Nostalgia and Phenomenon: Husserl and Patočka on the End of the Ancient Cosmos

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Abstract This essay argues that Jan Patočka's 'Galileo Galilei and the End of the Ancient Cosmos' goes beyond Husserl's fragmentary account of Galileo in *The Crisis of European Sciences and Transcendental Phenomenology* to present an account of the *a priori* eidetic structure of the foundation of a strand of the modern, scientific mathematisation of nature that is informed by *actual* history. In conjunction with this, Patočka adumbrates the eidetic structure of the concomitant limits on human meaning imposed by this historically dated conceptual foundation, insofar as the human being becomes a part of the mechanised world that Galileo's accomplishment makes possible.

Keywords Husserl • Patočka • François Vieta • Gottlob Frege • Analyticity • Algebra • Concept of number • Mathematical symbols • Phenomenology

A Priori and History in Transcendental Phenomenology

There is a view as widespread as it false that the appeal to history and the problem of existential meaning in Husserl's *Crisis* texts is the result of Heidegger's influence. Thus, so the story goes, Husserl finally recognised that the project to establish transcendental phenomenology as an empirically pure science was vulnerable to Heidegger's critique of his thought in the 1920s and early 1930s. As is well known, Heidegger's critique had two major foci: firstly, that intentionality as a phenomenon is derivative, in that it presupposes the ontico-ontologically more fundamental phenomenon of the historically determined facticity proper to human existence (*Dasein*); and secondly, that the eidetic structure of phenomenological cognition presupposes the historically driven and phenomenologically unsustainable meaning of Being that privileges a single and moreover derivative modality of time – the present – to the exclusion of time's more fundamental horizontal modes of the past and future. Husserl's appeals to *Existenz* and history in the *Crisis* texts are therefore supposed, by those subscribing to the view of Heidegger's influence on his thought,

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to originate in Husserl's recognition that the aspirations to both transcendental purity and eidetic universality in his phenomenology are vulnerable to the Heideggerian critique.

The falsity of the claim that Husserl was influenced by or otherwise responding to this critique in his *Crisis* texts is evident when his limited understanding of the seminal work of Heidegger that presents its groundwork, *Sein und Zeit*, is considered in conjunction with what Husserl actually says about human *Existenz* and history in those texts. Regarding the former, Husserl's marginal notes in his copy of *Sein und Zeit* make manifest how very little of the basic ideas of that work, beginning with the *Seinsfrage* and the *Daseinanalytik*, he was capable of grasping. So far as Husserl is concerned, formal ontology raises and answers satisfactorily the question of the meaning of Being, while what is behind the strange "word magic (*Wortezauber*)" (Cairns 1976: 107) of Heidegger's analysis of *Dasein* on his view is the "complicated formalities and unclarities, simply so as not to make use of intentionality" (Husserl 1997b: 382). Indeed, in a 1931 letter to Alexander Pfänder, Husserl confided that Heidegger's theories were "inaccessible" to his way of thinking and that Heidegger surrendered "both the method of my phenomenological research and its scientific character in general" (Husserl 1997a: 480). With respect to the latter, it would be surprising, given Husserl's own acknowledgment of his inability to penetrate Heidegger's thought in 1931, if some 4 years later he were to suddenly come to the realisation of the need to surrender his own commitment to the fundamentally *a priori* mode of phenomenological cognition in response to Heidegger's critique. Careful study of the *Crisis* texts discloses not only that this commitment remains intact, but also that the transcendental and eidetic universality of phenomenological cognition, far from being attenuated in response Heidegger's criticisms, is actually extended in those texts to include both the radical self-responsibility of the phenomenologist and the historical horizon that is now presented by Husserl as the driving force behind phenomenological self-reflection (*Selbstbesinnung*).

In a word, far from abandoning the *a priori* and eidetically universal pretentions of transcendental phenomenology in the *Crisis*, Husserl endeavours there to do what heretofore had never been imagined, let alone attempted in thought in the history of philosophy: namely, to unify *a priori* cognition with historical phenomena. Husserl does this in a manner that presents evidence for the historical origin of the *Sinne* that govern the apriority of the fundamental concepts of the exact science of mathematics; that is, of the ideal science that for Husserl provides the foundation for modern physics, the most rigorous of the contemporary (to Husserl) European sciences that in his view are in crisis. Such evidence also, *ipso facto*, amounts to the phenomenological–philosophical case for an expansion of the methodological scope of phenomenology's *epistemological* quest for foundational cognition, to include the *historical* horizon and origin of the *Sinne* of the basic concepts of mathematics that make modern physics possible.

The Genuine Husserlian Context of Patočka's Analysis of Galileo's Achievement, and Its Advance over Husserl's *Crisis* Analysis

The difficulty of thinking together what appear *conceptually* as opposites – the *concept* of the '*a priori*' and the *concept* of 'history' – is no doubt one factor behind the failure of most post-Husserl phenomenologists to follow what Husserl on his deathbed referred to as the "small beginning"[1] made in the *Crisis* texts, and thus to investigate the transcendental historicity of the origin of the *Sinne* of natural sciences' foundational concepts. Another factor is no doubt the fragmentary nature of the *Crisis* texts and the incomplete status of what their first editor, but not Husserl, referred to as the "intentional-historical problem" of the origin proper to the *Sinne* constitutive of the exact sciences (Fink 1939).[2] One thinker who, however, managed to follow Husserl's "small beginning", and thus to realise that not only is there no *conceptual* contradiction involved in *phenomenologically* thinking together the '*a priori*' with the 'historicity' of the *Sinne* determinative of the exact concepts of sciences, but also that the most rigorous phenomenological account of the *Sinne* in question demands precisely tracing back their *apriority* to origins manifest in history, was Jan Patočka. In his 'Galileo Galilei and the End of the Ancient Cosmos',[3] he goes beyond Husserl's fragmentary account of Galileo in the *Crisis* to present an account of the *a priori* eidetic structure of the foundation of a strand of the modern scientific mathematisation of nature that is informed by *actual* history. In conjunction with this, he adumbrates the eidetic structure of the concomitant limits on human meaning imposed by this historically dated conceptual foundation, insofar as the human being becomes a part of the mechanised world that Galileo's accomplishment makes possible.

But even while going beyond Husserl, Patočka remains indebted to him, because the guiding supposition behind Patočka's account of Galileo's accomplishment is that the *Sinn* constitutive of the *a priori* foundation of modern mechanics is inseparable from the historicity of the concepts that were presupposed as well as generated by this accomplishment. Moreover, clearly in the background of Patočka's analysis of Galileo's achievement is that it is philosophically worthwhile to "reactivate" the novel "anticipation" (*Vorhabe*) (Husserl 1970a: 356 [367]) of a

[1] Edmund Husserl, in a conversation reported by Adelgundis Jaegerschmid, OSB, in 1936, see Jaegerschmid 2001: esp. 346. For an extended discussion of Husserl's "small beginning", see Hopkins 2010: 5–6, 12, 170, 213–114, 251–152.

[2] Published as "Beilage III" in *Die Krisis der europäischen Wissenschaften und die transzendentale Phänomenologie. Eine Einleitung in die phänomenologische Philosophie* (Husserl 1954). English translation: "The Origin of Geometry", in *The Crisis of European Sciences and Transcendental Phenomenology* (Husserl 1970a). Henceforth, English and [German Husserliana Vol.] page numbers, respectively.

[3] Jan Patočka, 'Galileo Galilei and the End of the Ancient Cosmos', unpublished translation by Erika Abrams and Martin Pokorný (Patočka in press). Original publication: Patočka 1954.

mechanics of nature in Galileo's thinking, the anticipation out of which its novelty originated, against the background of the transformed understanding of the world and the human's place in that world it was to bring about. In other words, Patočka's analyses effectively uncover a layer of meaning "sedimented" in the foundation of modern mechanics, whose de-sedimentation is philosophically significant; because, among other things, it diagnoses in the basic supposition of this foundation a root condition of the crisis of European sciences. Finally, Patočka's account of Galileo shares with Husserl's a certain nostalgia for the unity of the cosmos and the human's place in that cosmos, which they both think is lost with the peculiarly modern mode of mechanistic mathematisation of the world begun by Galileo – or so I shall argue. I shall develop my argument by showing that this nostalgia is above all significant because it stands in the way of the detection of a technical problem in the basic concept of the modern mathematics that makes modern physics possible, that is far more responsible for the enduring crisis in the foundations of European sciences than the nostalgia for the experience of a unity that all of European humanity, alas, was born too late to remember.

Patočka's Galileo goes beyond Husserl's by attuning the analysis of Galileo's achievement to the actual historical record of his thought preserved in his writings, which contrasts with the historical liberties Husserl took by employing the name 'Galileo' as a collective noun for the impulses behind and achievements generated by early modern natural philosophy. Patočka's Galileo therefore reveals, sedimented in his thought, the supposition that his innovations were in the service of the Platonic conviction that the true order of the cosmos is essentially mathematical and that this order unifies the Aristotelian distinction between sub-lunar and celestial beings by articulating the theorems that allow this order's *lawful* mathematical deduction; a deduction, albeit, whose guiding natural supposition is that the natural, elementary and fundamentally perfect motion is circular.

The innovation of Patočka's Galileo thus lies not so much in his mathematisation of nature but in his conception of motion in a manner that "completely dissociates the law he formulates from the semi-animistic physics of 'impetus' [...] For Galileo, there is no mysterious quality that inspirits the moving body; the change in state of motion presupposes simply the impulse of a force, conceived as mere quantity in relation to other quantities" (Patočka in press). Galileo's use of this method, however, remains devoted to solving concrete problems, and Patočka's account of the reason why he never did what his followers (above all Newton) did – namely, to formulate his method's basic principles, above all the principle of inertia – is based in his account of the sedimentation in Galileo's thinking of "the idea of the cosmos, the perfect world order he started off with the idea of understanding and [that he] is attempting to formulate mathematically" (Patočka in press). In such a cosmos material bodies have gravity, and "all motion in the world comes, directly or indirectly, from gravity, so that uniform rectilinear motion is impossible". Galileo was therefore incapable of "abstractly representing such a possibility [of rectilinear motion], for a body [on his view] without gravity would not be a material body (which shows [sedimented in Galileo's thought] a survival of the ancient distinction between mathematical and physical bodies") (Patočka in press).

Thus, on Patočka's account, it remained for others to apply Galileo's method "consistently", that is, *generally*, which application "does away with all 'hierarchy', banishes from the world all 'values', all purposes, all teleology, putting all being whatsoever on a level: everything is equally an object" (Patočka in press). It is on the foundation laid but not explicitly followed by Galileo that the second *Sinn* is sedimented in his achievement – a sedimentation already uncovered by Husserl but given more precision by Patočka; that is, more precision insofar as he delineates that the relegation of all qualitative phenomena to subjectivity already noted by Husserl is coincident with something else. What it is coincident with is the fact that "the 'subject' becomes something that is not integrated in this mechanical world as an equal part: it becomes a mere image, a replica of objective being; knowledge of the world is a kind of contemplation, not an action taken within being, since the world of quality, subject, values, etc., is driven out of true, mathematical-physical reality" (Patočka in press).

That Galileo's successors would therefore employ the foundations prepared by Galileo in the service of achieving the "perfect unity and harmony of the cosmos" (Patočka in press) to the end of a universal science whose "emphasis on formalization and operation" (Patočka in press) led to the opposite result – that is, to the end of the ancient cosmos – is significant for Patočka; because, as already mentioned, for him the end of the ancient cosmos is coincident with the "demise" of "the cosmos itself" (Patočka in press).

Because Patočka's account of the *Sinne* sedimented in Galileo's achievement is, like Husserl's accounts in the *Crisis*, fragmentary, the philosophical payoff promised by their "de-sedimentation" is not at all clear. So far as I can tell, neither the foundational program of an ontology of the pre-scientific life-world, including the epistemological–phenomenological project of excavating the "ground" of the cognition of the various sciences in the pre-given life-world, nor the exhortation to initiate a cultural renewal by somehow returning to the life-world, are sufficient to address the deeply felt nostalgia for the 'unity' between human meaning and the "cosmos itself" that was lost forever with the end of the ancient cosmos. In Husserlian terms, these phenomenological responses to the crisis of European sciences are incapable of "apodictically conquering the will" (Husserl 1970b: 18 [16–17])[4] in a manner that would function as a norm capable of rationally pointing in *the* direction out of the crisis. In what follows I shall argue that in addition to the de-sedimentation of the geometrical suppositions behind the Galilean foundation of mechanics, what also needs to be uncovered and de-sedimented is the sedimentation constitutive of the modern concept of 'unity' and indeed of the modern concept of number itself; in order to disclose fully the phenomenological implications of the loss of the 'unity' of human meaning with the cosmos coincident with the end of the ancient cosmos itself. My argument shall take the form of a demonstration of the pre-modern concept of unity that is sedimented in its modern concept; as well as a demonstration of its consequent de-sedimentation. Above all,

[4] English and [German Husserliana Vol.] page numbers, respectively.

the demonstrations to follow are intended to show the arithmetical nature of what is most fundamentally at stake for phenomenological *philosophy*, in the loss of unity that both Husserl and Patočka identify as a crucial aspect of the diminution of human meaning inseparable from the conceptuality of European sciences; a conceptuality that each thinker maintains is responsible for those sciences' crisis. And, in anticipation of the results of these demonstrations, I shall conclude that the de-sedimentation of the pre-modern meaning of 'unity' sedimented in its modern concept identifies a philosophically fundamental technical error in the constitution of the modern concept of 'unity'; an error whose recognition is capable of "apodictically conquering the will" and thus adumbrating a normative response to the crisis of European sciences that is grounded in reason.

Overview: Unity and Number in Ancient Greek and Modern Mathematics

Ancient Greek arithmetic employed a different concept of unity and number than modern arithmetic employs; indeed, these concepts in their ancient and modern guises are so different that it's better to say that what the Ancient Greek mathematicians understood the unity of a number to be and what modern mathematicians understand it to be are radically different. To begin with, for Ancient Greek mathematicians numbers were sharply distinguished from *concepts*; whereas the quintessential modern definition of number understands it to be the property of a *concept* – or, better, the set of all concepts having a common property. This difference is buttressed by another fundamental difference in the mathematical understanding of the Ancient Greeks and the moderns: namely, their understanding of what mathematics is in general and what arithmetic is in particular. For the Greeks a μάθημα (*mathêma*) is something that can be learned and understood, and that, once learned, is *known*.Ἐπιστήμη (knowledge) is therefore closely connected with the Greek understanding of 'mathematics', and the idea of mathematics in this sense is the paradigm for all Greek philosophy and science. Greek arithmetic, as a learning matter, is concerned above all with two fundamental problems: what is the nature of things insofar as they are counted, and in what sense is the number of those things a unity? These problems are very remote from *our* arithmetic, which concerns the *practical* art of calculation. Nowadays these two questions raised by Ancient Greek arithmetic are dealt with by number *theory*; which brings me to a second crucial point, namely, the question of what is involved in the philosophical problem of *foundation* as it relates to mathematics.

That there *have* to be more profound *reasons* for the truth of what, in the case at hand, the science of mathematics claims and therefore pretends to know, was taken to be evident because of paradoxes or outright contradictions discovered by *reason* in the basic concepts of arithmetic. In the case of Ancient Greek arithmetic, the fact that number denotes both *many* things together with their *unity* as exactly so many was recognised to rest on a profound contradiction: namely, that of one and the same thing – number – being both many and one and therefore combining in its very *being* qualities that human speech must recognise for all time as uncombinable opposites.[5] In the case of modern number theory, the expansion of the number domain in 'universal analysis' or 'universal arithmetic' beyond natural numbers, to include irrational numbers, negative numbers, imaginary numbers, and so on, raised the problem of how to understand these non-natural numbers as *numbers* at all; that is, as units of measure – quantities – that provide an answer to the question: how many?

The Problem of Foundation in Pythagorean Arithmetic

An account of the problem of foundation in ancient arithmetic has to begin with those sixth-century B.C. mathematicians who were later referred to by the Greeks collectively as the 'Pythagoreans'. Contemporary philosophy and mathematics textbooks sum up their contribution to human thought as the theory that 'the essences of things are numbers'. So long as one understands numbers to be abstract concepts, this statement is meaningless. It is closer to what the Pythagoreans are reported to have thought to render their contention like this: 'everything that we see or hear can be counted'. This statement is as remarkable as it is false, although its falsity is noteworthy, because it is coincident with the discovery of incommensurable magnitudes (incommensurables). All things perceivable by the senses, especially visible things, were the things counted by the Pythagoreans. By counting they understood the process of adding one thing and another one and another one, and so on, until coming to a rest; when their number was expressed with words like five, seven, hundred, and so on. Each of these words expresses what the Greeks called an ἀριθμός (number), by which they understood *a definite amount of definite things*. This meaning of ἀριθμός didn't change for all subsequent Greek mathematics and philosophy, and until the sixteenth century it remained the meaning of the Latin word '*numerus*'.

Of the two things already mentioned as the concern of Greek arithmetic – the question of the nature of counted things, and the sense in which their number is a unity – the Pythagoreans focused on the second. The counted things signified by

[5] As the following quote demonstrates, Kurt Gödel likewise recognised the contradiction the Ancient Greeks saw at the heart of arithmetic: "A set is a unity of which its elements are the constituents. It is a fundamental property of the mind to comprehend multitudes into unities. Sets are multitudes which are also unities. A multitude is the opposite of a unity. How can anything be both a multitude and a unity? Yet a set is just that. It is a seemingly contradictory fact that sets exist. It is surprising that the fact that multitudes are also unities leads to no contradictions: this is the main fact of mathematics. Thinking [a plurality] together seems like a triviality: and this appears to explain why we have no contradiction. But 'many things for one' is far from trivial" (Wang 1996: 254).

their number are in every case *many* things while at the same time their multitude is comprehended by means of its number as composing *one* group – or as would be said today, ‘*one* set’ – of things. Precisely this, the *foundational* problem of what is responsible for *many* things being grasped as *one*, is what the arithmetic of the Pythagoreans sought to resolve. It did so by classifying numbers according to their εἴδη (Forms or Species), such as Odd and Even, square, cube – to cite some of the Forms discovered by Pythagorean arithmetic that remain a part of the terminology of arithmetic to this day. Unlike the many things that are determined by a number’s exact amount, the Forms of numbers are one in themselves: thus there is only one Form of the Odd, one Form of the Even, even though there are unlimitedly many odd and even numbers. In addition to these familiar Forms of numbers, the Pythagoreans classified numbers according to geometrical Forms made visible when each counted thing was represented by a pebble or dot, beginning with *one* such representation, to which various configurations of dots were added to produce similar figures of the following kinds: triangular, square, pentagonal, and so on. The numbers configured by these similar figures were called by these figures’ names – e.g. triangular numbers, square numbers, pentagonal numbers, etc. – and these figures were therefore understood be the cause of the many pebbles or dots nevertheless being comprehended as ‘one’. Thus, for instance, six things can be conceived as ‘one’ group, namely, as ‘six’, because the Form triangle causes these six things to be one. So, too, however, can ten things be conceived as ‘one’ group, namely, as ‘ten’, because the same triangular Form causes them to be one.

The Pythagorean attempt to solve the puzzle of the one-and-many composition of numbers thus introduced a distinction that is as crucial as it is fundamental, namely the distinction between the *being* of number – i.e. a multitude of things in the sense of their exact amount – and the *non*-numerical εἶδος (Form or Species) of that being, which, because it is itself precisely one and *not* many, is *not* numerical in its being. Using today’s terminology we could say that Pythagorean arithmetic distinguished *numbers* from the *concepts* of numbers, although this distinction becomes difficult to think by anyone who assumes that what numbers themselves really are is concepts. We’ll have occasion to return to this last crucial point, but for now need to stress two more important aspects of Pythagorean, and indeed of all, Greek arithmetic. The first aspect is that because they understood by ‘ἀριθμός’ an *amount* of something – that is, precisely how *many* of them there happen to be – ‘two’ is the first number in Greek arithmetic. Related to this is the second important aspect of Pythagorean arithmetic: that one is not considered to be a number but rather to be the ‘root’ (πυθμήν), the ‘source’ or ‘ruling beginning’ (ἀρχή)[6] of number.

[6] NB: The standard translation of *ἀρχή* as ‘first principle’ occludes the distinction, crucial not just for Greek arithmetic but for any science of numbers, made by the Pythagoreans between the Form of numbers and the numbers themselves. ‘One’, as the *ἀρχή* of number, is precisely *not* a concept or principle (first or otherwise) of number but its most basic element; as such, it belongs not to its Form but to its numerical *being*.

The Pythagoreans understood the different Forms or Species of numbers as their 'natural' order and they understood all things, and especially all visible things, to be numbers whose nature is the determinate Form responsible for their unity. Pythagorean 'arithmetic' was therefore not merely a 'mathematical' discipline in our sense of the word but also a science of the visible universe and thus a *cosmology*; the science of the unity and order of our universe. The Pythagoreans expanded their cosmological arithmetic further, to investigate the relations between the Forms of numbers and the numbers themselves, by relating all audible things and audible sounds to *ratios*, *proportions*, and to their forms and properties. Out of this arose what the Ancient Greeks called 'logistic', the science of ratios and proportions, which brings the numbers of things into relation with each other, and which remained the basis of all calculation until the invention of ancient and mediaeval algebra.

The Platonic Attempt to Solve the Problem of the Foundation of Arithmetic

The Pythagorean solution to the foundational problem of arithmetic, namely, to the problem of the unity of a number, is therefore the εἶδος. This solution is one of the sources of Platonic philosophy. Indeed, in one of Plato's dialogues his Socrates speaks of the "astonishing proposition that one is many and many are one" (Plato 1997b: 14c); a proposition he characterises "as a gift of gods to men" (Plato 1997b: 16c). But Plato went much further than the Pythagoreans in dealing with this problem. On the one hand, he took up the question of the nature of things that allows them to be counted, which, as we've seen, the Pythagoreans didn't focus on. On the other hand, he took issue with the supposition guiding the Pythagorean account of the Forms of numbers, that these Forms are capable of explaining the numerical *difference* between different numbers.

Regarding the first question raised by the understanding of number as a definite amount of definite things, Plato investigated what exactly the number itself is by means of which we count stars, cattle, soldiers, virtues, and so on. When we count 'four' stars, 'four' cattle, 'four' soldiers, 'four' virtues, Plato argued, this 'four' is obviously not limited to stars, cattle, soldiers, virtues – which is to say, the 'four' definite things are neither stars, cattle, soldiers, virtues, nor any other determinate things apprehended by any or all of our five senses. Our very ability to count, for Plato, must therefore presuppose that the numbers we use to count refer not to these determinate things with sensible qualities but to things that are only conceivable by our intelligence. That is, we can count any number of any kind of things, in this case, 'four' stars, cattle, soldiers, virtues, because another kind of number, composed of multitudes of things whose qualities are invisible to our senses and therefore *intelligible*, are already available to our intelligence before we begin counting the multitudes of those things that have sensible qualities. The multitudes of things composing intelligible numbers have the following qualities: changelessness, because unlike things with sensible qualities, intelligible things remain forever the same; absolute equality, because each intelligible thing in the multitude conceivable only by our intelligence is nothing but one; and indivisibility, because what is absolutely one cannot be partitioned, as dividing it would make it more than one. In a word, the human capacity to count is only possible if definite amounts of multitudes of 'pure' units, that is, 'pure numbers', are made available to the human soul before it begins to count the number of whatever kind of thing it happens to count. The Pythagorean foundation of arithmetic – the Forms of numbers – must therefore, according to Plato, have as its foundation the multitude of pure units that compose the source of the pure numbers presupposed by counting.

The new perspective on the nature of the things counted by arithmetic provided by Plato's 'purification' of the Pythagorean arithmetic leads, in turn, to a criticism of the Pythagorean answer to the question of how many things can form *one* number. For now the things in question are the 'pure' units, and so the question has to be reformulated: how can many pure units form one number? As we have seen, despite their function to unify the many sensible 'ones' that compose for them each number, the Forms of the Pythagoreans are alien to the numbers themselves. This is the case because as unitary, that is, as *one*, these Forms lack the multitude that is inseparable from the *being* of number. Thus these Forms don't explain the differences between the different numbers united by the same Form. For instance, as we have seen, in Pythagorean arithmetic, both the number 'six' and the number 'ten' are unities of six and ten things respectively, because when these things are represented by dots they have the Form triangle. For Plato, however, neither this Form nor the Form 'Even' can explain the nature of the difference between the form of unity of six pure units and the form of unity of ten pure units. This is because the arithmetical Forms (or arithmetical concepts) of 'unity' and 'multitude' cannot account for the *differences* in the unity of multitudes expressed by the different numbers: both 'six' and 'ten' are the unity of a multitude of pure units, but their *natures* as numerical unities are different, because 'six' is smaller than 'ten', and also because of this it is prior to it in the natural order of numbers. According to Plato, because the concepts of arithmetic cannot account for the *real* differences between numbers, arithmetic cannot sufficiently explain itself. That is, the concepts of arithmetic cannot explain its *foundation* as a science, because these concepts are incapable of explaining its most basic elements: numbers.

On Plato's view, only the concepts of philosophy can account for the scientific foundation of arithmetic; that is, for the true sources of the unity of any number. Thus while Plato thinks, like the Pythagoreans did, that these true sources are to be found in the unity of a multitude provided by εἴδη (Forms), the Forms that for Plato are the sources of numerical unity are not the different classes of numbers (e.g. Odd, Even, prime, square, etc.) but the very Forms of the numbers themselves. That is, for Plato, in addition to the unlimitedly many mathematical numbers there is a limited multitude of Ideal numbers that account for the mathematical being of the different unities of the multitudes composed by mathematical numbers. Therefore Plato's solution to the arithmetic puzzle of how number can be both many and one is to posit Ideal numbers that possess a differentiated one-and-many structure that provides the paradigm for the one-and-many structure of any mathematical number and therefore of each *different* number – and not the converse. In other words, for Plato only the concepts of philosophy are capable of providing the mathematical science of arithmetic with the foundation it needs in order to be scientifically complete.

Before considering in more detail how Plato understood Ideal numbers to provide the solution to the arithmetical problem of providing a foundation for the real difference between numbers, however, a word of caution needs to be sounded. There is a view, as widespread as it is false, that Plato's dialogues present a 'Theory of Ideas (of Forms)', and that this theory entails the thesis that there are two worlds, one of which is an other-worldly, intelligible world and the other the sensible world of physical things. According to this view, the things in the physical world are the pale and imperfect 'imitations' of their ideal exemplars in the intelligible world. Finally, according to this view, Plato's theory is fatally flawed, because it doesn't provide a satisfactory answer to the question of how exactly the physical things in the sensible world are related as 'images' to the Ideas in the intelligible world. This problem is known by a word that is indeed found in Plato's dialogues – 'participation' (μέθεξις) – but, as we shall see, it is understood in a way that fundamentally distorts what one really finds in those dialogues.

This mistaken view of Plato's understanding of Ideas has its basis in a superficial understanding of the criticism of Plato's philosophy advanced by Aristotle, in the presentation of his (Aristotle's) own philosophy of Forms. And while it is true that Plato's dialogues refer to the relation between sensible things and the Ideas of these things as the former's 'participation' in the latter, there is no instance in any work of Plato's where either the Forms are posited as existing *independently* or *separately* from the things in the sensible world, or where these latter, sensible things are characterised as existing in isolation from one another and therefore as being 'singular' or 'particular'. On the contrary, inseparable from Plato's account of Ideas is the problem of accounting for the unity of a *multitude* of things, whether those things are perceived by the senses – for instance, the unity of a swarm of bees – or apprehended only in thought – for instance, the unity of actions that are virtuous. That is, the very problem that we've already seen is at the root of Plato's account of the need to posit Ideal numbers as the foundation for arithmetic's most basic element – number – is also at the root of the participation problem in his account of Ideas. Thus not only is Plato's account of Ideal numbers his solution to the problem of the foundation of arithmetic; it also holds the key to solving the great problem of participation.

In Plato's dialogue the *Phaedo*, Socrates holds that the cause of ten things exceeding eight things is *not* the number 'two' but "multitude" (πλῆθος) (Plato 1967: 101a). He also holds that the adding of a one to a one is not the cause of the one becoming two but that there isn't "any other cause of it becoming two (δύο) than its participation in the dyad (δυάδ)" (Plato 1967: 101c). Moreover, Socrates maintains, "whatever's going to be one (ἓν) must participate in the monad (μονάδος)". In Plato's dialogue the *Greater Hippias* Socrates pursues the distinction made here in the *Phaedo* between 'two' and a multitude of ones, as he proves that it's possible in the cases of number and the Pythagorean Forms of number for something that is common to two things not to belong to either of them. Because for many things this is not the case, the mathematical nature of these exceptions will stand out. Non-exceptional cases include Socrates being just, healthy, wounded, golden, silver, and so on, and Hippias being just, healthy, wounded, golden, silver, and so on; as each would have these qualities in common and also as their individual possessions (Plato 1997a: 300e–301a). In the case of number and the Odd and the Even, however, what Socrates and Hippias have in common, neither possesses individually. Thus with regard to number, what *both* Socrates and Hippias are when considered together, neither is when considered separately. Only both are two, because each is exactly not two but only one. Thus the quality they share in common – *two* – neither alone possesses. Likewise, with respect to the Forms of number, because Socrates and Hippias are both two, they have the quality of Even in common, as they can be divided equally, without the source (ἀρχή) of Oddness – the one – being left over. However, because each is precisely not two but one, Socrates and Hippias considered individually are not Even but 'one' and therefore indivisible.

These exceptional cases of something in common *not* characterising the things that have them in common inevitably raises the question: *where* is this common quality? Is the 'two' something separate from the single things, as it were 'alongside' or 'outside' them? (We must remember that in asking about where the 'two' is we're not asking about where the mathematical symbol '2' is, since in itself this cipher is meaningless.) Plato's dialogue the *Sophist* presents the key to resolving this question, when its two interlocutors – an unnamed stranger from the city of Elea who is a philosopher, and a mathematician named Theaetetus – discover the paradigmatic case of a common quality shared by two things that neither taken singly possesses (Klein 1992).[7]

The investigation of the Philosopher and Mathematician, *both together* – an investigation that for Plato is paradigmatically 'dialectical' – points the way to the resolution of the question of where the common quality that composes number really is. It does so when their attempt to *count* the parts of Being fails because those parts are not analogous to the parts of arithmetical numbers; that is, to the multitude of 'pure' (intelligible) ones that, as we have seen, compose the mathematically 'pure' numbers presupposed by the science of arithmetic. This is made apparent by their discussion of what the Philosopher calls the "Greatest Forms", namely, Rest, Motion, and Being. Being is established as nothing but Rest and Motion, which raises the question whether the number of these Forms is two or three. Giving an account of the answer to one of the most fundamental questions of philosophy – 'what is Being?' – therefore turns out to enlist the service of numbers, the most basic elements of arithmetic.

[7] German edition: Klein 1936; 1934. See also Hopkins 2011: Ch. 19.

However, mathematical numbers don't prove to be up to the task of being able to enumerate Being and its parts, because when Rest is counted as one, Motion as another one, and then Being as a third one, their number adds up to three. But just this is completely impossible; namely, that Being count as another Form 'outside' of Rest and Motion. This is the case because whatever *is* has to be either at rest or in motion, and thus has to have the qualities of Rest and Motion, which are not three things but precisely 'two' – albeit they are 'two' in a manner unlike the manner two things in a mathematical number are two. As we have seen above, the numbers that are the foundation of arithmetic have as their parts identical ones. The parts of the Form of Being, Rest and Motion are not only not identical but are also completely opposite – even though they are still unities; because all resting and moving things have and therefore are identified, respectively, by their qualities. Nevertheless – and this is Plato's crucial discovery – just as the Form Being is not some third thing 'outside' of the Forms Rest and Motion, but precisely those Forms together, so, too, for instance, is the number two not some third thing 'outside' of the units it unifies as 'two', but exactly both units together. Plato's technical word for the way a mathematical number or the Form of Being unifies respectively the units or Forms that are their parts is "community" (κοινωνία).

The *community* structure of Being and mathematical number, which is the same insofar as Rest is not Being just as Socrates is not 'two', and Motion is not Being just as Hippias is not 'two', provides the basis of Plato's teaching that the Forms are Ideal numbers; a teaching whose details we know about mainly through Aristotle's criticism of it. And the difference between the parts of Ideal numbers and the parts of mathematical numbers provides the basis for Plato's teaching that the foundational problem of arithmetic has as its solution mathematical numbers' participation in Ideal numbers. The real difference between the different unities of the multitudes of the units that form each number is therefore accounted for by Plato on the basis of the structural community of the Forms with their parts, beginning with the community of Being with Motion and Rest. Because these parts – unlike the parts of mathematical numbers – are different from each other and indeed radically so since they are complete opposites, they are "incomparable (ἀσύμβλητοι)" (Aristotle 1941: M, 1080a 1019) and therefore unique. Thus the community of Being, with its unique parts, forms the Ideal number TWO, the *dyad*; which owing to the uniqueness of its parts provides the paradigm and thus foundation for the unity of the mathematical number 'two'. This is to say, any one among the unlimited mathematical twos that there happen to be possesses its specific unity as exactly 'two' – as opposed to 'three' or 'four' or any other number – on the basis of its relation to the paradigmatic Ideal TWO of Being.

Aristotle's Critique of the Platonic Solution to the Foundation of Arithmetic, and His Own Solution

According to Aristotle's report, Plato taught that there were nine Ideal numbers, with the *dyad* being the first Ideal number and the *decad* the last, since, as mentioned, one is not a number in the arithmetic of the Ancient Greeks. And, as also mentioned, Aristotle's report is embedded in his criticism of Plato's philosophical solution to the problem of the foundation of arithmetic. Aristotle's criticism has three foci, all of which have exerted tremendous influence in the history of human thought, including the thought behind Husserl's and Patočka's desedimentation of the Galilean mathematisation of nature.

The first focus concerns Plato's account of 'participation'. Aristotle does not, as is commonly but mistakenly thought, reject outright Plato's view that things participate in the Forms; but rather he rejects Plato's claim that these forms are "separate (χωρισμός)" from these things. Therefore, for Aristotle there is no "one-over-many (ἓν ἐπὶ πολλῶν)" (Aristotle 1941: 991a–992) unity of a Form, which means in the case of the Form of the *dyad* that the 'dual' is *common* to both the intelligible 'two' and the things that share in it. Aristotle accuses Plato's formulation of the relationship involved in participation as duplicating the world, because by employing the metaphorical language of 'image' and 'imitation' to characterise this relation, Plato introduces a duality – in the case at hand, the 'dual' of things that are 'two' and the 'dual' of the *dyad*, that is, the 'two itself' – where only the being-dual is *common* to both the *dyad* and any two things.

The second focus of Aristotle's criticism of Plato is related to this first, as Aristotle denies that there is any unity in a number of things. The word we pronounce when we've finished counting signifies *many* things and therefore isn't itself one at all. The 'community' of the multitude of the units counted doesn't mean that their number is itself a unity. The only unity connected with number is that of the unit that is repeated in the process of counting; i.e. one apple and one apple, which, in the case of two apples or six apples, is 'apple'.

Finally, the third focus of Aristotle's criticism of Plato is related to these first two. Not only is there no 'one-over-many' unity of the Forms in relation to the things that share in them or of the number in relation to the units that compose it, for Aristotle; but also the 'purity' of the intelligible units that Aristotle agrees are indeed the foundation of arithmetic does not consist in their being *separate* from the sensible things with which arithmetic also deals. Rather, the Platonic view of the separate existence of the pure units presupposed by the availability of numbers to the soul before it begins counting is the result of the soul being seduced by this advance availability into thinking that what follows from it is that these units exist independently of the counted things. The truth for Aristotle is rather that the applicability of these intelligible units to all sensible beings is the result of "abstraction (ἀφαίρεσις)". By "abstraction", however, Aristotle does not understand what it has come to be understood as, namely, a *psychological* account of the soul's supposed capacity to 'lift off' universal ideas from particular things or their images. On the contrary, Aristotle's account of abstraction (which is limited to mathematics) presents it, in the case of arithmetic, as a *logical* process of disregarding the properties of sensible things until all that is left *for thought* is their *arithmetical character* of 'being-one'.

Whatever the problems there are with Plato's account of the philosophical foundation of the mathematical science of arithmetic having its basis in Ideal numbers, it is apparent that Aristotle failed to see the problem that Plato was trying to solve: namely, that of the real difference between the different numbers. Just as the Pythagorean appeal to the Forms common to numbers is unable to account for the difference between different numbers unified by the same Form, so, too, Aristotle's claim that the only unity associated with number is that of the unit used in the counting that generates it is unable to account for the different numbers that would have that same unity. That is, just as the Pythagorean explanation of the unity of the numbers six and ten on the basis of their sharing the common figure triangle doesn't address the specific difference of the unity 'six' and the unity 'ten', so, too, Aristotle's explanation of the unity of two apples and six apples on the basis of the common unit 'apple' doesn't address the specific difference of these two numbers.

Philosophical Problems in the Foundation of Modern Arithmetic

Now, shifting the discussion to the problem of the foundation of arithmetic in modern arithmetic, what has to be established from the outset is that the modern understanding of the basic element of arithmetic – number – is inseparable from the historical origin of François Viète (Latin: Vieta) of Fonenay's invention of the "Analytic Art (*Artem Analyticen*)"[8] for Princess Mélusine (Catherine of Parthenay, 1554–1631) in 1591. To this day this 'art' functions as the *sine qua non* for the formalisation that makes modern mathematics possible and therefore composes its *foundation* (see Klein 1992; Hopkins 2011).

Vieta presented his analytical art as "the new algebra" and took its name from the ancient mathematical method of "analysis", which he understood to have been first discovered by Plato and so named by Theon of Smyrna. Ancient analysis is the 'general' half of a method of discovering the unknown in geometry; the other half, "synthesis", being 'particular' in character. The method was defined by Theon like this: analysis is the "'taking of the thing sought as granted and proceeding by means of what follows to a truth that is uncontested'". Synthesis, in turn, is "'taking the

[8] Francisci Vietae, *In Artem Analyticem* (sic) *Isagoge*, Seorsim excussa ab opere restituate Mathematicae Analyseo, seu, Algebra Nova (*Introduction to the Analytical Art*, excerpted as a separate piece from the *opus* of the restored Mathematical Analysis, or *The New Algebra* [Tours, 1591]). English translation: Vietae 1992.

thing that is granted and proceeding by means of what follows to the conclusion and comprehension of the thing sought'" (Vietae 1992: 320). The transition from analysis to synthesis was called "conversion", and depending on whether the discovery of the truth of a geometrical theorem or the solution ("construction") to a geometrical problem was being demonstrated (ἀπόδειξις), the analysis was called respectively "theoretical" or "problematical".

Vieta's innovation involved understanding a novel form of arithmetical analysis found in the recently rediscovered third-century text (titled simply *Arithmetic*) of Diophantus of Alexandria as a procedure that is completely parallel to geometrical analysis. This permitted Vieta to treat the sought-after and therefore unknown numbers – understood as unities of multitudes of units – as already granted in their *species*. By the *species* of numbers he followed Diophantus's designations in his *Arithmetic*, e.g. square, cube, square-times-cube and cube-times-cube. To the species of each of these unknown and therefore indeterminate quantities as well as to the species of every known quantity he assigned what he called an "everlasting and very clear symbol" taken from the alphabet (vowels to the known and consonants to the unknown). This allowed both the *possibility* of there being given a determinate amount of units (that is, a number in the pre-modern and therefore non-formalised sense) to be apprehended in a manner that functioned as if it were *actually* given and it also allowed known numbers to be expressed by their species. With this, the arithmetical need for an analogue to the second part of the geometrical method of analysis, the theoretical or problematic conversion of the synthesis that proved a *particular* theorem or solved a *particular* problem, was dispensed with by Vieta, which made possible for the first time the "analytic" – that is, indeterminate and therefore 'general' – solution to arithmetical problems. Three significant results follow from Vieta's innovation. Firstly, the geometrical distinction between the kind of object presented in a theorem and in a problem falls away, such that in the analytic art theorems are equated with problems and with this the synthetic distinction between the 'theoretical' and 'problematical' dissolves. Secondly, the exclusive calculation with the *species* of known and unknown numbers made possible by Vieta's analytic art – what he terms "*logistice speciosa*" – is employed by him in the service of "pure" algebra, and therefore applied indifferently to finding unknown numbers and to finding unknown geometrical magnitudes (which are measured by numbers). And, thirdly, because the *logistice speciosa* has but a small interest in the determinate results of the solutions to its calculations – what Vieta terms the "*logistice numerosa*" – the artful procedure of Vieta's analytic method is conceived as a general auxiliary method whose purpose is not to solve problems singly but to solve the problem of the general ability to solve problems. Characterised by Vieta as "the art of finding, or the finding of finding", the general analytic is an *instrument* in the realm of mathematics analogous to the sense in which Aristotle's *Prior and Posterior Analytics* are presented as an *organon* in the realm of all possible knowledge. In this regard, Vieta's conclusion to his *Analytic Art* is telling: "[T]he analytic art...appropriates to itself by right the proud problem of problems, which is: TO LEAVE NO PROBLEM UNSOLVED" (Vietae 1992: 353, capital letters in original).

Vieta's method is recognised by historians of mathematics to be coincident with the invention of the mathematical formula and the first modern axiom system, whereby the syntactical rules of mathematical analysis 'define' the object to which they apply. But it is also coincident with something about which historians of mathematics and philosophers alike remain to this day ignorant: the transformation of both the mode of being of the foundational concept of arithmetic – number – and with this, the transformation of the mode of being of the objects of mathematics in general, together with the transformation of the process of abstraction that generates the formal concepts operative in the system of knowledge in general.

Vieta's innovation contains three interrelated and interdependent aspects. Firstly, there is its *methodical* innovation of making calculation possible with both known and unknown indeterminate (and therefore 'general') numbers. Secondly, there is its *cognitive* innovation of resolving mathematical problems in this general mode, such that its indeterminate solution allows arbitrarily many determinate solutions based on numbers assumed at will. Thirdly, there is its *analytic* innovation of being applicable indifferently to the numbers of traditional arithmetic and the magnitudes of traditional geometry.

The philosophical significance of this first innovation is the *formalisation* of number and thus of its concept; such that number no longer signifies, as we have seen that it did in Greek arithmetic and in mathematics generally prior to Vieta's innovation, a "multitude composed of units" (Euclid, Book VII, def. 2); but rather number now signifies the *concept* of such a multitude in the case of known numbers and the *concept* of a multitude as such (or in general) in the case of unknown numbers. The formalisation of number and of its concept can be grasped neither by Aristotelian abstraction nor by Platonic dialectic.[9] This is because *formalised* number is neither the product of the abstraction that yields the unit that functions to measure a multitude of items, as it is for Aristotle; nor the Ideal unity of such a multitude that is grasped by dialectic as being irreducible to the items it unifies once the sensible suppositions of the mathematicians are left behind, as it is for Plato. Rather, number for Vieta is the result of the conceptual process of ascending from the mind's unmediated and therefore direct relation to multitudes of items to its relation to its own apprehension of this unmediated and direct relation; while simultaneously identifying these two modes of relation. This simultaneous identification of heterogeneous 'relations', namely of (1) the real relation to a multitude of concrete things and (2) the cognitive relation to the concept of this multitude, is exhibited by the meaning assigned by Vieta both to ordinary number signs and to his algebraic letters. And it was exhibited and therefore manifest for him as it is for us every time a sense-perceptible letter is intuited *as* – and not simply as *signifying*

[9] Thus the attempt, for instance Patočka's, to capture the difference between the ancient and modern concepts of number in terms of "the much more abstract character" (Patočka in press) of the modern concept falls short of the mark of the difference in question; which, as we have seen, cannot be measured in terms of degrees of abstraction but only captured in terms of the transformation of the basic unit of arithmetic from a determinate multitude to the *concept* of such a multitude.

– the general concept in question; whether that concept be of this or that number – for instance, *the concept of any 'two' in general* or the *concept of any 'number' in general*. What is manifest in this intuition of *at once* a sensible mark and a general concept is precisely Vieta's invention of the mathematical *symbol*.

The foundational problem that follows from the analytic innovation of Vieta's method concerns the derivation of the syntactical rules that govern the axiom system and establish the systematic context that defines the indeterminate objects to which they apply. Vieta established these rules on the basis of the "*logistice numerosa*" and thus in calculations with determinate amounts of monads, which is to say, in calculations with the 'natural' and therefore non-symbolic numbers dealt with by Ancient Greek arithmetic. This is what allows letter signs with no numerical properties to nevertheless have a numerical significance in the *logistice speciosa* and in the new algebra for which it is the foundation. Vieta, however, conceptualises these multitudes composed of units at the *same time* from the perspective of their symbolic presentation. One significant result of this is that both number and its general concept attain an equivocal status in mathematics and the philosophy of mathematics, oscillating between its indeterminate and therefore general symbolic significance as 'number in general' and its pre-formalised *natural* significance as a multitude composed of units. This equivocity is perhaps nowhere more evident than in the schematism in Kant's critical philosophy, where 'number' provides the first illustration of a schema understood as "a general procedure of the imagination for providing a concept with its image". Thus, for Kant, the empirical image of number, for instance, points in a row – five in the case of the number five (. . .) – is distinguished from its schema in the thinking of "a number in general, which could be five or a hundred". One cannot find a better articulation of the equivocity of number in question here than in Kant's claim that the latter "thinking [of number in general] is more the representation of a method for representing a multitude (e.g. a thousand) in an image in accordance with a certain concept than the image itself" (Kant 2000: 180).[10] The irony of Kant's appeal to an instance of intensive magnitude – which is determined by the sliding scale of "more and less" – to characterise the transcendental mode of being of the paradigm of exact quantity cannot be formalised, let alone quantified, but it is nevertheless very real and runs deep (see Hopkins 2013).

It is precisely this temporally dated and therefore historically conditioned equivocity of number, and therefore the historically conditioned equivocity of the *apriority* inseparable from this most basic concept of arithmetic and indeed mathematics generally, that Husserl sought to resolve in his first major work, *The Philosophy of Arithmetic* (Husserl 2003). He did so by attempting to account for the origin of the logical objectivity of the formalised general concept of number in an idealising extension of the psychological activity that generates non-formalised numbers (i.e. the determinate amounts of units that defined number in pre-modern mathematics).

[10] German edition: Kant 2001: A140/B179.

As is well known, Husserl's attempt to ground the logical objectivity of the concept of number employed in the symbolic calculus of universal analysis in the psychological phenomenon of "collective combination" ended in self-acknowledged failure. However, less well known is the fact that Husserl never managed to resolve either of the mathematical problems that *The Philosophy of Arithmetic* failed to resolve psychologistically: namely, that of the foundational nature of the unity of both determinate and general numbers; and the foundational nature of their mathematical relationship (Hopkins 2006). The common view, that the pure logic developed in Husserl's *Logical Investigations*, and more specifically, the descriptive phenomenological foundation of this logic in categorial intuition, is capable of providing logical foundations for either of these mathematical relations, is wrong. In the case of the foundation of the 'collective unity' of pre-formalised numbers, the categorial distinction between logical 'species' and their 'instances' merely substitutes one philosophically unsuitable solution – psychologism – for another one – logicism. This is the case because so long as the 'collective unity' of a multitude is held to have its foundation in the *concept* of the 'species' of that multitude, the problem of the unity of a non-conceptual manifold is in no way resolved but simply shifted to a higher level; i.e. to the problem of providing the foundation for the conceptual unity of the manifold presupposed in the extension of the concept of 'species' (or the logically equivalent concept of class) supposedly providing the foundation of the unity of the original manifold. Thus with respect to the problem at hand: to say, as Patočka says in following Husserl's *Logical Investigations* and in thinking thus that he is resolving it, that "the number five is not my counting to five, nor someone else's counting, nor is it my or some else's conceptualization of five; it is a *species*, a generality, an *ideality* which is realized or given in the individual *instance* of a class of five members" (Patočka 1996: 35) doesn't resolve the issue. And, again, it doesn't do so because the foundation of the unity of the manifold composed by the "class of five members" is in no way provided but simply presupposed as unproblematically given in this account (Hopkins 2011: Ch. 32).

A second significant result of the equivocity of numbers and their concepts in Vieta's foundational innovation of the analytic method occurs when Gottlieb Frege attempts to solve the problem that the equivocity of numbers presents to the foundational problem of arithmetic, by completely doing away with non-formalised numbers in the logical reformulation of arithmetic. With this, number and the concept of number become identical, as number itself is now defined as an assertion about a concept; or more precisely, it is defined in terms of the structure of certain conceptual relations – an thus in terms of a 'syntactical' definition. The *real* problem that Frege's numbercide gives rise to, however, is how does the one-to-one correspondence between the elements of two sets that for him is foundation of the definition of number, what he calls "equinumerosity" (Frege 1996: §§71–72), account for the *real* difference between numbers?

The 'numerical' property that defines number as a predicate of a concept – for instance, 'nine' as the number of the concept of planets in our solar system – is understood as the property of being *instantiated* 'nine' times. Because not only the concept of planets has this property, but also the concepts of inning, holes on a par three golf course, and so on, the number nine is defined as the *set* of all concepts with the 'equinumerous' property of being instantiated 'nine' times. But to the question of what it is in the different one-to-one correspondences of the elements in the sets that compose the difference between the different numbers – i.e. what it is in the conceptual *quality* of being equinumerous that determines the difference between the numerical properties of six and ten as the *quantitative* properties of having just, for instance, six or ten items that 'fall under' the concepts in question – the conceptual definition of number can provide no answer. This is because the one-to-one mapping that defines equinumerosity *presupposes* rather than establishes the properties of, in the case at hand, being instantiated just *six* or just *ten* times.

The de-sedimentation of the meaning sedimented in the modern concept of number thus establishes that the problem Plato saw ancient arithmetic was unable to solve on its own terms – namely, that of how to account for arithmetic's foundational supposition that the unities of different numbers *really* are different – remains unresolved in the modern attempt to logicise number. Therefore the most basic concept of the mathematics that made modern physics *possible* and in so doing brought about the *actuality* of the end of the ancient cosmos – and thus of that world in which the human being was recognised as having the power to function as an integral part – remains to this day without a foundation. This de-sedimentation has also identified the philosophically fundamental technical error behind the failure of the conceptuality of the modern logic born from the thigh of Vieta's innovation in mathematics to be able to account for the 'unity' of the most basic concept of the mathematics that is responsible for the loss of that world: the error, namely, of the unwitting substitution of the *concept* of a multitude for its (the multitude's) determinate *being* as the basic 'unit' of arithmetic. As we have shown, the technical nature of the error involved here is strictly speaking not mathematical but philosophical, since with the identification of the 'concept' of a very specific 'being' – the determinate being of a multitude – with that being itself, the philosophical need to provide an account of the foundation of the being in question (determinate multitude) is *seemingly* obviated. But as we have also shown, the philosophically foundational problem of the 'unity' of a determinate multitude does not go away just because the mathematics at issue is incapable of recognising this as a problem. That the solution to this problem – the problem of the true nature of the 'unity' that binds at first two and then a multitude of things into a unity – is not something that was known prior to the advent of Galilean science but rather is something that then as now awaits a solution should be sufficient to accomplish the following: to cure us of any nostalgia for the 'unity' of the ancient cosmos, the 'demise' of which Galileo unwittingly prepared the self-styled 'true' modern science of nature to bring about. And the *knowledge* that the 'unity' in question remains unknown should be sufficient to conquer apodictically our wills with the sole rational norm capable of pointing in the direction of *the* solution to the crisis of the European sciences: that is, to the norm born of the unshakable phenomenological recognition that the nature, let alone the source, of the 'unity' presupposed by all human pretension to know is – at least so far in human history – something that remains fundamentally *beyond* what that pretension is capable of realising.

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