 \dots n$. Since these operators are not bounded it is convenient to pass to bounded functions of them. To this end we introduce the Weyl operators

$$W(\vec{\alpha}, \vec{\beta}) = U_1(\alpha_1) \dots U_n(\alpha_n) V_1(\beta_1) \dots V_n(\beta_n) e^{-i \frac{<\vec{\alpha}, \vec{\beta}>}{2}}$$

with $\vec{\alpha} = (\alpha_1 \dots \alpha_n)$, $\vec{\beta} = (\beta_1 \dots \beta_n)$ $\qquad \alpha_i, \beta_j \in \mathbf{R}$

$$U_i(\alpha) = e^{i\alpha Q_i} \qquad V_i(\beta) = e^{i\beta P_i} .$$

Then the Weyl form of CCR

$$W(\vec{f})W(\vec{g}) = e^{iIm<\vec{f}, \vec{g}>} w(\vec{g}) w(\vec{f}) ,$$

where $\vec{f} = \vec{\alpha} + i\vec{\beta} \qquad \vec{g} = \vec{\alpha}' + i\vec{\beta}'$ is equivalent to ($**$). As our C^*-algebra $\mathcal{A}$ we take the C^*-algebra generated by all $W(\vec{f})$, $\vec{f} \in \mathbf{C}^n$. We call it the algebra of the CCR over $\mathbf{C}^n$ and denote it by $\mathcal{A}(\mathbf{C}^n)$. In this case the representation problem is easy. Indeed the celebrated Stone-von Neumann theorem asserts that all irreducible representations Π of $\mathcal{A}(\mathbf{C}^n)$ are unitarily equivalent and we have

$$\mathcal{M} = \Pi(\mathcal{A}(\mathbf{C}^n))'' = \mathcal{B}(\mathcal{H})$$

where $\mathcal{H}$ is an infinite dimensional separable Hilbert space. We recover the familiar framework of Quantum Mechanics for systems with finitely many degrees of freedom.

Every normal state on $\mathcal{B}(\mathcal{H})$ is given by $\varphi(A) = tr(\rho A)$ for some unique density matrix ρ acting on $\mathcal{H}$, i.e. $\rho = \rho^* \geq 0 \quad \rho \in \mathcal{B}(\mathcal{H})$ such that $tr(\rho) = 1$. A special case is constituted by the vector states $\varphi(A) =< \psi, A\psi >= tr(P_\psi A)$, with $P_\psi = |\psi >< \psi|$ the one-dimensional projector onto in (ψ).

2.4 Algebraic Probability

2.4.1 Algebraic Classical Probability

We first remind the reader that a classical probability space is a triple $(\Omega, \mathcal{F}, P)$ consisting of a set Ω the sample space, a σ-algebra of subsets $\mathcal{F}$ of Ω and a σ-additive map $P : \mathcal{F} \to [0, 1]$ such that $P(\Omega) = 1$, the probability measure. The points of Ω represent the outcomes of a random experiment, a subset $E \subset \Omega$ lying in $\mathcal{F}$ is called an event; if $\omega \in E$ is the outcome of the random experiment, then we say that the event E has taken place, and $P(E)$ is the probability of E. To formulate the classical probability space $(\Omega, \mathcal{F}, P)$ in an algebraic way we first associate to this triple a von Neumann algebra. Let $\mathcal{A}$ be a commutative C^*-algebra, then a character ω of $\mathcal{A}$ is a state on $\mathcal{A}$ such that $\omega(xy) = \omega(x)\omega(y)\ \forall x, y \in \mathcal{A}$. Let $\Omega(\mathcal{A})$ be the subset of $\mathcal{A}^*$ consisting of all characters on $\mathcal{A}$.

Gelfand's theorem says that with the relative weak* topology inherited from $\mathcal{A}^*$, $\Omega(\mathcal{A})$ is a locally compact Hausdorff space and that $\mathcal{A} \to C_0[\Omega(\mathcal{A})]$ given by $x \to \hat{x}$ with $\hat{x}(\omega) = \omega(x)$ for any character is an isomorphism of $\mathcal{A}$ onto $C_o[\Omega(\mathcal{A})]$; it is called the Gelfand isomorphism.

The basic result of classical algebraic probability theory is the following.

Theorem 2 *Let $(\Omega, \mathcal{F}, P)$ be a classical probability space. Then $\mathcal{A} = L^\infty(\Omega, P)$ is a commutative von Neumann algebra acting on $\mathcal{H} = L^2(\Omega, P)$ and*

$$x \in \Omega \to \varphi(x) = \int_\Omega x(\omega) dP(\omega) \qquad (*)$$

is a faithful normal state on $\mathcal{M}$. Conversely given a commutative von Neumann algebra $\mathcal{M}$ and a faithful normal state φ on $\mathcal{M}$ there exists a classical probability space $(\Omega, \mathcal{F}, P)$ such that $\mathcal{M}$ is isometrically isomorphic to $L^\infty(\Omega, P)$ and φ is given by $(*)$.

2.4.2 Algebraic Quantum Probability

To obtain a noncommutative or "quantum" generalization of some mathematical structure one casts the axioms of the structure in terms of properties of an appropriate commutative algebra of functions with some extra structure. Then one generalizes from commutative to noncommutative algebras to obtain the quantum version. This strategy has been successfully applied to probability (see the following), topology (C^*-algebras can be viewed as a noncommutative version of space), groups generalized by quantum groups and differential geometry.

A state φ is called faithful if $x \geq 0$ and $\varphi(x) = 0$ imply $x = 0$. A state φ is called pure if for any other state φ' such that $\varphi \leq \varphi'$, i.e. $\varphi(x) \leq \varphi'(x)\ \forall x \geq 0$ it follows that $\varphi = \lambda\varphi'$ for some $\lambda \in [0, 1]$. A state φ is called normal if and only if

$$\varphi(\sum_{i \in I} p_i) = \sum_{i \in I} \varphi(p_i)$$

for any family $\{p_i\}_{i \in I}$ of mutually orthogonal projectors.

Definition: A pair $(\mathcal{M}, \varphi)$ consisting of a von Neumann algebra $\mathcal{M}$ and a normal state φ on $\mathcal{M}$ is called a quantum (or noncommutative) probability space.

To identify the algebraic counterparts of the main concepts of classical probability we use the relation between $(\Omega, \mathcal{F}, P)$ and $(\mathcal{M}, \varphi)$, $\mathcal{M} = L^\infty(\Omega, P)$ given in Theorem 2. The random variables (the observables of the random experiment) become associated to the self-adjoint elements of the algebra. For $E \in \mathcal{F}$ the indicator function χ_E corresponds to the projection in $L^\infty(\Omega, P)$. Thus the events $\mathcal{F}$ correspond to the projections $\mathcal{P}(\mathcal{M})$. The σ-additivity of P corresponds to the normality of φ.

Let $E_1, E_2 \in \mathcal{F}$, the relation $E_1 \subseteq E_2$ (i.e., $E_1 \implies E_2$) corresponds to the ordering $p_1 \leq p_2$ of projectors and exclusive events $E_1 \wedge E_2 = \phi$ corresponds to orthogonality $p_1 \perp p_2$. Finally the event "E_1 and E_2," i.e. $E_1 \cup E_2$ corresponds to $p_1 \vee p_2$. A new feature in the noncommutative case is that we can have $[p_1, p_2] = p_1 p_2 - p_2 p_1 \neq 0$ for $p_1, p_2 \in \mathcal{P}(\mathcal{M})$. This non-commutativity has no counterpart in classical probability. In case of $[p_1, p_2] = 0$ the two projections are compatible or noninterfering. In the commutative case all random variables are compatible.

Consider a von Neumann algebra $\mathcal{M}$ describing a physical system together with a normal state φ. Let $\mathcal{M}$ acting on a separable Hilbert space then any maximal abelian subalgebra $\mathcal{N}$ of $\mathcal{M}$, i.e. $\mathcal{N} = \{a\}''$ with a self-adjoint describes a classical probability model. For any bounded Borel function f we have $f(a) = \int_{\sigma(a)} f(\lambda) dP(\lambda)$ where P is the spectral measure of a.

If we consider another abelian algebra $\mathcal{N}_0 \subseteq \mathcal{M}$ different from $\mathcal{N}$, then the two classical probability models are in general not the same. Noncommutating observables cannot be represented on the same probability space as marginals of a single distribution. However if an observable is a member of two different maximal abelian subalgebras, it produces the same distribution but it is represented in each case by different random variables on different sample spaces. This property is called contextuality.

2.4.3 Bell's Inequality

Let $\mathcal{A}$ be a C^*-algebra and let $p, q, p', q' \in \mathcal{P}(\mathcal{A})$. Assume that $[p, p'] = [q, q'] = 0$ and define the self-adjoint operators $a = 2p - \mathbf{1}$ $b = 2q - \mathbf{1}$ $a' = 2p' - \mathbf{1}$ $b' = 2q' - \mathbf{1}$ which all take the value ± 1. Next define

$$c = a(a' + b') + b(b' - a')$$

then we have

$$c^2 = 4 + [a, b][a', b'] = 4 + 16[p, q][p', q'] \, .$$

If the observables p, q, p', q' are described by a classical probability distribution (and therefore generate an abelian algebra), we find for any state φ on $\mathcal{A}$

$$|\varphi(c)| \leq 2$$

which is Bell's inequality. In a system with quantum character the Bell's inequality is violated, i.e. there exists a state ψ such that $|\psi(c)| > 2$. The choice of p, q and p', q' has to be non-contextual in the sense that the random variables representing p' and q' do not depend on whether p or q is being measured. The violation of Bell's inequality is experimentally testable and implies that there exists no underlying classical probability model whatsoever.

3 Open Systems and Decoherence

3.1 Open Systems

We consider a closed quantum system described by a von Neumann algebra $\mathcal{N}$ containing the observables of the system together with a reversible time evolution given by a one parameter group $\{\alpha_t\}_{t\in\mathbf{R}}$ of automorphisms. We consider a subsystem, described by a subalgebra $\mathcal{M} \subseteq \mathcal{N}$ including the observables of the subsystem. In this situation we can define the reduced dynamics by

$$T_t(x) = Eo\alpha_t(x) \qquad x \in \mathcal{M} \quad t \geq 0 \, .$$

E being a normal conditional expectation $E : \mathcal{N} \to \mathcal{M}$ onto $\mathcal{M}$. T_t is the time evolution an observer whose experimental capabilities are restricted to the observables of $\mathcal{M}$ would witness. In general T_t is no longer reversible.

3.1.1 System-Environment Models

In this class of models $\mathcal{N}$ is given by the tensor product $\mathcal{N} = \mathcal{M} \otimes \mathcal{M}_0$ acting on $\mathcal{H} \otimes \mathcal{H}_0$, where $\mathcal{M}_0$ describes the environment. The time evolution of the system and environment is Hamiltonian, i.e. $\alpha_t(x) = e^{itH} x e^{-itH}$ with

$$H = H_1 \otimes \mathbf{1} + \mathbf{1} \otimes H_2 + H_I$$

where H_1 and H_2 are the Hamiltonians of the system and environment. The conditional expectation E_ω is given with respect to a reference state of the environment, i.e.

$$\varphi \otimes \omega(x) = \varphi[E_\omega(x)]$$

for all $x \in \mathcal{N}, \varphi \in \mathcal{M}_*$. Let ρ be a density matrix on $\mathcal{H}$, the reduced time evolution is given by

$$T_t(\rho) = tr_{\mathcal{H}_o}[e^{-itH}(\rho \otimes \omega)e^{itH}] \, .$$

The reduced dynamics $T_t : \mathcal{M} \to \mathcal{M}$ is a normal completely positive linear map for every $t \geq 0$.

In many physically relevant situations it is a good approximation to assume that T_t is Markovian (memory free)

$$T_t \circ T_s = T_{t+s} \quad \text{for all } s, t \geq 0 \, .$$

In this case $\{T_t\}_{t\geq 0}$ is a quantum dynamical semigroup and T_t can be described in infinitesimal form through its generator Z which is defined by

$$Zx = \lim_{\epsilon\to 0} \frac{1}{\epsilon}[T_\epsilon(x) - x] \ .$$

We formally write $T_t = e^{tZ}$.

3.2 Decoherence in the Algebraic Framework

We are now able to introduce our notion of decoherence.

Definition $\{T_t\}_{t\geq 0}$ is said to display decoherence if there is a decomposition

$$\mathcal{M} = \mathcal{M}_1 \oplus \mathcal{M}_2$$

such that

1. $\mathcal{M}_1$ is a T_t-invariant von Neumann subalgebra
2. $\mathcal{M}_2$ is a T_t-invariant and *-invariant ultraweakly closed subspace of $\mathcal{M}$
3. $\mathcal{M}_1$ is the largest von Neumann subalgebra on which the restriction of $\{T_t\}_{t\geq 0}$ extends to an automorphism, i.e. $T_t \restriction \mathcal{M}_1 = \beta_t$, $\{\beta_t\}_{t\in\mathbf{R}}$ being a group of automorphisms of $\mathcal{M}_1$.
4. $\lim_{t\to\infty} \varphi(T_t(x)) = 0$ for any $x \in \mathcal{M}_2$ and any normal state φ on $\mathcal{M}$.

Let us interpret this definition. For every observable $x \in \mathcal{M}$ there exists a unique decomposition $x = x_1 + x_2$, $x_1 \in \mathcal{M}_1$ $x_2 \in \mathcal{M}_2$ such that $\lim_{t\to\infty} \varphi(T_t(x_2)) = 0$ for all normal states. In other words the $\mathcal{M}_2$ part is beyond experimental resolution after decoherence has taken place. For very large times the system behaves effectively like a closed system described by the von Neumann algebra $\mathcal{M}_1$ with reversible time evolution $\{\beta_t\}_{t\in\mathbf{R}}$. It is therefore natural to call $\mathcal{M}_1$ the algebra of effective observables. By looking at the structure of $\mathcal{M}_1$ and $\{\beta_t\}_{t\in\mathbf{R}}$ we may classify different scenarios of decoherence. Sufficient conditions for decoherence have been established [8, 19].

3.3 Scenarios of Decoherence

Let a physical systems display decoherence with an algebra $\mathcal{M}_1$ of effective observables and reversible time evolution $\{\beta_t\}_{t\in\mathbf{R}}$ on $\mathcal{M}_1$. By looking at the structure of $\mathcal{M}_1$ and $\{\beta_t\}_{t\in\mathbf{R}}$ we will now give an exhaustive list of possible emergent scenarios due to decoherence.

- *Pointer states:* If $\mathcal{M}_1$ is commutative and $\{\beta_t\}_{t\in\mathbf{R}}$ is trivial (i.e., $\beta_t = \mathbf{1}$ for all t), we speak of pointer states . $\mathcal{M}_1$ contains the observables associated to the pointer positions of the apparatus. The commutativity implies that we obtain a classical probability over the pointer positions and the triviality of the dynamics means that the pointer positions are immune to the interaction with the environment.

  If $\mathcal{M}$ acts in a separable Hilbert space and $\mathcal{M}_1$ is generated by minimal projections $\{p_n\}_{n\in\mathbf{N}}$, then $\sum_n p_n = \mathbf{1}$ and

$$\mathcal{M}_1 = \bigoplus_{n=1}^{\infty} p_n\mathcal{M} \quad \mathcal{H} = \bigoplus_{n=1}^{\infty} p_n\mathcal{H}_n \ .$$

  Each $x \in \mathcal{M}_1$ can be written as $x = \sum_{n=1}^{\infty} p_n x p_n$ and the dynamics is automatically trivial. See [5] for examples. Finally if $\mathcal{M} = \mathcal{B}(\mathcal{H})$, then $\mathcal{M}_1$ is always generated by minimal projections and therefore we will always obtain discrete pointer states if we start from a quantum system with finitely many degrees of freedom. To get continuous pointer states we must therefore start from an infinite system. In [6] we present a Spin-Boson model of an infinite spin $\frac{1}{2}$-chain coupled to a Bose gas for which decoherence takes place and we show that $\{\beta_t\}_{t\in\mathbf{R}}$ is trivial and $\mathcal{M}_1$ is given by

$$\mathcal{M}_1 = L^{\infty}(C, \mu)$$

  where C denotes the Cantor set and μ is a probability measure on C.
- *Superselection Rules* If $\mathcal{M}_1$ is noncommutative but has a nontrivial center, i.e. $\mathcal{Z}(\mathcal{M}_1) = \mathcal{M}_1 \cap \mathcal{M}_1' \neq \mathbf{C1}$, we speak of superselection rules and $\mathcal{Z}(\mathcal{M}_1)$ contains the superselection observables. They are simple to describe if $\mathcal{M}_1$ is totally atomic: Then $\mathcal{M}_1$ can be written as a (countable if $\mathcal{H}$ is separable) direct sum of type I-factors

$$\mathcal{M}_1 = \bigoplus_{i=1}^{\infty} \mathcal{B}_i$$

  and

$$\mathcal{H} = \bigoplus_{i=1}^{\infty} \mathcal{H}_i$$

  with $\mathcal{B}_i$ acting on $\mathcal{H}_i$. Relative phases between different sectors $\mathcal{H}_i$ and $\mathcal{H}_j$ $i \neq j$ are unobservable: Let $\psi_i \in \mathcal{H}_i$ and $\psi_j \in \mathcal{H}_j$ be normalized, then

$$< \psi_i, x\psi_j >= 0 \quad \text{for any } \ x \in \mathcal{M}_1 \ .$$

Define $\psi_\alpha = \psi_i + e^{i\alpha}\psi_j$ with $\alpha \in \mathbf{R}$. It follows that $< \psi_\alpha, x\psi_\alpha >$ is independent of α. In other words if $\psi = \alpha_1\psi_i + \alpha_2\psi_j$ where $|\alpha_1|^2 + |\alpha_2|^2 = 1$, then the state $\omega_\psi(x) =< \psi, x\psi >$ and $\omega(x)$ with

$$\omega(x) = |\alpha_1|^2\omega_{\psi_i}(x) + |\alpha_2|^2\omega_{\psi_j}(x)$$

agree, so superposition between different superselection sectors corresponds to mixtures: superselection observables are classical, taking a definite value in each sector and therefore correspond to realistic properties associated to superselection observables. See [5] for examples showing discrete as well as continuous superselection rules.

- *New Quantum System* If $\mathcal{M}_1$ is again a factor, then after decoherence the system still has a pure quantum character. However the new pure quantum system may be smaller than the original system. An example showing this behavior is given in [7]. In this scenario the pair $(\mathcal{M}_1, \{\beta_t\}_{t\in\mathbf{R}})$ describes a pure quantum system which is immune to decoherence and hence it might be useful in quantum information theory where it is necessary to have systems available which are immune to decoherence.
- *Classical system* If $\mathcal{M}_1$ is commutative, then decoherence induces a classical structure and the system can be described in terms of classical probability. However a classical physical system has more structure. For example, the underlying probability space and the dynamics associated to $\{\beta_t\}_{t\in\mathbf{R}}$ need not come from a classical dynamical system or more precisely from the Hilbert space representation of a classical dynamical system with a time evolution given by a flow on phase space Ω. We refer to [5] and [21] for details and examples.
- *Ergodicity* If $\mathcal{M}_1$ is trivial, i.e., $\mathcal{M}_1 = \{\mathbf{C1}\}$ we say that decoherence induces ergodic properties, for example see [2].

4 Some Messages of Quantum Science

Quantum Theory has enjoyed an incredible number of success since its formulation in the first third of the twentieth century. Today it is considered as the most fundamental physical theory available, has explained the structure and interactions of atoms, nuclei and elementary particles and given rise to many revolutionary technologies. At the same time it is amazing that a lot of physicists still question whether or not Quantum Mechanics is the ultimate description of nature [20].

In classical mechanics after fixing the equations of motion, the initial and final conditions are not independent and only one can be chosen. In Quantum Mechanics the relationship between initial condition and final condition after measurement can be one to many. Indeed two identical particles can exhibit different properties under identical measurements. The assumption of time asymmetry claims that measurements have consequences only for the past of the system. Aharonov, Bergmann, and Lebowitz showed that the information obtained from measurement was also relevant for the past, not just for the future of the system [1]. This result suggests that two identical particles stop to be identical if we use information coming from the future.

When the laws of Quantum Mechanics are directly applied to macroscopic objects contradictions arise, the most famous case being the Schrödinger cat, which is a superposition of the "dead" and "alive" state of the cat. The program of environment induced decoherence provides an answer to these problems. It contends that quantum mechanics is universally valid but one has to take into account that macroscopic systems are strongly interacting with their environment. This interaction implies that the time evolution of the system becomes irreversible and this irreversibility is able to dynamically generate classical properties. The entanglement between system and environment limits the superposition principle.

The algebraic framework is an alternative formulation of quantum physics which is more general than the traditional Hilbert space formulation; in particular, it permits the rigorous discussion of systems with infinitely many degrees of freedom occurring in quantum field theory. This approach shows in which way quantum probability generalizes classical probability and how the algebraic formulation of physics can be understood in terms of quantum probability.

In [23] Penrose distinguishes in quantum mechanics two kinds of mysteries, which he calls X (like in paradox) and Z (like puzzles) mysteries. The X mysteries are the ones resulting from a theory that is not confined to our classical macroscopic world, and which we have finally to accept. Z mysteries suggest that something is missing. Decoherence is an X mystery. For Omnés [22] "decoherence is a mystery because it is intimately related with the deepest mystery of physics, namely the relation between mathematical theories and empirical reality. Newtonian physics led mankind to assume an identity between them, or rather a strict correspondence. . . ... Perhaps we should also say that our theories are human constructs. Our task is to make sure of their agreement with facts and their logical consistency."

In 1932 von Neumann (see [20]) introduced an assumption about the reduction of the wave function (R-process) within the Born probabilistic interpretation of quantum mechanics: As a consequence of the measurement of one observable A the state vector ψ characterizing the measured system undergoes an instantaneous change, a discontinuous quantum jump R; after the measurement, the system is described by one of the possible eigenfunctions of A: The R-process describes the problem of objectification that can be loosely defined as the relation between quantum theory and the uniqueness of empirical reality. One possible answer is to say that quantum theory is basically probabilistic and therefore should not be concerned with the actualization of a definite datum.

Many people believe in some physical objectification process [20].

Decoherence exists, has been observed, and is extremely efficient in eliminating the superposition principle [15, 18]. Decoherence suppresses every observable consequences of a quantum superposition

$$\psi = \sum_i \lambda_i \psi_i \qquad \sum |\lambda_i|^2 = 1 \qquad < \psi_i, \psi_j >= \delta_{ij}$$

replacing the pure state ψ by the density matrices

$$\psi \approx \rho_\psi = \sum_{i=1}^{n} |\lambda_i|^2 P_{\psi_i}$$

a result that is perfectly valid for all practical purposes (FAPP). After decoherence the state is practically a diagonal matrix $(p_1 \ldots p_n)$, i.e. a classical probability measure, the $p_i = |\lambda_i|^2$ being the respective quantum probabilities for the n possible results. Suppose now that the R process enters and ends with the pointer in position 1. Quantum mechanics must start again from the diagonal matrix $(0, 1, \ldots, 0)$. The R process has to be a random dynamical process with giving final probabilities.

The FAPP equivalence between ψ and ρ_ψ raises the question of the meaning of extremely small probabilities. Borel [10] maintained, as a unique principle for the interpretation of probability theory, that an event with a too small probability should be considered as never happening. Too small a probability means getting us outside empirical science. This principle can also be used to define the algebra $\mathcal{M}_1$ of effective observables in Sect. 3.

We are at the end of our journey. We have seen that Quantum Theory caused the greatest revision in our conception of the nature of the physical world since Newton and is one of the most outstanding intellectual achievements of the last century. Compared with the quantum revolution the great discoveries of special and general relativity can be viewed nevertheless as very clever and extremely deep variations on themes of classical physics.

Decoherence is a way to understand the Classical Physics as emergent within the Quantum formalism. The Classical world C sits no longer in opposition to the Quantum one Q but is demanded by Q. Decoherence can serve to make some quantum probabilities look more like classical probabilities but it does not make the same. Measurements are a chain of correlated consequences and decoherence does not explain why a particular event is realized on a particular measurement of a particular measuring process. All classical notions (events, facts, …) are idealizations and apply only FAPP.

It is worth noting that statements FAPP are not the privilege of Quantum Physics. Physicists idealize almost always and, for example, phase transitions in statistical physics occur only FAPP after taking the thermodynamical limit. The spectacular success of Quantum Theory in providing accurate predictions is indisputable and undisputed. For this reason there is almost no doubt that the computational part of the orthodox interpretation is essentially correct. Copenhagen offers therefore a pragmatic though not fully logical approach. The classical world C of our everyday experience is a projected shadow in a quantum universe. C is only Q viewed through the lens of decoherence. We recover as metaphor of present scientific knowledge again the Plato's cave!

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Present and Future in Quantum Mechanics

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Abstract After a short overview over the questions of time, permanence, and change in the philosophical tradition, the concept of time in physics is discussed. The fact is emphasized that the usual real parameter t is not sufficient, in some cases, to solve conceptual problems of physics. Sometimes it becomes necessary to consider the "full" concept of time with present, past, and future. This can be seen already with the concept of objectivity, which is intimately connected with predictions. It comes out very clearly especially in probability considerations: The concept of probability can be best understood when it is identified with predicted relative frequency. This insight is used to recall a solution of the problem of the "time arrow" in statistical thermodynamics. It is applied mainly to quantum mechanics, where it is shown that there are rather simple solutions, e.g., to the problem of the "collapse of the wave function" and the "EPR" problem; there the "spooky actions at a distance" are unmasked to be no actions at all.

What makes the present so particularly interesting that a whole volume of papers is devoted to it? Let me take the key word "present" as shorthand for time in general. The structure of time as present, past, and future tends to be "forgotten" in the natural sciences, especially in physics.

Why is that?

1 Time, Space, and Change

Science is interested in "laws of Nature," i.e., in structures we can describe, and which for that very reason have to be in a sense permanent, "eternal" in the extreme case. But still we want to describe changes; we want to be able to predict what will happen under certain circumstances, etc. As long as physics, e.g., is successful, it uses equations to describe changes, movements. Such equations contain a parameter

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usually named *t* that is supposed to represent time. This *t* is a real number parameter; we could imagine it representing in reality the position of the hand on a clock, or of a planet on the sky. So we actually use a spatial representation of time in order to be able to deal with it in an equation in order to describe change.

But the equation itself does not change; it is supposed to be of "eternal" validity. A law of nature is a "general" law; what it describes is change that can occur at any time. In this sense an equation, a physical theory as a whole, is "objective," i.e., it depends neither on the individual who applies it nor on the time when it is applied. That is why time appears only in the form of that notorious parameter *t*. Usually we imagine time as represented by a horizontal line on paper with an arrow to the right: Again a spatial representation of time. We cannot expect that this line represents all aspects of time. And apparently there is no feature of a horizontal line nor of equations that could stand for the present—and for that much, there is no feature for past and future either.

Physicists tend to consider the aspect of time we describe as present, past, and future as something that isn't objective: Thus, in the language of physicists, it is subjective. And consequently physics is not supposed to deal with it.

Still, occasionally there arises a necessity to talk about aspects of time that cannot be represented by that parameter *t*; we will consider such occasions later. In order to have an opportunity to talk and to think of the structure of time in view of the objectivity of physics, physicists grew accustomed to using words like "flow of time" or "arrow of time." But when you look a bit closer you can see that this is rather misleading: A river can flow (through space!) *in* time, as we are accustomed to say; so time cannot be the stuff that flows as well. But even that image of a river flowing in time seems to be rather queer: How can anything be *in time*? Is time something like a container?—We see that even this seemingly harmless metaphor transforms temporal relations into spatial ones thereby distorting them.

Thus, even though for many purposes a spatial representation of time is practical, we have to admit that the character of time is fundamentally different from the character of space. This tends to be forgotten in the context of natural science—hence "the forgotten present."

2 Time in Ancient Philosophy

2.1 Parmenides

Time was a favorite subject of philosophy since philosophy began in sixth century (BC) Greece. Parmenides, after whom our hosting society is named, gives that discussion a rather strong start in denying that time really existed at all; time belongs, according to Parmenides, to "δόξα" (*doxa*), to the realm of appearance and illusion. I think it is important for understanding the history of philosophy to see the truth that is in the enigmatic text of Parmenides. Let me give you a short and rather bold account of what I think I understand of it: The subject Parmenides is talking about is "being," in Greek he says "εἶναι," "ἐστίν," or "ἐόν." A characteristic sentence in his didactic poem is, to my advice: "οὔτε γάρ ἂν γνοίης τό γε μὴ ἐὸν (οὐ γὰρ ἀνυστόν) οὔτε φράσαις."[1]: "For you cannot know what is not – that is impossible – nor utter it." This sounds rather self evident, at first glance. But Parmenides—and following him, a lot of classical philosophy—took "being" very seriously as being in eternal presence, as not at all changing; and hence, many concluded like Parmenides that it is impossible to think or to talk in truth of anything but the eternal. Thus the question how to describe change became a major problem for Greek philosophy.

This problem seems to be solved in modern physics by the introduction of equations that include the parameter t already mentioned. Still from time to time we hit on questions in physics which show that the problems of time are not solved entirely with equations.

Now this touches philosophy. Time is a major theme through all of philosophy. The article "Time" in the Historical Encyclopaedia of Philosophy [2] extends over 78 columns. So there is no chance to cover this here. But let me indicate a few highlights.

2.2 Plato

We started with Parmenides, who practically denies the "real" existence of change, i.e., of time altogether. Plato greatly esteemed Parmenides—he devoted one of his dialogues to Parmenides' philosophy; but still, Plato tries to cope with the problem of change. He gives his famous definition of time:

> εἰκὼ δ' ἐπενόει κινητόν τινα αἰῶνος ποιῆσαι, καὶ διακοσμῶν ἅμα οὐρανὸν ποιεῖ μένοντος αἰῶνος ἐν ἑνὶ κατ' ἀριθμὸν ἰοῦσαν αἰώνιον εἰκόνα, τοῦτον ὃν δὴ χρόνον ὠνομάκαμεν. (*Plato*, *Timaeus 37d*)

> In English: But he took thought to make, as it were, a moving likeness of eternity; and, at the same time that he ordered the Heaven, he made, of eternity that abides in unity, an everlasting likeness moving according to number—that to which we have given the name time.[2]

[1]Parmenides [1]. I am quoting in Greek knowing that many readers will not readily understand the quotation. But I want to emphasize the importance of referring to the original text since every translation is an interpretation. If one really wants to find out what the text says there is no better way than studying the original.

[2]Cornford [3], p. 98.

Plato uses "moving" (or, more literally, "going") in order to define time: A rather unusual approach, in our modern thinking, since we would consider time as more fundamental than movement. But Plato with his approach seems to be closer to modern physics than to modern philosophy, especially with the other ingredient of his definition, namely number: For time he considers essential counting the revolutions—of the sun or of planets. Thus he apparently formed already the image of time we use now in physics. Heisenberg, in his dialogue-book "Der Teil und das Ganze,"[3] suggests that Plato had already anticipated in his Timaeus dialogue the fundamental nature of mathematics for physics as we understand it today.

Plato describes his thoughts, not quite as mythical as Parmenides, but still in the form of a myth: the "He" he is talking about is the Demiurge, the god-like craftsman who composes the universe from primitive materials. This seems to be Plato's way of describing the structure of the universe, in telling a tale.

2.3 Aristotle

You will notice the contrast between Plato's text and texts we have from Aristotle's works. This contrast is partly accounted for by the difference between the addressees: Plato writes for a broader public, and he emphasized that it is impossible to write the truth directly—hence, I think, the myth. Aristotle's text, by contrast, might have been notes for his lectures or notes taken by one of his students from a lecture: Short, very sober outlines of a line of argument. But it is, in my impression, not only the difference in addressees, but also a difference in the style of thinking between Plato and Aristotle: Plato was an aristocrat, mainly interested in good governance for his state, and a poet; whereas Aristotle was, from his roots, a biologist, a scientist, who later led a large research institution—to put it in modern terms.

Aristotle introduces time, much like Plato does, dependent on change (κίνησις). And change, in Aristotle's system, is derived from possibility (δύναμις), which in turn is part of the fundamental pair, according to Aristotle, actuality–possibility (ἐνέργεια—δύναμις). Aristotle defines change, depending on possibility, as follows: "ἡ τοῦ δυνάμει ὄντος εντελέχεια, ᾗ τοιοῦτον, κίνηςίς ἐστιν." ("The actuality of that which potentially is, *qua* such, is change." Phys. 201a10–11).[4] I am not going to dwell any further on that very intricate formulation and its afterlife in Aristotle exegesis. The essential feature of Aristotle's argument is the fundamental role of possibility, not of time. But let me still quote his definition of time: "τοῦτο

[3]Heisenberg [4].

[4]Hussey [5], p. 2; In German cf. Wieland [6], 298^{25}.

γὰρ ἐστὶν ὁ χρόνος, ἀριθμὸς κινήσεως κατὰ τὸ πρότερον καὶ ὕστερον.” (“For that is what time is: a number of change in respect of the before and after.” Phys. 219b2–3).[5] So we see that his definition is closely akin to Plato’s.

But Aristotle does more than that. He talks about the present as well! He introduces his chapter on time with a very interesting consideration, whether time “is” at all:

> Ἐχόμενον δέ τῶν εἰρημένων ἐστὶν ἐπελθεῖν περὶ χρόνου· πρώτον δὲ καλῶς ἔχει διαπορῆσαι περὶ αὐτοῦ καὶ διὰ τῶν ἐξωτερικῶν λόγων, πότερον τῶν ὄντων ἐστίν ἢ τῶν μὴ ὄντων, εἶτα τίς ἡ φύσις αὐτοῦ. ὅτι μὲν οὖν ἢ ὅλως οὐκ ἔστιν ἢ μόλις καὶ ἀμυδρῶς, ἐκ τῶνδέ τις ἂν ὑποπτεύσειεν. τὸ μὲν γὰρ αὐτοῦ γέγονε καὶ οὐκἔστιν, τὸ δὲ μέλλει καὶ οὔπωἔστιν. ἐκ δὲ τούτων καὶ ὁ ἄπειρος καὶ ὁ ἀεὶ λαμβανόμενος χρόνος σύγκειται. τὸ δ’ ἐκ μὴ ὄντων συγκείμενον ἀδύνατον ἂν εἶναι δόξειε μετέχειν οὐσίας. Aristotle, Physics 217b29–218a3

> English: “After what has been said, the next thing is to inquire into time. First, it is well to go through the problems about it, using the untechnical arguments as well [as technical ones]: whether it is among things that are or things that are not, and then what its nature is. That it either is not at all or [only] scarcely and dimly is, might be suspected from the following considerations. Some of it has been and is not, some of it is to be and is not yet. From these both infinite time and any arbitrary time are composed. But it would seem to be impossible that what is composed of things that are not should participate in being.”[6]

This is a nice specimen of the style of Aristotle’s texts. In his system there follows a longer consideration where he gives the definition of time quoted above, and then he adds that in this case he means the number that is counted, not the number by which we count. For our consideration of the forgotten present, the only part that seemed to me to be helpful is his question whether time, being present, past or future, “is” at all.

2.4 Augustine

We find that very same question in Augustine’s famous essay on time in his “Confessions.” But Augustine’s solution is quite different from Aristotle’s: He finds past and future “being” in my memory or in my expectations, respectively. Augustine, therefore, is considered the first philosopher of “Subjectivity.” The following quotation from “Confessions” gives a good idea of Augustine’s thinking and writing: In his philosophical argument he is, at the same time, praying, arguing with God. And one still sees his tradition of a classical rhetorician[7]:

[5] Hussey [5] p. 44.

[6] Hussey [5], p. 41.

[7] Augustinus [7] ch. 11.18.23.

> "Sine me, domine, amplius quaerere, spes mea; non conturbetur intentio mea. si enim sunt futura et praeterita, volo scire, ubi sint. quod si nondum valeo, scio tamen, ubicumque sunt, non ibi ea futura esse aut praeterita, sed praesentia. nam si et ibi futura sunt, nondum ibi sunt, si et ibi praeterita sunt, iam non ibi sunt. ubicumque ergo sunt, quaecumque sunt, non sunt nisi praesentia. quamquam praeterita cum vera narrantur, ex memoria proferuntur non res ipsae quae praeterierunt, sed verba concepta ex imaginibus earum quae in animo velut vestigia per sensus praetereundo fixerunt. pueritia quippe mea, quae iam non est, in tempore praeterito est, quod iam non est; imaginem vero eius, cum eam recolo et narro, in praesenti tempore intueor, quia est adhuc in memoria mea. utrum similis sit causa etiam praedicendorum futurorum, ut rerum, quae nondum sunt, iam existentes praesentiantur imagines, confiteor, deus meus, nescio. illud sane scio, nos plerumque praemeditari futuras actiones nostras eamque praemeditationem esse praesentem, actionem autem quam praemeditamur nondum esse, quia futura est." –

> Permit me, Lord, to seek further. O my hope, let not my purpose be confounded. For if times past and to come be, I would know where they be. Which yet if I cannot, yet I know, wherever they be, they are not there as future, or past, but present. For if there also they be future, they are not yet there; if there also they be past, they are no longer there. Wheresoever then is whatsoever is, it is only as present. Although when past facts are related, there are drawn out of the memory, not the things themselves which are past, but words which, conceived by the images of the things, they, in passing, have through the senses left as traces in the mind. Thus my childhood, which now is not, is in time past, which now is not: but now when I recall its image, and tell of it, I behold it in the present, because it is still in my memory. Whether there be a like cause of foretelling things to come also; that of things which as yet are not, the images may be perceived before already existing, I confess, O my God, I know not. This indeed I know, that we generally think before on our future actions, and that that forethinking is present, but the action whereof we forethink is not yet, because it is to come.

This was to show from the philosophy of antiquity how time was treated at least in some sense different from space. Our glance into classical philosophy might also serve to see a bit clearer the same problem in the way it has been renewed by modern physics.

3 Time in Modern Physics

We'll do a large jump now from Augustine to physical thought of modern age. There I will not deal with "time" in general, but with time in the framework of physics.

3.1 Laws of Nature

Physics, as I mentioned above, deals with "laws of Nature," mostly in the form of equations; and those equations are eternal in the sense that they do not change in time. Time is represented in the equations by the parameter t.

But what does a law of Nature, represented by an equation, mean?—It is in some way a description of the inner workings of Nature; it gives us an objective picture of reality. How does it do that?—"Objective" in this context means that it is valid at any time at any place, independently of individuals. I can always verify (pace Karl Popper!) its truth by looking in reality, by looking in an experiment whether what the law says really comes out. That means that the law of nature can give me predictions about what will come out when I perform a certain experiment. So this ability to make predictions from a law of Nature is indispensable for its character of being objective.

A law of Nature gives me also the possibility to use it for predictions in order to get a result that is useful for me. That is, I can use it for a technical application; namely when I am able to manipulate the situation of applying the law of Nature in such a way that the predicted result is what I wanted to achieve.

So prediction is a decisive feature of any law of nature—and there it is, the structure of time: Predicting means saying something about the future. So we can conclude that, even though it does not look like that, and even though physicists usually do not talk about it, the structure of time beyond that parameter t lies at the basis of modern science. If we ever "forget" the present, this is only a subjective event; the present still forms an important part of the fundament of the building of science.

3.2 "Classical" Ontology

I might use the equations, e.g., of astronomy, as well to "retrodict" certain events: Astronomy works for the past as well as for the future. It has been calculated, e.g., that the solar eclipse Thales of Miletus is supposed to have predicted occurred on May 28, 585 BC (according to our modern calendar). This seduced classical astronomers to assuming that "in themselves" all events were predetermined. P.S. Laplace, the great astronomer and mathematician, considers that assumption in his work on probability. He says that only we, limited humans we are, depend on probability considerations. A superhuman spirit could do without[8]:

> Une intelligence qui, pour un instant donné, connaîtrait toutes les forces dont la nature est animée, et la situation respective des êtres qui la composent, si d'ailleurs elle était assez vaste pour soumettre ces données à l'analyse, embrasserait dans la même formule,

[8]Laplace [8], p. 2.

> les mouvements des plus grand corps de l'univers et ceux du plus léger atome: rien ne serait incertain pour elle, et l'avenir comme le passé, serait présent à ses yeux.
>
> English: An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

Please note the last words: "... would be *present* before its eyes." That point of view of classical astronomy would really abandon time in reality—it might be kept as some subjective superstition—; everything would be drawn into the present. There, you see it, the present is by no means forgotten. But, not any better for time, the universe consists of the present alone.

Our considerations above have shown that Laplace cannot maintain his view consistently: If there were only the present, no predictions existed; thus objectivity would break down, and with it the whole nice construction of Laplace's intellect.

3.3 Probability

So we must now turn Laplace's argument around, it works the other way: Since there is future, probability is one possible way to deal with it.

The past is factual. We cannot change facts any more, they are henceforth eternal. So in some respect we can deal with past facts like with mathematical truths. But for the future it is different, the future is open. For the future there are many possibilities; future "facts" are facts only potentially. Thus predictions may have the form: "This and that will happen." But predictions may also have the form: "This possibility may become a fact, but that other possibility may as well become a fact. We might only be able to predict the relative frequency of occurrences of one or the other." And that is what probability is when it is applied in natural science: Predicted relative frequency.

With this definition in mind, we can solve several puzzles of probability theory.

First, the definition of probability we have given here: For a long time it seemed impossible to define probability. All attempts at a definition seemed to fail, from Laplace's "classical" one with his "ratio of the number of cases favorable, to the number of all cases possible," to Richard von Mises' limit of relative frequency. Kolmogorov's axiomatic was so successful because he explicitly avoided any attempt at defining probability in its use in science; his theory is a purely axiomatic system, and he leaves the hard questions to the "application" of his theory.

It is true, our definition does not look very mathematical. And with its term "predicted," it looks awfully subjective to any physicist. But if you followed my argument so far, you should not be really surprised: Predictions are in the foundations of physic, in any case.[9]

There is a serious problem with that definition that has, I suppose, kept people from adopting it so far: it cannot give exact values to probability that would correspond to exact measurements. But this is a problem of probability itself, not of talking about it or of defining it. This impossibility lies in the concept of probability. In Kolmogorov's axiomatic system as well as in any other serious theory of probability it is possible to derive positive probabilities for different possible outcomes of a test series of probabilities—i.e., for different relative frequencies. Thus the theory itself excludes the possibility of an exact definition, analogous to the definition of length or charge. And this is another difficulty of the concept of probability: Almost all propositions about probability use the concept of probability again. Thus there is in probability a kind of infinite recursion of probability of probabilities. But again this is not a fault of the definition of probability but a feature of probability itself. We can understand more of it when we seriously make use of the structure of predictions. Again I must end this discussion here, in order not to be too long. One can find more detail in the texts referred to above.

On these grounds one may ask whether probability and whether time is objective at all. A first answer to this question is given by the facts: Objective science is working very successfully with probability. But probability theory itself gives us good arguments why this is so: In spite of the recursive structure of "probability of probabilities," one can always cut off the infinite process and get measurable frequencies in a good approximation. This might not be satisfactory for a mathematician or logician, but that's the way physics is; approximation is at its roots! Thus probability is an objective property of physical systems in so far as probability predictions can be verified independently of time, space, and subject.

And is time itself, in its structure of present, past, and future, objective? Time is, as we know, a fundamental concept. The concept of time is more fundamental than the concept of objectivity: A proposition is objective if it can be corroborated empirically, i.e., if a prediction derived from it can be verified. Thus we presuppose the concept of prediction in order to define what we mean by "objective." So it is not really possible to ask whether time itself is objective.

This is not the right place to delve into the subject any more. I mentioned it to hint at an example where we run into trouble when we "forget the present," i.e., when we try to stick to the description of time solely as that real parameter t.

[9] cf. the treatment of that definition in Drieschner [9]. For more detail cf. Drieschner [10].

3.4 Statistical Thermodynamics

We run into real trouble as well with the question how time asymmetry comes in, when we deal with statistical thermodynamics.

Statistical thermodynamics is a wonderful achievement of nineteenth century physics: There was classical (Newtonian) mechanics, which was considered the fundamental theory of everything—as we saw in quoting Laplace's intellect. And there was thermodynamics, originally a theory of steam engines, that turned out to be of interesting mathematical elegance and generality. The achievement of statistical thermodynamics was the proof that thermodynamics can be reduced to mechanics, namely to the mechanics of a large ensemble of molecules, in using statistical methods. In a quantity of gas that can be treated by humans—say, a few liters—there are as many as about 10^{23} molecules. This is a huge number, much larger than anybody could imagine. Statistics of such huge numbers is rather precise. Statistical thermodynamics turned out to be an extremely successful story.

But there remained a fundamental problem that haunts foundational research till today: Mechanics is a reversible theory. That means: if you have a solution to a mechanical problem, i.e., a function that describes the change of your system correctly, then there is always another solution under the same circumstances that would be correct as well, namely the reverse order of states with the reverse direction of changes. For example, for the system of planets revolving around the sun it would be an equally good possibility to revolve the other sense. This is what "reversible" means: you can reverse the order and still have a valid solution according to the theory. But thermodynamics is irreversible. When you leave your cup of hot coffee on the table for awhile, it will cool down until it has acquired room temperature; but when you leave a cup of cold coffee on the table, it will never become hot by itself. This is represented in thermodynamics: The temperature of bodies in contact will equalize, according to thermodynamics, the pressure of amounts of gas that are connected will equalize, etc. This is a fundamental feature of thermodynamics, deeply rooted in its equations. Now the big question: How is it possible that thermodynamics, which is "really" mechanics, according to statistical thermodynamics, becomes irreversible? How can a reversible theory just by not being looked at so closely (namely by using statistics) become irreversible?

Already Ludwig Boltzmann, one of the "fathers" of statistical thermodynamics, proposed a solution to that problem in using the possibility of fluctuations within a system at equilibrium. His solution has been reproduced through the decades again and again, e.g., in the famous treatise by Adolf Grünbaum,[10] until recent textbooks on the subject. Boltzmann expressed it so nicely that I cannot but quote it here—again in the original German and in an English translation[11]:

[10]Grünbaum [11].

[11]Boltzmann [12]; especially vol. II; § 90 (pp. 256–259).

> Man kann sich die Welt als ein mechanisches System von einer enorm grossen Anzahl von Bestandteilen und von enorm langer Dauer denken, so dass die Dimensionen unseres Fixsternhimmels winzig gegen die Ausdehnung des Universums und Zeiten, die wir Aeonen nennen, winzig gegen dessen Dauer sind. Es müssen dann im Universum, das sonst überall im Wärmegleichgewichte, also todt ist, hier und da solche verhältnissmässig kleine Bezirke von der Ausdehnung unseres Sternenraumes (nennen wir sie Einzelwelten) vorkommen, die während der verhältnissmässig kurzen Zeit von Aeonen erheblich vom Wärmegleichgewichte abweichen, und zwar ebenso häufig solche, in denen die Zustandswahrscheinlichkeit gerade zu- als abnimmt. Für das Universum sind also beide Richtungen der Zeit ununterscheidbar, wie es im Räume kein Oben oder Unten giebt. Aber wie wir an einer bestimmten Stelle der Erdoberfläche die Richtung gegen den Erdmittelpunkt als die Richtung nach unten bezeichnen, so wird ein Lebewesen, das sich in einer bestimmten Zeitphase einer solchen Einzelwelt befindet, die Zeitrichtung gegen die unwahrscheinlicheren Zustände anders als die entgegengesetzte (erstere als die Vergangenheit, den Anfang, letztere als die Zukunft, das Ende) bezeichnen und vermöge dieser Benennung werden sich für dasselbe kleine aus dem Universum isolirte Gebiete, "anfangs" immer in einem unwahrscheinlichen Zustande befinden. Diese Methode scheint mir die einzige, wonach man den 2. Hauptsatz, den Wärmetod jeder Einzelwelt, ohne eine einseitige Aenderung des ganzen Universums von einem bestimmten Anfangs- gegen einen schliesslichen Endzustand denken kann.
>
> English: One can think of the world as a mechanical system of an enormously large number of constituents, and of an immensely long period of time, so that the dimensions of that part containing our own "fixed stars" are minute compared to the extension of the universe; and times that we call eons are likewise minute compared to such a period. Then in the universe, which is in thermal equilibrium throughout and therefore dead, there will occur here and there relatively small regions of the same size as our galaxy (we call them single .worlds) which, during the relative short time of eons, fluctuate noticeably from thermal equilibrium, and indeed the state probability in such cases will be equally likely to increase or decrease. For the universe, the two directions of time are indistinguishable, just as in space there is no up or down. However, just as at a particular place on the earth's surface we call "down" the direction toward the center of the earth, so will a living being in a particular time interval of such a single world distinguish the direction of time toward the less probable state from the opposite direction (the former toward the past, the latter toward the future). By virtue of this terminology, such small isolated regions of the universe will always find themselves "initially" in an improbable state. This method seems to me to be the only way in which one can understand the second law—the heat death of each single world—without a unidirectional change of the entire universe from a definite initial state to a final state.

I suppose that you feel, similarly as I did when I first read this proposal, that something must be wrong with it. Closer inspection shows, again, that the point is the structure of time: Boltzmann explicitly draws on an analogy with space ("up or down" with "two directions of time"). But that makes no sense: If you start out with fluctuations (in time), what could it mean that "a living being will . . . distinguish the direction of time . . . "? Should a living being live "backwards" in time? More recent authors don't express that idea in such naïve terms, but you always find the distinction of "beginning" and "end," that was supposed to come out of the argument, introduced by hand in some hidden way.

There is a really convincing solution introduced by C.F. v. Weizsäcker in 1939, which does not seem to have been recognized much[12]: It is not that time asymmetry *comes out* of using statistics, but we introduce that asymmetry ourselves—apparently without noticing it—in going over from mechanics to statistical thermodynamics. The point is that we introduce probability in that process. And the natural area of application of probability is predictions. This is probably the reason that it went almost unnoticed that in the argument for statistical thermodynamics probability is applied only to the future, but not to the past. Small wonder, thus, that the result bears an asymmetry between past and future.

The ingenious Josiah Willard Gibbs noted in his work on statistical thermodynamics as early as 1902 a faint suspicion that this might be the reason for the much discussed puzzle. He wrote[13]:

> But while the distinction of prior and subsequent events may be immaterial with respect to mathematical fictions, it is quite otherwise with respect to the events of the real world. It should not be forgotten, when our ensembles are chosen to illustrate the probabilities of events in the real world, that while the probabilities of subsequent events may often be determined from the probabilities of prior events, it is rarely the case that probabilities of prior events can be determined from those of subsequent events, for we are rarely justified in excluding the consideration of the antecedent probability of the prior events.

Still there are occasions where we give past events a probability. One field of such occasions is history. For instance we could say that it is highly probable that the apostle Jacob went to Spain. What does that mean? It is quite certain that, in fact, he went to Spain or he went not. The uncertainty arises only that we do not know for sure. So actually we can again refer that probability to the future, namely to the possible event that somebody will find out how it really was. Another field of application of probability to past events is in statistical thermodynamics itself: We might know (or we might suppose) that the system considered is in thermal equilibrium, i.e., that there is no permanent change in its state. Then the only possible changes are fluctuations caused by the "statistical" movement of the molecules. Now let me say that, to make it short, in a bit more technical terms: If we find a state that does not have maximal entropy, we can conclude with high probability that it is the extreme of a fluctuation. In that case looking backward in time gives the same result as looking forward in time, namely that entropy probably was lower than at present, and probably will be lower than at present. But this is a very intricate statistical argument. It can be discussed rather clearly with the non-realistic model that has first been proposed by Paul and Tatjana Ehrenfest in 1906 and has been used many times since.[14] The point of the argument is that for a system

[12] Weizsäcker [13].

[13] Gibbs [14].

[14] Ehrenfest [15].

in equilibrium there is no asymmetry of time; conclusions for the future are just as valid for the past. But if there is no thermal equilibrium, we do predict for the future, but there is no sense in "predicting" the past. Since this would become too lengthy, let me again refer to Drieschner loc. cit.

3.5 Quantum Mechanics

After that long run-up let me turn, finally, to Quantum Mechanics. The run-up was necessary in order to make clear the role of time for the interpretation of probability. Since quantum mechanics is indeterministic, in fact the first truly indeterministic theory in history, probability is the one concept that is most intimately connected with the new features of quantum mechanics. The notorious interpretation problems of quantum mechanics turn out to be for the most part connected with interpretation problems of probability.

Quantum mechanics is fundamentally indeterministic. Before the invention of quantum mechanics probability was already used in physics; we saw it in the example of statistical thermodynamics. But in classical (i.e., pre-quantum) physics one could always think of an underlying deterministic theory so that the use of probability became only necessary when it was too hard or too laborious to get an exact description. We saw that in Laplace's description of his use of probability, and this is usually supposed for statistical thermodynamics: The processes could in principle be described with the mechanics of 10^{23} molecules, but practically we depend on probability.

This is different in quantum mechanics. Quantum mechanics is a *fundamentally* probabilistic theory. Even if one knows all that can be known about a quantum mechanical object, according to its theory, there remain always more than one possibility for the further development; the most one can do about that is, attaching a probability value to each possibility. The situation is fundamentally different from the situation in statistical thermodynamics. For if you assume that there is an underlying deterministic theory in quantum mechanics as well, you run into serious trouble.

Since the invention of quantum mechanics in 1925, there have been attempts at finding "Hidden Parameters" of a deterministic theory for quantum mechanics, but the success of those attempts is rather doubtful. This is not the place to describe the long and tedious story of Hidden Parameters. One remark only: There is a way to introduce a deterministic theory with hidden parameters into the way of speaking about quantum mechanics; David Bohm invented it as in 1950. But the consequences of that way of speaking about quantum mechanics are rather queer and contradict principles that have been well established, e.g., the principle that the speed of light is the maximum speed for the movement of particles. So the overwhelming majority of scientists and philosophers of science consider Bohm's experiment just a curious side effect of the discussion about the consequences of the great discovery of the quantum world.

One motive for seriously discussion Bohm's and similar proposals has always been the fact that the interpretation of quantum mechanics seems so difficult. Quantum mechanics has so many features that contradict traditional ideas of classical physics that sheer desperation may lead physicists to think of rather strange ways out. But I shall try to show that the culprit for many problems is "the forgotten present." It must suffice here to pick out the two most serious examples, namely the "Collapse of the wave function" and the "EPR paradox."

3.5.1 Collapse of the Wave Function

Usually the dynamics of a quantum mechanical system is described as following two entirely different laws:

One law is the Schrödinger equation that describes the development of the wave function in a deterministic way, just like any other field equation, e.g., of electrodynamics.

The other law is the "collapse of the wave function." The latter describes the effect of a measurement: Before the measurement several outcomes are possible, with probabilities implied by the wave function; and after the measurement the one outcome, that was unpredictable before, determines which wave function describes the further development. That means that the measurement induces a sudden change in the description of the system that does not conform to the Schrödinger equation. In a measurement of position this would mean that the wave function, which was spread out in space before the measurement, is concentrated in a small volume afterwards—hence the name "collapse of the wave function."

Let us look at this description a bit closer: The wave function or, more generally, the "state" of the system under consideration, represents a catalogue of probabilities for all possible measurements of the system. So, according to the description above, it is a collection of predictions. This state develops in time according to the Schrödinger equation. This development is deterministic; the indeterminism comes in through the fact that what develop are probabilities. In general, none of the predictions bears probability 1, which would mean certainty. But there are in general several possible outcomes of the measurement; it is not predetermined which one of the possibilities will come true: This is the indeterminism of quantum mechanics.

Most theorists express regret about the fact that not all developments can be described by the Schrödinger equation. Some of them even try to develop the description of some interaction of the system under consideration with its environment that takes care of *all* changes in the framework of the Schrödinger equation, including the "collapse." In the light of the considerations above we can unmask those considerations as founded in a misunderstanding: If you accept an indeterministic theory at all, it is the unavoidable consequence that you will have two entirely different descriptions of the development.

For if a theory is not deterministic, the best you can have from it are probabilities. So the dynamics of the theory must consist in a development of the probabilities. This dynamics might even be indeterministic itself, but it can be deterministic as well, as in the case of quantum mechanics. Probability means prediction of relative frequency. So what the dynamics of the theory gives us is a prediction of relative frequency in the outcomes of like measurements. If the predicted frequency is positive but less than one, the single outcomes *must* be unpredictable. So if you continue predictions—the dynamics of the system—after the measurement, you can either continue the original dynamics, keeping all possible outcomes of the experiment with their respective probabilities within your scope. Or you take the result of the experiment into consideration. That means that from the experiment on you drop whatever could follow from the other results that were possible before the experiment, and follow only the consequences of the result that really came out. But since this very result was unpredictable, according to our basic assumption that the theory is indeterministic, there *cannot* be a way to derive that result, i.e., the further dynamics, from a theory.

This means that, in an indeterministic theory, something like the collapse of the wave function must necessarily occur. The collapse of the wave function is a necessary ingredient of any indeterministic theory.

How can we incorporate this consequence into our understanding of physical theories? In early discussions of quantum mechanics there was a strong tendency towards a subjectivist way of description. C.F.v.Weizsäcker e.g., the most philosophical thinker of the traditional ("Copenhagen") school of interpretation,[15] says in an early essay: "Dies wird besonders deutlich durch den allgemeinen Formalismus der Quantenmechanik. Er beschreibt *unser Wissen* über ein Objekt durch die Angabe einer abstrakten 'ψ-Funktion'" [17]. In English: "This becomes especially clear through the general formalism of quantum mechanics. It describes *our knowledge* about an object through an abstract 'ψ-function'." This tendency culminates in an entirely subjective interpretation of quantum mechanics by London and Bauer [18], which was not supported, though, by "Copenhagenians."

Calling the wave function ("ψ-function") a description of our knowledge is possible, if you interpret it in the right way. But you can use a more "objective" language as well. Because the wave function (the state of the system) is actually a collection of probabilities, and probability, being predicted frequency, is as objective as any prediction: If the theory is correct then you will be able to corroborate the prediction quite objectively.

Applying this argument to the case of the collapse of the wave function, we have the following situation:

- You can continue using the state before the measurement, calculated according to the Schrödinger equation, in order to predict the probabilities that apply after the

[15]The "Copenhagen interpretation of quantum mechanics" is, to my mind, still the only acceptable way of talking about quantum mechanics, mainly because of its modesty: it does not try to give more than it has. Cf. Drieschner [16].

measurement. You will then corroborate the relative frequencies implied by that state within the ensemble of all single systems you had before the measurement.

- But you might as well apply the collapse of the wave function after measurement, keeping only those single systems for further predictions of relative frequency that belong to a certain result of the measurement. Then you use a smaller ensemble, and you will corroborate the predictions for the "collapsed" state within this smaller ensemble.

Thus it becomes clear that the notorious collapse of the wave function is nothing but a *decision* of the one who makes the predictions. And usually he will act wisely in taking the result of every measurement into account for his predictions, i.e., to apply the collapse of the wave function. But nobody *has* to do that!

3.5.2 EPR[16]

The authors "EPR" give an example of a quantum mechanical correlation of two objects that have interacted before, but are separated afterwards. In 1951 David Bohm gave a simpler example that is usually discussed instead of the one by EPR because it is easier to understand and can be (and has been) realized experimentally[17]: Take a physical system with spin 0 that decays into two subsystems ("particles") with spin ½ each. The two subsystems have to have their spins oriented in opposite directions to conserve angular momentum. So when one measures the angular momentum of one of the particles one can conclude what the angular momentum of the other one is, even if the particles have moved apart in the meantime for a distance of light years. This is the same in quantum mechanics as in classical mechanics. But now a quantum mechanical specialty comes in: The orientation of the spin cannot be measured as some "objective" property, as in classical mechanics. What can only be measured is, whether the orientation of the angular momentum (spin) is parallel or antiparallel a certain direction fixed by the measuring apparatus.[18] Thus the result of the measurement will to a large part depend on a decision of the experimenting physicist, namely on the decision how he orients his measuring apparatus.

Let us, in order to facilitate communication, call the experimenter at one measuring apparatus "Alice," and the one at the other apparatus "Bob" (a quite common practice). The conservation of angular momentum implies that, if the measuring apparatuses are parallel, the orientations measured must be opposite. But, what sounds quite strange, not only will Bob find the opposite orientation when he orients his measurement in the same direction as Alice, but at any orientation of his experiment he will find that very frequency distribution that follows from the result

[16]The acronym refers to the paper Einstein et al. [19].

[17]Bohm [20].

[18]Stern and Gerlach [21, 22].

of Alice's experiment. Quantum mechanically, the state of Bob's particle always is anti-parallel to the state Alice has measured, even if Alice decides only milliseconds before her particle arrives how she wants to orient her experiment—or rather, even if she decides a time so short before that it is impossible that a signal can reach Bob before his own measurement. Einstein called this "spooky action at a distance."[19]

Actually this is not a spooky action, but it is no action at all. In order to see this we have to look a bit closer at that experiment. Bob, e.g., will see as results of his experiment (approximately) equal numbers of outcomes on both sides of his apparatus, whichever orientation he gives it, and whichever orientation Alice gives to her apparatus. What EPR talk about is the *correlation* between the results of Alice and Bob, i.e., something that somebody can find out only when he has information from both, Alice's and Bob's experiments: He has to *compare* the results. Thus, without knowing a lot about Alice's experiment, Bob cannot find out anything about it from the results of his experiment. Nothing happens on his side of the world that would depend on what Alice does on her side.

Whence, then, the whole question?

The reason for the trouble with EPR is the fact that the *state* of Bob's particle is changed by what Alice finds out.

The state is the collection of all predictions for possible measurements. But if these predictions change, why is it that Bob cannot measure that change?

The predictions are probability propositions. Probability presupposes, as we know, a certain ensemble from which the single cases for the measurement of relative frequency are taken. Let us assume for our case that Alice's apparatus is oriented vertically. Thus Bob's particle would assume the state "up" the instant when Alice measures "down" at her particle. But Alice's results are a mix of ups and downs, with an about equal number of both. So if Bob wants to do any statistics on his state "up," he can do so only if he selects the cases where Alice found "down." If he does not use information about the sequence of Alice's results, he sees nothing but his random sequence of ups and downs, about equally distributed.

Thus the change of the state of Bob's particle by Alice's measurement is actually a change of the ensemble under consideration. And Bob can effect this change only when he uses information about Alice's experimental setup and, most important, about the sequence of Alice's single results.

Thus that change of state is no action at a distance; it is no action at all. It is rather a decision, again, of the experimenters.

Physicists who call themselves realists—mainly the Bohmian school[20]—regret very much that the state of a system is not "real" in their sense. In German there is the beautiful word "Wirklichkeit" for reality. It is related to the verb "wirken,"

[19]"spukhafte Fernwirkungen": Born and Einstein [23], letter 84, p. 210.

[20]cf. the very interesting book Passon [24].

which means “to act” in English. In English you can imitate this relationship by the words “act” and “actual world”: Something is actual if it *acts* somehow. And since there is no action in the EPR effect, the EPR effect is not “actual”; it cannot be a *real* effect.

4 “Timeless”?

Concepts are “timeless,” eternal. In order to have concepts, there must be something that bridges time and connects present with past and future. Without that, experience would not be possible. Plato introduces that with his world of ideas, which he considers the only *real* world.

CFv Weizsäcker gives an approach from the other side. He considers time as fundamental. According to Weizsäcker, logic is fundamentally temporal logic; the “timeless,” mathematical logic we are used to is a theory that is derived from temporal logic.

Now, to be sure, we are using *concepts*, which are time bridging, and we are talking about the *structure* of time, and structures are time bridging; and we are talking about *physical theory*, which is time bridging. Truth is time bridging. So the emphasis of philosophy on the eternal is quite all right. But sometimes—i.e., at certain *times*—it is necessary to bethink oneself of the fundamental role of time, e.g., in order to understand what the eternal physical theory tells us.

What time is has been a great question of philosophy through the ages. Although this seems an abstract, rather dry subject, fundamental philosophical themes underlie our discussions of truly practical matters. So an important point in the political discussions about the environment, about sustainable economy, and similar subjects, is the question how we see the world around us, our “reality.” And for our image of reality it is decisive how we understand the picture quantum mechanics draws of this reality, although it seems so remote at first glance. And in order to understand that, we must not forget the present.

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Quantum Physics and Presentism

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Abstract This paper argues that the case of presentism is open both from the physical and the metaphysical point of view. It is open from the physical point of view, since we do not have an elaborate account at our disposal of how quantum non-locality can exist in the space–time of special relativity, without presupposing an objective foliation of space–time into spatial hypersurfaces that are ordered in time. The GRW flash ontology is the proposal in the current debate that to a certain extent comes close to such an account, but meets with serious reservations. The case of presentism is open from a metaphysical point of view as well, since an ontology of matter in motion implies endurantism and thereby, as one can argue, presentism. Again, we do not have a precisely worked out proposal at our disposal that replaces an ontology of matter in motion with an ontology of properties existing at space–time points in a block universe, and any such proposal meets with serious reservations.

1 Introduction

Presentism is the view that only what is present exists. What is past no longer exists, and what is future does not exist as yet. On the most widespread understanding of presentism, it is the view that only what there is at a certain time exists, but not the view that only what there is in a certain region of space exists. That is to say, presentism is not the solipsistic stance that maintains that only what there is at a certain space–time point (i.e. the point where I am now) exists.[1] Presentism, thus construed, presupposes objective simultaneity. More precisely, it takes for granted that there is exactly one global, objective foliation of four-dimensional space–time into three-dimensional spatial hypersurfaces that are ordered in time.[2] It is the view

[1]But see Harrington [27] for the defence of such a view.

[2]But see Fine ([22], ch. 8, § 10, pp. 298–307) for a view that relativizes existence to inertial frames.

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that these hypersurfaces come into and go out of existence such that always only one such hypersurface exists—the present one. Monton ([38], p. 264) characterizes this view as "Heraclitean presentism", because its central tenet is the reality of change in the sense of events coming into being and going out of being. Presentism thus is opposed to eternalism according to which everything that there is in space–time simply exists.

The claim of this paper is that the case of presentism is open. The structure of the paper is as follows: I first recall the standard argument from the special and the general theory of relativity that refutes presentism (Sect. 2). I then show that this argument is strongly challenged by the experimentally proven fact of quantum non-locality (Sect. 3). That is why the case of presentism is open from the physical point of view. I then argue that the issue of presentism vs. eternalism goes deeper than the question of the compatibility of our two current main fundamental physical theories (quantum physics and general relativity theory), concerning the general metaphysics of objects and properties (Sect. 4). That is why the case of presentism is open from the metaphysical point of view as well.

2 The Argument Against Presentism from the Special and the General Theory of Relativity

The special theory of relativity [18] is built on the following two principles:

1. All inertial reference frames are equivalent for the description of physical phenomena.
2. The velocity of light is a constant, being independent of the state of motion of its source and thus the same in all inertial reference frames.

Principle (1) is taken over from pre-relativistic physics, going back to Galilei. Principle (2) is a consequence of the field solution to the action-at-a-distance problem in Newton's theory of gravity: according to the field solution, interactions propagate from a space–time point to its neighbouring points and thus with a finite velocity (local action). In fact, the velocity of light is the upper limit velocity for the propagation of effects. This principle implies that we have to replace the Galilean transformations with the Lorentz transformations when switching from one inertial reference frame to another one. The latter unify space and time in the following sense: only the four-dimensional, spatio-temporal distance between any two events is an invariant. This is the reason for the claim that according to the special theory of relativity, space and time are not separate entities, but unified in a four-dimensional space–time. Both these principles apply also to the general theory of relativity. In particular, both the special and the general theory of relativity entail that there is no privileged or objective foliation of space–time into three-dimensional, spatial hypersurfaces that are ordered in time.

The contradiction between the conjunction of these two principles and presentism consists in the fact that presentism as characterized above presupposes an objective foliation of space–time into three-dimensional, spatial hypersurfaces that are ordered in time. In other words, presentism takes for granted that for any one event that exists, there are indefinitely many other events that exist as well and that are simultaneous with the event in question, constituting a spatial hypersurface of the universe. But the special and the general theory of relativity imply that there is no objective simultaneity, because any two events that are simultaneous in one inertial reference frame are not simultaneous in other inertial reference frames, and all inertial reference frames are equivalent; in other words, there is no objective foliation of space–time into spatial hypersurfaces that are ordered in time.[3]

Special and general relativity therefore suggest eternalism, that is, the view that the whole of space–time with all its content simply exists. Wüthrich ([51], sections 3–6) examines various strategies to avoid this conclusion and argues that none of these strategies is convincing. His assessment is correct to my mind. Hence, to put it in a nutshell, if the special and the general theory of relativity told the whole story of fundamental physics, the only reasonable position to take would be eternalism, and the case of presentism in a metaphysics based on science would be closed.

3 Quantum Non-locality and the Case for Presentism

However, the general theory of relativity does *not* tell the full story about what happens in space–time. John Bell, in one of his last papers entitled "La nouvelle cuisine" (1990), formulates a principle of local causality: "The direct causes (and effects) of events are near by, and even the indirect causes (and effects) are no further away than permitted by the velocity of light" (quoted from [3], p. 239). No particular notion of causation is implied here (see [3], p. 240). The idea is that whatever events whose occurrence contributes to determining the probabilities for a given event to happen at a certain space–time point are located in the past light-cone of that event. This is one way of formulating the principle of local action that is implemented in classical field theories and that overcomes Newtonian action-at-a-distance. Relativity physics endorses this principle. That is why relativity physics can waive the commitment to an objective, global temporal order of events and thus the commitment to an objective simultaneity of events: whatever contributes to determining a given event is situated in its past light cone; consequently, there is no need to settle for an objective temporal order of events that are situated outside each others light cones (Fig. 1).

[3] See notably Saunders [44] and Wüthrich ([51], section 2) for a clear exposition of this contradiction.

Fig. 1 Illustration of Bell's definition of local causality; figure copied from http://www.scholarpedia.org/article/Bell's_theorem

In view of considering quantum physics, Bell makes his definition more precise in the following manner; "local beable" is Bell's neologism for whatever exists as localized in space–time according to the theory under consideration:

> A theory will be said to be locally causal if the probabilities attached to values of local beables in a space–time region 1 are unaltered by specification of values of local beables in a space-like separated region 2, when what happens in the backward light cone of 1 is already sufficiently specified, for example by a full specification of local beables in a space-time region 3. (Quoted from [3], pp. 239–240)

Bell's theorem from 1964 (reprinted in [3], chap. 2) proves that any theory that complies with the experimentally confirmed predictions of quantum mechanics has to violate Bell's principle of local causality: in some cases, specifying the local beables in region 2 changes the probabilities attached to values of local beables in region 1, although the beables in the backward light cone of region 1 are already specified. Switching from quantum mechanics to quantum field theory does not alter that issue: also in quantum field theory, in some cases, specifying the local beables in region 2 changes the probabilities attached to values of local beables in region 1, although the beables in the backward light cone of that region are already specified. On this basis, Maudlin ([34], chaps. 1–6) convincingly argues that quantum non-locality implies the existence of superluminal influences and thereby the existence of superluminal causation (again, no specific theory of causation is presupposed).

However, quantum non-locality does not permit sending superluminal signals. The reason is that one cannot control the relevant local beables in space–time region 2 (i.e. the outcomes of quantum mechanical measurements made in this region). Consequently, one cannot employ the local beables in space–time region 2 to send superluminal signals to space–time region 1. Nonetheless, on the ontological by contrast to the operational level, there is a conflict between quantum theory (quantum mechanics, quantum field theory) and relativity theory (special relativity, general relativity), since what happens in a space–time region 2 that is separated from a space–time region 1 by a space-like interval nevertheless contributes to determining the probabilities of what happens in region 1, and *vice versa*.[4]

[4]See notably Bell ([3], p. 172), Albert [1], Norsen [42] and Seevinck [45].

This conflict does not automatically imply that we have to give up one of the two principles on which the special theory of relativity is built. First of all, (1) the mentioned determination does not mean that there is a signal travelling from space–time region 2 to space–time region 1 with a velocity that is much higher than the velocity of light. There is no precisely formulated version of quantum theory that includes superluminal signals, although one can contemplate models of quantum non-locality that are built on the idea of superluminal signals, as notably Chang and Cartwright ([11], section III) do. Secondly, (2) there is a straightforward way to solve the conflict, but it comes at a high price: if one countenances backward causation, one can contemplate acknowledging a signal that travels from space–time region 2 backwards in time to the region where the past light cones of region 2 and region 1 overlap and from that region then forwards in time to region 1.[5] Apart from well-founded general reservations that one can voice against backward causation, the problem in our context is that such models are committed to closed causal loops, as Berkovitz [7, 8] convincingly argues. Thirdly, (3) there is the possibility to explain quantum non-locality in terms of some sort of a common cause that is not located in the intersection of the past light cones of space–time region 1 and space–time region 2. All precisely worked out versions of quantum theory pursue this strategy (insofar as one interprets them in causal terms, again without presupposing a specific theory of causation).

If one endorses this strategy, one is not committed to any sort of direct superluminal interaction, since one searches for a common cause of the correlation between the local beables in space–time region 1 and space–time region 2 instead of explaining that correlation in terms of a direct interaction between these beables (signal travelling with superluminal velocity). Furthermore, it is not excluded that it may turn out to be possible to respect in this framework the principle according to which there is no privileged foliation of space–time into spatial hypersurfaces that are ordered in time as well.

The most promising proposal in this respect is the ontology that Bell ([3], chap. 22) puts forward for the version of quantum theory developed by Ghirardi et al. [25] (GRW). According to Bell, the spontaneous localizations of the quantum mechanical wave-function in configuration space that the GRW dynamics introduces describe the local beables in space–time. That is to say, whenever a spontaneous localization of the wave-function in configuration space occurs, there is a local beable centred around a point in physical space–time, and these *local beables are all there is in space–time*. Tumulka [47] proposes to call these local beables "flashes". Thus, all that exists in space–time is a sparse distribution of flashes at space–time points.

Tumulka [47] sets out to show that the flash ontology does not have to commit itself to more space–time structure than the special theory of relativity admits. In other words, it does not have to presuppose a privileged foliation of space–time into

[5] See notably Price ([43], chap. 8 and 9), Dowe ([15], chap. 8), as well as the papers in *Studies in History and Philosophy of Modern Physics* 38 (2008), pp. 705–840.

spatial hypersurfaces that are ordered in time.[6] As it stands, Tumulka's proposal does not include interactions. On the one hand, one can maintain that the point at issue just is whether one can account for interactions in quantum physics without presupposing a privileged foliation of space–time. On the other hand, one can retort that we do in general not have a relativistic quantum theory at our disposal that provides a precise dynamics for interactions.

Be that as it may, one can already raise reservations about the Lorentz invariance of Tumulka's proposal as it stands. If one considers space–time as a whole, this proposal can describe the distribution of flashes in space–time without presupposing a preferred foliation, and its dynamical law does not rely on there being a particular foliation of space–time. However, we have good physical and philosophical reasons for maintaining that there are concrete physical relations of entanglement instantiated in space–time.[7] That is why quantum physics is commonly regarded as being incompatible with David Lewis' famous thesis of Humean supervenience according to which there are only local matters of particular fact occurring at points in space–time.[8] Applied to the flash ontology, that is to say that insofar as there are flashes occurring at space–time points, there are correlations among these flashes existing as relations instantiated in space–time, constituting certain structures of correlated flashes.

However, one can retort that it is not mandatory to recognize relations of entanglement. As Bell remarked in "The theory of local beables" (1975), one can put forward an ontology of quantum physics that admits only the distribution of the local beables in space–time (see [3], p. 53)—that is, an ontology that acknowledges only the Humean mosaic of local matters of particular fact, these being the flashes in the case of the GRW flash theory. On this view, the quantum-mechanical wave-function and its temporal development according to a dynamical law (the Schrödinger equation, or the GRW equation) is a mere instrument of economical book-keeping of the distribution of the local beables (the flashes).

Nonetheless, if one takes the physical and philosophical reasons for a richer ontology that includes relations of quantum entanglement to be convincing, it is reasonable to regard these relations as being dynamically relevant. Putting the matter in causal terms (again without presupposing any specific theory of causation), that is to say in the case of the GRW flash theory that structures of correlated flashes cause the occurrence of further correlated flashes, being their common cause.[9] The wave-function and the dynamical law in which the wave-function figures describe how they do so. The wave-function thus is not a mere instrument of economical book-keeping, but refers to something that there is in space and time over and above the distribution of the local beables. One illustration of this view is the claim

[6]See Maudlin ([33, 34], chap. 10) for a discussion of that proposal.

[7]See e.g. Esfeld [19].

[8]See e.g. Lewis ([31], pp. ix–x). See Darby [12] for a recent assessment of this conflict.

[9]See e.g. Esfeld [20].

that the correlated flashes under consideration include as a whole the disposition or propensity to bring about further correlated flashes.[10] In any case, these relations are not limited to time-like separated flashes, but connect space-like separated flashes. That is to say, structures consisting in correlations among space-like separated flashes are dynamically relevant for the occurrence of further correlated, space-like separated flashes (e.g. by including the disposition or propensity to bring about such correlated flashes).

It is not clear whether and how such an ontology could be spelled out by working only with the space–time structure of special relativity theory, that is, without presupposing an objective foliation of space–time. If there are correlations among space-like separated flashes and if the existence of such correlated flashes determines, via the wave-function and its dynamics, the occurrence of further correlated and space-like separated flashes, then it seems that an objective temporal order of both the initial and the subsequent correlated and space-like separated flashes is required. In general, hence, as soon as one admits relations of quantum entanglement existing in space–time and takes these relations to be dynamically relevant, it is not clear how one could achieve an ontology of quantum physics that is Lorentz invariant, even if one recognizes only sparsely distributed flashes as the local beables of one's quantum theory.

Over and above the issue of correlations existing among space-like separated flashes and their dynamical relevance, there are further problematic aspects of the flash ontology. The theory is formulated in terms of particles, assuming that there is a fixed number of particles (at least as long as quantum field theory is left aside). However, there are no particles in its ontology. There are only flashes, each flash being an event occurring at a space–time point. There are no continuous sequences of flashes that could be considered as worldlines of particles, since the distribution of flashes in space–time is sparse. There are only occasionally flashes occurring at a space–time point.

Although the distribution of flashes is sparse, let us suppose for the sake of the argument that there are enough flashes to account for macroscopic objects, not going into the reservations that Maudlin ([34], pp. 257–258) voices as regards this point. Consider what the flash ontology tells us about typical quantum mechanical experiments. In the double slit experiment with one particle at a time, according to the flash ontology, there is one flash at the source of the experiment and one flash at the screen, but nothing at all in between, apart from the macroscopic object with two slits; the question of whether the quantum system travels through one slit or through both slits does not make sense in this ontology, since there is nothing at all in the space between the source and the screen (apart from the macroscopic device with two slits). By the same token, in the EPR-Bohm experiment, there are two flashes at the source, and then one flash in each of the two wings of the experimental set-up, corresponding to the two measurement events, but again nothing at all in between.

[10] See the dispositionalist ontology for GRW that Dorato and Esfeld [14] propose.

This fact is troublesome, for the story that the GRW dynamics tells about measurement does not make sense on the flash ontology: in the EPR-Bohm experiment, that story says that the measuring device in one wing of the experiment interacts with the quantum system, so that the state of the quantum system becomes entangled with the state of the measuring device. That entanglement is extremely rapidly reduced, for one among the enormous number of particles that make up the measuring device immediately undergoes a spontaneous localization so that all the other particles, including the quantum system, are localized as well. But this story does not make sense on the flash ontology for there is nothing with which the measurement device could interact. There is no particle that is absorbed by it, and no field or wave in physical space that stretches out to it either. In sum, there are good reasons to have reservations about the flash ontology and to be sceptical as to whether this ontology can really achieve a peaceful resolution of the conflict between quantum theory and relativity theory.

If one admits local beables that are continuous in space–time (i.e. do not leave gaps between them, space–time may be discrete), then, as things stand, one is in any case committed to an objective foliation of space–time into spatial hypersurfaces that are ordered in time (although we can in principle not know that foliation). Apart from the GRW flash ontology, there are two other precise proposals for local beables of quantum physics, namely Ghirardi's proposal for a mass density ontology developing according to the GRW equation [24, 37] and Bohm's quantum theory in terms of particles moving on definite trajectories in space–time (going back to [9]). Both these proposals are committed to a privileged foliation of space–time.[11]

Nonetheless, even if one is committed to settling for *a* particular foliation of space–time, one can rescue the principle according to which *all* foliations are equivalent by maintaining that what exists depends on a particular foliation, and that what exists is not unequivocal (nothing simply exists), but depends on the specification of a particular foliation of space–time.[12] This claim is on a par with the anti-realist claim according to which what exists is relative to an observer in the sense of a conscious subject, a language, a discourse, a conceptual scheme, etc. The problem with all these proposals is that they presuppose that the observer, the language, the discourse, the conceptual scheme, or the foliations of space–time for that matter all do exist without their existence being relative to anything.[13]

In sum, if one does not endorse the flash ontology and goes for a quantum theory that admits local beables that are continuous in space–time, then, as things stand, it seems that one cannot reasonably avoid the commitment to accepting that there is a preferred foliation of space–time, although we cannot know which one that preferred foliation is. This then is the basis on which one can make a case for presentism in quantum physics: if there is an objective foliation of space–time into spatial hypersurfaces that are ordered in time, then one can maintain that these

[11] See Maudlin [33] for an explanation of why this is so.

[12] See Fleming [23] and Myrvold [39, 40]. See also the view of Fine [22] mentioned in note 2.

[13] See Heil ([28], in particular chap. 1.1) against any such relativism.

hypersurfaces come into and go out of existence so that only one such hypersurface exists. Note that admitting an objective foliation of space–time is only a necessary and not a sufficient condition for endorsing presentism as characterized at the beginning of this paper: an eternalist can also recognize an objective foliation of space–time and then maintain that *all* the temporally ordered spatial hypersurfaces simply exist. But the eternalist has no motivation to be keen on recognizing an objective foliation of space–time, thereby provoking a conflict with special and general relativity theory, whereas the presentist has to face that conflict and can draw on quantum non-locality in order to argue that special and general relativity theory do not tell the full story about what there is in space–time. Nonetheless, the presentist then has to develop further arguments to justify the step from there being an objective foliation of space–time to the commitment to presentism.

Let us briefly consider in concrete terms how the case for presentism can be made by going into Bohmian mechanics, the contemporary dominant variant of Bohm's theory that is also the most elaborate version of a quantum theory with local beables.[14] Following Bohmian mechanics, the local beables are particle positions. These particle positions develop in time according to a law that is known as the guiding equation:

$$\frac{dQ}{dt} = v^{\psi_t}(Q) \qquad (1)$$

In this equation, Q stands for the configuration of N particles in three-dimensional, physical space at a time t, and Ψ_t is the quantum mechanical wave-function of this particle configuration at t. The wave-function itself develops in time according to the Schrödinger equation. Its role in Bohmian mechanics is to fix the velocity v of the particles at t given their position Q at t. In short, the guiding Eq. (1) takes as input the particle positions at t and yields as output the velocities of the particles at t by means of the wave-function of the particle configuration. To be precise, the guiding equation takes as input the positions of *all* the particles in the universe at t, and the wave-function figuring in it accordingly is the universal wave-function of the configuration of all the particles in the universe at t. That is how Bohmian mechanics accounts for quantum non-locality, namely by making the temporal development of the position of any particle, its velocity, dependent on strictly speaking the position of all the other particles in the universe. (Nonetheless, Bohmian mechanics is operational, for it is possible to derive effective wave-functions that describe sub-systems of the universe by abstracting from the rest of the universe). Bohmian mechanics thereby admits relations of quantum entanglement in the form of correlations among space-like separated particles that are dynamically relevant (cf. the remark about the flash ontology above).

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[14] See the papers in Dürr et al. [16] as well as Dürr and Teufel [17] for a textbook presentation.

That is also how Bohmian mechanics accommodates presentism: this theory commits us to accepting the particles' positions at t and the wave-function at t. The ontological status of the wave-function is a controversial matter: the wave-function is a mathematical object defined on configuration space. The controversy about its status is beyond the scope of this paper. Suffice it here to mention that one can regard this mathematical object as representing a holistic and dispositional property of all the particles taken together that determines their form of motion by determining their velocity—in other words, that determines the temporal development of the particle configuration.[15] What is crucial for the purpose of this paper is that on whatever reading of Bohmian mechanics, this theory is committed only to entities that exist at a given point of time. The theory describes the temporal development of these entities, but in doing so it does not require a commitment to entities that exist at more than a point in time.

Bohmian mechanics therefore accommodates presentism even more easily than Newtonian classical mechanics: Bohmian mechanics is a first order theory, whereas Newtonian mechanics is a second order theory. That is to say, Bohmian mechanics accepts only the position of the particles as primitive and derives their change of position in time (i.e. their velocity) by means of the wave-function. Newtonian mechanics, by contrast, accepts both the position and the velocity of the particles as primitive and derives the change of velocity in time (i.e. the acceleration of the particles) by means of their inertial mass and external forces. However, acknowledging velocity as primitive implies that one endorses as primitive a quantity that is strictly speaking not defined at a point in time, but only for an arbitrarily small interval. By contrast, both the position of particles and their wave-function are well defined at a point in time.

Let us note a few points in order to assess this result: (1) On the one hand, we have found no argument that goes as far as claiming that a certain version of quantum theory entails presentism. The result is only that some versions of quantum mechanics are compatible with presentism. At most, one can say that these versions accommodate presentism, as illustrated by considering Bohmian mechanics. (2) On the other hand, in order to make a case for quantum theory excluding presentism—in the same sense as the special and the general theory of relativity exclude presentism—, one would as a necessary (but not sufficient) condition have to develop an account of quantum non-locality that does not presuppose an objective foliation of space–time. Such an account has not been worked out hitherto, and there are important reservations against the account that comes closest to fulfilling this condition (Tumulka's further development of Bell's flash ontology). Therefore, the case of presentism is open from the physical point of view.

(3) As mentioned above, moving from quantum mechanics to quantum field theory does not change that matter, since quantum non-locality as given by the violation of Bell's theorem concerns quantum field theory in the same way as quantum mechanics. Furthermore, it seems premature to take the search for a

[15] See Belot ([4], pp. 77–80) and Esfeld et al. [21].

quantum theory of gravity into account in this context. Monton [38] maintains that there may be a prospect for a quantum theory of gravity that is based on an objective foliation of space–time, but Wüthrich [50] objects that none of the more advanced approaches to quantum gravity admit a privileged foliation of space–time. This debate seems premature, since as a prerequisite for a sensible discussion of the relationship between quantum non-locality and relativity physics, one has to elaborate on what one takes to be the local beables of the quantum theory in question, and none of the more advanced approaches to quantum gravity has as yet spelled out the local beables to which it is committed. Moreover, some of these approaches suggest that space–time does not belong to the ontology of fundamental physics, but emerges from non-spatio-temporal elements of reality. However, this conception of emergence is left entirely vague. As Lam and Esfeld [30]) have shown, none of the precise notions of emergence is applicable in this case.

(4) Even if one subscribes to presentism on the basis of an account of quantum non-locality in terms of a privileged foliation of space–time, there is no empirical conflict with special or general relativity, since the principle of the equivalence of all inertial reference frames and the principle of the equivalence of all foliations of space–time do not have empirical consequences. All empirical phenomena can be formulated in one reference frame or foliation, whichever one chooses (and whichever happens to be the objective one, if there is an objective one). Furthermore, presentism is compatible with the central tenet of general relativity theory according to which space–time is itself dynamical instead of being a background structure. Even if there is a privileged foliation of space–time, the spatial and temporal distance between events may depend on their physical properties such as their mass. Moreover, in this context, one is not committed to going back to endorse anything like an ether that serves as the privileged inertial frame. On the contrary, one can maintain that the distribution of mass in the universe fixes the objective foliation of space–time.

To put the matter in other words, given the fact that there is a conflict between quantum theory and relativity theory and given that, as things stand, the only precisely worked out proposal for a peaceful resolution of this conflict meets with serious reservations, one can resolve this conflict by giving up the relativistic principle of there being no objective foliation of space–time without facing empirical consequences. But one cannot resolve this conflict by giving up quantum non-locality, since quantum non-locality in the sense defined at the beginning of this section is an empirical fact.

(5) Even if there is a privileged foliation of space–time, all the versions of quantum theory that are committed to such a privileged foliation imply that we cannot know which one is the objective foliation of space–time. Callender [10] formulates on this basis a coordination problem between the unknowable privileged foliation of space–time to which some versions of quantum theory such as Bohmian mechanics are committed and presentism as based on common sense, more precisely as based on the experience of a particular foliation of space–time. Nothing guarantees according to Callender [10] that these foliations coincide. However, it is doubtful whether there is the experience of one particular *global*

foliation of space–time in common sense, given the limited scope of common sense experience and given in particular the fact that the velocities with which we are familiar in common sense are very small in comparison to the velocity of light.

One can with reason maintain that the point at issue in the support that presentism can draw from common sense is temporal becoming: the block universe view (eternalism), to which one is committed if one accepts the principle of the equivalence of all foliations or inertial reference frames, rules out temporal becoming, since everything that there is in space and time simply exists. One can argue that (a) common sense and in particular our experience of ourselves as acting beings in the world are based on the view that our future gradually comes into existence and that (b) it is this commitment to temporal becoming that drives the common sense support for presentism. Consequently, in order to do justice to common sense, an ontology that admits temporal becoming is required. But it is of no importance which one is the privileged foliation of space–time and whether or not we have an epistemic access to that privileged foliation.

Furthermore, subscribing to presentism on the basis of recognizing an objective foliation of space–time in the ontology of physics makes room *only* for accommodating temporal becoming in the ontology, thus fulfilling what one may take to be one requirement in the theory of human agency. However, if one maintains that agency implies free will and that furthermore free will is incompatible with determinism, one can draw no support from physics. The dynamics of Bohmian mechanics is deterministic. The dynamics of the GRW theory is indeterministic, but includes probabilities that are completely fixed by physical variables alone—no agent that stands outside the laws of physics could manipulate the GRW probabilities. Thus, endorsing presentism based on an objective foliation of space–time is no means to alleviate the conflict between physical laws and free will, if one assumes that there is such a conflict.

In sum, one can compare the argument for an objective foliation of space–time from quantum non-locality to Newton's famous bucket argument. Newton postulates more space–time structure than is observable, namely absolute space and motion with respect to absolute space. By means of the famous bucket argument, he argues that we have to endorse absolute space in order to accommodate rotation, since rotation cannot be considered as relative motion. He gives a precise account of how rotation can be conceived as motion in absolute space. We have to recognize absolute space although doing so contradicts the well-established metaphysical principle of the identity of indiscernibles; but the denial of this principle does not lead to a conflict with empirical results, whereas rotation is a form of observable motion that has to be accounted for.[16]

By the same token, one can argue that in the context of today's physics, we have to postulate more space–time structure than is observable, namely an objective foliation of space–time, in order to accommodate quantum non-locality as manifested in the EPR-Bohm experiment. Once one endorses an objective foliation

[16] See Maudlin ([35], chap. 2) for a forceful reconstruction of Newton's argument.

of space–time, there are full and precise accounts of quantum non-locality in space–time available (Bohm's theory, as well as the GRW mass density ontology in the framework of a collapse dynamics). We have to recognize an objective foliation of space–time although doing so contradicts one of the theoretical principles on which the special and the general theory of relativity are founded; but the denial of this principle does not lead to a conflict with empirical results, whereas quantum non-locality is an empirical fact that has to be accounted for.

Consequently, the philosopher-physicist who is not willing to admit Newtonian absolute space is committed to giving a full and precise account of rotation that does not presuppose absolute space, which neither any of Newton's contemporary critics did nor Mach accomplished.[17] By the same token, the philosopher-physicist who is not willing to admit more space–time structure than is recognized in the special and the general theory of relativity is committed to giving a full and precise account of quantum non-locality in space–time that is Lorentz-invariant. As in the case of a relational account of rotation, so in the case of a Lorentz-invariant account of quantum non-locality, this is of course not to say that such an account cannot be achieved, but only that there is a challenge to be met that should not be underestimated.

4 The Deeper Issue: The Ontology of Physical Objects and the Case for Presentism

Classical mechanics as well as Bohmian mechanics are theories that propose an ontology of matter in motion: the fundamental physical domain consists in moving particles. The behaviour of complex objects is to be explained in terms of their composition by particles and the movement of these particles. Thus, Newton famously writes at the end of the "Opticks" (1704):

> … it seems probable to me, that God in the Beginning form'd Matter in solid, massy, hard, impenetrable, moveable Particles …; no ordinary Power being able to divide what God himself made one in the first Creation. … the Changes of corporeal Things are to be placed only in the various Separations and new Associations and motions of these permanent Particles. (Question 31, p. 400 in the edition [41])

The particles do not have spatial parts: they are located at points in space. They do not have temporal parts either: each particle is wholly existing at the point in space where it is located at a given time. It moves in the sense that the whole particle changes its position, being located at another point of space at another time. By moving the particle creates a continuous trajectory in space–time. The trajectory occupies a region in space–time, more precisely a worldline. The worldline has spatial as well as temporal parts, but the particle has no parts at all. Thus, employing

[17] The most elaborate account today is Barbour [2].

technical philosophical vocabulary, the particle persists in time by *enduring*, that is, by wholly existing at the location where it is at a given time. A worldline, by contrast, persists by *perduring*, that is, by having spatial as well as temporal parts, so that only a proper part of it exists at any given point in space at a given time.

One can with reason maintain that endurantism implies presentism.[18] The main argument is, in brief, this one: if an object x *wholly* exists at point p_1 in space at time t_1, then it cannot *wholly* exist at point p_2 in space at time t_2 AND it being true that whatever exists in space and time, simply exists, existence not being dependent on time. If x wholly exists at p_1 and if x wholly exists at p_2, then existence depends on time in the sense that when the object exists at p_2, it *no longer* exists at p_1. The argument against solipsism—that is, in this context, the argument against only the space–time point designated as "here-now" existing—then leads us to presentism, that is, the view that a spatial hypersurface ("the present") exists, whereby the spatial hypersurfaces are ordered in time such that they continuously come into and go out of existence. If, by contrast, an object x exists at point p_1 in space at time t_1 and if the same object x exists at point p_2 in space at time t_2 AND if both t_1 and t_2 exist, then x has spatial as well as temporal parts (in short, spatio-temporal parts, like its worldline).[19] Furthermore, one can develop a similar argument to the effect that the existence of motion implies temporal becoming and thereby an ontology that ties existence to time, as does presentism: if an object x wholly exists at point p_1 at time t_1, then it can exist at point p_2 at time t_2 only by its being at p_2 at t_2 coming into existence, as its being at p_1 at t_1 goes out of existence.

An analogous reasoning applies to any ontology that is committed to the motion of something. For instance, if one replaces the commitment to particles with a commitment to waves or fields that move by occupying ever more space in time, then one can defend instead of presentism also a view that recognizes both the past and the present as existing. Such a view is known as the "growing block universe" view. It of course also implies an objective foliation of space–time into spatial hypersurfaces that are ordered in time. The view then is that fields or waves move by growing in space, as what exists grows in time.

If, by contrast, one holds that there is no objective foliation of space–time, then one is committed to the position that the whole of space–time including all there is in space and time simply exists, that is, the block universe metaphysics (unless one sympathizes with the above-mentioned solipsism). The content of the block universe then consists in events in the sense of the properties that occur at space–time points. One can reconstruct what we take to be particles as continuous sequences of similar events, the so-called genidentical events: there are continuous regions of space–time that are distinct from their environment by there being similar properties instantiated in them. Thus, instead of starting with particles and getting worldlines from the movement of particles, one starts with regions of space–time that one can designate as worldlines due to the similarity of the properties instantiated

[18] But see Sider ([46], pp. 80–87).

[19] See e.g. Dorato [13]. Cf. also Benovsky [6].

in them and reconstruct what we take to be particles on this basis. A similar reconstruction can be applied to what we take to be waves or fields expanding in space–time. Consequently, there is nothing that moves or changes in time. But one can reconstruct what we take to be the motion or the change of something on the basis of the variation of events in space–time, that is, on the basis of the variation of the properties that occur at points in space–time.[20]

One may be inclined to take the view that relativity physics simply settles this issue in favour of the ontology of the block universe with events as its content and forces us to reconstruct everything that a physical theory takes itself to be committed to on this basis. However, the metaphysical debate about endurantism vs. perdurantism and, accordingly, an ontology of substances such as particles (or waves or fields expanding in space) vs. an ontology of events remains open. One can with reason say that it does not remain open because metaphysicians tend to ignore physics or are wedded to common sense. One can raise at least two objections against settling for the block universe metaphysics that are based on physics in general and that are independent of the above-mentioned issue of quantum physics (quantum non-locality) vs. relativity physics.

The first objection concerns the experimental evidence for any physical theory: that evidence consists in general in particles and their motion, more precisely, in evidence for changes in the state of motion of particles. To mention just two examples, the evidence for quantum field theory derives from various sorts of particle detection measurements. Furthermore, the evidence for the curvature of space–time posed by general relativity theory consists in the first place in tidal effects on the motion of particles.

The second objection concerns the question of what the properties are that the events located at space–time points instantiate which constitute the basic ontology according to the block universe metaphysics. If one asks what the properties are that make up a particle that moves in space and time, one mentions properties such as mass and charge. If one asks for a physical characterization of what these properties are, then one gets a reply of the type that these are dispositions to change the state of motion of particles as spelled out by the laws in which these properties figure—mass as a disposition that leads to acceleration of particles in the form of universal attraction, charge as a disposition that leads to a certain form of attraction or repulsion of charged particles via the creation of a field, etc. In short, the characteristic properties of particles are described in terms of how these properties change the state of motion of particles.

However, if there are no particles and if there is no motion, but a variation of the properties instantiated at space–time points with all these properties simply existing in a block universe, then one cannot use such characterizations in replying to the question of what the properties are that occur at space–time points in a block universe. In other words, one then has to elaborate on another reply to the

[20] See e.g. Lewis ([32], pp. 202–204), Heller [29], Sider ([46], chap. 4.6). See Benovsky ([5], first part) for an excellent overview of the debate.

question of what properties such as mass and charge are (if one wishes to retain these properties in one's ontology). There is one reply worked out in the literature that is based on the fact that one can develop an ontology of general relativity theory that identifies gravity with the metrical field and that, furthermore, takes the metrical field to consist in geometrical properties of space–time points (more precisely, space–time regions that can be pointlike, that is, arbitrarily small). The original programme of geometrodynamics of John A. Wheeler from the 1950s and 1960s is the ambitious project of reducing all physical properties to geometrical properties of space–time points or regions.[21] However, that programme failed and was subsequently abandoned by Wheeler.[22]

Nonetheless, the failure of a specific programme does not imply that it is impossible to develop a reply to the above-mentioned question. But as things stand, the only route open to elaborate on such a reply is of the type that Wheeler envisaged, namely the reduction of all physical properties to some sort of geometrical—or, more generally speaking, mathematical—properties (as given in geometrical, algebraic, or group theoretical characterizations, etc.). The general worry that one can raise against any such project is the old anti-Pythagorean and empiricist one that consists in saying that the essence of physical existence is missed by identifying it with mathematical existence. This, of course, is a debate that reaches far beyond the scope of this paper. But it shows that the issue of presentism concerns much more than just the question of an ontology that suits relativity physics well. And it shows that the case of presentism is open, both from the metaphysical point of view (as argued in this section) as well as from the physical point of view (as argued in the preceding section).

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[21] See Wheeler ([48], in particular pp. XI–XII, 8–87, 129–130, 225–236). For a brief overview, see Wheeler [49]. For a philosophical characterization, see Graves ([26], chaps. 4–5, in particular pp. 236, 312–318).

[22] See Misner et al. ([36], § 44.3-4, in particular p. 1205).

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Now, Factuality and Conditio Humana

Hartmann Römer

Abstract The relationship between inner and outer time is discussed. Inner time is intrinsically future directed and possesses the quality of a distinguished "now." Both of these qualities get lost in the operationalized external physical time, which, advancing towards more fundamental physics, tends to become more similar to space and even fade away as a fundamental notion. However, inner time as a constitutive feature of human existence holds its place in the heart of quantum theory and thermodynamics.

1 Introduction

Time lies next to the hot focus of our mode of existence as conscious beings and has always been a permanent subject of human thinking and philosophy. Our world and even our own mind is given to us in the inexorably temporal form of a movie-like course of appearances rather than as a simultaneous panoramic picture. All our reflections about this temporality are caught inside the genuinely temporal structure of a stream of consciousness. In fact, trying to escape temporality by an effort of thought is an extremely delicate task at the verge of paradox and unthinkability.

Over the centuries, much philosophical activity was devoted to a detailed analysis of the temporal mode of human existence. Prominent names as Augustine, Kant, Hegel, Bergson, and Heidegger are witnesses of this endeavor. Employing the terminology of Mc Taggart [1], internal existential time of man is an *A-time*, characterized by future directedness and in particular by the existence of the temporal quality of a distinguished *"now."* The window of this "now" moves forward into the future leaving past behind it. The feeling of this flow is what remains, if all sensory input is neutralized. According to Heidegger the driving force behind this motion of the "now" is not a push from the past but rather a pull from the future originating in the fundamental structural feature of *"worry"*(German: "Sorge") of human existence.

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One particularly important fruit of the persistent philosophical concern about time is the emergence, clarification, and sharpening of the concept of outer or physical time. Here time has been tamed, harnessed, and subdued to measurability by clocks. External physical time differs from internal time in several respects. Again using Mc Taggart's terminology, it can be denoted as a *B-time*, a scale time representable by the set of points on a line or a set of real numbers. There is no quality of a distinguished "now," all time points are equivalent just as points on the line. Moreover, future directedness is not necessarily related to physical B-time. The concept of physical time has been spectacularly successful. Its power manifests itself in the omnipresence of clocks which hold all of us under their sway. There is a common tendency to accept physical B-time as the only exact and in a sense only real notion of time and to reduce internal A-time to a subordinate or even illusionary status. Investigating the relationship between internal and physical time will be the main subject of this note.

2 "Spacialization" and Evaporation of Physical Time

Along with the advancement towards more fundamental levels of physics, physical B-time shows a tendency to become more and more similar to space. In addition, there are strong indications, that, together with space, it might disappear altogether as a fundamental notion, when pushed to the extremes of cosmology and quantum gravity.

From the outset, the lack of a privileged "now" in B-time parallels the absence of a privileged "here" in space. In this respect, B-time is more similar to space than A-time. This opens up the possibility to represent processes in time in a diagrammatic atemporal way by introducing a spacial time axis.

Let us first look at the motion of a point particle. The presence of this particle in a point x at (or very close to) time t is an example of a *point event* with a precise localization in time and space, which can be described by four coordinates, one temporal coordinate and three spacial coordinates. Then the motion of the point particle is completely described by its *world line*, a one-dimensional set of point events giving the position of the particle at every time (see Fig. 1). This world line, as it stands, is a line in space, for instance on a sheet of paper and as such entirely timeless. Reference to time is only given by its interpretation as a representation of a motion.

This possibility of a timeless representation of processes is not restricted to physical motions. For instance, the phylogeny of man can be adequately represented by a totally timeless family tree (see Fig. 2). There is no evident way to decide, which representation is more correct or "real."

Passing from Newtonean space-time to space-time in Special and General Relativity, one observes that time becomes more and more similar to space.

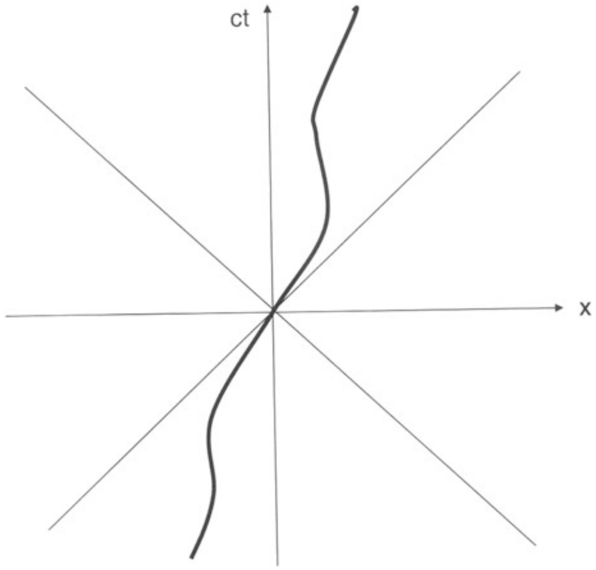

Fig. 1 Motion of a point particle in the timeless world line representation. Two space dimensions are suppressed

From the perspective of Kant's philosophy this is not a complete surprise: According to Kant time and space are forms of intuition (German: "Anschauungsformen"), through which everything has to pass, which reaches our mind. Time is the form of the inner sense and space the form of the outer sense. (The Kantian distinction between inner and outer sense is akin to the Cartesian distinction between res cogitans and res extensa.) As physics is concerned with the outer world, externalized physical time can be expected to become more similar to space.

Let us now follow the increasing "spacialization" of time from Newtonean mechanics to General Relativity Theory in more detail.

(a) In Newtonean space time the transition between the coordinates of point events in different inertial systems is performed by Galilei-transformations. It is possible to define a global Newtonean world time attributing a time coordinate to every point event such that time differences $\tau_{i,j} = t_i - t_j$ between any two point events are independent of the inertial system. This implies in particular that the simultaneity $\tau_{i,j} = 0$ of two events is an invariant notion. Moreover, one immediately obtains

$$\tau_{1,2} + \tau_{2,3} = \tau_{1,3}. \tag{1}$$

Space behaves differently under Galilei-transformations. Spacial coincidence is a relative, system dependent notion: Point events occurring at the same space point in one inertial frame will in general occur at different space points in another inertial frame. Only for *simultaneous* events the spacial distance

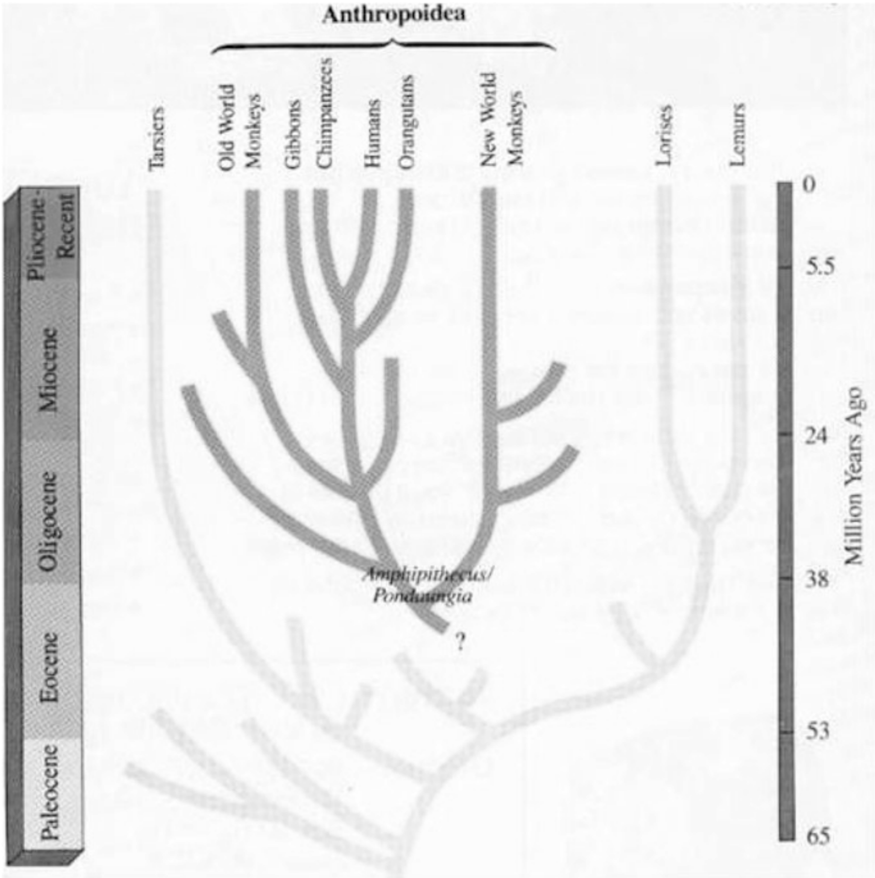

Fig. 2 Development of hominides as a timeless family tree

$r_{i,j} = |\vec{x}_i - \vec{x}_j|$ has an invariant meaning. Instead of Eq. (1) one has the *triangular inequality*

$$r_{1,2} + r_{2,3} \geq r_{1,3}. \tag{2}$$

(b) In Special Relativity Theory the transition between different inertial systems is performed by proper orthochronous Poincaré-transformations. Time differences are no longer invariant but depend on the inertial system just like spacial distances. In particular, simultaneity becomes a relative, system dependent notion. To be more concrete, the situation in Special Relativity Theory is as follows:

Light propagates with the same velocity c in all inertial systems, and this velocity c is the highest possible velocity for any signal. Two different point events e_i and e_j are called *relative timelike*, if a subluminar signal can be exchanged between them, *relative lightlike*, if only a luminal signal can be exchanged between them and *relative spacelike*, if they cannot be connected by a signal. It turns out that the temporal order of relative timelike or lightlike signals is independent of the inertial systems. In other words, the sign (but not magnitude) of the time coordinate difference of such signals is invariant. This assures that the temporal order of cause and effect is the same in every inertial system as it should. On the other hand, the temporal order of relative spacelike events is system dependent. It is always possible to change the sign of time coordinate differences of such events by a change of the inertial system, and there is always an inertial system, in which relative spacelike events are simultaneous. (Of course simultaneous events are relative spacelike, because evidently a signal connecting them would have to have infinite velocity.)

For relative timelike events e_i and e_j there is an invariant measure of their temporal distance: $\tau_{i,j}$ is defined as the so-called *proper time* measured along a geodesic, i.e. straight world line connecting e_i and e_j. If e_2 lies in the future of e_1 and e_3 in the future of e_2, then rather than the equality (1) an inequality

$$\tau_{1,2} + \tau_{2,3} \leq \tau_{1,3}. \tag{3}$$

holds, a triangular inequality similar to the spacial inequality (2) but with a $\leq$ instead of a $\geq$ sign. (This $\leq$ sign is the origin of the famous twin paradox: The proper time difference for a worldline connecting two relative timelike events is maximal for a geodesic.) For relative lightlike events $\tau_{i,j} = 0$, and (3) remains valid, if we allow some of the proper time differences in it to vanish.

For relative spacelike events e_i and e_j it is possible to define an invariant spacial distance $r_{i,j}$, which is simply the spatial distance in an inertial system in which e_i and e_j are simultaneous. The spacial triangular inequality (2) remains valid as long as e_1, e_2, and e_3 lie in one spacelike plane or, equivalently, if all three events are simultaneous in an appropriate inertial system. So, we notice yet another similarity between time and space in Special Relativity Theory: Both time- and space- differences fulfill triangular inequalities.

(c) The unification of space and time goes even further in General Relativity Theory. Global inertial coordinate systems are no longer definable, but an observer, freely falling in a gravitational field, can at least realize a *local inertial system*, in which the laws of Special Relativity are valid for small deviations from the origin of the space time coordinates. This is just a reformulation of Einstein's *equivalence principle*. The concept of signals and the distinction between relative timelike, lightlike, and spacelike events remains valid. Space and time are fused in the geometric concept of a pseudoriemannian space-time manifold M and point events are points in M. On M a metric tensor $g_{\mu\nu}$ is defined, which allows to measure the length of world lines in M. Also the concepts of geodesics and geodesic distances remain well defined and the inequalities (2) and (3) still hold (at least if e_1, e_2, and e_3 are not too far apart). Timelike and lightlike geodesics are the worldlines of massive particles and photons respectively in a gravitational field.

In sharp contrast to Newtonean physics and Special Relativity, space-time is no longer the external arena on which the play of physics is performed, but the arena takes part in the play: The curvature of the space-time metric, which measures its deviation from the "flat" geometry of Special Relativity depends on the distribution of energy and momentum of matter in the physical universe. The view of the universe as suggested by General Relativity is a timeless geometric structure often referred to as a *block universe*. The dynamics of particle motions appears to be frozen in a concept of world lines in the space-time manifold M, which does not suggest any natural definition of a time coordinate. Also fields like the electromagnetic field or the gravitational field appear in the timeless geometric mathematical form of sections in certain bundles over the space-time manifold M.

Physical B-time, as opposed to internal A-time is not necessarily directed. In fact, neither Newtonean time nor time in Special Relativity nor the concept of pseudoriemannian space-time manifolds in General Relativity is connected to any notion of a preferred time direction. Moreover, the fundamental laws of physics are invariant under time inversion. A tiny violation of time inversion invariance in weak interactions may look like a possible exception, but this asymmetry is probably due to spontaneous symmetry breaking: It results not from an asymmetry of the fundamental equations of physics but from an asymmetry of their solution. This also applies to the apparent asymmetry imposed by the space-time metric of the expanding universe, which is a time asymmetric solution of the symmetric Einstein equations of General Relativity. The directedness of time coming from the second law of thermodynamics will be discussed later.

One of the virtues of General Relativity Theory is that it is able to predict borders of its validity. The singularity theorems of Hawking and Penrose prove that certain world lines will inevitably terminate in space-time singularities, where curvature quantities become infinite. This, for instance, occurs, when black holes are formed. Also, all time- or lightlike worldlines of the visual parts of our universe originate in a gigantic "big bang" singularity. No time parameter can be extended before this singularity. Moreover, adding some elements of quantum theory, one sees that the "spacialization" of time turns into an abdication and successive loss of both time and space. In fact, time and space lose their meaning for distances of the order of Planck's time $t_P = \sqrt{\hbar G/c^5} \approx 5.4 \cdot 10^{-44} s$ or Planck's length $l_P = ct_P = \sqrt{\hbar G/c^3} \approx 1.6 \cdot 10^{-35} m$, where G is the gravitational constant and $\hbar$ Planck's quantum of action. This can be seen in the following way: In order to resolve spacial distances l by a measurement, according to Heisenberg's uncertainty principle one has to provide momentum of order $\hbar/l$ in a volume l^3. By Einstein's equations of General Relativity this gives rise to a change of space-time curvature characterized by a curvature radius L with $1/L^2 = Gp/l^3c^3 = \hbar G/l^4c^3$. The measurement of l certainly becomes ill-defined if the resulting curvature change L is of the order of magnitude of l. Equating l and L gives $l = L = l_p = \sqrt{\hbar G/c^3}$.

As a consequence, the classical General Relativity Theory becomes unreliable in a neighborhood of order l_P or t_P of the singularities predicted by this theory, including the big bang singularity.

Although quantum field theory on fixed pseudoriemannian space-time manifolds is by now well developed, quantum theory of space-time itself is still in its infancy. The preceding argument shows that quantum effects become dominant at distances of the order of magnitude of Planck's length and time. Also the distinction between timelike, spacelike, and lightlike separation becomes blurred by quantum indeterminacies. The problem of finding a Quantum Gravity Theory [2], i.e. a quantum theory of space-time is unsolved and perhaps the greatest challenge of fundamental theoretical physics. Describing, comparing, and assessing the various current approaches towards this problem would at least require a book and be way beyond the scope of this note. We restrict ourselves to some short remarks about three different approaches.

(a) *String Theory* [3] starts out from quantizing the motion of a one-dimensional string like object vibrating in a background space-time manifold B. Planck's length is closely related to the tension of this string. In fact, the string has to be supersymmetric and carries spin degrees of freedom. There are good arguments that the quantization of the string leads to a massless particle of spin 2 to be identified with the quantum of the gravitational field. (However, the emergence of curved four-dimensional space-time is not quite clear.) In addition, the claim is, that String Theory describes all particles and interactions of physics. Later on, it turned out that for the sake of consistency the quantum theory of the string should also contain higher dimensional vibrating objects called *branes*. The technical and, more seriously, conceptual difficulties of String Theory are formidable. So far, no precise formulation of its theoretical framework is available. It is not yet clear how to get rid of the undesirable background dependence of string theory. For consistency, the background manifold B must be ten-dimensional, and six space dimensions must be "compactified," i.e. curled up at a small scale in order not to be directly visible. The precise form of this compactification is undetermined and seems to be highly arbitrary. (According to rough estimates there are more than 10^{500} different compactifications.) The particle content and the interactions of the resulting theory radically depend on the compactification, leading to a loss of predictive power as long as no principle for identifying preferred compactifications can be given. To make things worse, no known compactification reproduces the well-established standard model of elementary particle theory.

(b) *Loop Quantum Gravity* [4, 5] belongs to a class of theories which assume that for very small scales of the order of magnitude of l_P space-time or space loses its continuous manifold character and becomes discontinuous and discrete like a lattice. On larger scales the number of lattice points should become so large that in the average a *continuum limit* is a good approximation, such that a smooth manifold structure is regained. So, the smooth space time of Newtonean theory and Special and General Relativity is conceived as a derived, approximate notion. The virtue of such theories is that singularities at small scale are excluded from the outset. However, it is still not clear, how and under what conditions the smooth continuum limit is obtained.

(c) *Canonical Quantum Gravity* tries to apply the procedure of canonical quantization to Einstein's theory of gravitation. In its most popular form, it leads to the *Wheeler- de Witt equation* [6], an equation of the structure

$$H(g^{(3)}, \delta/\delta g^{(3)})\Psi(g^{(3)}) = 0. \tag{4}$$

Here, the "Schrödinger wave function" $\Psi(g^{(3)})$ is a functional depending on metric $g^{(3)}$ on a three dimensional manifold, and the Hamiltonean $H(g^{(3)}, \delta/\delta g^{(3)})$ depends on $g^{(3)}$ and the functional derivatives with respect to $g^{(3)}$. Unlike a normal Schrödinger equation in quantum mechanics, the Wheeler- de Witt equation contains no time derivative of the wave function and is, in this sense, completely timeless. This is related to the absence of any natural time coordinate in General Relativity. Something like time may be recuperated in a quasiclassical approximation (after all, the space-time we are living in is classical) of the type

$$\Psi \sim \exp(iS/\hbar). \tag{5}$$

In this case, the support of the action functional S is decomposed into one-dimensional families of three-metrics, which can be interpreted as metrics on four-dimensional manifolds. Depending on the solution Ψ, the signature of the four-metric may be pseudoeuclidean, and the parameter along such a one-dimensional family can be interpreted as a time coordinate. The Big Bang singularity may appear in the form of a boundary condition on Ψ. We see that also in this approach, time only appears as a secondary, derived and approximate notion. In addition, there are also solutions of the Wheeler- de Witt equation which do not allow for the introduction of a time coordinate. It must also be admitted that the precise mathematical definition of the Hamiltonean and the wave functional in the Wheeler- de Witt equation is not yet clear.

These examples for Quantum Gravity theories clearly show that the tendency of a successive abdication of physical time is continued, if quantum effects on space-time are taken into account.

3 The Revival of A-Time in Quantum Theory and Thermodynamics

The progressive spacialization of physical time becomes also evident in a comparison of nonrelativistic quantum mechanics, quantum field theory on Minkowskian space, and quantum field theory on curved background space-time manifolds.

Nonrelativistic quantum mechanics is the quantum version of Newtonean mechanics. Space and time play quite different roles: Whereas space is represented by position operators acting on the Hilbertspace of quantum states, time is present as a classical parameter t of B-type. In the Schrödinger picture of quantum mechanics, the state vector $\psi(t)$ depends on this parameter. Alternatively, in the Heisenberg picture all observables depend on t.

In quantum field theory the Heisenberg picture is conceptually favored and both space and time are reduced to the same status of classical parameters such that the field operators $\Phi(t, \vec{x})$ are functions of t and $\vec{x}$. It is still formally possible to construct spacial and temporal localization operators from these field operators [7, 8] but with a lot of arbitrariness. The similarity of the time and space parameters is further enhanced in special relativistic quantum field theory and even more so in quantum field theory on curved background manifolds. The parametric space time dependence is of paramount importance for the axiomatic formulation of local quantum field theory [9]. The concept of locality has to be revised, if also the geometry of time and space is subject to quantization and loses its fundamental meaning as we saw in the preceding paragraph.

In view of the spacial character of physical (B-)time and its eventual disappearance, time is sometimes assumed to be unreal and illusionary. Attributing such an ontologically subordinate status to time signals a strong physicalistic and reductive attitude connected to the claim that, at least in principle, everything should be describable in terms of physics. We shall argue that this creed is of low plausibility. Right here we should notice that physics is a highly developed, sophisticated and spectacularly successful method to build up a mathematical model of the world. But by definition a model concentrates on features amenable to its framework, and it is certainly a methodological mistake to identify a model with what it modelizes.

Classical physics silently supposes an external observer merely registering phenomena of the observed system without influencing it in an essential and non-negligible way. This supposition is no longer tenable in quantum theory, where measurement will in general change even pure states. The role of the observer becomes an active rather than a merely registrating one, and time in the form of the A-time of the observer has its place right in the heart of quantum theory:

A measurement result is factual and appears via the "now" of the observer. The directedness of the observer's A-time is encoded in the non-commutative structure of quantum theory. The order of successive measurements is vital, and in general only the result of the last and latest measurement can claim factual status, whereas the factuality of previous measurement results is destroyed by subsequent measurements. Notice that the composition $P_1 P_2$ of two proposition observables P_1 and P_2 means that P_1 is applied and measured *after* P_2.

A complete physical description of a measurement process in terms of quantum dynamics is not available. In quantum dynamics, time development is given by unitary transformations of states or observables. A measurement results in a non-unitary reduction of the quantum state. Of course, a measurement process is accompanied by a quantum physical process, but this physical description does not seem to be exhaustive. In fact, no clear purely physical criterium lends itself to qualify a physical process as a measurement process. In addition to being a physical event, a measurement is also an act of cognition. In the subtle role of the measurement process, quantum physics has an open door to cognition and epistemology, and such philosophical considerations cannot be ignored as easily as in classical physics. Unitary time development of quantum dynamics is completely deterministic, and nondeterministic randomness enters only through measurement. Also the causal closure of pure quantum dynamics is broken by the measurement process and the freedom of a choice of the observable to be measured. (This also applies to classical theory but becomes more conspicuous in quantum theory.)

The thermodynamic time arrow pointing towards an increase of entropy is another case, where an inroad of directed A-time into physics can be observed. The notion of entropy rests on an incomplete description of a system in terms of *macroscopic states*, where a complete description in terms of *microscopic states* is unfeasible. The transition from a microscopic to a macroscopic description is performed by the application of a *coarse graining* procedure, which associates a certain well-defined mixed statistical product state to the system and thereby a definite value to its entropy. Normal microscopic physical dynamics (unitary in quantum theory, symplectic in classical theory) does not change the entropy of this state. An increase of entropy only results when coarse graining is again applied to the time developed state. Thereby inaccessible correlation information between the different subsystems corresponding to the coarse graining is discarded and entropy is increased. So far, the reasoning was still time symmetric, because the microscopic physical time development could have been taken in both time directions. The directedness of A-time enters, because an observer can only *first* register a macroscopic state and *then* discard correlation information and not vice versa.

For an investigation of the relationship between inner A-time and outer physical B-time we need a comprehensive framework including both mental and material systems. *Generalized Quantum Theory*(GQT) [10–12] provides a general system theory, which arose from physical quantum theory in its algebraic form leaving out those features which only pertain to physical systems.[1] The resulting formalism is still rich enough to allow a controlled and formally well-defined application of quantum theoretical notions like complementarity and entanglement far beyond the realm of physics. Mental systems, in particular the human mind as seen from an inner first person perspective are well inside the scope of GQT and are even of paradigmatic importance in this framework.

4 Generalized Quantum Theory

In this section, we give a brief account of the vital structural features of GQT in order to ease the understanding of our argumentation in the subsequent sections. For a full account of GQT we refer to the original publications [10–12]. References

[1]Originally it was called “Weak Quantum Theory,” but this led to misunderstanding by non-mathematicians. Admittedly, the term “Generalized Quantum Theory” is somewhat unspecific and equivocal and has been used in different senses by other authors.

for numerous applications of GQT, which have been worked out in more or less detail, can be found in [12] and [13]. In [14] a new application of GQT to order effects in questionnaires is described. GQT takes over from quantum physics the following four fundamental notions:

1. *System*: A system is anything which can be (imagined to be) isolated from the rest of the world and be subject to an investigation. In the sequel we shall consider systems containing also conscious individuals. In contradistinction to, e.g., classical mechanics the identification of a system is not always a trivial procedure but sometimes a creative act. In many cases it is possible to define *subsystems* inside a system.
2. *State*: A system must have the capacity to reside in different states without losing its identity as a system. One may differentiate between *pure states*, which correspond to maximal possible knowledge of the system and *mixed states* corresponding to incomplete knowledge. In the most general form of GQT, the set of states has no underlying Hilbert state structure. For some applications (see, e.g., [14–17]) one may want to enrich the minimal scheme of GQT, for instance by adding this additional structure.
3. *Observable*: An observable corresponds to a feature of a system, which can be investigated in a more or less meaningful way. *Global observables* pertain to the system as a whole, *local observables* pertain to subsystems.
4. *Measurement*: Doing a measurement of an observable A means performing the investigation which belongs to the observable A and arriving at a result a, which can claim factual validity. What factual validity means depends on the system: Validity of a measurement result for a system of physics, internal conviction for self-observation, consensus for groups of human beings. The result of the measurement of A will in general depend on the state z of the system before the measurement but will not be completely determined by it. In GQT as well as in physical quantum theory, the notion of measurement contains an element of idealization, because measurement is not described as a temporal process, rather the focus of attention lies on the factual result obtained eventually.

In addition to these definitions the following structural features of GQT are of particular importance, generalizing essential properties of physical quantum theory. To every observable A we associate its *spectrum*, a set Spec A, which is just the set of all possible measurement results of A. Immediately after a measurement of an observable A with result a in Spec A, the system will be in an *eigenstate* z_a of the observable A with *eigenvalue* a. The eigenstate z_a is a state, for which an immediate repetition of the measurement of the same observable A will again yield the same result a with certainty, and after this repeated measurement the system will still be in the same state z_a. This property, which is also crucial in quantum physics justifies the terminology "eigenstate of an observable A" for z_a and "eigenvalue" for the result a. We repeat that this is an idealized description of a measurement process abstracting from its detailed temporal structure.

Two observables A and B are called *complementary*, if the corresponding measurements are not interchangeable. This means that the state of the system depends on the order in which the measurement results, say a and b, were obtained. If the last measurement was a measurement of A, the system will end up in an eigenstate z_a of A, and if the last measurement was a measurement of B, an eigenstate z_b will result eventually. For complementary observables A and B there will be at least some eigenvalue, say a, of one of the observables for which no common eigenstate z_{ab} of both observables exists. This means that it is not generally possible to ascribe sharp values to the complementary observables A and B, although both of them may be equally important for the description of the system. This is the essence of quantum theoretical complementarity which is well defined also for GQT.

Non-complementary observables, for which the order of measurement does not matter, are called *compatible*. After the measurement of compatible observables A and B with results a and b, the system will be in the same common eigenstate z_{ab} of A and B irrespective of the order in which the measurements were performed.

In quantum physics, *entanglement* is normally explained by the existence of non separable Hilbert space states, which are linear superpositions of separable tensor product states. But entanglement can also be defined in the framework of GQT, which contains no reference to a Hilbert space of states [10–12, 18]. It may and will show up under the following conditions:

1. Subsystems can be identified within the system such that local observables A_i pertaining to different subsystems are compatible.
2. There is a global observable A of the total system, which is complementary to local observables A_i of the different subsystems.
3. The system is in an *entangled state*, for instance in an eigenstate of the above-mentioned global observable A, which is not an eigenstate of the local observables A_i.

Given these conditions, the measured values of the local observables will be uncertain because of the complementarity of the global and the local observables. However, so-called *entanglement correlations* will be observed between the measured values of the local observables A_i pertaining to different subsystems. These correlations are non-local and instantaneous.

In physical quantum theory, the singlet state of a two-spin system is a standard example of entanglement. In this case, the total spin

$$S^2 = (s_1^{(1)} + s_1^{(2)})^2 + (s_2^{(1)} + s_2^{(2)})^2 + (s_3^{(1)} + s_3^{(2)})^2 \tag{6}$$

is the global observable, the individual spin observables $s_3^{(1)}$ and $s_3^{(2)}$ take over the role of the local observables complementary to the global observable, and the entangled singlet state is the eigenstate of S^2 with eigenvalue zero. We see how our generalized definition captures the essentials of quantum theoretical entanglement.

In physical quantum theory it is not difficult to show that entanglement correlations cannot be used for signal transmission or controlled causal influences. In order to avoid intervention paradoxes this must be postulated for GQT [18–20].

In view of the possibility of entanglement correlations in GQT the problematic and anything less than trivial character of the act of identification of a system becomes even more acute.

5 Observables, Partitions and Epistemic Cut

In classical or quantum mechanics, observables like positions and velocities seem to be given in a very direct and unproblematic way by the system itself. The example of entropy[2] shows that even for physical systems the identification of observables need not be so trivial but sometimes means a major discovery. For very general systems like those considered in GQT, observables are not so directly given by the system and read off from it like location and velocity in a mechanical system. On the contrary, as already suggested by the name of an "observable," the identification of an observable may be a highly creative act of the observer, which will be essentially determined by his horizon of questions and expectations. This marks a decidedly epistemic trait of the notion of observables in GQT even more so than in quantum physics. Moreover, the horizon of the observer will change, not the least as a result of his previous observations adding to the open and dynamical character of the set of observables. In humanities, this iterative and constitutive process is sometimes called the hermeneutic circle.

What has just been said about observables also applies to *partitioning* a system into subsystems. G. Mahler [21] vigorously pointed out that the identification of subsystems in a complex system may be a highly creative act. In general, subsystems do not preexist in a naïve way but are in a sense created in the constitutive act of their identification. Partitioning may be considered as a special case of constituting observables, because partitioning is achieved by means of *partition observables* whose different values differentiate between the subsystems. Partition observables and, hence, the associated partitions may be complementary, resulting in an incompatibility of different partitions. Two such incompatible partitions cannot be overlaid in order to arrive at a common refinement of both of them. A simple physical example of such a situation are partitions according to position or momentum of a quantum multiparticle system. In physical systems, the position observable Q is a privileged partition observable, which differentiates between subsystems by their different locations. The paramount importance of the position

[2]In physical quantum theory, as opposed to GQT, entropy is not an observable in the technical sense.

observable Q is reflected in the fact that the realm of physics is often identified with the range of applicability of the position observable. This is reminiscent of Descartes' denotion of the material world as "res extensae." Indeed, position and locality play a vital role in physics down to small distances of the order of the Planck length l_P.

The first partition, prior to and prerequisite for every act of measurement or cognition is the split between observer and observed system. In quantum theory this is referred to as the *Heisenberg cut*. In the wider framework of GQT we call it the *epistemic split.* Just as the Heisenberg cut, the epistemic split is movable but not removable, because every cognition accessible to us is the cognition of someone about something. The epistemic cut may be far outside the observer, if a remote quasar is observed, or run right through a person's mind in the case of self-observation, but it is never absent. It is conceivable that there are partitions and observables incompatible with a given or even any epistemic split. Quantum theoretical uncertainties are closely related to the epistemic split. As already mentioned above, in the quantum theory of physical measurement the time development in the large system containing both observer and observed is completely deterministic. Stochasticity enters into quantum theory as a result of the epistemic split into the subsystems "measured system" and "observer" or "measurement apparatus" and by the subsequent projection onto the latter system and the interpretation of the result obtained there as a statement about the measured system. In quantum physics, there is a symmetry between observer and observed. Projection onto the measured system yields the same probability distributions as the above-mentioned projection onto the measurement device. One may wonder [18] to what extent this symmetry has a generalization to GQT in the form of a certain "inside-outside" symmetry.

Coming back to observables in quantum physics and GQT, it is important to stress that they already presuppose the epistemic split. Observables neither exclusively pertain to the observed system nor to the observer, rather they are located astride on the epistemic split. Related to this, the observer plays an active role in the constitution of observables, in the choice of the observables to be measured and in the factual establishment of the measurement results.

One might be tempted to associate observables with properties of systems, which, like position or velocity, correspond to substantial entities best expressed by nouns. In [22] we argue that this would point in the direction of a one-sided preference for an ontology of substances. Process observables associated to transitions and changes of a system and typically better expressible by verbs are of equal importance, and a balance between substance and process ontology [23, 24] seems to be desirable. We shall have to say more about this in the following section.

For what follows we should also keep in mind that in GQT inner observables occurring in introspective self-observation are fully legitimate objects of consideration.

6 The Emergence of Physical Time

We already mentioned several times that, as opposed to internal A-time, external physical B-time lacks an intrinsic directedness as well as the quality of a distinguished “now.” The physics of time direction [25] endeavors to establish a physical basis for the evident directedness of time encountered everywhere in the world we live in. In other words, a physical mechanism is sought, which endows B-time with directedness. (The equally fundamental problem of the missing “now” is normally not dealt with.) The investigations related to this task proceed in the following way: First various “arrows of time” are described, in which time shows directedness, such as the thermodynamic time arrow for the increase of entropy, the cosmological time arrow for the expansion of the universe, the retardation time arrow for the emission of radiation from a spatially localized source or the evolutionary time arrow for the formation of structures in our world. Then arguments are given, that the various time arrows are aligned, i.e. point into the same direction. Finally, one of the time arrows is identified as primary and a physical mechanism is proposed to account for it. Usually, either the cosmological or the thermodynamical time arrow is placed in the pivotal position. Here, the cosmological time arrow appears in the role of a boundary condition for cosmological evolution, whereas a complete formal deduction of the second law of thermodynamics from the laws of physics is still missing.

In these physical approaches to the directedness of time, the evolutionary time arrow is interpreted as a reflection of the thermodynamic time arrow. This is also offered as the explanation for the directedness of internal A-time, referred to as the “psychological time arrow” in this context.

In this note, we propose to place internal A-time in the primary position as a categorial constitutive feature of the human mode of existence, prior to any physical modelling of the world. Physical B-time would then result secondarily together with a loss of the qualities of directedness and “now.” One evident advantage of this view is that it is certainly easier to understand a loss of features than the emergence of completely new features of time. Anyhow, as explained in Sect. 2, A-time is present in the heart of quantum theory and thermodynamics.

Our scenario for the constitution of physical B-time has been described in detail in [19]. GQT offers itself as a suitable formal framework, because it allows to treat material and mental systems on an equal footing. (For a related but in many respects rather different quantum approach to temporal matter-mind systems, see [26].) Here we list the most important steps on this way, referring to [19] for details:

1. After an epistemic split, internal time observables T_i arise in subsystems corresponding to individuals and the time bounded mode of their conscious personal existence.
2. The partial synchronization between different internal time observables T_i as well as between many “clock” observables T_I identified in the outer physical world is certainly not an effect of causal interactions, but an acausal parallelism which can be described by entanglement correlations in the sense of GQT between many subsystems.

3. External time is successively sharpened and operationalized in a long and sophisticated process of optimizing physical clock observables such that the aforementioned entanglement correlations become as strict as possible. (For a nice account, see [27].) At present, the best clocks are given by atomic systems with oscillations between sharp energy levels.
4. In view of what was said in Sect. 2, time becomes more and more similar to space in this process of externalization. After all, space is the form of the exterior sense. In the course of "spacialization" of time both the notion of "now" and the future directedness of time get lost, because space neither has a privileged "here" nor a natural directedness.
5. We saw that, pushed to the extremes, sharpening of the concept of physical time eventually will lead to its deconstruction with a loss of its fundamental meaning. So, the course of events can be summarized in the following way: establishment of internal time, establishment of physical time along with a fading of characteristic features of inner time, deconstruction of physical B-time.

The distinction between substance and process observables mentioned at the end of Sect. 5 can be formalized using the time observables T_i [22, 28]. A substance observable C is an observable which commutes with T_i: $T_i C = C T_i$. This means that either C is compatible with a sharp localization in time or bears no reference to time whatsoever. The position observable Q or the angles of an triangle are simple examples of substance observables. Process observables can be defined as complementary to T_i: $T_i D \neq D T_i$. By definition, a temporal process has no precise temporal localization. The complementarity of substance and process observables was proposed in [22, 28] as a resolution of Zeno's paradox: A flying arrow seems to freeze in its motion, when attention is focused on its momentary position at any given instant of time: The arrow never occupies more space than given by its length. The position of the arrow at a given time is a substance observable, whereas its total flight is described by a process observable. The incompatibility of process and substance in this case is similar to the vanishing of the concept of the orbit in the description of motion in quantum mechanics.

7 Conditio Humana

Evidently, the world is never given to us directly but only as it appears on our inner screen. This trivial fact, which in philosophical terminology is just the phenomenal character of the world, when taken seriously, has far reaching consequences. Everything we sensually or intellectually conceive of our world is shaped and conditioned in a categorial way by the mode of our existence as conscious individuals. Naive realism asserts that the world appears to us more or less "like it really is." Sometimes our categorial cognitive structure is compared to a pair of colored sunglasses, which can be taken off to allow a look at the real world. But also this optimistic belief underestimates the inexorable phenomenality of our existence, which must be the starting point of every reflection about the way we orient ourselves in our world. In particular, physics cannot lay its own foundations but has to be aware of the categorial prerequisites imposed by our cognitional system and our mode of existence. In this spirit we already mentioned in Sect. 3 that a measurement should not entirely be conceived as a physical process but also as an act of cognition. This also prevents a complete causal closure of physics. Of course, the physical process accompanying measurement has to be investigated and consistency with the possibility of cognition must be guaranteed. A strict physical reductionism, trying to reduce "everything" to physics, is unaware of the phenomenal character of the world and, hence, of its own foundations. Moreover, as already mentioned, it runs into the naive methodological mistake to identify the model with what is modelled. Everetts's many-world interpretation of quantum theory illustrates the bizarre consequences of an extreme physicalism in the interpretation of measurement: The whole universe, conceived as a purely physical system, is assumed to split into several branches as the result of each measurement.

The main structural features of the phenomenal mode of human existence have already been mentioned in passing. We briefly collect them here, a more detailed analysis can be found in [29]. (Prauss [30] contains a comprehensive and deep discussion.)

- *The figure of oppositeness*. In every act of cognition we experience ourselves as an observer, different and set apart from what we observe. This is sometimes referred to as the *egocentricity* [31–33] of human existence. The epistemic cut between observer and observed is never absent.
- *Temporality*. Human existence is inescapably temporal in the sense of a future-directed A-time with a privileged "now."
- *Factuality*. We live in a world of facts rather than a world of potentialities. Everything which appears to us primarily touches us in the form of a fact. In particular, the "now" carries the imprint of prototypic factuality.

These basic existential features are deeply encoded in the structure of quantum theory and GQT [29]. The naturalness and, in a way, a priori structure of quantum theory has been observed by many authors and has, for instance, been expressed in full clarity by M. Bitbol [34].

- The epistemic cut is present in the very special and fundamental role attributed to measurement in quantum theory and GQT. We saw that observables are located right on the epistemic cut. Standard reductive physicalism ignores the importance of the observer and the epistemic cut in favor of the outside world. In this sense, it is as one-sided and implausible as a solipsistic world view, which ignores the outside in favor of the inside world.
- In Sect. 3, we saw how deeply A-time is encoded into quantum theory, GQT (and thermodynamics).
- Factuality is intimately related to quantum theoretical measurement, which basically amounts to a transition from potentiality to a measurement result of factual validity.

The categorial scheme of human existence is, of course, the product of a long development. The temporality of primitive animals is a total subjection to the undivided factuality of a simple "now." Memory and the possibility of preparing actions open up the horizon of temporality eventually resulting in a differentiation between past, present, and future. Causality and personal freedom, which are often considered to be in contradictory relationship, actually rely on one another and are in fact offshoots of the same root of such a developed and differentiated temporality. This phylogenetic process is repeated in quick motion in the ontogenesis of every human individuum. Related to the unfolding of temporality is an emancipation from the tight binding to primitive factuality. Free exploration of the space of possibilities comes into sight with the capacity for hypothetical and contrafactual thinking. Along with this emancipation goes a deepening of the epistemic cut. The precise form of human existence undergoes a process of varied cultural evolution and also shows large individual differences. Development goes on: Man is always rebellious against his categorical limitations. Philosophy, science, and arts grant visions on timeless structures. Utopianism challenges factuality and integrative world views embedding man into an comprehensive universe try to alleviate the egocentricity of the epistemic cut.

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