Chapter 11
Taking Design to Task: A Critical Appreciation

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11.1 The Evolution of Task

While task design has long been a central concern of mathematics education, it is only recently that an organized community has emerged in which task design has been linked with design research. Together, the editorial introduction to this book (Chap. 1) and Chap. 2 provide a useful historical sketch of the ground-laying for such activity, the emergence of several energetic groups, and the development of this international community, leading to the preparation of this book.

Using *Educational Studies in Mathematics* as a convenient historical section provides a simple means of tracing the penetration of talk about *tasks* into the mainstream of mathematics education research. One finds that *task*—used in the sense of a stipulation for some unit of mathematical activity—has been present since the inception of the field in the late 1960s (c.f. the use of *discovery task* in Scandura, Barksdale, Durnin, & McGee, 1969). More specialized terms followed, such as *task sequence* in the mid-1970s (c.f. Scandura, 1975) and *task-based interview* in the mid-1980s (c.f. Presmeg, 1986), but the term *task design* itself did not surface until the late 1990s (c.f. the title of one of the references in Noss, Healy, & Hoyles, 1997).[1] By comparison—as the traces revealed by this trail suggest—*task design* was already established in the psychological literature in the 1960s in relation to the design of diagnostic and other assessment tasks (c.f. its use in connection with multiple choice test items in Tversky, 1964).

[1] The next use of "task design" in *Educational Studies in Mathematics* occurred in 2001, and over 20 articles employed the term during the subsequent decade. By comparison, the earliest use of "task design" in the *Journal for Research in Mathematics Education* occurred in 1983, with only one further use before 2000.

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The emergence of *task design* within the field of mathematics education has seen a reshaping of the originally psychological framing to accommodate a long-standing tradition of mathematical popularization. This tradition seeks to find relatively self-contained and accessible "activities" which are exemplary of some topic within mathematics and/or form of mathematical activity. For example, introducing an early paper on "geometrical activities for the upper elementary school", Engel wrote:

> In this paper I shall present a selection of activities which I have used in grades 5 to 7 for the past 16 years… At this level I prefer topics which are not treated later, but which are still interesting, important and challenging… These examples will show that, even at an early age, one can reach rather deep results in a short time and starting from scratch. (Engel, 1971, p. 353)

Although it makes passing reference to pedagogical considerations, Engel's paper analyses the tasks which it presents in primarily mathematical terms, doubtless reflecting the way in which the author's craft knowledge (developed through his considerable practical experience of working with these tasks) was framed (c.f. Ruthven, 2011, p. 92).

Equally, though, a contemporary paper by Egsgard on "some ideas in geometry that can be taught from K-6" illustrates a more explicitly theorized and psychologically influenced approach to pedagogical design. This approach appeals, in particular, to Piaget's model of developmental stages and Dienes' typology of intrinsic motivation to guide the pedagogical model of *directed discovery* which frames the discussion of the student "activities" or "assignments" presented in the paper:

> In using [the directed discovery] technique opportunities and activities are provided so that a child can make his own discoveries. Careful planning of the sequence and pace of activities is essential to ensure that the child understands and learns the concepts. As children are led to discover a concept, discussion with the teacher, the class or another individual must be allowed. The primary role of the teacher is to question, to guide, to stimulate and to assess the progress made so that time is used wisely. (Egsgard, 1970, pp. 481–482)

Egsgard's paper also recognizes a need to locate tasks within a larger curricular framework: it concludes with a diagram which suggests how, over the years of elementary education, and in preparation for secondary education, such activities could underpin a systematic development of key concepts along interconnected lines of development of geometrical thinking.

This example illustrates how, from the very inception of the research field of mathematics education, task design has often formed part of a larger enterprise of *curriculum development*, a term which has figured consistently in *Educational Studies in Mathematics* since its start. Exploring potential contrasts in the connotations of task design and curriculum development calls attention to a spectrum of scale in which the micro-level of task can be differentiated from the macro-level of curriculum, with perhaps an intervening meso-level corresponding to the unit or module. Although design can refer to either process or product, development more typically refers to process: nevertheless, as an iterative conception of design research has become influential, "design" has become more strongly associated with this cyclic process of "development", as the marriage of *design as intention* and *design as implementation* discussed in Chap. 2 makes clear.

Over the last half-century, the field of mathematics education has undoubtedly become more ambitious in its aspirations to coordinate the design of teaching with the formulation of theory through a development process more closely guided by research-based techniques. The clinical interview and its evolution into the teaching experiment have been particularly influential as paradigms of method for design research in mathematics education. However, herein lies one plausible reason why attention has shifted over this period from the design of larger-scale curriculum towards smaller-scale task: within the amounts of time and levels of resourcing normally available to designers, the complexity and cost of a research-based approach to the development process renders it feasible only on a limited scale. Certainly, my own experience of conducting design research at the meso-level within such constraints is that this called for very careful focusing of the research with an eye to the core feature of the design and the associated line of theory development, leaving more peripheral aspects of the design—but ones potentially crucial to its success—to be handled more informally (Ruthven & Hofmann, 2013).

11.2 A Scheme for Design

The core of Chap. 2 surveys a range of intermediate frameworks that have been employed in task design. Each intermediate framework represents some working synthesis of grand theory and/or craft knowledge for the specific purpose of designing instructional tasks and sequences. In particular, the chapter presents each framework in a manner which articulates its central principles for design and relates these to an illustrative case of design. As the chapter notes, only some aspects of a design can be attributed to the guiding framework; art is involved as well as science. Equally, the types of learning goal on which such intermediate frameworks focus and the educational values which they express are not uniform.

Nevertheless, Chap. 2 does identify some common underlying assumptions across the intermediate frameworks presented: notably, that learning mathematics takes place through doing mathematics; that the mathematical task posed must take account of students' current understandings; and that learning depends on development of representations and models. It may be valuable, then, to push a little further by developing an organizing scheme for the analysis of intermediate frameworks and their design principles. As the chapter makes clear, these frameworks generally relate not just to the task itself, but to its staging (in the sense of *mise-en-scène*) in the classroom. Any framework for task design, then, comprises one or more of the following elements.

11.2.1 A Template for Phasing Task Activity

For example, within Japanese Lesson Study, the template for lesson activity on a problem-solving task consists of phases of teacher introduction (*donyu*), student investigation (*jiriki-kaiketsu*), class comparison (*neriage*), and teacher summing-up (*matome*). Likewise, within the Theory of Didactical Situations, the template for staging an adidactical situation calls for an opening teacher-led "didactical" phase of *devolution*, followed by "adidactical" phases of *action*, *formulation*, and *validation*, concluding with a further teacher-led "didactical" phase of *institutionalization*. Similarly, within Formative Assessment for Developing Problem-Solving Strategies, an initial phase of intuitive student work on a problem is followed by teacher analysis of strategies before the next lesson; that lesson then starts with a phase of student evaluation of teacher-provided samples of student work which have been chosen and annotated so as to provoke reflection on earlier strategies, leading to a phase of student modification or refinement of their own earlier strategy, and concluding with a phase of whole-class comparison of the revised strategies and reflection on them.

11.2.2 Criteria for Devising a Productive Task

As already noted, the frameworks surveyed generally stipulate that a task should pose some kind of problem to be solved; a problem which admits a range of solutions; solutions typically differing in their level of mathematical sophistication, including a level which ensures that students will be in a position to propose some kind of initial solution. Beyond this, there are some differences in the types of criteria specified within different frameworks. One type of criterion concerns the realism and potency of the task to students; for example, its origin in some out-of-school practice, which is "crucial and alive" for students, and which can be transposed into the educational system as exemplary of target mathematical concepts (Anthropological Theory of the Didactic); or its relation to a "realistic" problem situation that affords students opportunities to attach meaning to the mathematical constructs it serves to develop (Realistic Mathematics Education). Another type of criterion concerns the potential of the task to foster productive conceptual reorganization, for example, by eliciting misconceptions from students (Conceptual Change Theory); by affording students a starting approach which turns out to be unsatisfactory (Theory of Didactical Situations); or by triggering responses which highlight common mistakes (Formative Assessment for Developing Problem-Solving Strategies). By contrast, Conceptual Learning through Reflective Abstraction represents an approach to concept development which seeks to afford students the opportunity to build an abstraction from already available activity, rather than through triggering cognitive conflict and reconstruction.

11.2.3 Organization of the Task Environment

One aspect of such organization concerns the instrumentation which mediates a task. For example, in the Proof Problems with Diagrams framework, the diagram provided with a proof problem plays a crucial role in scaffolding students' search for counterexamples or non-examples, and in supporting their deductive guessing. The choice of particular representational tools plays a similar mediating function within Realistic Mathematics Education, supporting the raising of students' level of conceptualization through emergent modelling. Likewise, within Conceptual Change Theory, external representations and bridging analogies fulfil this mediating function. Another important aspect of the organization of task environment is the form of social interaction. For example, within many of the intermediate frameworks, dialogic group or class discussion is intended to support reflection on task strategies and reformulation of them. Such discussion, in turn, calls for the creation of particular interactional norms, as noted in the account of the Cognitive Apprenticeship for Productive Problem-Solving framework. Within the Theory of Didactical Situations, the organization of the task environment is conceived in terms of "the creation of a (material and social) milieu that provides students with feedback conducive to the evolution of their strategies". A more unusual variant—on the boundary between instrumentation and interaction—is the provision of annotated work samples (Formative Assessment for Developing Problem-Solving Strategies).

11.2.4 Management of Crucial Task Variables

This is a prominent aspect of the Variation Theory framework in which analysis of the variation space associated with a task or task type leads to the identification of crucial (structural rather than superficial) dimensions, and to the development of a task sequence intended to be optimally efficient in creating an enacted variation space. A similar process of analysis of variation to create a well-tempered sequence of tasks is apparent in the Conceptual Learning through Reflective Abstraction framework where:

> The task sequence starts with word problems and context-free tasks to elicit and reinforce the diagram drawing strategy. Once the student is using the intended strategy, the task sequence provokes the anticipated abstraction. For this purpose, larger numbers for the denominators and invited mental runs of diagram drawings were used.

Likewise, design within the Theory of Didactical Situations framework depends on the identification and judicious tuning of key didactical variables which influence the particular character of a task, the approaches available to it, and the pathways through which these unfold.

Thus, the "design" that emerges from a development process of this type is much more than the "task" alone—in the sense of the manifest form of the task presented to students. This design encompasses, first, any template for phasing task activity and any organization of the task environment; then the rationale for judging the task productive, for phasing activity on the task, for organizing the task environment, and for managing task variables. As the examples presented in Chap. 2 show, few if any intermediate frameworks explicitly address all these aspects of task design. However, these aspects must figure—even if only implicitly—in the design process. Thus, although intermediate frameworks and design principles serve to articulate the *science* behind a design, they are often silent on the *art* or *craft* that also contributed to it, and which may indeed have shaped crucial assumptions about the manner in which it should be staged.

11.3 Design Continues in Use

Hence, because a design is much more than the overt task, and because a design presumes more than the intermediate framework and design principles make explicit, the dissemination of task designs is far from straightforward. Recent research on patterns of use in mathematics education of textbooks (Remillard, 2005) and dynamic software (Ruthven, Hennessy, & Deaney, 2008) has shown the degree of *interpretative flexibility* that such tools afford, resulting in their being employed by teachers and students in ways very different from those envisaged by their developers. In effect, the process of design continues in use, as users appropriate the tool to address their particular purposes and adapt it to their specific context.

Accordingly, Chap. 3 focuses on the role of the teacher in (re)designing and implementing tasks. Some contributions note that—subject to teachers appreciating the rationale of its design—their adaptation of a task may enhance the result of its implementation. Exercising such a role thoughtfully calls, of course, for the teacher to (re)address—with a specific context in mind—issues that exercised the original developer. Thus, the five dilemmas and six criteria presented in Chap. 3 can be related to the scheme of design elements that has already been introduced as a means of organizing the concerns of the various intermediate frameworks and illustrative designs presented in Chap. 2. The dilemmas of context, language, structure and distribution all relate to devising a productive task, as do the criteria of epistemic, cognitive, affective and ecological suitability. Likewise, the dilemma of interaction and the criteria of interactional and mediational suitability relate primarily to organizing the task environment. The agendas provided by these dilemmas and criteria usefully enlarge on those components of the organizing scheme of design elements.

Equally, the examples presented in Chap. 3 emphasize how the interpretation and redesign of tasks by teachers are shaped by their pedagogical orientations and so by the value and instrumental rationalities informing these. At the same time, the aspiration of many researchers, developers and teacher educators involved in task design appears to be to effect particular changes in such orientations and rationalities: they intend tasks—and whatever support materials accompany them—to be "educative" for teachers (Davis & Krajcik, 2005). However, as any reader of this book will quickly appreciate, it can be hard to keep track of the differing rationales and expectations associated with a multiplicity of tasks of differing provenance, let alone to integrate them. Indeed, when every design team appears to proceed from its own distinctive position, and to operate closer to the micro-level of task rather than to the macro-level of curriculum, it is the teacher who is left to integrate the results. This challenge is all the greater if we accept the argument that teachers should display "relentless consistency" of approach in order to establish ways of working mathematically with students. This argument raises questions about achieving a sound balance and a clear relationship between—on the one hand—the often highly specified design of tasks and—on the other hand—more generic ways of working. This is one of the central challenges confronting a "re-sourcing" movement in mathematics education which advocates that schools and teachers devise their own local schemes of work through assembling, adapting and structuring materials from a variety of sources (Ruthven, 2015).

Of course, one answer to such questions is to restrict oneself—as designer and/or teacher—to a particular intermediate framework which provides—for example—an explicit and consistent phasing of task activity and organization of task environment into which students can be inducted. For example, my own experience of conducting "redesign" research to develop curricular modules—capable of implementation at scale within known systemic constraints—points to the importance of a substantial introductory module in which the task sequence specifically aims to induct teachers and students into generic ways of working (and the rationale behind them) that are then reinforced systematically in subsequent topic modules: in this case, norms and practices of "dialogic teaching" (Ruthven & Hofmann, 2013). Likewise, achieving this kind of balance appears to be a characteristic of Japanese Lesson Study, which—as Chap. 9 reports—keeps one eye on developing tasks and lessons "that bring broad educational values to life in the classroom" while the other eye attends to the mathematical topic of immediate concern.

For the teacher, then, as for the designer, one approach is to filter any task through the lens of the generic ways of working mathematically that one seeks to develop. This appears to be what is taking place in the example—described in Chap. 3—in which teachers work together to "turn a lesson upside down" in order to ensure that it will prompt certain ways of working: in this case, concerned with generalizing and justifying. There is clearly an argument, then, for placing greater emphasis on developing systems of generic heuristics to guide the staging of tasks. For example, Chap. 3 refers to a repertoire of strategies developed to guide teacher interventions while students are working on a mathematical task: these address issues such as whether or not to intervene; how to initiate an intervention; whether to withdraw or proceed with the intervention; and how to intervene to support students experiencing difficulty.

11.4 Scope for User Agency

Chapter 3 concludes that the role of the teacher in adapting tasks to context and in managing their unfolding in the classroom is unavoidable. Translating any task—however tightly specified—into classroom activity calls for a degree of interpretation and elaboration on the part of the teacher and this already grants them some scope for agency. Moreover, the teacher will seek to integrate any task into some larger system of classroom practice, and may judge it necessary to adapt the task in doing so. And finally, as Chap. 4 notes, lines of thought and action that emerge during classroom activity may encourage the teacher to further modify or extend tasks or to create new ones. There can be little doubt, then, about the scope for agency that resides with the teacher.

But the processes through which users interpret and redefine tasks are not confined to teachers. Just as between the original designer and the teacher user, many of the assumptions and expectations underpinning the design of a task—and its implementation—remain implicit, so too between the teacher proposer of a task and the student actor in response. This leads Chap. 4 to argue for the importance of accounting for student perspectives in task design, in order to understand how to reduce the gap between the intentions of teachers and the activity of students. In particular, the chapter clearly illustrates how the mathematical socialization of designers and teachers may render alternative student interpretations of a task unimaginable and incomprehensible to them: this "expert blind-spot" (Nathan, Koedinger, & Alibai, 2001) all too easily leads to a "bifurcated situation".

In searching for ways of avoiding or retrieving such situations of mismatch between teacher and student interpretations of a task, Chap. 4 examines a number of options. First, it argues that one apparent option—for the teacher to state more explicitly their expectation of the type of approach or response to the task—may be counterproductive where the aim is for students to develop creative, flexible and independent mathematical thinking. That is indeed plausible, but one might also ask whether many of the tasks presented in this book truly have—or actually realize—such an aim; often it appears that the designer already has a particular path of "creativity", "flexibility" or "independence" firmly in mind. Indeed, the chapter itself points out how intermediate frameworks tend to assume (as do classroom norms) that the knowledge to be learned through tackling a task and the anticipated learning trajectories through which that knowledge will be constructed have been determined in advance. That leaves only the option of manipulating one or more of the critical didactical variables for the task in order to manoeuvre student interpretations into closer alignment with those of the teacher and/or designer.

However, Chap. 4 explores one further option in some depth, reflecting approaches to task design which emphasize the significant role of student agency and voice in the development of mathematical thinking. One such approach seeks to bring out the utility of mathematical ideas through tasks which give students some clear and immediate purpose within the context of the lesson (in the sense of a purpose distinct from learning target mathematical ideas) but which also require students to form and/or use those target mathematical ideas in a meaningful way. The criterion for a successful design is that it achieves a strong coordination between, on the one hand, the interpretations that students form of the purposeful task and its aim, and so their approaches and responses to it, and, on the other hand, the specific teaching intentions for mathematical development that underlie the design of the task. This Design for Purpose and Utility framework appears to share the perspective of those intermediate frameworks—reviewed in Chap. 2—which emphasize the need for students to experience tasks as realistic and authentic, through resonances of the task itself with students' interests and/or a managed process of devolution in which the task is made the students' own.

11.5 An Apparatus for Design

It goes without saying that this book makes reference to many resources that—as presented more fully in their original sources—can be treated as tools for design: in particular, each of the intermediate frameworks and its design principles, but equally each task case with its potential to serve as a generic example. Nevertheless, confronted with such a diversity of intermediate frameworks, design principles, and exemplary cases, one is likely to feel a need to identify some larger order within which these can be mapped and potentially used in a more coordinated way: the organizing scheme that I set out in an earlier section represents the very simple device which I formulated in a first attempt to do just this.

I see the development of modular "design tools" as probably the most potent way to populate such an organizing scheme. This notion of a design tool was briefly referred to in Chap. 2. Its definition in the source paper (Ruthven, Laborde, Leach, & Tiberghien, 2009) is largely ostensive, through presenting and comparing examples of such tools. Nevertheless, the paper positions the design tool as one component in a system moving from the level of *grand theory* through *intermediate framework* to *design tool*, so that the latter is distinguished by its proximity to the design process and its sharpness of focus within that process. Equally, the paper characterizes a design tool in terms of its capacity to identify and address some specific aspect of task design in order to support both the initial formulation of a design and its subsequent refinement in the light of implementation. For example, the design tool of Communicative Approach identifies and addresses the specific issue of finding a suitable combination of authoritative or dialogic and interactive or noninteractive discourse in each phase of the staging of a task while the design tool of Modelling Relations identifies and addresses the specific issue of managing, over the course of a task sequence, the compound relations between everyday and disciplinary languages and between observational/ostensive and theoretical/non-ostensive worlds.

Moreover, bearing in mind the way in which design continues in use, such tools might also prove more accessible to users than intermediate frameworks and more effective in guiding their implementation and adaptation of tasks, by virtue of their sharp focus on a particular aspect of task design and/or staging. Here, I find myself in sympathy with the position taken in the editorial introduction to this book to the effect that theory in task design should be clear and give meaning to phenomena in classrooms, while also having practical meaning for teachers and designers.

In my view, then, the chapters that I have discussed—and this book as a whole—provide a valuable staging post for the continuing development of task design as an area of systematic enquiry in mathematics education. By bringing together, summarizing and comparing such a rich collection of conceptual frameworks and exemplary cases, my hope is that this book will motivate ongoing analysis and further synthesis, as well as stimulating the development of more comprehensive organizing schemes to guide the process of task design.

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