

One Problem, Two Structures, Six Solvers, and Ten Years of Personnel Scheduling

Louis-Martin Rousseau

CIRRELT, École Polytechnique de Montréal, Montréal,
C.P. 6128, succ. Centre-ville, Montréal, H3C 3J7, Canada
`louis-martin.rousseau@polymtl.ca`

The shift-scheduling problem was originally introduced by Edie in 1954 [8] in the context of scheduling highway toll booth operators. It was solved a short time later, by Georges Dantzig [6], using a set covering formulation. However, the Multi-Activity Shift Scheduling (MASSP) version of that problem, where one not only needs to schedule when employees are working or resting, but more precisely, what activity they are performing, still remains a challenge. During this invited lecture, we will recall the turning points of this 60-year journey, focusing particularly on the efforts of the last decade to solve MASSPs.

The first breakthrough came from Constraint Programming (CP), with the introduction of the Regular Language Membership Constraint [13,1], which enabled us to specify shift regulations through Deterministic Finite Automata. Two years later, the Context-free Grammar Constraint [15,18] was introduced, shortly followed by both a decomposed formulation [16] and incremental filtering algorithm [11]. From these constraints it is possible to identify two network structures (paths in a layered directed acyclic graph for *Regular* and hyper-paths in a hyper-graph for *Grammar*).

Using these graph structures, Mixed Integer Programming (MIP) models were initially proposed [3] to address the MASSP. Thanks to Orbital Shrinking [9], a new MIP formulation [4], and hybrid CP-MIP branch and bound [17] were proposed which allowed us to solve these models more efficiently.

Dynamic Programming (DP) algorithms were also developed to optimize (find the shortest paths and hyper-paths) for both *Regular* and *Grammar* given that marginal costs are associated with performing certain activities at a given time. These costs can be estimated manually during a Large Neighbourhood Search (LNS) [14] or obtained through dual values in the context of a Branch-and-Price approach [7,5]. Finally Lazy-Clause Generation (LCG) within CP [10] has shown to produce very good results for a subset of the benchmark originally introduced in [7].

From a practical point of view, the concepts of [5] were implemented into commercial software (*Planora*), while the models using the decomposition of *Regular* were used in case studies involving a major fashion retailer [2] and Hydro Québec's large call center [12].

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