Chapter 1
Themes and Issues in Mathematics Education Concerning Task Design: Editorial Introduction

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1.1 Rationale

This study was initiated to produce an up-to-date summary of relevant research about task design in mathematics education and to develop new insights and new areas of relevant knowledge and study. Attention to task design is important from several perspectives in mathematics education research and practice. From a cognitive perspective, the detail and content of tasks have a significant effect on learning; from a cultural perspective, tasks shape the learners' experience of the subject and their understanding of the nature of mathematical activity; from a practical perspective, tasks are the bedrock of classroom life, the "things to do." Recently, there has been growth of research and publication activity about the work of designers in mathematics education: some of it oriented around teams that work globally; some of it focusing on the affordances of digital technologies. There has also been a growth of research activity arising from international comparisons of classroom characteristics, including tasks and task adaptation, and from comparisons of textbooks. It is interesting that globalization in mathematics education research, practice, and policymaking has led us to focus more closely on the minutiae of tasks in mathematics teaching, as well as on the more predictable issues to do with culture, local policy, and technological advances. Task design is also a core issue in research about learning; whether this research takes place through clinical interviews or authentic classroom practice, the detail of the tasks and the way they are presented

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is often reported sketchily, and without full justification, yet tasks have a major influence on assumed findings about capability.

The state of play before this study conference included the following strands of activity:

- Effects of task design on learning and assessment (e.g. Anderson & Schunn, 2000; Runesson, 2005)
- Improvement of communication between designers and researchers, with more exchange about research and principles and practice (e.g. Schoenfeld, 2009; International Society for Design and Development in Education (ISDDE) http://www.isdde.org/isdde/index.htm)
- Inclusion of Topic Study Groups in task design as a regular feature of ICME conferences (Mexico, 2008 http://tsg.icme11.org/tsg/show/35; Korea, 2012; Germany, 2016)
- Publication of tasks, principles of design, and research on effects and implementation by long-standing design teams (e.g. Shell Centre, UK; Freudenthal Institute, the Netherlands; QUASAR, USA; Connected Mathematics, USA)
- Changes in task design at implementation stage (e.g. PME research forum; Tzur, Sullivan, & Zaslavsky, 2008)
- The process of didactic engineering and the influence of tasks on teaching (e.g. Margolinas et al., 2011)
- International textbook comparisons that draw attention to differences in task design (e.g. Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002)
- Tasks in teacher education (e.g. Tirosh & Wood, 2009; Zaslavsky & Sullivan, 2011; *Journal of Mathematics Teacher Education*, volume 10 (4–6))

However, we recognize that these represent only what came to the attention of the International Programme Committee. Hence, they are restricted to what is available internationally and mainly in English and cannot reflect the working practices of myriad groups of teachers and textbook writers worldwide.

1.2 Structure of the Book

The chapters of the book are organized in four parts. The first part consists of this introductory editorial. In the second part are five chapters which reflect the five organizing themes of the ICMI study conference. The initial themes were identified from a reading of existing research:

- Theme A: Tools and representations
- Theme B: Accounting for student perspectives in task design
- Theme C: Design and use of text-based resources
- Theme D: Principles and frameworks for task design within and across design communities
- Theme E: Features of task design informing teachers' decisions about goals and pedagogies

Reflective work by the International Programme Committee led to new titles and a new sequence for this book that represents more closely the scholarly work undertaken at the conference and subsequently.

The third part of the book consists of chapters by four invited plenary speakers who provide examples of the design process relating to underlying principles and practices. These processes vary widely not only in the ways in which individuals describe their work but also in theoretical perspectives relevant to their working context.

The final part of the book consists of two commentaries, one from Michèle Artigue and one from Ken Ruthven. We invited them to comment as senior scholars in mathematics education who have themselves been intimately involved in the processes of task design and implementation.

1.3 Editorial Overview of Chapters from the Thematic Groups of the Study

1.3.1 Frameworks and Principles for Task Design

Chapter 2 is the longest in the book because it presents a way of thinking about the multiple frameworks and sets of principles that arise in the literature on task design. We are deeply grateful to the participants and authors who contributed and hope that future researchers, research students, and designers who wish to publish their practices in scholarly journals find it useful to focus or structure their work according to the ideas in this chapter. It offers a significant theoretical step forward in the field; to a great extent, Chaps. 3–6 depend on Chap. 2 for their theoretical background.

Frameworks and principles for task design are identified as addressing three theoretical *grain sizes*, although a specific set of principles might incorporate different sizes. *Grain size* descriptions are intended to be descriptive tools for thinking in a structured way about task design, rather than being prescriptive. The grain sizes identified are *grand frames*, *intermediate frames*, and *domain-specific frames*. Grand frames present theories about learning in and out of educational settings at a general level. Intermediate frames present the complex interactions between task, teacher, teaching methods, educational environment, mathematical knowledge, and learning so that the purposes and implications for task design are always understood within the total structure of practice. Intermediate theories take time to develop, and applications of them can lead towards elaboration of the theory as well as developments in the practices of both teachers and designers. Communities can develop around both grand and intermediate theories in which there are shared language, shared materials and resources, and shared research studies and conferences. Presentation of work developed within intermediate theories to the outside world cannot always restate the complex background principles, so some background work has to be expected of readers and reviewers. An extra dimension of intermediate frames is that they are based in teaching as craft knowledge and arise from teachers' actions and interactions. Domain-specific frames focus on particular areas of mathematical knowledge or activity and may not be generalizable across mathematics.

A particularly useful contribution from this chapter is the passage on the history of task design in mathematics education. Very often in our field, people only refer to recent research and recent experiences of practice. Where practice and research are based on a mature accumulated body of knowledge, this is not a problem. However, because much of our practice is influenced by policy, ideology, and (at the time of writing) the international testing regime, it is tempting to refer to those as the basis for critical academic work rather than our own past research. It can also be quite difficult for researchers to access past work if it has been locked into the relationships between design, curriculum, and teaching and has not been specifically researched and reported. Globalization of the field helps in this process to some extent: for example, some practices in Singapore can be traced back to work in the UK in the 1970s, even though the influence is now in the opposite direction. This is an example of how worthwhile practices might disappear for some time in one part of the world but be alive and well in another part. More effortful attention to the history of ideas in task design would enable consolidation of the field: there is no need to reconstruct from scratch; there is no need to ignore elements that have been important in the past. Often important ideas continue in practice but are not recognized by researchers, and current recognition by a researcher does not mean that an idea is "new."

1.3.2 The Relationship Between Task Design, Anticipated Pedagogies, and Student Learning

Chapters 3 and 4 between them address the relationships between tasks, teaching, and learning. Sometimes people refer to "gaps" between what is intended by the designer and enacted by the teacher or what is intended by the teacher and perceived by the learner. Such gaps can also be seen as "interactions" which are inevitable in the teaching-learning process. These two chapters are like two sides of a coin, represented by the task as stated and presented to students. On the one side are the teacher's decisions about the nature of mathematics, the collection of students who are being taught, and many emanating practical considerations. On the other side, without the intimate feedback that is available in one-to-one clinical situations, the teacher has to have theoretical support to anticipate and understand students' experiences.

Chapter 3 is very practical. The authors posed questions about the influence on teachers' decision-making of task features and had available to them a wide range of reports about how teachers put given tasks, designed by external sources initially, into practice embedded in pedagogical variables. The authors use three tasks from the study conference as exemplars around which the discussions in the chapter are oriented, raising issues which can then be used to think about any task. They elaborate on how to analyse the thinking that went on in the design, decisions about suitability and applicability, and how these are influenced by teachers' views of the nature of mathematics, the prevailing school and classroom culture, and the relative emphases on mathematical content and broader epistemological goals a teacher may be pursuing with the students. Each of the example tasks is presented as a complex situation, in which the dividing line between task design and implementation, and lesson design and implementation, is not easy to draw and may even be unnecessary. The teacher's knowledge of mathematics pedagogy and ability to anticipate students' responses is critical in all these decisions. The authors had originally intended to address related issues about educating teachers in the processes of using tasks, but these vary so widely between cultures and between teacher education programmes that we cannot do it justice in this volume and refer readers instead to the *Journal of Mathematics Teacher Education*, in particular volume 10 (4–6).

Finally, there is discussion about how to present a task to students initially so that they are motivated, have access to the task and can get started, and also understand what the teacher would like them to be thinking about. These final ideas lead naturally on to Chap. 4, in which learners' perceptions are considered from a different point of view, including the notion of *interest-dense* tasks for sustaining both effort and learning. Questions that arise from Chap. 3 also arise from other chapters, so we shall incorporate them in our final remarks below.

1.3.3 Accounting for Student Perspectives in Task Design

Chapter 4 constructs a theoretical view of the interactions between students, teachers, and tasks in the classroom; it is not possible to infer how the learners "see" a task merely from their actions and their written or verbal products. Merely doing what the teacher hopes and expects is evidence for a certain form of compliance, but might not constitute evidence of learning or evidence of understanding the purpose of the task or even evidence of having the same perception of the task as that of the teacher. The question the authors of this chapter wanted to address is how learners answer this question: "what is this task asking me to do?" The authors became aware of a general dearth of research in this area, and yet knowledge of how learners perceive a task is crucial to planning effective lessons as well as to designing effective tasks. *Perception* therefore has to be imagined, and in some cultures the expertise of the teacher is seen in terms of the accuracy of that process of imagination. We welcome the authors' decision to stay firmly with what can be *known* about the learners' perspective and not be deviated into what might be *assumed*.

The chapter firstly reflects on the literature about word problems, which draws attention to differences in students' perceptions of the purpose of the task and, maybe, designers' intentions, although this is sometimes achieved through inferential reasoning from students' productions. These kinds of differences can arise in any mathematics teaching situation, not only with word problems. So, the authors offer the construct of *didactical situation* as a structure within which to consider the *didactical contract* and *milieu* (terms related to theories discussed in Chaps. 2 and 8) of the learner and, hence, how they might be viewing the nature and purpose of a mathematical task. This chapter includes a rare example of phenomenographic research identifying students' perceptions of a statistical task and what this reveals about their understanding of the purpose of the task and of statistics as a field of study.

The second part of the chapter proposes ways in which various educators, teachers, and researchers have sought to reduce any gaps between the teacher's intentions and the learner's perceptions. In doing so, the authors present the importance of the quality of teachers' expectations of students, the importance of reflective redesign, the idea of *emergent task design*, and considerations of openness. Some of these ideas arise also from other perspectives. For example, teachers' expectations are a component of the discussions in Chap. 3 and also a central aspect of teachers' professional learning described in Chap. 9. Reflective redesign is routinely undertaken with others in Japanese Lesson Study and is also a component of a well-wrought design process for teams and individuals. A degree of openness to allow and also encourage student agency is a key aspect of many problem-solving task design initiatives in which learners' contributions are valued and discussed, and to add to this we would draw attention to the idea of listening *to* learners (Davis, 1996) rather than listening *for* particular solutions. Indeed, in some cultures, generating several methods for approaching a question is a key feature of a lesson. Whereas in Western literature some students' responses might be described as *misconceptions*, in other traditions these are *alternative ways of seeing* and are valuable for the learning of both teachers and students. A welcome development of this is given in the chapter, which offers emergent task design as a process arising from ideas of learners, in the course of a lesson, which are made into tasks by teachers in the moment.

Another way to look at the issues in both Chaps. 3 and 4 would be to consider the work of teaching in Valsiner's terms of aligning the zone of free movement (ZFM) (i.e. what is possible in the situation), the zone of promoted action (ZPA) (i.e. how the teacher directs learners towards particular actions), and the zone of proximal development (ZPD) (i.e. the learning a child can be expected to achieve in that educational situation). Ideally, according to Valsiner, the ZPA matches the ZPD for optimal learning (Valsiner, 1997, p. 198). However, teachers rarely know accurately the ZPD of all their students in school situations, so teachers need to engineer a balance between defining the boundaries of the ZFM through task and situation design and providing loose enough boundaries for the ZPA to allow optimal overlap with their students' relevant ZPD.

1.3.4 Design Issues Related to Text-Based Tasks

This theme group set out originally to focus on the design of tasks in textual format, textbooks more generally, downloadable materials, and other forms of text-based communication designed to generate mathematical learning. The group was aware of differences in the order, development, representation, and presentation of content between textbook series and also between countries and cultures. The group hoped to consider how to analyse the content of individual questions or sequences of questions. Another way to look at tasks would be to view them as the shapers of the curriculum rather than merely presenting a given curriculum and hence consider the differences between author and teacher intentions. The questions originally posed for the study conference concentrated on issues of text design and use.

In practice, few papers were submitted to the theme group that addressed these questions. Instead, the theme received many papers about designed tasks or collections of tasks that were based on clearly enunciated principles and showed how these worked out in practice, generally addressing overarching aspects of students' mathematical learning, such as proof, interdisciplinary perspectives, reasoning, problem-solving, and values. Some papers were on specific examples of textbook issues: involvement of teachers as digital authors and the need for ancillary materials such as assessment tasks. A small number addressed specific details: why this diagram, why these numbers, why this questioning sequence, and so on. Meanwhile, the ICMT *International Conference on Mathematics Textbook Research and Development 2014* signalled an increase in international comparison, cooperation, and knowledge exchange about mathematics textbooks, their design, development, use, and analysis—a field of study focusing on textbooks in particular. The expectations for this conference freed the working group to focus on issues raised by the conference papers and others that could not be addressed at the level of textbook production and use.

The chapter offers a triangular, mutually interactive relationship between the nature and structure of the task, the intended mathematical activity, and the pedagogic purpose. It refers throughout to tasks that are free-standing or situated within *learning management systems*, meaning published textbooks, task banks, programmed systems, and so on. The triangular relationship is relevant for free-standing tasks, home-made task banks, and textbooks, whether digitally delivered or paper based; and tasks created during lessons. Discussions led to the formation of a focus on the learners' perspective when presented with a task, as in Chap. 4, and how the task influences their subsequent mathematical activity, their learning, and their view of mathematics. This perspective is never constructed in isolation from their whole mathematics educational experience, which could include textbook design and use, but is also influenced strongly by pedagogy and presentation. One section of the chapter proposes a detailed consideration of visual appearance and layout as influences on learning.

This chapter focuses on tasks without dynamic or interactive content, while the following chapter addresses tool use, which includes the full range of digital tools.

1.3.5 Designing Mathematics Tasks: The Role of Tools

This theme concerns designing teaching-learning tasks that involve the use of tools in the mathematics classroom and consequently how, under such design, tools can represent mathematical knowledge. This aspect of task design research is currently "coming of age" through combinations of various and widely available digital tools and a Vygotskian understanding of relationships, through semiotic mediation, between artefact and learning. The issue for designers is how to relate the tool-specific discourse representation to mathematical knowledge. There has been an international conference relating to digital technologies since 2004, the *International Conference for Technology in Mathematics Education*, journals, and several special issues of mathematics education journals. For our study, the submitted papers were mainly concerned with practical and theoretical issues of task design in dynamic digital environments, but usefully included papers on physical tool use, thus allowing theories developed in digital environments to be expanded for nondigital tools.

The chapter starts by outlining the practical considerations of tool use in the mathematics classroom and then moves to consider relevant theoretical perspectives that connect instruments and didactics. It then presents various ways in which contributors to the study conference had enacted these connections and introduces the idea of *discrepancy potential of tool*, which is the space between the feedback a learner might experience from using the tool and the mathematics concept, combined with the need for the tool user to make decisions. In this space, unanticipated disturbances might take place, but also the teacher can intervene to introduce disturbances.

The chapter closes with a synthesis of the issues that any task design heuristics need to address: complementarity of feedback and mediation, relationships between pragmatic and epistemic considerations, symbiosis of mathematics and pedagogy, multiplicity of tools, and the discrepancy potential previously described. The design of the task—what the learner is supposed to do with the tool—needs to take account of, bridge, and coordinate these aspects of the activity. The final remark is about the importance of the teacher's perception of the nature of mathematics, particularly as tool-based tasks can challenge the nature of mathematical activity, and hence the nature of mathematical knowledge and competence.

1.4 Overview of the Plenary Chapters

Chapters 7–10 are written by the invited plenary speakers at the study conference. These speakers were selected to represent well-formed examples of task design in practice. Michal Yerushalmy opens this section with an example of *domain-specific* design (as defined in Chap. 2) in which she describes the theory and design of a digital resource that focuses on various features of functions for secondary students. One interesting feature of this resource, and the reason she was chosen to give a plenary, is that she embraces the facility of digital technology to provide flexible sequencing of tasks. In order to use the resource, the teacher (or even student) has to make her own decisions about what to do and when. This is not, therefore, a digital learning management system but rather a digital task world, and she describes the structuring of such a world in terms that can be useful for other designers.

Chapter 8 presents two examples to illustrate the manifestation in practice of the *intermediate frame* of didactics founded by Brousseau. The description of this as intermediate arises from Chap. 2, that is, theories which provide methods of application across mathematics. Berta Barquero and Marianna Bosch illustrate how the theory of didactic situations has been used at a primary level to establish the measurement of quantities, and then they demonstrate the more complex world of didactic engineering in an anthropological development of the original theory. There are also *domain-specific* principles in their descriptions and also craft knowledge, and a disturbing account of how the product of careful longitudinal design research can be subverted by practitioners who do not share the theoretical commitment.

In Chap. 9, Toshiakira Fujii describes an aspect of Japanese Lesson Study, *kyozaikenkyu*, that can be overlooked in some Western adaptations of the process. The disciplined process of Japanese Lesson Study can be seen as an example of a craft-based frame, as described in Chap. 2, and this is becoming widely recognized outside Japan. Typically, each lesson is oriented around one task, which may be one calculation (he gives "12-7" as an example) or may be a conceptual problem (e.g. classifying triangles). The selection and design of one task and how to use it is the focus of teachers' regular professional development activity, and this creates a deep repertoire of "good" tasks that are also reflected in the contents of the authorized textbooks and a pedagogic repertoire for teachers. This approach, where task design is the central focus for teachers' planning and development, runs counter to the comments of Wittmann (1995) who argued for task design to be in the hands of specialist designers, and it poses challenges for teacher knowledge and training and also for the value of externally designed tasks. Each lesson study report uses domain-specific frames alongside, or even as a basis for, generic considerations.

The final plenary chapter also presents a contradiction, this time to the whole book in some respects. Jan de Lange is an experienced designer whose work at the Freudenthal Institute has been influential throughout the world, particularly in the Netherlands, South Africa, and the US reform process. His description of the design process is down-to-earth and practical, setting high standards for the use of intuition and insight as starting points, with a focus on students' learning. He claims, and illustrates, an approach he calls *slow design* that arises from knowledge of mathematics, the environment, teaching, classrooms, and children. While he describes the actions necessary for a designer to take in order to test and improve the design (e.g. not relying on the designer's own teaching), he also challenges the academization of the design process. For him, the nature and direction of research about task design is in danger of moving the emphasis away from direct experience of children and mathematics in classrooms and towards theorization. A series of online articles, *A Designer Speaks* published by ISDDE (http://www.isdde.org/isdde/index.htm), is worthy of mention here, giving alternative insights into designers' working practices.

1.5 Final Comments and Recent Developments

As conveners of this study, and editors of this volume, we have been excited by the breadth and diversity of contributions and impressed by the immense work that has gone into the resultant chapters. We are both actively involved in schools and teacher education as well as in mathematics education research, and our multiple perspectives have helped us consider the different roles of theory in relation to task design. While preparing this volume, Minoru has been involved in several lesson studies and has also been a school principal, while Anne has been teaching mathematics in the UK year 7 and leading teacher workshops. Our main concern in leading this study has been to accelerate the growth of attention to task design given by researchers in their work and their written artefacts. We note that several papers that contributed to the conference have now been expanded and developed and published elsewhere and have included these where possible in the relevant reference lists. A volume about task design with digital technologies follows the study (Leung & Baccaglini-Frank: *Digital technologies in designing mathematics education tasks: Potential and pitfalls* forthcoming from Springer) as does a special issue of the *Journal of Mathematics Teacher Education* edited by Keith Jones and Birgit Pepin. A research forum took place in 2014 on the relationships between task and students (Clarke, Strømskag, Johnsen, Bikner-Ahsbahs, & Gardner, 2014).

Research reports rarely give sufficient detail about tasks for them to be used by someone else in the same way and hence build on knowledge by extending the domain of application. Few studies justify task choice or identify what features of a task are essential and what features are irrelevant to the study. In some intervention/treatment comparison studies to investigate cognitive development, the intervention tasks are often vague, as if the reader can infer what the learning environment was like from a few brief indications. Alan Schoenfeld commented similarly some time ago (Schoenfeld, 1980). As an example of how the task can be *invisible* to researchers, we could look at the commentaries about a well-known and widely accessible video of a mathematics lesson for the TIMSS study (http://www.timssvideo.com/67). In the commentaries reported on the website, it is only the teacher who mentions connections between the task design, its presentation, and students' participation; the researcher talks only generally about the social and structural features of the lesson. Yet the task is central to the success of the lesson in terms of lesson structure and learning and has been included in every official Japanese textbook for at least 40 years, and the diagram that goes with it is one the students are already familiar with. These features of the task are, we believe, crucial to understanding the lesson and the students' mathematical responses, but are hardly mentioned.

To some extent, task design issues are addressed in the literature using design research in which the result of the research is a designed product to fulfil a desired role. Task design also emerges in the growing field of international comparison (Shimizu, Kaur, Huang, & Clarke, 2010). Although there have been two recent edited collections of the use of tasks in mathematics teacher education as previously mentioned, we comment that, perhaps strangely, as yet no comparable international "go-to" collections for thinking about task design for mathematics classrooms have been published, although there have been research foci at ICME and PME in the last decade. We hope this volume will go some way towards filling that gap.

Meanwhile, teams of designers who are now well established have been producing and publishing tasks consistently for decades. Tasks initially invented and disseminated by Alan Bell (1993), Hans Freudenthal (1973), and Guy Brousseau (1997) and their colleagues are widely and effectively used throughout the world. The work of designing, trialling, and publishing often took priority over reporting the design research processes in an internationally accessible way, or researching their own practice, and the degree to which they expected teachers to understand their background theoretical justifications varied. Teachers all over the world might be familiar with the task of graphing the heights achieved by filling bottles of various shapes, or the task of estimating the size of the giant given the dimensions of the handprint, or the task of enlarging the drawing of a rectilinear animal. Teachers use these tasks not because they are committed to the precise background theory that led to their invention nor because their use has been researched and theorized in some other classroom or country. Rather, teachers use these tasks because they match the practices involved in local coordination of curriculum demands, classroom practices, intended mathematical outcomes, and anticipated participation of particular individuals and groups of students, using a craft-based frame as described in Chap. 2.

So we ask ourselves to what extent this book provides a go-to place for thinking about task design in mathematics classrooms. Because of our regular school-based experience, we both have the view that theory in task design should be clear and give meaning to phenomena in classrooms while also having practical meaning for teachers and designers. In Chap. 2, the distinction is drawn between theories as resource and theories as product: *theory for* and *theory of*. The intermediate level frames, as categorized in Chap. 2, combine theoretical structures that are well founded in theories of learning and classrooms with the practical, local theorizing that teachers do on a day-to-day basis. The technical terms used in academic writing might be seen as an obstacle (e.g. milieu; didactic contract in Chap. 4), but the underlying ideas would be familiar to many teachers. By contrast, the notion of instrumental genesis (see Chap. 6) is more abstract and less likely to relate to teachers' day-to-day thinking, although they would see evidence of a *utilisation scheme* in practice. Nevertheless, the concept of instrumental genesis has much to offer designers of both tasks and mathematics software, as well as in research. These thoughts are conjectural, but based on our own recent experience of teaching and talking with teachers in English and Japanese cultures of practice.

Returning to Schoenfeld's paper, which is entitled "On useful research reports," we would therefore ask "useful for whom?" We agree with the closing remarks made in Chap. 2 that there is a need for detailed research reports that are not unhelpfully limited in length and can fully report studies of design and use of tasks as well as pedagogy. We agree that details of tasks and the likely effects of task design features, as well as pedagogy, should be included more frequently in research about classrooms and learning. We point also to a need for researchers to distinguish between theories *of* their observations and theories *for* designers and teachers and to consider drawing on teachers' and learners' situated perspectives when theorizing in either case. In these respects, Chaps. 3–6 can all provide starting points. As for theories of task design, evaluations of effectiveness are always going to take place in natural contexts consisting of specific classrooms, teachers, constraints, and cultures, so it is inevitably the case that empirical studies will not be extensively generalizable, but can be illuminative and give rise to conjectures.

Finally, we pose some areas for further research that arose from the study, sometimes from several theme groups:

- How learners/teachers make sense of, and understand the purpose of, different kinds of tasks
- How different design principles reflect or generate different perceptions of mathematical concepts
- How different combinations of tasks and pedagogy influence learners' perceptions and mathematical activity
- How visual features of task presentation affect activity
- The design and implementation of task sequences
- The professional learning of prospective and practising teachers about task design, sequencing, and adaptation
- The role of task design in promoting equity and other values
- Task design and individual learner differences
- The effectiveness of forms of collaboration and communication between task designers, classroom teachers, educators, and policymakers.

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