Comparison of Performance Indices for Tuning of $PI^{\lambda}D^{\mu}$ Controller for Magnetic Levitation System

Waldemar Bauer, Tomasz Dziwiński, Jerzy Baranowski,
Paweł Piątek, and Marta Zagórowska

AGH University of Science and Technology,
Department of Automatics and Biomedical Engineering,
Al. Mickiewicza 30, 30-059 Kraków
{bauer,tdz,jb,ppi,zagor}@agh.edu.pl

Abstract. Control of active magnetic bearings is an important area of research. The laboratory magnetic levitation system can be interpreted as a model of a single axis of bearings and is a useful testbed for control algorithms. The mathematical model of this system is highly nonlinear and requires careful analysis and identification. In this paper authors compare performance indices for tuning of $PI^{\lambda}D^{\mu}$ controller for this system. It is a part of an ongoing research on non integer controller tuning rules.

1 Introduction

Magnetic levitation systems have many varied uses such as in frictionless bearings, high-speed maglev passenger trains, levitation of wind tunnel models, vibration isolation of sensitive machinery, levitation of molten metal in induction furnaces and the levitation of metal slabs during manufacture, see [5]. Much interest is recently focused on active magnetic bearings. These bearings are considered to be superior over conventional bearings because the friction losses are significantly reduced due to contactless operation. The bearings can also give high speed and are also able to eliminate lubrication and moreover, operation will be quiet, see [1]. Magnetic bearings are increasingly used in industrial machines such as compressors, turbines, pumps, motors and generators. Very interesting are also their applications in artificial hearts. Also important, especially in current popularity of "green" energy solutions, is the flywheel energy storage system.

Flywheel energy systems are now considered as enabling technology for many applications including space satellite low earth orbits, hybrid electric vehicles (see [6]), and many stationary applications. Such mechanical batteries normally consist of a high speed inertial composite rotor, a magnetic bearings support and a control system, an integral drive motor/generator, power electronics for electrical conversion, and so on. One of the advantages over chemical batteries is that the design life has no degradation during its entire cycle life, and current testing indicates that flywheels are not damaged by repetitively deep discharge. Also,

K.J. Latawiec et al. (eds.), *Advances in Modeling and Control of Non-integer Order Systems*,
Lecture Notes in Electrical Engineering 320, DOI: 10.1007/978-3-319-09900-2_12

the contactless nature of magnetic bearings brings up higher energy efficiency, lower wear, longer life span, absence of lubrication and mechanical maintenance, and wider range of work temperature. Moreover, the closed-loop control of magnetic bearings enables active vibration suppression and on-line control of bearing stiffness (see [18]).

Control of magnetic levitation system was analysed by many researchers focusing on different approaches. A linearising feedback control was considered among the others by Barie and Chiasson (see [5]), Joo and Seo (see [10] and [14]). Different approach to feedback linearisation of mag-lev (see [2]). The comparison of this approach with Takagi-Sugeno fuzzy control (see [9]). The cascade variant of the linearising feedback was also discussed by Baranowski and Piątek (see [3]). Real time neural feedforward control was considered by Bloch (see [11]). Practically efficient results were also obtained by Piątek (see [13]) with very fast linear control based on FPGA circuits. Piłat in [15] considered a non-integer order PD controller.

In this paper we discuss an application of tuning non-integer $\mathrm{PI}^{\lambda}\mathrm{D}^{\mu}$ controller, when control signal is not disturbed and disturbed. This is a continuation of authors earlier works (see [3,8,16]).

1.1 The Mathematical Model of the System

We consider the magnetic levitation system consisting of the electromagnet, the ferromagnetic sphere (which is later referred to as the "ball"), the current driver and the position measurement system.

To construct the mathematical model of the plant we will rely on a basic relation of Newton's second law, in this case:

$$m\ddot{x}_1(t) = F_l(x_1(t), x_3(t)) + mg \tag{1}$$

where $x_1(t)$ is the gap between the ball and the electromagnet, $x_3(t)$ is the electromagnet coil current, $F_l(x_1(t), x_3(t))$ is the force generated by the electromagnet, m is the mass of the ball and g is the gravitational acceleration. It is widely known that the force generated by the electromagnet is given by the following relation

$$F_l(x_1(t), x_3(t)) = \frac{1}{2} \cdot \frac{\mathrm{d}l(x_1(t))}{\mathrm{d}x_1(t)} x_3^2(t) \tag{2}$$

where $l(x_1(t))$ is the electromagnet inductance. Commonly, the inductance is considered for cuboidally shaped gaps as a hyperbolical function, as for an example (time argument was omitted)

$$l(x_1) = l_1 + \frac{\mu l_0}{\mu + x_1} \tag{3}$$

where l_0, l_1 and μ are positive constants. Expressions of this type were considered among the others by Barie and Chiasson (see [5]). What should be noted is that levitation systems such as considered have gaps of a different shape because a levitating object is round. That is why we consider the approximation developed by Piłat (in [14]) in a form of the following exponential function

$$l(x_1) \approx a \exp\left(-\frac{x_1(t)}{b}\right) \tag{4}$$

where a and b are positive constants. This approximation was obtained and verified experimentally and leads to very good results. Parameters a and b were determined by analysis of series of steady state points of the system with a closed stabilising feedback loop. Exponential function was fitted into these points through a least squares minimisation. For details see [14].

The coil current in the system usually is influenced by many factors like changes in inductance, velocity and others. However, our system includes a current driver, which has its own feedback loop. This solution is very popular (see [7]) because it leads to either lower order or simpler model structure. In optimal situation the driver should allow full current control, however in real situations it introduces its own dynamics. For considered system, this dynamics can be sufficiently modelled by a first order dynamical system given by the following equation

$$\dot{x}_3(t) = \frac{1}{T_s}(k_s u(t) - i_s - x_3(t)) \tag{5}$$

where $u(t)$ is the control voltage, k_s is the gain of current controller, T_s is the time constant of the current driver and is i_s the zero error of current driver.

Velocity of the ball x_2 is the first derivative of position, so we can construct the state space model. Let us introduce state space vector $\mathbf{x}$ given by

$$\mathbf{x} = [x_1 \; x_2 \; x_3]^T \tag{6}$$

which can be used to formulate the model of the system as the following system of first order differential equations

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), u(t)) \tag{7}$$

where

$$\mathbf{f}(\mathbf{x}(t), u(t)) = \begin{bmatrix} x_2(t) \\ -\frac{a}{2mb}\exp\left(-\frac{x_1(t)}{b}\right) x_3^2 + g \\ \frac{1}{T_s}(k_s u(t) - i_s - x_3(t)) \end{bmatrix} \tag{8}$$

1.2 Nonlinear Feedforward

It is a known fact that the linear controller can operate properly in the neighbourhood of a chosen steady state. Performance of classical PID can be strongly improved, if the appropriate reference control value corresponding to a reference value is added to the generated control signal. Authors tested this solution with non-integer $PI^{\lambda}D^{\mu}$ controller.

Let us consider control structures presented in figures 1. Let us assume, that set point signal is piecewise constant. This goal can be satisfied then function $\Psi(w_r)$ have form:

$$\mathbf{f}(\mathbf{x}_r, \Psi(w_r)) = 0 \tag{9}$$

where $x_r = [w_r \; 0 \; x_{3r}]^{\mathsf{T}}$, w_r is constant value of $w(t)$, $\mathbf{f}$ is given by (8) and x_{3r} is the value of current corresponding to $w(t)$. Such function (along with x_{3r}) can be obtained by solving (9) and is given by the following formula:

$$\Psi(w_r) = \frac{1}{k_s}\left(i_s + \sqrt{\frac{2mbg}{a} \cdot \mathrm{e}^{\dfrac{w_r}{b}}}\right) \tag{10}$$

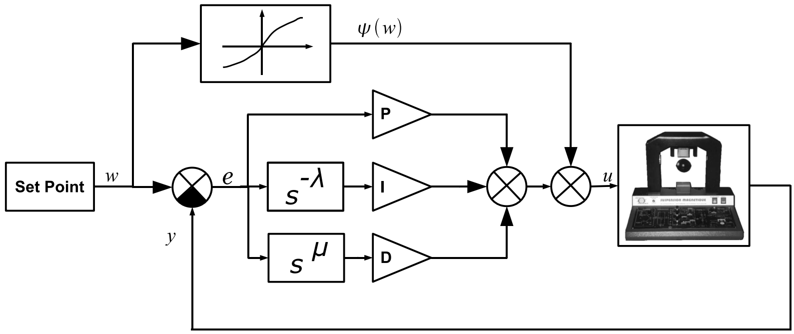


Fig. 1. Magnetic levitation with $PI^{\lambda}D^{\mu}$

2 Non-integer $PI^{\lambda}D^{\mu}$

This section describes a more generalized structure for the classical PID controller. Podlubny proposed a generalization of the PID, namely the $PI^{\lambda}D^{\mu}$ controller, involving an integrator of order λ and a differentiator of order μ. In time domain the equation for the $PI^{\lambda}D^{\mu}$ controller's output has the form (see [17]):

$$u(t) = K_p e(t) + K_i \, {}_0^C\mathrm{D}_t^{-\lambda} e(t) + K_d \, {}_0^C\mathrm{D}_t^{\mu} e(t) \tag{11}$$

Where:

- K_p is proportional gain
- K_i is integral gain
- K_d is derivative gain
- $e(t)$ is control deviation in time t
- λ, $\mu > 0$

And the transfer function formula is given by the equation:

$$G(s) = K_p + K_i s^{-\lambda} + K_d s^{\mu} \tag{12}$$

As can be observed, when $\lambda = 1$ and $\mu = 1$ we obtain a classical PID controller, similar when $\lambda = 0$ and $\mu = 1$ give PD, $\lambda = 0$ and $\mu = 0$ give P, $\lambda = 1$ and $\mu = 0$ give PI.

All these classical types of PID are the particular cases of the fractional PI$^\lambda$D$^\mu$. However, the PI$^\lambda$D$^\mu$ is more flexible.

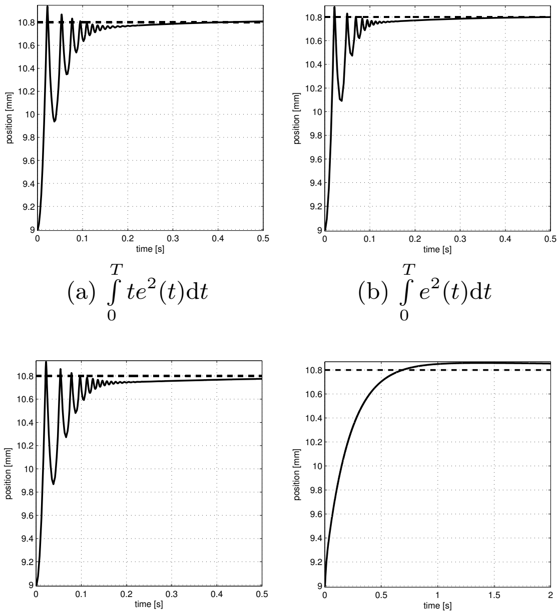
For all numerical experiments the Simulated Annealing optimization method has been chosen for tuning PI$^\lambda$D$^\mu$ controller parameters. In this case we can define the decision variables as: K_p, K_i, K_d, λ and μ. The tests will be conducted for the following quality index:

Table 1. Result of tuning system without disturbance

Quality index	K_p	K_i	λ	K_d	μ	Quality value
$\int_0^T te^2(t)\mathrm{d}t$	517.017	116.408	0.917	20.6418	0.6796	$2.69 \cdot 10^{-3}$
$\int_0^T e^2(t)\mathrm{d}t$	475.1759	63.0862	0.2555	4.7824	0.7788	$3.69 \cdot 10^{-2}$
$\int_0^T \lvert e(t)\rvert\mathrm{d}t$	553.146	91.828	0.786	4.336	0.77	0.112
$\int_0^T \left(e^2(t) + x_2^2(t)\right)\mathrm{d}t$	498.241	68.415	0.777	66.5197	0.997	$1.33 \cdot 10^{-2}$

Table 2. Result of tuning with load disturbance

Quality index	K_p	K_i	λ	K_d	μ	Quality value
$\int_0^T te^2(t)\mathrm{d}t$	481.202	291.560	0.0104	38.773	0.6429	$7.95 \cdot 10^{-2}$
$\int_0^T e^2(t)\mathrm{d}t$	582.31	118.476	0.087	47.023	0.585	$5.42 \cdot 10^{-2}$
$\int_0^T \lvert e(t)\rvert\mathrm{d}t$	491.63	54.99	0.0237	57.378	0.59	0.313
$\int_0^T \left(e^2(t) + x_2^2(t)\right)\mathrm{d}t$	553.333	−38.514	0.997	94.882	1	9.88



(a) $\int_0^T te^2(t)\mathrm{d}t$ (b) $\int_0^T e^2(t)\mathrm{d}t$

(c) $\int_0^T |e(t)|\mathrm{d}t$ (d) $\int_0^T e^2(t) + x_2^2(t)\mathrm{d}t$

Fig. 2. Result of tuning system without disturbance for differing quality index

- $\int_0^T te^2(t)\mathrm{d}t$
- $\int_0^T e^2(t)\mathrm{d}t$
- $\int_0^T |e(t)|\mathrm{d}t$
- $\int_0^T \left(e^2(t) + x_2^2(t)\right)\mathrm{d}t$

where $e(t) = w_r - x_1(t)$.

The controller was implemented with Oustalup method. For the fractional-order operator $G(s) = s^\alpha$, the continued fraction expansion can be written as (see [12]):

$$G_t(s) = K \prod_{i=1}^{N} \frac{s + \omega_i'}{s + \omega_i} \tag{13}$$

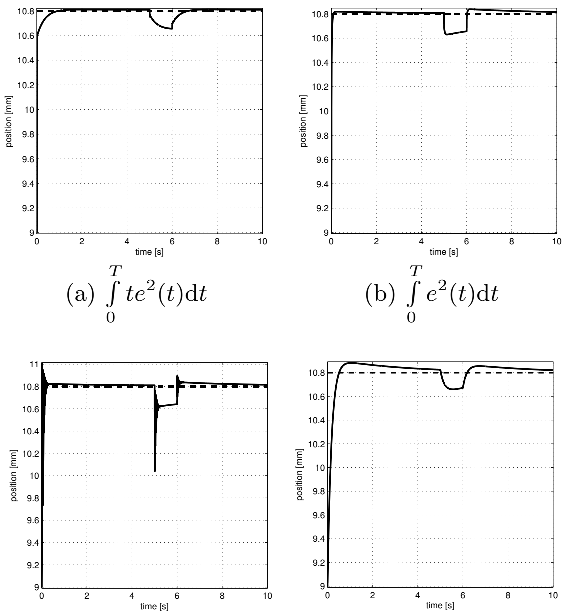
where:

$$\omega_i' = \omega_{\min}\omega_u^{(2i-1-\alpha)/N} \tag{14}$$

$$\omega_i = \omega_{\min}\omega_u^{(2i-1+\alpha)/N} \tag{15}$$

$$K = \omega_{\max}^{\alpha} \tag{16}$$

$$\omega_u = \sqrt{\frac{\omega_{\max}}{\omega_{\min}}} \tag{17}$$



(a) $\int_0^T te^2(t)\mathrm{d}t$ (b) $\int_0^T e^2(t)\mathrm{d}t$

(c) $\int_0^T |e(t)|\mathrm{d}t$ (d) $\int_0^T e^2(t) + x_2^2(t)\mathrm{d}t$

Fig. 3. Result of tuning system with disturbance in control signal for differing quality index

2.1 Results

In all experiments, values of approximation parameters are:

- $N = 3$,
- $\omega_{\mathrm{min}} = 10^-6$,
- $\omega_{\mathrm{min}} = 10^6$,

and initial points have value:

- $K_p = 500$
- $K_i = 100$
- $K_d = 6$
- $\lambda, \mu = 0.5$

The optimal PI$^\lambda$D$^\mu$ settings for the system without disturbance are collected in table 1 and for the system with load disturbance settings are collected in table 2. Position states of the magnetic levitation were shown in figures 2 and 3.

How can see the best results have been achieved when quality indices of form $\int_0^T e^2(t)\mathrm{d}t$ or $\int_0^T te^2(t)\mathrm{d}t$ have been used (see figures 2(b) and 3(b)) (see figures 2(a) and 3(a)).

3 Conclusion and Further Research

It has been shown that fractional-order PI$^{\alpha}$D$^{\mu}$ controller is suitable for control of magnetic levitation systems. The paper has shown that simulated annealing optimisation method could be helpful in the tuning process. The authors tested also some quality indices for tuning the controller.

The further research is planned to implement PI$^{\alpha}$D$^{\mu}$ controller in digital real-time environment, based on RT-DAC board and MATLAB/RT-CON library, and to conduct experiments on physical plant.

Acknowledgement. Work realised in the scope of project titled "Design and application of non-integer order subsystems in control systems". Project was financed by National Science Centre on the base of decision no. DEC-2013/09/D/ST7/03960.

References

1. Agarwal, P.K., Chand, S.: Fault-tolerant control of three-pole active magnetic bearing. Expert Systems with Applications (36), 12592–12604 (2009)
2. Bania, P.: Stabilizujące sprzężenie zwrotne dla systemu magnetycznej lewitacji. Automatyka 4(2), 117–139 (2000)
3. Baranowski, J., Piątek, P.: Nonlinear dynamical feedback for motion control of magnetic levitation system. In: Proceedings of 13th International Power Electronics and Motion Control Conference EPE-PEMC, Poznań, September 1-3, pp. 1469–1476 (2008)
4. Baranowski, J., Piátek, P.: Observer-based feedback for the magnetic levitation system. Transactions of the Institute of Measurement and Control 34(4), 422–435 (2012)
5. Barie, W., Chiasson, J.: Linear and nonlinear state-space controllers for magnetic levitation. International Journal of Systems Science 27(11), 1153–1163 (1996)
6. Beno, J., Thompson, R., Hebner, R.: Flywheel batteries for vehicles. In: Proceedings of the 2002 Workshop on Autonomous Underwater Vehicles, pp. 99–101 (2002)
7. Duan, G., Wu, Z., Howe, D.: Robust control of a magnetic-bearing flywheel using dynamical compensators. Transactions of the Institute of Measurement and Control 4(23), 249–278 (2001)
8. Dziwiński, T., Bauer, W., Baranowski, J., Piątek, P., Zagórowska, M.: Robust non-integer order controller for air heater. In: 2014 19th International Conference on Methods and Models in Automation and Robotics, MMAR (accepted 2014)
9. Grega, W., Piłat, A.: A comparison of nonlinear controllers for magnetic levitation system. In: 5th World Multiconference on Systemics, Cybernetics and Informatics, Orlando-Florida, July 22-25, pp. 188–193 (2001)
10. Joo, S., Seo, J.: Design and analysis of the nonlinear feedback linearizing control for an electromagnetic suspension system. IEEE Transactions on Control Systems Technology 5(1), 135–144 (1996)
11. Lairi, M., Bloch, G.: A neural network with minimal structure for maglev system modeling and control. In: Proceedings of the 1999 IEEE International Symposium on Intelligent Control/Intelligent Systems and Semiotics, 1999, pp. 40–45 (1999)

12. Oustaloup, A., Levron, F., Mathieu, B., Nanot, F.M.: Frequency-band complex noninteger differentiator: characterization and synthesis. IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications 47(1), 25–39 (2000)
13. Piątek, P.: Wykorzystanie specjalizowanych architektur sprzętowych do realizacji krytycznych czasowo zadań sterowania Application of Specialized Hardware Architectures for Realization of Time-critical Control Tasks. PhD thesis, Akademia Górniczo-Hutnicza im. Stanisława Staszica, Wydział Elektrotechniki, Automatyki, Informatyki i Elektroniki, Kraków, Promotor: W. Grega (2007)
14. Piłat, A.: Sterowanie układami magnetycznej lewitacji. PhD thesis, Akademia Górniczo-Hutnicza im. Stanisława Staszica, Wydział Elektrotechniki, Automatyki, Informatyki i Elektroniki, Kraków, Promotor: W. Grega (2002)
15. Pilat, A.: A comparative study of $pi^{\lambda}d^{\mu}$ controller approximations exemplified by active magnetic levitation system. In: Mitkowski, W., Kacprzyk, J., Baranowski, J. (eds.) Theory & Appl. of Non-integer Order Syst. LNEE, vol. 257, pp. 231–241. Springer, Heidelberg (2013)
16. Piątek, P., Zagórowska, M., Baranowski, J., Bauer, W., Dziwiński, T.: Discretisation of different non-integer order approximations. In: 2014 19th International Conference on Methods and Models in Automation and Robotics, MMAR (accepted 2014)
17. Podlubny, I.: Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications. Mathematics in Science and Engineering. Elsevier Science (1998)
18. Zhu, K., Xiao, Y., Optimal, R.K.Y.: control of the magnetic bearings for a flywheel energy storage system. Mechatronics (19), 1221–1235 (2009)