

Off-Line Crack Detection in Rotors

Erfan Asnaashari and Jyoti K. Sinha

Abstract This paper intends to extend the application of the residual operational deflection shape (R-ODS) to a simplified model of the LP turbine of a steam turbo generator. The numerical model consists of a stepped shaft which accommodates some balance disks to represent the blades at different stages of the LP turbine. The ability of the R-ODS method in detecting cracks in beam-like structures has already been presented. The method removes the effect of the first harmonic component (1x) from the ODSs at higher harmonic components to highlight the non-linear behaviour of cracks. In this paper, single and multiple cracks are introduced at different locations to the finite element model of the rotor. It is assumed the rotor can be taken out from the rig and mounted on a simply supported boundary condition. After carrying out the modal analysis, the rotor is excited at its first resonant frequency and acceleration responses along the rotor are computed to construct the ODSs. The R-ODS method uses the normalised ODSs for the purpose of crack detection. The curvature of R-ODS method is also presented for further improvement in indication of crack locations especially for multiple-crack rotors.

Keywords Cracked rotor · Crack detection · Crack breathing · Residual operational deflection shape

1 Introduction

Shafts play an important role in power transmission in rotating machines. Initiation and propagation of fatigue cracks are very common in rotating shafts since they are constantly subjected to fluctuating stresses and broad variations in temperature and

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environmental conditions. Early detection of fatigue cracks in the shaft of a rotor system is vital to prevent fatigue failure and consequent potential damage to other equipment and huge costs in down time.

Crack development changes the dynamic characteristics of the shaft. These changes are the main idea of many vibration-based methods for crack detection. A comprehensive overview of the vibration-based crack detection in shafts is presented in [1]. Breathing is the key feature of fatigue cracks which produces non-linear behaviour. Pennacchi et al. [2] presented a model-based identification method for transverse breathing cracks in industrial machines. They simulated the crack effect on static and dynamic behaviour of the rotor in the frequency domain by applying equivalent forces of the cracked beam element to the rotor. The crack was then identified by an external force identification procedure. Chasalevris and Papadopoulos [3] investigated the cross-coupled bending vibrations of a rotating shaft with a breathing crack and resilient bearings. A continuous model for the cracked rotor-bearing system was offered to detect the existence of the crack by monitoring the change in the stiffness due to the change of crack local compliance. Darpe [4] proposed a detection method based on the breathing of the crack. The method uses the presence or absence of bending and torsional vibrations coupling at various orientations of the crack. Darpe applied a transient torsional excitation for a very short period of time and investigated the lateral vibrations. The transient features of the resonant bending vibrations were revealed by wavelet transforms in order to detect the crack. Sinou [5] used higher harmonic components of exciting frequency and the crack-unbalance interactions to identify the presence of a single breathing crack in a rotor system. By means of numerical simulations, it was demonstrated that for a given crack depth, the unbalance affects the amplitude of the $\frac{1}{2}$ and $\frac{1}{3}$ sub-critical resonant peaks as well as the $1x$ component which could be used to identify the crack. Sawicki et al. [6] presented the modelling and analysis of rotors with breathing cracks. They utilised sinusoidal excitations to the shaft through active magnetic bearings (AMB). These excitations generated response frequencies combining the rotor spin speed and excitation frequency. The system was finally analysed using an approach based on the harmonic balance method and could be used with inverse methods for the purpose of damage detection.

Recently, the residual operational deflection shape (R-ODS) method has been proposed for crack detection in beam-like structures [7]. The R-ODS method removes the effect of first harmonic component ($1x$) from the ODSs at higher harmonics ($2x$, $3x$, etc.) to highlight the non-linear behaviour of the crack. This study intends to extend the application of the method by using a very simplified model of the LP turbine of a steam turbo generator. The model consists of a stepped shaft which accommodates some balance disks to represent the blades at different stages of the LP turbine. The reliability of the R-ODS method in detecting the location of single and multiple cracks in the shaft is investigated.

2 Finite Element Simulation

A rotor consisting of a stepped shaft with mounting balance disks is considered. The density and Young's modulus of the rotor are $7,850 \text{ kg/m}^3$ and 210 GPa respectively. Figure 1 shows the dimensions of the rotor with a single crack. Here, it is assumed the rotor can be taken out from the rig and mounted on a simply supported boundary condition. Single and multiple cracks are introduced at different locations with depth ratio of 20 % ($\frac{d_c}{D} = 0.2$). Table 1 shows the details of all the investigated cases.

A finite element model was created for the rotor using Euler-Bernoulli beam elements. There are two nodes defined per element and each node has two degrees of freedom; translational displacement and bending rotation. Stiffness proportional damping is obtained using the constructed stiffness matrix. The approach proposed in [8] is used here to introduce cracks to the finite element model of the rotor. This approach modifies the local flexibility in the vicinity of the crack within Euler-Bernoulli beam elements based on the concept explained in [9]. It assumes that the flexibility from uncracked to cracked section of the beam varies linearly and considers triangular reduction in the stiffness of cracked element. If this reduction happens over more than one element, the nodes of the model will be moved so that the crack effect remains within a single element.

The breathing of the crack(s) is modelled through sign recognition of nodal displacements of the cracked element(s). This means that when the displacement of nodes of the cracked element is greater than zero, the crack is open. Otherwise, the crack is assumed to be closed. The equation of motion for the cracked rotor under external excitation is:

$$M\ddot{x}(t) + C\dot{x}(t) + K(t)x(t) = F(t) \quad (1)$$

where M , C and K are the mass, damping and stiffness matrices for the rotor respectively and F is the external force.

In Eq. 1, the stiffness matrix is time dependent because it changes with opening and closing of the crack(s) during the external excitation. When the crack is closed, a healthy (global) stiffness matrix is used in Eq. 1. However, the existence of cracks reduces the stiffness of the cracked elements and consequently the stiffness of the whole rotor. In order to consider the stiffness reduction corresponding to each open crack in the global stiffness matrix, the number and location of all open cracks are determined at every time step. The stiffness matrix in the presence of open cracks can be written as:

$$K_c = K - \sum_{p=1}^n \Delta K_{c,p} \quad (2)$$

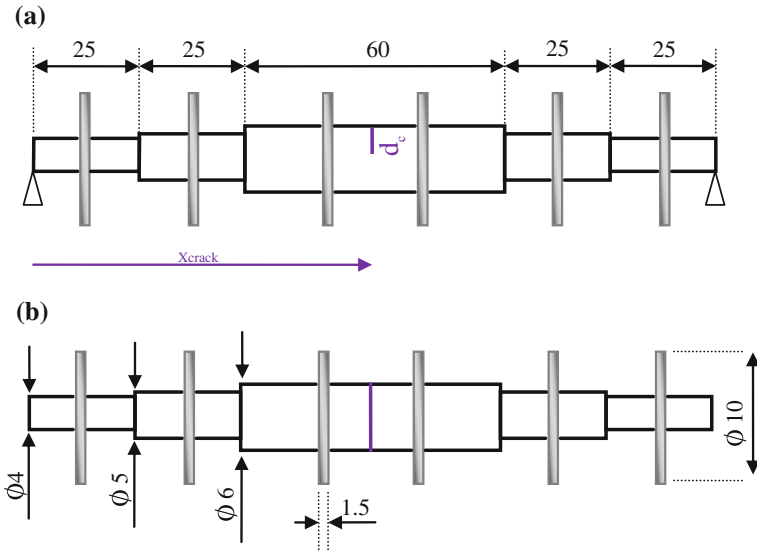


Fig. 1 Schematic view of the simply supported cracked rotor, **a** side view, **b** top view

Table 1 Number and locations of cracks used in the rotor for numerical simulations

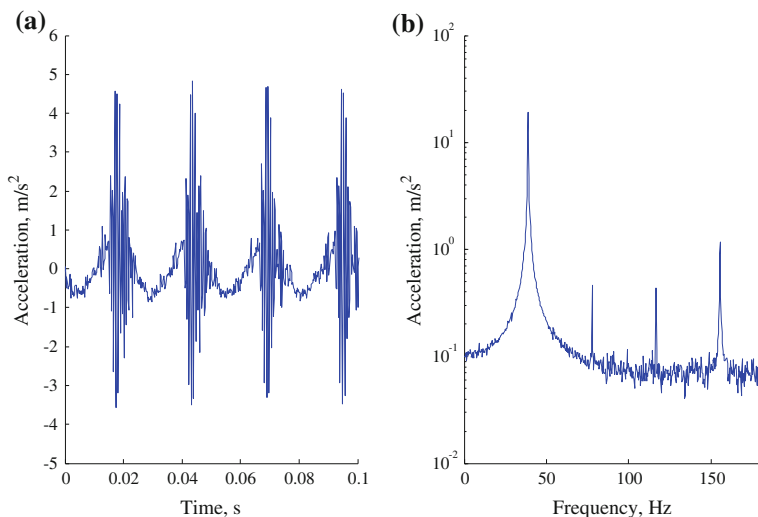
	No. of cracks	Crack location (m)	Crack depth (cm)
Case 1	1	0.35	1.0
Case 2	1	0.85	1.2
Case 3	2	0.4, 0.9	1.0, 1.2
Case 4	2	0.4, 1.2	1.0, 1.0

where K_c is the stiffness matrix for the cracked rotor, K is the stiffness matrix for the healthy rotor (global), n is the number of open cracks and $\Delta K_{c,p}$ is the maximum reduction in the stiffness due to p -th open crack.

Equation 1 is solved numerically using the Newmark- β method with time steps of $200 \mu s$ assuming that the initial displacement and velocity is zero. Modal analysis is carried out to compute the natural frequencies of the rotor assuming all the existing cracks are open. The first three calculated natural frequencies of the rotor in the vertical direction for all the cases are shown in Table 2. The rotor is then excited by a sinusoidal input at its first natural frequency. The calculated acceleration responses are polluted with normalised random noise of 20 dB in order to simulate experimental condition. By using the Fast Fourier transform (FFT) the time-domain acceleration responses are converted into the frequency-domain with sampling frequency of 5 kHz. Figure 2 shows a typical noisy acceleration response in time domain and its frequency spectrum from a specific node at mode 1 where the higher

Table 2 Computed natural frequencies

	Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)
Case 1	38.88	137.22	321.76
Case 2	38.77	138.81	315.49
Case 3	38.35	136.95	311.59
Case 4	38.50	134.67	314.66

**Fig. 2** Calculated acceleration response with noise in **a** time domain and **b** frequency domain for the cracked rotor at mode 1

harmonics of the exciting frequency can be observed clearly. These higher harmonics confirm the breathing of the crack during external excitation.

3 The Curvature of R-ODS Method

The curvature of R-ODS method is proposed in order to magnify the effect of crack in the R-ODS signals which is expressed in Eq. 3:

$$d^2(R - ODS_j) = \frac{(R - ODS)_{j+1} - 2(R - ODS)_j + (R - ODS)_{j-1}}{(\Delta x)^2}, \quad j = 2, 3, \dots \quad (3)$$

where j refers to each R-ODS data point and ΔX is the difference between adjacent data points.

In fact Eq. 3 is the second derivative of the R-ODS data which removes the effect of curvature for better indication of discontinuities related to the crack(s) in the R-ODS signal. The R-ODS data can be obtained as follows:

$$(R - ODS)_{p,m} = (NODS)_{p,m} - (NODS)_{1,m}, \quad p = 2, 3, 4, \dots \quad (4)$$

where $(NODS)_{p,m}$ is the normalised ODS at p -th harmonic for mode m .

Generally, as stated in Eq. 4, the R-ODS method eliminates the effect of the exciting frequency ($1x$) from the normalised ODSs at the higher harmonic components. Therefore, The R-ODS only contains the non-linearity due to the crack(s) breathing. Indices p and m in Eq. 4 are equal to 2 and 1 respectively in this paper because firstly the R-ODSs are calculated for $2x$ data and secondly the cracked rotors are excited at their first resonant frequency.

4 Results and Discussion

Figures 3 and 4 demonstrate the obtained R-ODS and their second derivatives for shafts with single and multiple cracks respectively. The method explained in [10] was used to obtain normal ODSs at the exciting frequency and its higher harmonic components. In the case of the rotor with a single crack (Fig. 3a, c) the R-ODS plot itself indicates the crack location clearly and the curvature magnifies the

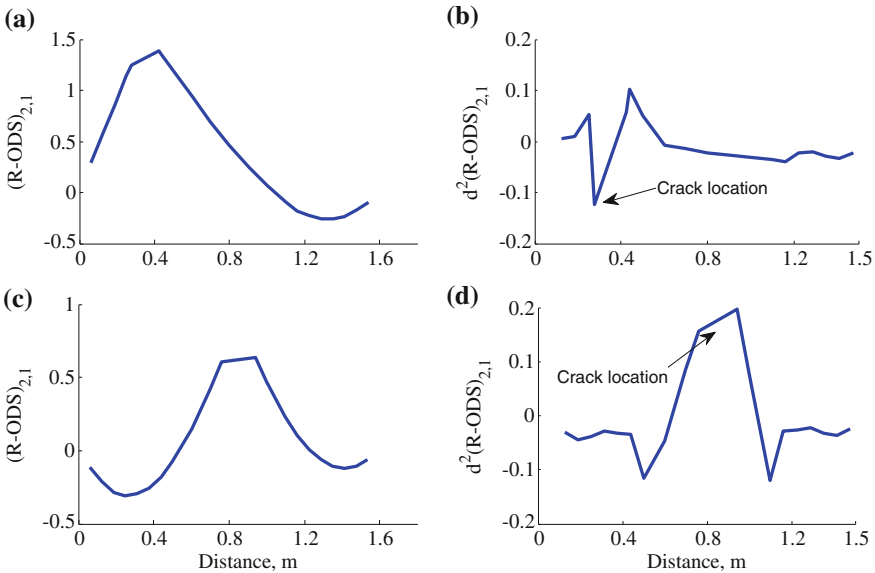


Fig. 3 The $(R-ODS)_{2,1}$ for Cases 1 and 2 (a, c) and their $d^2(R-ODS)_{2,1}$ (b, d) respectively

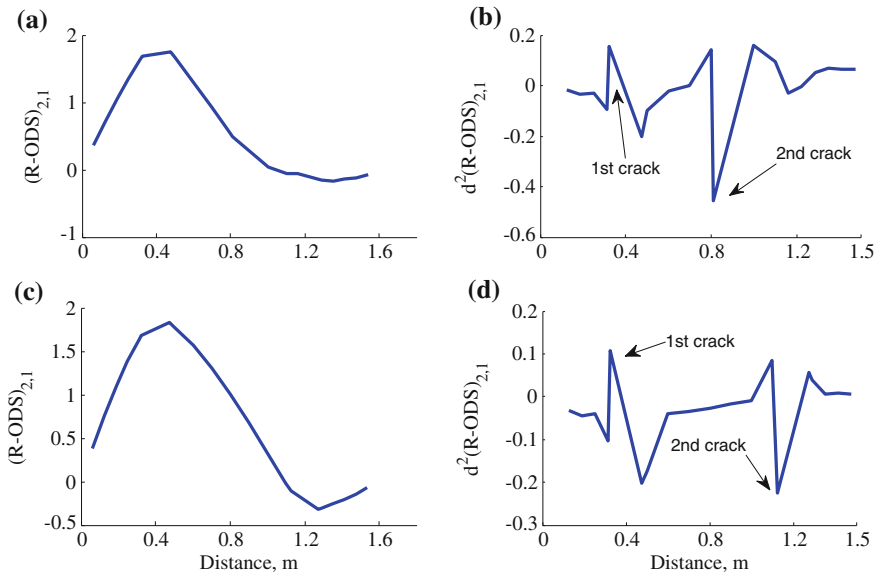


Fig. 4 The $(R-ODS)_{2,1}$ for Cases 3 and 4 (a, c) and their $d^2(R-ODS)_{2,1}$ (b, d) respectively

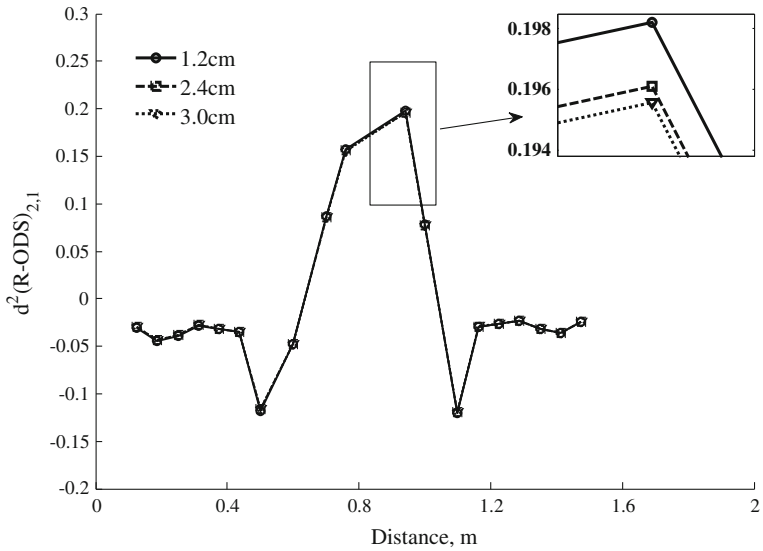


Fig. 5 Effect of crack size on the curvature of RODS

discontinuity even further. However, when there is more than one crack in the rotor the R-ODS plot may not provide a clear indication of all the crack locations, since the amplitude of discontinuity depends on the breathing intensity of each crack.

This fact can be seen in Fig. 4 for the rotor with multiple cracks. In such cases, the curvature of the R-ODS (Fig. 4b, d) plots presents encouraging results for detecting the crack locations.

The rotor in Case 2 is used here to assess the ability of the curvature of R-ODS method to detect the severity of cracks. For this purpose crack sizes of 1.2, 2.4 and 3.0 cm were used in the rotor. As Fig. 5 demonstrates, increase in the crack depth reduces the magnitude of the R-ODS curvature.

5 Conclusion

The application of the curvature of R-ODS method to the simplified LP turbine of a steam generator is presented in this paper. The numerical model consists of a stationary stepped shaft with multiple balance disks representing the blades of the turbine. The details of modelling the breathing fatigue crack(s) are discussed. Shafts with different numbers of cracks at various locations were excited at their resonant frequency. Afterwards, the acceleration response at different locations along the shafts was computed to construct the ODS of the cracked shafts. Normalised ODSs were then utilised to obtain the R-ODSs. The numerical results illustrated that the R-ODS method indicates the location of single-crack shaft by showing a sharp discontinuity. However, when multiple cracks exist in the shaft, the amplitudes of discontinuity relating to each crack may not have the same magnitude in the R-ODS plot. In other words, depending upon the intensity of breathing, the discontinuity generated by one crack might be significantly smaller than the other crack in the R-ODS plot. Therefore, the second derivative is used to remove the curvature of R-ODS plots so that all the discontinuities relating to the crack locations can be observed clearly.

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