Chapter 3
Formalization and Intuition in Husserl's *Raumbuch*

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3.1 The *Philosophie der Arithmetik* and the Origins of the *Raumbuch*

The *Philosophie der Arithmetik* was published in 1891, but it marks the convergence point among contrasting theoretical issues dating from 1886, when Husserl moved to Halle and started collaborating with Carl Stumpf in order to obtain his *Habilitation*. During this time, Husserl's ideas start to diverge from those inherited by his former master Karl Weierstrass.[1] Husserl, indeed, declares his intention to carry on Weierstrass' work using the theoretical tools that he inherited from his second master, Franz Brentano.[2] So Husserl dedicated the first part of the *Philosophie der*

This paper deals with the methodological issues that Husserl encountered when he was developing his first space theory. In particular, the present paper tries to throw some light on the impact of intuition and formalization on early Husserl's geometrical studies. In *Il problema dello spazio nel primo Husserl* ("Rivista di Filosofia," vol. CIV, n. 2. agosto 2013), instead, I offer an historical overview on the *Raumbuch*, mostly focusing on the psychological issues, like Husserl's critique of Helmholtz's space theory.

[1]Cf. Miller, J.P. 1982. *Number in presence and absence. A study of Husserl's philosophy and mathematichs,* 11. The Hague/Boston/London: Nijhoff.

[2]Cf. HUA XII pp. 294–295. The *Philosophie der Arithmetik* reflects on the foundation of arithmetic that, in the first part of the book, is defined as "science of number," a subject based on the concept of positive integer. Indeed, this definition is originated from Weierstrass's studies. Claudio Majolino explains in which way Brentanian psychology answers to an exigency of intuitive researches that Weierstrass left unfulfilled. Cf. Majolino, C. 2004. Declinazioni dello spazio, sul rapporto tra spazialità percettiva e spazialità geometrica nel primo Husserl. *Paradigmi* XXII(64/65): 223–238.

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G. Lolli et al. (eds.), *From Logic to Practice*, Boston Studies in the Philosophy and History of Science 308, DOI 10.1007/978-3-319-10434-8_3

Arithmetik to develop a set of psychological analysis aiming to discover the intuitive roots of the concept of number. Actually, after a few chapters, it appears obvious that this inquiry layout rests on a theory of representation that cannot handle those numbers lacking of corresponding intuition (e.g., imaginary and irrational numbers).

After the failure of this first research trend, the second part of the *Philosophie der Arithmetik* contains the ideas that Husserl developed during his lectureship as *Privatdozent* in Halle.[3] Here, the arithmetical method is presented as a computational technique (*Rechenkunst*) that "can break completely free of the conceptual substrata" focusing on the mode of relation (*in der Weise der Beziehung*). In this sense, it is a "formal processing method, i.e., algorithmic," that is

> a system of formal rules by means of which mathematical problems can be solved in purely mechanical operations, i.e. we can find unknown numbers and numerical relations starting from known ones.[4]

The *Rechenkunst* is a valuable method because, filtering all kinds of numbers within the same algorithmic system, it can deal with all conceivable types of numbers: so, it avoids the *impasse* affecting the first section of the book.[5]

In the preface to *Philosophie der Arithmetik*, Husserl alludes to a second volume that should contain logical researches on the arithmetical algorithm and a philosophical theory of Euclidean geometry, both sharing the same principles (*Grundgedanken*).[6] It is possible that he devises to study the applicability of algorithms to the geometrical field, being interested in a frame of research connecting theory of geometry, formal arithmetic, and manifold theory.[7]

In the same time, Husserl plans "[. . .] to communicate more detailed investigations concerning symbolic representations and the methods of cognition grounded thereon"[8] in an appendix to the second volume. Both algorithms and general psychological representation fall within the domain of symbolic representations

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For an overall view on Husserl's juvenile years cf. Rollinger, R.D. 1999. *Husserl's position in the school of Brentano*, Phaenomenologica, 15–21. Dordrecht: Kluwer.

[3]Douglas Willard hypothesize that this second perspective was influenced by Schröder's algebra of logic: indeed, Husserl was writing a (negative) review on his *Vorlesungen über die Algebra der Logik* during the composition of the *Philosophie der Arithmetik*, last chapter. (cf. D. Willard, D. 1984. *Logic and the objectivity of knowledge. A study in Husserl' early philosophy,* 109. Athens: Ohio University Press.).

[4]HUA XII p. 132; cf. also pp. 258, 346. For example, numbering is a mechanical operation that "[. . .] substitutes the names for the concepts, and then by means of the systematic of names and a purely external process, derives names from names, in the course of which there finally issue names whose conceptual interpretation necessarily yelds the result sought" (HUA XII p. 239). On this matter, cf. also Sinigaglia, C. 2000. *La seduzione dello spazio,* 64–66. Milano: Unicopli.

[5]Cf. HUA XII p. 283.

[6]Cf. HUA XII pp. 7–8.

[7]Cf. HUA XXI pp. 244–249, 396. Cf. also Sinigaglia, C. *La seduzione dello spazio*, 61, op. cit.

[8]HUA XII p. 193.

because they are both inauthentic representations.[9] The link between arithmetic and psychology becomes even more clear considering that *arithmetica universalis* is part of *formal logic*; the latter is still conceived as a Brentanian *Kunstlehre* – the art that detects the proper judgment on the base of psychological categories.[10]

The convergence between arithmetical and psychological studies appears quite clearly in geometry because space can be described by special algorithms or analyzed as a psychological representation. At first, Husserl will deal with the formal side of the space problem elaborating a critique of analytical geometry. The psychological perspective, instead, will be developed in the later *Raumbuch*. For these reasons, the *Raumbuch* can be regarded as the last outcome of the philosophical theory of Euclidean geometry that should have been presented in the second volume of the *Philosophie der Arithmetik.*[11]

3.2 The *Raumbuch* Affair

Before the nineteenth century, geometers believed that Euclidean geometry was based on intuitive space: they only argued on the origin of space representation. Husserl gives a brief account of the old *Raumproblem* (space problem) in some notes dated back to 1893: he distinguishes the apriorist faction – according to which space concepts are already present before any experience (i.e., Kant, König, Baumann, Sigwart), from the empiricist one, according to which space concepts are abstractions or idealizations of empirical spatial figures (i.e., Comte, Mill, Taine, Beneke).[12]

Then, the spread of non-Euclidean geometries complicated the connection between geometrical and intuitive space, showing that geometrical concepts may not be grounded on intuition.[13] This situation urged to perform a new philosophical

[9]In *Zur Logik der Zeichen*, Husserl explores the wide range of symbolic representations. Among them he numbers the artificial signs (*Künstliche Zeichen*) of general arithmetic and those conceptual second class signs (*symbolichen Vorstellungen der Zweiten Klasse*) standing for things that cannot be properly represented; cf. HUA XXI pp. 349–350, 354–356.

[10]Cf. HUA XXI p. 248. On the relation between Husserl and the brentanian logic, cf. De Boer, T. 1978. *The development of Husserl's thought,* 91–93. Den Haag: Nijhoff.

[11]Cf. Argentieri, N. 2008. Matematica e fenomenologia dello spazio. In *Forma e materia dello spazio, dialogo con Edmund Husserl*, ed. P. Natorp, 246, edited by N. Argentieri. Napoli: Bibliopolis, Corrado Sinigaglia proposes a close analysis of the relations between the *Philosophie der Arithmetik* and the *Raumbuch*; cf. Sinigaglia, C. 2001. La libera variazione delle forme. Husserl lettore di Riemann. In *Logica e politica. Per Marco Mondadori, Fondazione Arnoldo e Alberto Mondadori*, edited by M. D'Agostino, G. Giorello, and S. Veca, 377–403. Milano: il Saggiatore.

[12]Cf. HUA XXI pp. 285–286.

[13]Non-Euclidean geometries deny the parallel postulate. This postulate states that if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the angles are less

foundation of geometry, since it offered new perspectives on the issue, therefore updating the classical dispute on the *Raumproblem*. Indeed, non-Euclidean ideas were born within a mathematical frame of discussion but reverberated through the scientific world stimulating new interpretation of space representation from biological, psychological, and philosophical points of view. Thus, some philosophers dealt with non-Euclidean geometries, opening the way to a common research field for geometry and philosophy. For example, both Hermann Lotze's and Hermann von Helmholtz's investigations conclude that our space representation does not reflect the external space. According to Lotze, space is the form through which mind perceives the external space acting upon the mind itself: since that form coincides with Euclidean space, a non-Euclidean intuition is just impossible.[14] According to Helmholtz too, space representation is consistent with Euclidean law since it originates from the properties of an Euclidean world affecting our nerves. Nevertheless, we can suppose that a stimulus generated by a non-Euclidean world could induce a non-Euclidean intuition.[15]

Having this debate in the background, Husserl has been interested in geometry at least since 1886, when he writes a note on homogeneous and heterogeneous *continua*. During the 1889–1890 winter semester, he also delivers lectures on the *Grundproblem der Geometrie*, so at the time, he is already dealing with space from a mathematical point of view.[16] Between 1892 and 1894, he writes down a collection of notes in which he deals with the psychological/philosophical side of the *Raumproblem*, outlining his first theory of space. This work is approximately planned in a brief draft called "Spacebook diary" (*das Tagebuch zum Raumbuch*); following this clue, Ingeborg Strohmeyer gathered the recommended passages and shaped them in a quite organic treatise that has been published in Husserliana XXI volume.[17]

than the two right angles. Cf. M. Kline, M. 1972. *Mathematical thought from ancient to modern times*, vol. I, p. 59, vol. III, p. 865. New York/Oxford: Oxford University Press. On non-Euclidean geometries paternity, cf. Kline, M. *Mathematical thought from ancient to modern times*, vol. III, 869–870, op. cit.

[14]Cf. Torretti, R. 1984. *Philosophy of geometry from Riemann to Poincaré,* 285–291. Dordrecht: Reidel. On the fracture between things and representation, cf. Lotze, H. 1899. *Microcosmus: An essay concerning man and his relation to the world,* 344–353, 573–578. Edinburgh: T. & T. Clark.

[15]Cf. Helmholtz, H. 1867. *Handbuch der Physiologischen Optik,* 194. Leipzig: Voss; Helmholtz, H. 1876. The origin and the meaning of geometrical axioms. *Mind* 1(3): 316–318. On the relation between Lotze and Helmholtz, cf. Gehlhaar, S. 1991. *Die Frühepositivistsche (Helmholtz) und phänomenologische (Husserl) Revision der Kantischen Erkenntnislehre,* 30. Cuxhaven: Junghans-Verlag.

[16]The note can be found in Ms. K I 50/47a. The *Grundproblem der Geometrie* is published in HUA XXI pp. 312–347.

[17]Cf. HUA XXI pp. 262–311. The *Raumbuch* structure is presented in a note published in HUA XXI pp. 402–404. For an historical panorama on the *Raumbuch* birth and on the *Tagebuch zum Raumbuch*, cf. the Textkritische Anmerkungen published in HUA XXI pp. 469, 485–486; HUA D. I pp. 36–37; Mohanty, J. N. 1999. The development of husserl's thought. In *The Cambridge*

Actually, Husserl never finishes nor publishes the *Raumbuch*. Moreover it is impossible to find a direct reference to the *Raumbuch* in later published books, whereas private notes and letters only report indirect quotes.[18] Probably, obscure concepts and contradictions condemned the *Raumbuch* to a *damnatio memoriae* and distracted the attention of scholars from a juvenile work that, instead, deserves a careful investigation. In the *Raumbuch*, in fact, Husserl deals with some core themes of the forthcoming phenomenology, giving answers that will be a (negative) paradigm for future investigations.

This paper tries to clarify how the relation between intuition and formalization changes in the context of the first Husserlian space theory. This would also throw light on the reasons that led Husserl to modify his approach to space representation in mature phenomenology.

3.3 Non-Euclidean Geometries: Husserl Against the Analytical Way

In the early 1890s, Husserl opts for a formal approach to geometry. In his expectation, it should have granted a deep understanding of both Euclidean and non-Euclidean spaces, because it should have highlighted spatial structures instead of material qualities. This approach was originally developed by Descartes' analytical geometry – a science that solves geometrical problems reducing them to algebraic equations. It actually translates the intuitive properties of figures into a formal/algorithmic language that describes space within the quantitative frame of coordinates.[19] Here emerges a connection between what is formal and what is analytical that will be further developed in the *Logische Untersuchungen* and in the *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie*. In these works, Husserl defines "formalization" (*Formalisierung*) as the procedure eliminating any material content from the proposition. In the end, we obtain a formal structure such that we can replace all material contents with an empty formal "whatever" without altering the logical form of the proposition.[20] Despite the early notion of "formal," still overlaps the notion of "algorithmic" inherited by the *Philosophie der Arithmetik*, and despite the word "formalization" has not been coined yet, Husserl already conceives the first step towards a formal representation

companion to Husserl, edited by B. Smith and D. W. Smith, 51. Cambridge: Cambridge University Press.

[18]For example, cf. HUA D. III/5 p. 80.

[19]Husserl uses the expression "analytical geometry" in a standard geometrical way; cf. HUA XXI pp. 232, 323.

[20]Cf. HUA XIX p. 259. On the opposition between analytical and synthetical truths, cf. HUA 3-1 pp. 22, 30.

as an elimination of any material content. He describes the geometrical structure resulting from this process in a letter to Paul Natorp:

> I also conceived the plane as a peculiar double continuous series, the space as a peculiar triple continuous series. [. . .] Anyway, in my opinion too, distance and direction were essential elements of the topoidal manifold.[21]

He remembers that at the time he was looking among complex numbers for arithmetic expressions (*arithmetische Ausdrucke*) properly describing the order of the relations in a plane surface (*Ordnungsverhaltnisse der Ebene*). Although Husserl left no coherent and complete treatise on the matter, we can find some interesting hints in notes, lectures, and letters.[22] In the abovementioned letter, for instance, he pays homage to Hermann Grassmann's *Ausdehnungslehre* (extension theory), a sort of geometrical calculus that, without coordinates and intuition, deals with space as an *n*dimensional vector space.[23] He also acknowledges the influence that Carl Friedrich Gauss' study on complex numbers exerted on him.

Gauss, indeed, plays an important role in Husserl's mathematical formation. An early Husserl's lecture on the history of geometry presents Gauss' theory of curvature as the "handhold" (*Handhabe*) of Bernhard Riemann's geometry.[24] In Gauss' geometry, surface properties are determined in an *analytical* and *intrinsic way,* i.e., without considering the surrounding space. Thus, instead of conceiving the surface as an outline of a body, this approach studies surface as a body with one indefinite small dimension. Surface metric (lengths, angles, areas) is determined only by its curvature that, in turn, is defined by an equation describing a geodetic line. Riemann and Beltrami deduced from Gauss' studies that since surface metric is not affected by the surrounding (Euclidean) metric, then such a surface can possibly display a non-Euclidean metric.[25]

In *Über die Hypothesen, welche der Geometrie zugrunde liegen*, Riemann formulates a concept of space that, according to him, should grant an ultimate understanding of space. The *multiply extended magnitude* is a totally abstract concept "in which space magnitudes are included"[26] as lower order concepts.

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[21] HUA XXI pp. 396–397.

[22] Cf. HUA XXI p. 396. Cf. Hartimo, M.H. 2007. Towards completeness: Husserl on theories of manifolds 1890–1901. *Synthese* 283; Hartimo, M.H. 2008. From geometry to phenomenology. *Synthese* 162: 226–227. Selected manuscripts have been published in HUA XXI pp. 234–243, 312–347.

[23] Husserl reads carefully and annotates his 1878 reprint of the 1844 version of the *Ausdehnungslehre* (Grassman, H. 1878. *Die lineale Ausdehnungslehre*. Leipzig: Otto Wigand). Cf. Hartimo, *From geometry to phenomenology,* op. cit., pp. 225–233.

[24] Cf. HUA XXI p. 323. For an historical account on non-Euclidean geometry, cf. HUA XXI pp. 322–347.

[25] On this matter, cf. Sinigaglia, C. *La seduzione dello spazio*, 24–25, op. cit.

[26] Cf. Riemann, B. 1868. Über die Hypothesen, welche der Geometrie zugrunde liegen. In *Abhandlungen der Königlichen Gesellschaft der Wissenschaften in Göttingen,* Vol. XIII, 133. Göttingen.

By a process of mathematical determination, we can shape a multiply extended magnitude according to Euclidean or non-Euclidean metrics: in the first case, we produce a specific Euclidean magnitude, and in the second case, we obtain a non-Euclidean space configuration. In this sense, Euclidean space is "only a particular case of a triply extended magnitude," and therefore, it has no logical priority over other spatial configurations.[27]

Husserl expresses ambivalent opinions on Riemann's work: although in the *Prolegomena zur reinen Logik* he recognizes Riemann's theory as a preliminary step towards pure logic, he raises doubts about that same theory in notes dated back to 1890–1891.[28] For instance, he states that flat manifolds have a logical priority over curved ones because we think curvity as a variation of flatness and, therefore, we can think flatness without thinking curvity.[29] So, if non-Euclidean systems are artificial constructions and mere *variations* of Euclidean geometry, then Riemann's theory inverts premise (flatness) and consequence (curvity).[30] Moreover, it has no philosophical nor descriptive value since it does not acknowledges that the *logical* priority of Euclidean geometry is founded on an *ontological* priority: Euclidean plane geometry perfectly describes real space because *it embodies* the logical structure organizing real space.

In 1892, Husserl still lingers on Riemannian theories: he now focuses on the Riemann-Helmholtz's definition of a rigid body as an ideal system of moving points described by a fixed equation.[31] According to Husserl, this definition implicitly reduces the distance between two points to the equation describing the distance, to numbers.[32] Such a definition mistakes "intrinsic" (*innere*) relations for "extrinsic" (*äußere*) ones. In the latter, connections are mediated by a third element whose genus (*Gattung*) differs from the genus of the two terms (e.g., the mathematical expression that binds together two geometrical points): according to Riemann and Helmholtz, geometrical distance belongs to this kind of relation. In intrinsic or continuous relations, instead, all the elements belong to the same genus: according to Husserl, geometrical distance belongs to this kind of relation, because both the

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[27] *Ibidem.* On this matter, cf. Kaiser-El-Safti, M. Fenomenologia trascendentale versus iletica. Psicologia e fenomenologia in Husserl e Stumpf. In *Carl Stumpf e la fenomenologia dell'esperienza immediata*, edited by S. Besoli and R. Martinelli, Discipline Filosofiche, Anno XI, numero 2, 247. Macerata: Quodlibet.

[28] HUA XVIII p. 252. Cf. also HUA D. III/1 p. 11. Cf. Parrocchia, D. 1994. La forme générale de la philosophie husserlienne et la théorie des multiplicités. *Kairos* 5: 137–140.

[29] Cf. HUA XXI p. 345.

[30] Cf. HUA XXI p. 344. For a deeper examination of Husserl's remarks on Riemann's arguments, cf. L. Boi, *Le problème mathèmatique de l'espace*, Springer, Berlin/Heidelberg, 1995, pp. 241–243; Sinigaglia, C. *La libera variazione delle forme. Husserl lettore di Riemann*, 387–388, op. cit.; Hartimo, M. H. *From geometry to phenomenology,* op. cit., p. 228.

[31] Cf. Helmholtz, H. 1921. Über die Thatsachen, die der Geometrie zu Grunde liegen (1868). In *Schriften zur Erkenntnistheorie*, edited by M. Schlick and P. Hertz, 55. New York: Springer. Cf. also Torretti, R. *Philosophy of geometry from Riemann to Poincaré*, 156–157, op. cit.

[32] Cf. HUA XXI p. 409.

mediating term (i.e. the distance) and the points are spatial elements. To be more precise, distance is a "magnitude moment" (*Größenmoment*), a *spatial* aspect of a continuous relation: space, in fact, is a continuous manifold, a "manifold in which each couple of elements is connected by a continuous relation."

So, Riemann-Helmholtz's analytical description of space totally misses the essential features of space and reduces it to a numerical manifold – that is a mere analytical analogue of a spatial manifold.[33] Husserl seems actually to agree with Nicolaj Lobačevsky who, in his *New foundations of geometry*, wished that an extensive deploy of intuition supplanted the formal approach of the *analytic way*, reestablished the role of intuition in geometry, and opened a new *synthetic way*.[34] As Husserl explains in *Geschichtlicher Überblick über die Grundprobleme der Geometrie*, synthetic geometry has been developed by Lobačevsky as an empirical *Naturwissenschaft* relying on intuition.[35] Similarly, in the early nineties, Husserl starts to develop psychological studies concerning spatial intuition: he aims to throw light on those spatial sensations grounding geometrical concepts.[36] At this point, Husserl's investigations can no longer remain within the mathematical domain because "mathematicians are satisfied when they can calculate or build. The philosopher wants to understand too."[37] And the analytical method does not grant such a philosophical understanding of space.

Then, after a season of studies approaching space from a formal/analytical point of view, Husserl starts a new kind of analysis inspired by Brentano's descriptive psychology and by synthetic geometry: he leaves the analytical way and he takes the "other way,"[38] the one focusing on the intuitive contents of space representation.

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[33]Cf. HUA XXI pp. 348, 407–410. Leaving aside the debate about Husserl's theory of manifold, it would be useful to refer to the other definitions of manifold proposed in the *Prolegomena* (cf. HUA XVIII p. 249) and in the *Philosophie der Arithmetik* (cf. HUA XII p. 81). Husserl himself, in a footnote of the *Ideen*, provides a brief historical-critical examination of the concept of manifold in his former works: cf. HUA III/1. On the riemannian *Zahlenmannigfaltigkeit*, cf. Brisart, R. 2003. Le Général et l'abstrait: sur la maturation des Recherces Logiques de Husserl. In *Aux origines de la phénoménologie*, edited by D. Fisette e S. Lapointe, 39. Paris: Vrin; Majolino, C. *Declinazioni dello spazio, sul rapporto tra spazialità percettiva e spazialità geometrica nel primo Husserl*, 228–229, op. cit.; Sinigaglia, C. *La seduzione dello spazio*, 57–58n, op. cit.

[34]Lobačevsky, N. 1898. Neue Anfangsgründe der Geometrie mit einer vollständigen Theorie der Parallellinien. In *Zwei geometrische abhandlungen aus dem russischen uebersetzt, mit anmerkungen und mit einer biographie des verfassers*, edited by F. Engel and P. Stäckel, 80–82. Leipzig: Teubner.

[35]Cf. HUA XXI pp. 312–314, 322–323.

[36]This last perspective represents the core idea of the *Raumbuch* and seems to echoes Lobačevsky's opinion about the importance of synthesis in mathematics, whose "constructive procedure" has to clarify "those representations that are directly connected to the early concepts of our mind" (Lobačevsky, N. *Neue Anfangsgründe der Geometrie mit einer vollständigen Theorie der Parallellinien*, 80–81, op. cit.).

[37]HUA XXI p. 411.

[38]HUA D. III/1 p. 11.

3.4 Mereology, Material *a Priori*, and Idealization: The Other Way

Several passages of the *Raumbuch* witness the essential role that intuitive qualities play in the descriptive analysis of space – an aspect shared with Stumpf's *Über den psychologischen Ursprung der Raumvorstellung.*[39] And just like Stumpf, Husserl conceives space as a multisided whole.

Husserl, in fact, notices that the "space of the world" (*Weltraum*) is composed by a variety of places connected through a network of symbolic cross-references[40]: for instance, the observation of the wall naturally leads to perceive the room; the exploration of the room reminds that this last one is part of the flat that, in turn, is a portion of the house located in the neighborhood and so on until the process reaches the space that contains every places, i.e., the space of the world. This connection between places is not only possible but also necessary because each place neither can *be*, nor can *be perceived*, nor can *be thought* without surrounding places. Therefore, each intuitive representation of space contains a symbolical reference to the surrounding space.[41]

The smallest part of this spatial mosaic is "spatiality" (*das Räumliche*), a basic extent that is the "abstract substratum" of every intuitive quality.[42] This argumentation was previously deploid by Stumpf against the Kantian thesis according to which space would be the form of sensibility organizing phenomenical multiplicity.[43] According to Stumpf and Husserl, the concept of space simply highlights the structure of real space, the organization of raw empirical data instead of shaping it. For instance, both extent and visual/tactile qualities display their own configuration: they are "abstract elements" or "grounded contents" because we cannot conceive

[39]C. Stumpf, C. 1873. *Über den psychologischen Ursprung der Raumvorstellung*. Leipzig: Verlag von S. Hirzel.

[40]Cf. HUA XXI p. 281.

[41]This argument may remind a Kantian thesis, but, actually, the *Raumbuch* displays an anti-Kantian perspective on space. Kant wanted to prove the priority of spatial form over spatial object showing that object cannot be displayed without a surrounding space, whereas space itself can be conceived as object-free. Instead, Husserl speaks in terms of extension: the single fraction of space is an extension as well as the world space. Obviously, the first extension is part of the second one, but – here it is the difference from kantianism – the single place cannot be conceived without conceiving its surrounding places as well as the world space cannot be conceived without its composing parts. On this subject, cf. Kant, I. *Kritik der reinen Vernunft,* A24, B39. This thesis anticipates, in a different theoretical context, an idea that Husserl will elaborate in the *Logischen Untersuchungen*. There he notices that every representation has both intuitive and symbolical sides, each one contributing in a different degree to the whole representation. Cf. HUA XIX pp. 610–614.

[42]HUA XXI p. 276.

[43]Cf. Kant, I. *Kritik der reinen Vernunft*, A 99, 107, 120n; B 201-2n, 218-9, 129–130, 134–135. Victor Popescu highlights the subtle differences between Stumpf's and Husserl's mereologies: cf. Popescu, V. 2003. Espace et mouvement chez Stumpf et Husserl, une approche méréologique. *Studia Phaenomenologica* III(1–2): 115–133.

extent without color in such a way "that the suppression of the former implies the suppression of the latter."[44] Besides, extent and qualities are connected in a subtle way. For example, colors fade or shine depending on surface illumination and according to *a priori* material laws; on the other hand, when color is obscured, surface disappears as well. These relations do not concern formal consistence between parts (*contra* Kant) and they are neither grounded on habits (*contra* Empiricism). Instead, they express an objective configuration that does not change depending on the intentional subject.[45]

At this basic stage of perception, we can only sense sides of things. In order to perceive an object, we have to synthesize the separated extent into a stable composition of visual sides. Each perceived side contains symbolical references pointing to the other side; the synthetic act binds these symbolic references to the first intuition and crystallizes them into an object (e.g., the room is composed following the references to the adjacent walls that are contained in the perception of the first wall). This gradual composition is a "teleological process" because it aims to give a complete representation of the object, i.e., to "accomplish" this object perception linking all its sides into a complete whole.[46]

Sensible objects exhibit intuitive qualities in a greater or lesser degree of perfection: e.g., a straight line could be more or less straight and a point could have more or less extent. We can appreciate qualitative differences because each quality value is disposed on a teleological scale leading to an ideal perfection, to an unperceivable "limit" (straightness, redness). As Husserl will point out in the *Ideen* – within a different theoretical context – geometrical concepts as "ideas in a Kantian way" express "something invisible."[47] In order to conceive these concepts, we should execute an "idealization" (*Idealisierung*) – a process that reiterates endlessly an "almost induction" (*Quasi-Induktion*) and accentuates a material content until perfection.[48] Indeed, idealization "starts from what is intuitively given and implied

[44]Cf. HUA XXI pp. 281, 307. Cf. also Stumpf, C. *Über den psychologischen Ursprung der Raumvorstellung*, 107–109, op. cit. This distinction will be further developed in the *Psychologische Studien zur elementaren Logik* (cf. HUA XXII pp. 97–98) and in the *Logische Untersuchungen* (cf. HUA XIX pp. 231–240, 272–274). Cf. Kaiser-El-Safti, M. *Fenomenologia trascendentale* versus *iletica. Psicologia e fenomenologia in Husserl e Stumpf*, 236, op. cit.; Majolino, C. *Declinazioni dello spazio, sul rapporto tra spazialità percettiva e spazialità geometrica nel primo Husserl*, 230–231, op. cit.

[45]According to Stumpf, those judgments describing objective relations are necessary by nature and universally valid. Starting from those judgments, we can develop a set of *a priori* material laws. Cf. Stumpf, C. 1982. Psychologie und Erkenntinistheorie. In *Abhandlung der Königlich Bayerischen Akademie der Wissenschaften*, I Classe, 19, 2, München, 494–495. On this subject cf. De Palma, V. 2001. *L'a priori del contenuto. Il rovesciamento della rivoluzione copernicana in Stumpf e Husserl.* In: *Carl Stumpf e la fenomenologia dell'esperienza immediata*, edited by S. Besoli and R. Martinelli, Discipline Filosofiche, XI, 2, 316–318. Macerata: Quodlibet.

[46]HUA XXI p. 284. Pursuing this strand of research, in the *Dingvorlesung*, Husserl will deal with the problem of the tridimensional circularity of the real object.

[47]Cf. HUA III/1 p. 138.

[48]Cf. HUA XXI p. 286.

into the nature of a thing"[49] and simply enhances the material content. This happens, for example, when we detect a median point between two points that are gradually getting closer: when these two points became indistinguishable, the process can be further protracted beyond "the limits of the operating potentiality of our measuring instruments."[50] In this way, we gain the "best conditions of sight,"[51] and since our intentional activity has been freed from any subjective defect, we can finally conceive the geometrical concept in its ideal and universal objectivity. Thus, for instance, we conceive the concept of point subtracting extent to the point until it becomes a dimensionless geometrical entity. The real point and the concept of point are both dimensionless to different degree: the real object is linked to the concept through a shared content (e.g., being dimensionless). This shared content legitimizes the bond of continuity between object and concept, and therefore, it highlights the intuitive roots of the concept. This kind of link is further confirmed by the continuity of the idealization process: indeed, idealization connects concept to intuition by an uninterrupted and iterative induction. For example, the concept of point is a product of an uninterrupted process that subtracts extent to the point.[52] Thanks to this double line of continuity, intuition and concept are so "similar" that "intuitions symbolize concepts, the former are not the object of concepts but symbols, more precisely hieroglyphs of the concepts."[53]

The symbolic relation between concept and intuition makes conceptual work simpler since it allows to translate a conceptual problem into intuitive terms. Nevertheless, intuitive evidence is not exact as formal evidence and an intuitive demonstration is not as rigorous as a formal demonstration: by interpreting topics of pure geometry in terms of intuitive figures we may oversimplify the issue.[54] Thus, in notes dated 1894, Husserl reconsiders the differences between the analytic and the "other way," and this time, he underlines the merit of the analytic side. According to the "other way," intuition and concept should be reconnected by idealization – a procedure enhancing similarities: actually, many passages clearly deny there is such a similarity. For instance, as noted down in 1893, sensible space and ideal space have totally different features since, while we can perceive the former, we can only think the latter. To be more precise, pure geometry is a formal domain of contentless objects that "has to expels errors from the same foundations by a purely

[49]HUA XXI p. 308.

[50]HUA XXI p. 296. This passage reminds Lobačevsky's *New principles of geometry*: "[. . .] it will be possible to form any body by means of composition, reaching an identity degree beyond which our senses stop perceiving imperfections. [. . .] although we get our first concepts from it [the nature] we owe the rigor of the former to our senses imperfection" (Lobačevsky, N. *Neue Anfangsgründe der Geometrie mit einer vollständigen Theorie der Parallellinien*, p. 81, op. cit.).

[51]HUA XXI p. 287.

[52]*Ibidem*.

[53]Cf. HUA XXI pp. 289–290, 294. Not only single objects but the entire intuitive space may be used as a symbolic surrogate of pure geometrical space.

[54]Cf. HUA XXI p. 295.

formal procedure and rigorous axioms and [has to] show the intuitive procedure in its own limits [...]."[55] It deals with a pure concept of space that shares almost nothing with the empirical concept of space studied by physical geometry: we cannot subsume the latter under the former because there is no continuity between a formal concept, devoid of any contents, and a sensible concept, still characterized by material contents.[56] Husserl further develops this idea, and in a letter he writes to Natorp in 1897, he numbers three concepts of space differing in formal purity. The spatial manifold is the most formal concept; from it we deduce the tridimensional Euclidean manifold by formal determination. The third and less formal concept is the concept of intuitive space that cannot be derived by formal determination because it is enriched by material contents. Thus, there are two kinds of space concept – the formal one and the material one. Moreover, these two kinds of concept cannot be linked through a single act of mind – neither formal determination nor idealization.[57] This last process, in fact, reveals its uselessness when it pretends to conceive formal concept – devoid of any content – by a continuous enhancement of intuitive contents.

3.5 Representation, Intuition, and Symbolization

Such a methodological issue implies that when Husserl was planning the *Raumbuch*, he did not clearly distinguish between formal and material concepts.[58] First, he needed to clarify which kind of intellectual act could conceive concepts, that is he needed to discover that we can visualize some kinds of concepts through an intuition (*Anschauung*).[59] In the *Raumbuch*, instead, he still relies on a slightly modified version of the theory of representation introduced in the *Philosophie der Arithmetik*: he makes a few distinctions, but he still contrasts concept and intuition.

> [...] we should ask ourselves if the real representation that each time we have, has the character of intuition or symbolization (*Repräsentation*) and, in this last case, if it has the character of an intuitive or non-intuitive symbolization (proper or improper) of what we call space. In the case of non-intuitive representations we have to investigate if they have [...] the character of conceptual representation, which relation they have with corresponding intuitions, if they can be grounded on these last ones or if [...] they necessarily lack of corresponding intuition.[60]

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55 HUA XXI pp. 271, 295–296.

56 Cf. HUA XXI p. 296. This passage anticipates the distinction between physical and pure geometry in the *Prolegomena*; cf. HUA XVIII p. 251.

57 Cf. HUA D. III/5 pp. 53–54.

58 Cf. Brisart, R. *Le Général et l'abstrait: sur la maturation des Recherces Logiques de Husserl*, 39–40, op. cit.

59 This idea shows up in the *Psychologische Studien zur elementaren Logik*, cf. HUA XXII p. 104.

60 HUA XXI p. 262.

Husserl still defines intuition and symbolization according to the guidelines of the *Philosophie der Arithmetik*:

> If a content is not directly given that which it is, but it is only indirectly given through signs that univocally characterize it, then, instead of having an authentic representation, we have a symbolic representation of it [. . .].[61]

So, because of symbolization mechanics, we can represent concepts through intuitions standing for them: for example, a real point may stand for the concept of point because watching the former we catch a symbolic link to the latter. This connection implies that intuitions and concepts are both different and similar in a way that Husserl does not further clarify.

Furthermore, symbolization (whose content *is not* directly given to us) is defined as a mere negation of intuition (whose content *is* directly given to us), and therefore, its representational domain is reduced to what *is not* intuitive. As a consequence, symbolization has not an autonomous representational status.[62] Such a feeble demarcation of the conceptual domain can be interpreted as a symptom of veiled and impending psychologism. If "what is intuitive" is determined by subjective configurations and if symbolization is simply "what is not intuitive," then these psychological faculties will define "what is not intuitive" too. Thus, psychological faculties circumscribe the symbolical domain (and the conceptual one within it) by defining what they are not. For example, empirical concepts are presented as what is beyond "the limits of the operating potentiality of our measuring instruments."[63] Leaving aside the classical debate about Husserl's supposed psychologism, it seems that the domain of symbolization shrinks depending on the extent of the intuition domain.[64] Nevertheless, we find various *Raumbuch* passages implying that concepts have an objective and defined status. For instance, an impossible concept cannot be represented, no matter the subject; in another note, he says that a concept of space based on material *a priori* determines the conditions of possibility of experience; elsewhere, idealization is presented as a procedure purifying the psychic process from subjective defects.[65] In the end, it seems that the *Raumbuch* representational theory is quite fuzzy.

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[61] XII p. 193. This definition has many similarities with the one that Husserl gives in HUA XXI p. 272. It is worth noticing a minor semantic sliding: the "proper representations" in the *Philosophie der Arithmetik* are named "intuitions" in the *Raumbuch*.

[62] Cf. HUA XXI pp. 295–296. Husserl will define the representational status of concepts when he will deal with the categorial intuition in the *Logische Untersuchungen*. There he also dismantles the intuition/symbolization dichotomy that structures the *Raumbuch* representational theory.

[63] HUA XXI pp. 295–296.

[64] That reminds an idea from the *Philosophie der Arithmetik*, where arithmetic is presented as a tool dealing with sets that cannot be intuited because of subjective inability; consequently, since powerful subjectivities, like angels or God, need not to develop arithmetic to handle large sets, then their arithmetical domain (objects and procedures) is almost empty. At the end, "results" are the same: both man and angel represent the same large sets, but the former uses a tool (arithmetic), whereas the latter need it not. Cf. HUA XII pp. 191–192.

[65] Cf. HUA XXI pp. 262, 296, 287.

Moreover, the internal distinctions between the various kinds of symbolization are not even always respected, at least not during the psychological analysis of the "everyday life space." According to Husserl, the "everyday life space" is an ideal formation: a everyday life object, in fact, is an "ideal object" formed by an intellectual synthesis binding together all the sides of the object.[66] No object, indeed, can show us all its sides at the same time, but, nevertheless, we intend the complete object with all its sides when we conceive the object. For example we know that a dice has six faces but we cannot *see* six faces at the same time. We know, however, that the three manifest faces hide other three faces and that all those faces together form the dice. When Husserl says that every real object is an ideal object, Husserl confuses the symbolic link between the intuitive symbol (the manifest three sides) and the symbolized object (the hidden three sides) with a symbolic link between the intuitive symbol (the three sides) and the symbolized concept (the concept of dice).

In general, this theory lacks of balanced composition of its founding concepts. For instance, non-intuitive symbolization (concept) is defined as alternative to intuition, despite non-intuitive symbolization (concept) has a direct contrary, i.e., intuitive symbolization (intuitive signs). Such a definition has two consequences. First, we do not know what symbolization is *per se*, but we only know that it is not intuition. Second, the various species of symbolization are not defined as reciprocal alternatives but, all together, as alternatives to intuition.

The conceptual representation *depends* on intuition because Husserl tries to ground concepts into sensible experience relying on immature psychological methods. This effort becomes evident when he analyzes real bodies following a method inspired by solid geometry. He adopts the technical terminology of geometry when he explains how "division" (*Teilung*) decomposes the "physical body" (*Körper*) into geometrical entities as "surfaces" (*Flächen*), "lines" (*Linien*), and "points" (*Punkte*).[67] The transition from the esthetical to the geometrical dimension is witnessed by a synonymical overlap of the terms "physical body" and "figure" (*Gebilde*) – an overlap justified by the fact that we can extract "forms" (*Formen*) and "corporeal figures" (*körperlichen Figuren*) from every physical body.[68] At the base of this consideration, there is a major confusion between external experience (physical/esthetical body) and internal one (geometrical form). Thus, the *Raumbuch* betrays the first rule of immanentism according to which, since external experience data are untrustworthy, analysis should be focused only on the immanence of conscience – where the features of intentional objects can be ascertained once

[66] Cf. HUA XXI pp. 281–283.

[67] Cf. HUA XXI pp. 278–279.

[68] Cf. HUA XXI pp. 278–279, 286. Stumpf explains that many space theories of his time incorrectly mix two strands of research that should be kept separated: epistemology, focusing on immediately evident truths, merges with descriptive psychology, focusing on the genesis of concepts. Thus, the researches on the origin of spatial representation overlap the studies on the nature of geometrical axioms. For this reason, the spatial analysis of the early Husserl displays a geometrical nuance. Cf. Stumpf, C. *Psychologie und Erkenntinistheorie*, 484, op.cit.

for all.[69] In order to achieve an optimal description of the spatial representation, analysis should keep spatial sensations and spatial things separated; moreover, this separation should be actively maintained. Husserl will satisfy these two requisites by developing the *epoché* in the *Ideen*. This procedure neutralizes the presuppositions threatening the purity and the independence of phenomenological analysis: for instance, we should "put between brackets" the belief in the existence of the natural world in order to focus on sensations rather than on things, unlike what happen within the *Raumbuch* frame.[70] Moreover, a radical philosophical investigation should avoid concepts, methods, and practices that have been derived from other sciences. A pure psychological analysis should also start from the bottom, highlighting those primordial structures that ground the edifice of science: in this sense, we should not aim to justify a scientific idea since that would adjust analysis and distort results. Actually, the *Raumbuch* analysis is influenced both by implicit geometrical categories and by a previously established aim, i.e., the justification of Euclidean geometry.[71]

Thus, because of a methodological immaturity, the first Husserlian theory of space collapses within a few years. The *Raumbuch* is one of those experiences pushing Husserl to conceive the *epoché* – a phenomenological method that will redefine the relations between space, geometry, and experience:

> If the province of phenomenology were presented with such an immediate obviousness as the province pertaining to the natural attitude in experiencing, or if it became given in consequence of a simple transition from the latter to the eidetic attitude as, for example, the province of geometry becomes given when one starts from what is empirically spatial, then there would be no need of circumstantial reductions with the difficult deliberations which they involve.[72]

3.6 Conclusion

Although just after the publication of the *Philosophie der Arithmetik* Husserl plans to develop a formal approach to geometry, in the notes from the early 1890s, he criticizes the formal method of analytical geometry for not being able to grasp the essence of space. So he chooses to investigate the intuitive side of space through

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[69]Husserl inherits immanentism from Brentano, and when he is working on the *Raumbuch*, he still adopts this intentional theory. For example, he stresses the distinction between immanent object (*immanente Objekt*) and real object; he defines the metaphysical space – i.e., the real space – as transcendent space (*transzendent Raum*). Cf. HUA XXI pp. 262, 265–266, 270, 305. Paradoxically, he makes the same mistake that he highlights in Helmholtz's empiricist space theory: according to him, Helmholtz confuses the inner psychological experience with the real external one. Cf. HUA XXI p. 309.

[70]Cf. HUA III/1 pp. 59–60, 108, 115–116.

[71]Cf. HUA III/1 pp. 112–115. It is no accident that several sciences presuppose the axiomatic method called *mathesis universalis* — whose first model was Euclidean geometry.

[72]HUA III/1 pp. 115–116.

the *Raumbuch* psychological analysis; anyway, in the *Raumbuch* last notes, he approves formalization again. Such a theoretical mutability may be explained if we consider that in the early 1890s, Husserl is still defining the representational status of conceptual representations; as a consequence, he has not conclusively established if geometry is a formal or a material science or both. For these reasons, he cannot formulate an ultimate description of the geometrical method.

The first step towards a solution can be found in the *Prolegomena* where Husserl clarifies the relation between intuitive geometry as "phenomenical space science" and formal geometry as "categorial form of geometrical theory."[73] Although this distinction has already been sketched in the *Raumbuch* note dealing with the differences between physical and pure geometries, in the *Prolegomena*, it is associated with a pondered reflection about methods that redefine the various kinds of concepts. This process is completed in the *Ideen*, where Husserl coins two procedures that elaborates formal and material concepts, each one within its own research field. The first procedure is "formalization" and replaces contents with contentless variables in order to reduce a material field to a manifold whose formal objects are defined solely by the form of their connections with other objects.[74] The second procedure is called "generalization" and explores all the possible manifestations of a material content by means of imagination; it investigates which features are essential and which ones are not, until the eidetic essence of the content emerges as the invariable core of every possible manifestation.[75]

This methodological reorganization redefined the relations between geometry and space theory. The phenomenological space theory, as developed in the *Dingvorlesung*, abandons geometrical categories and adopts esthetical ones: it primarily focus on the transcendental constitution of the real space rather than on our geometrical representations. In the same time, geometry deals no more with intuitive space, contents and qualities: as specified in the *Ideen*, geometry should be an axiomatic-deductive system dealing with exact concepts. Once its proper object has been detected, geometry finally finds a place among the other sciences as an "exact science" based on formalization.[76]

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[73] Cf. HUA XVIII p. 252.

[74] Cf. HUA III/1 pp. 133–136; HUA XVII pp. 79–80; HUA XVIII pp. 247–248.

[75] Cf. HUA III/1 pp. 131–132.

[76] Cf. HUA III/1 pp. 133–139.

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