An Aspiration Set EMOA Based on Averaged Hausdorff Distances

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Abstract. We propose an evolutionary multiobjective algorithm that approximates multiple reference points (the aspiration set) in a single run using the concept of the averaged Hausdorff distance.

Keywords: Multi-objective optimization · Aspiration set · Preferences

Background. In the following we consider unconstrained multiobjective optimization problems (MOPs) of the form $\min\{f(x) : x \in \mathbb{R}^n\}$ where $f(x) = (f_1(x), \ldots, f_d(x))'$ is a vector-valued mapping with $d \geq 2$ objective functions $f_i : \mathbb{R}^n \to \mathbb{R}$ for $i = 1, \ldots, d$ that are to be minimized simultaneously. The optimality of a MOP is defined by the concept of *dominance*.

Let $u, v \in F \subseteq \mathbb{R}^d$ where F is equipped with the partial order $\preceq$ defined by $u \preceq v \Leftrightarrow \forall i = 1, \ldots d : u_i \leq v_i$. If $u \prec v \Leftrightarrow u \preceq v \wedge u \neq v$ then v is said to be *dominated by* u. An element u is termed *nondominated* relative to $V \subseteq F$ if there is no $v \in V$ that dominates u. The set $\mathsf{ND}(V, \preceq) = \{u \in V \mid \nexists v \in V : v \prec u\}$ is called the *nondominated set* relative to V.

If $F = f(X)$ is the objective space of some MOP with decision space $X \subseteq \mathbb{R}^n$ and objective function $f(\cdot)$ then the set $F^* = \mathsf{ND}(f(X), \preceq)$ is called the *Pareto front* (PF). Elements $x \in X$ with $f(x) \in F^*$ are termed *Pareto-optimal* and the set X^* of all Pareto-optimal points is called the *Pareto set* (PS). Moreover, for some $X \subseteq \mathbb{R}^n$ and $f : X \to \mathbb{R}^d$ the set $\mathsf{ND}_f(X, \preceq) = \{x \in X : f(x) \in \mathsf{ND}(f(X), \preceq)\}$ contains those elements from X whose images are nondominated in image space $f(X) = \{f(x) : x \in X\} \subseteq \mathbb{R}^d$.

If we are not interested in finding an approximation of the entire PF a reference point method [8] can be used to find a solution that is closest to a so-called reference point gathering the user-given level of aspiration for each objective. A modified version [1] does not only offer a single solution but also some additional solutions in its neighborhood, whereas multiple reference points can be

P.M. Pardalos et al. (Eds.): LION 2014, LNCS 8426, pp. 153–156, 2014.
DOI: 10.1007/978-3-319-09584-4_15

used to approximate larger parts of the PF by running the original method in parallel for each reference point [3]. Here, we propose an alternative method to approximate only desired parts of the PF (which we call *aspiration set*) that is a marriage between a set-based version of the original reference point method [8] and the *averaged* Hausdorff distance [6] as selection criterion. The value $\Delta_p(A,B) = \max(\mathsf{GD}_p(A,B), \mathsf{IGD}_p(A,B))$ with $p > 0$,

$$\mathsf{GD}_p(A,B) = \left(\frac{1}{|A|} \sum_{a \in A} d(a,B)^p \right)^{1/p} \text{ and } \mathsf{IGD}_p(A,B) = \left(\frac{1}{|B|} \sum_{b \in B} d(b,A)^p \right)^{1/p}$$

is termed the *averaged Hausdorff distance* between sets A and B, where $d(u,A) = \inf\{\|u-v\| : v \in A\}$ for $u, v \in \mathbb{R}^n$ and a vector norm $\|\cdot\|$. In our previous work [2,4,5,7] we successfully used the concept of the averaged Hausdorff distance in designing EMOAs that find an evenly spaced approximation of the PF.

Algorithm. The AS-EMOA was designed for approximating the aspiration set: We applied a weighted normalization for each candidate solution,

$$\tilde{f}(x)_j = \frac{f(x)_j - \min_j}{\max_j - \min_j} \cdot w_j,\ j \in \{1,2\} \text{ with } w_1 = \frac{\max_1 - \min_1}{\max_2 - \min_2} \text{ and } w_2 = 1/w_1,$$

in objective space during Δ_1 computation in order to focus on the given aspiration set and to avoid biases due to its orientation in objective space. Here, $\min_j$ and $\max_j$ denote the minimal and maximal value attained for objective f_j over all elements in the aspiration set. The value $p = 1$ is recommended due to its robustness to outlier points [4].

AS-EMOA

Require: aspiration set R
initialize population P with $|P| = \mu$
$P = \mathsf{ND}_f(P, \preceq)$
while termination criterion not fulfilled **do**
  generate offspring x by variation of parents from P
  $P = \Delta_1$-update$(P, x; R)$
end while

Δ_1-update (line 8: ties are broken at random)

Require: archive set A, new x, aspiration set R
1: $A = \mathsf{ND}_f(A \cup \{x\}, \preceq)$
2: **if** $|A| > N_R := |R|$ **then**
3:   **for all** $a \in A$ **do**
4:     $h(a) = \Delta_1(A \setminus \{a\}, R)$
5:   **end for**
6:   $A^* = \{a^* \in A : a^* = \mathsf{argmin}\{h(a) : a \in A\}\}$
7:   **if** $|A^*| > 1$ **then**
8:     $a^* = \mathsf{argmin}\{\mathsf{GD}_1(A \setminus \{a\}, R) : a \in A^*\}$
9:   **end if**
10:   $A = A \setminus \{a^*\}$
11: **end if**

Experiments and Results. The AS-EMOA has been evaluated for four well known bi-objective test problems (SPHERE: convex, $n = 2$, DTLZ2: concave, $n = 10$, DENT: convex-concave, $n = 2$, ZDT3: disconnected, $n = 20$) [4]. Aspiration sets were generated in the utopian objective space ("before PF") and in the dominated objective space ("behind PF"), see Fig. 1. AS-EMOA was executed 20 times per test problem and considered aspiration sets for 50,000 function evaluations (FE) with SBX crossover ($p_x = 0.9$) and polynomial mutation ($p_m = 1/n$).

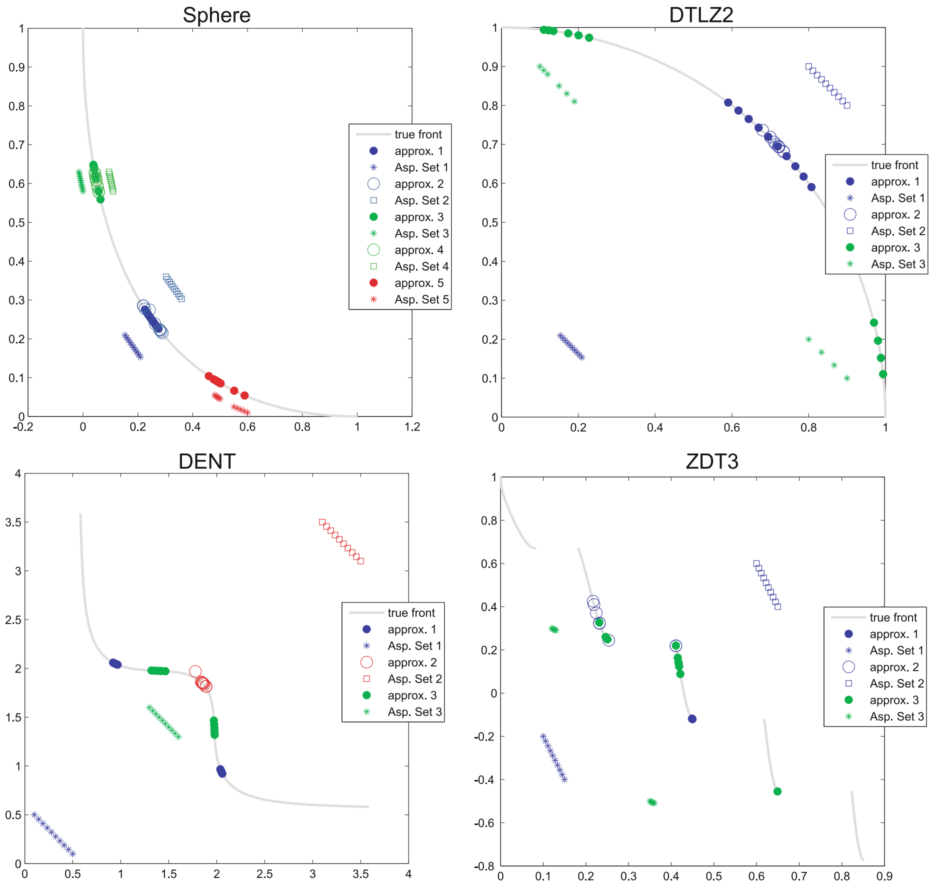

Fig. 1. Exemplary approximation results for applying AS-EMOA to different bi-objective test problems using various reference sets.

Each plot in Fig. 1 aggregates the results for all applied aspiration sets. The AS-EMOA closely approximates the aspired region of the PF while reflecting original structures of the aspiration set, see e.g. Asp. Set 5 in the SPHERE case and Asp. Set 3 in DTLZ2. Even placing an aspiration set behind the true PF leads to a good approximation. Depending on the position of the respective set in objective space, different regions of the true PF come to focus due to the distance-based selection pressure induced by the Δ_p indicator: for the DENT case two separate sets form the best approximation results for Asp. Sets 2 and 3 in the concave part of the true PF. In fact, the extremal members of the aspiration set have the smallest distance to the solution sets. In order to comment on the stability of the proposed approach, we computed the coefficients of variation for the Δ_p values of aspiration sets and approximated solutions which are all in the range from $2.26 \cdot 10^{-11}$ to 0.2 with a single outlier of 0.4 for the disconnected PF (see Table 1). Furthermore, depending on the test problem, AS-EMOA only needed between 400 and 2,500 FE to reach a good and stable quality level.

Table 1. Coefficients of variation for all problems and aspiration sets based on 20 experiments each.

Problem	Asp. Set 1	Asp. Set 2	Asp. Set 3	Asp. Set 4	Asp. Set 5
SPHERE	$2.26 \cdot 10^{-11}$	$1.48 \cdot 10^{-2}$	$7.99 \cdot 10^{-2}$	$1.05 \cdot 10^{-2}$	$1.98 \cdot 10^{-1}$
DTLZ2	$3.07 \cdot 10^{-8}$	$6.20 \cdot 10^{-3}$	$1.19 \cdot 10^{-7}$	–	–
DENT	$1.76 \cdot 10^{-2}$	$5.30 \cdot 10^{-3}$	$2.50 \cdot 10^{-9}$	–	–
ZDT3	$1.88 \cdot 10^{-1}$	$1.53 \cdot 10^{-1}$	$4.08 \cdot 10^{-1}$	–	–

Conclusions. Within the experiments the AS-EMOA successfully approximated the aspiration sets for different front shapes in 2D. Even suboptimal aspiration sets do not hinder the AS-EMOA from reaching the true Pareto front. The approach shows promising perspectives for higher dimensions as well; a suitable normalization within the Δ_p update procedure is a matter of current research.

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