Synchronicity Assessment Using a Non-parametric Dynamic Dissimilarity Measure

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Abstract In this paper, we introduce a non-parametric dynamic dissimilarity measure (DDM) of synchronicity based on recurrence plots, which is particularly suited to use in small samples. The measure attempts to capture the dissimilarity of the topology of the dynamics of time series, based on an epoch analysis of the cumulative sums of data series. The measure is applied to US State macroeconomic data and is used to assess how synchronous US State business cycle variables are with US aggregates.

1 Introduction

Social scientists often refer to variables as being synchronized if they exhibit co-movement. But generally co-movement in social science is measured from a long term perspective, using relatively large datasets, and employing simple measures such as maximal windowed correlations to indicate synchronization, or more complex techniques such as cointegration and concordance measures from factor models (see [1, 2]), if data permits. Otherwise, if only small samples are available, then most social scientists resort to simple correlations as a measure of synchronicity. Macroeconomic researchers in particular are typically faced with small data sets (relative to those found in natural and environmental sciences) and so more often than not, when these conditions arise they appeal to regression analysis or basic correlation to assess synchronization.

Macroeconomists in particular are also concerned specifically with the synchronization of business cycles (the boom and bust cycles that are a stylized fact in economics), and so are particularly focused on correlations of economic growth patterns across countries or regions as a way of measuring economic integration across

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regions or geographic areas that have some facet of economic policy in common (such as a common currency). But when applying synchronization measures economists often face major problems in interpreting these measures (such as with accounting for lag or lead effects), and also the distributional assumptions of these measures might not be appropriate.

In this study we therefore introduce a simple non-parametric dynamic dissimilarity measure (DDM) that can be applied to economic and other data series to overcome some of the issues encountered when using small datasets. We present this DDM for assessing synchronization, which is derived from recurrence plot techniques, as a relatively simple approach particularly suited to small sample stochastic empirical data. In the paper, for illustrative purposes, we apply this technique to US State macroeconomic data.

The paper is organized as follows: Sect. 2 presents two different ways of assessing the degree of synchronization with small data samples, while Sect. 3 looks at an application using the methodology employed in this study. Section 4 then concludes.

2 Measurement of Synchronization

The topic of synchronization is vast, with probably the best reference on the subject being [3], which details the myriad forms of synchronization in nonlinear science. In this section we first explore the cross-recurrence methodology for synchronicity detection, and then we introduce the new measure, both of which are specifically applied to small sample measurement of synchronization.

2.1 Cross-Recurrence Methodology for Synchronicity Detection

The first measure of synchronization presented here is based on recurrence plots. Recurrence plot analysis is now over 20 years old (see [4] for the first contemporary application) and the quantification of these plots is much more recent (see [5, 6]) but the notion of recurrence has a much longer pedigree in mathematics (see [7]). Recurrence plots first originated from work done in mathematics and physics but now has a considerable following in a variety of fields.[1] There are several excellent introductions available to RQA and recurrence plots, not least those by [8, 9]. There are very few papers that apply recurrence plot techniques to macroeconomic issues, the notable exceptions being [10–13].

In terms of the mathematical background, a recurrence plot is calculated from a phase space trajectory, that can be reconstructed using Takens' embedding theorem (see [14]). However, here we simplify the usual exposition of recurrence plots and

[1] Norbert Marwan's website catalogues all the articles published using recurrence plots and RQA, and is a veritable mine of information on this topic. See http://www.recurrence-plot.tk.

consider just the dynamics of the raw data x_i ($i = 1, \ldots, N$). To derive the recurrence features of the series then every point in the series x_i is tested to see whether it is close to another point, i.e., the distance $D_{i,j}$ between these two points i and j

$$D_{i,j}(x, y) = \left\| x_i - y_j \right\| \tag{1}$$

is less than a specified threshold ε. In this case the value one (a black dot in the recurrence point) is assigned to this point in a $k \times k$-array (the auto-recurrence plot):

$$R_{i,j} = \Theta(\varepsilon - D_{i,j}(x, x)) \tag{2}$$

where x_i and y_i in (Eq. 1) are two series such that $x = y$ in the auto-recurrence case, and ε is the predefined "threshold" and Θ is the Heaviside function. Following [15] the cross recurrence plot is defined by considering two different time series x and y:

$$CR_{i,j} = \Theta(\varepsilon - D_{i,j}(x, y)) \tag{3}$$

where in this instance x_i and y_j in (Eq. 1) are two series such that $x \neq y$. This gives a thresholded cross recurrence matrix $CR_{i,j}$ which, dependent on the value of ε contains either 0s (the white areas in the plots) or 1s (the black areas in the plots).

In an auto-recurrence plot, the main diagonal is always present, as every point in the series is identical to the same point in the series, so there will always be a diagonal line (1's down the main diagonal of the thresholded $R_{i,j}$ matrix), once all points in the series are considered. In the cross recurrence plot if line-like patterns that correspond to diagonal lines on the leading diagonal in the recurrence plot appear, the two series are identical, but this is obviously a special case. If line-like patterns that correspond to diagonal lines in the recurrence plot appear in other parts of the cross-recurrence plot, it implies similar dynamics, but these implying short lived periods of synchronization or phasing of the two cycles. This line, if it can be identified, is termed the "line of synchronization" or LOS [15]. These observations also hold when not applying the thresholding but using the distance matrix (Eq. 1) itself (instead of lines we consider then the line-like patterns formed by the lowest distance values in the matrix).

Next, complexity measures can be derived to characterize the cross-dynamics of a given series. For two series these will be characterized as diagonal lines (not necessarily on the main diagonal), which demonstrate similar dynamics maybe at different points in time. Following [16] the distributions of the diagonal line lengths can be written as $P_t(l)$ for each diagonal parallel to the main diagonal, where $t = 0$ denotes the main diagonal, $t > 0$ denotes diagonals above the main diagonal (a lead) and $t < 0$ denotes diagonals below the main diagonal (a lagged dynamic). RQA was initiated by [5] and has now been introduced into mainstream physics through the study of nonlinear dynamics. A good summary is available in [9].

The starting point in this paper is the analysis conducted in [12] with cross recurrence plots. Here we take the example of the real Gross State Product (GSP) growth rate for Texas and the real GDP growth rate for the US, and display the unthresholded

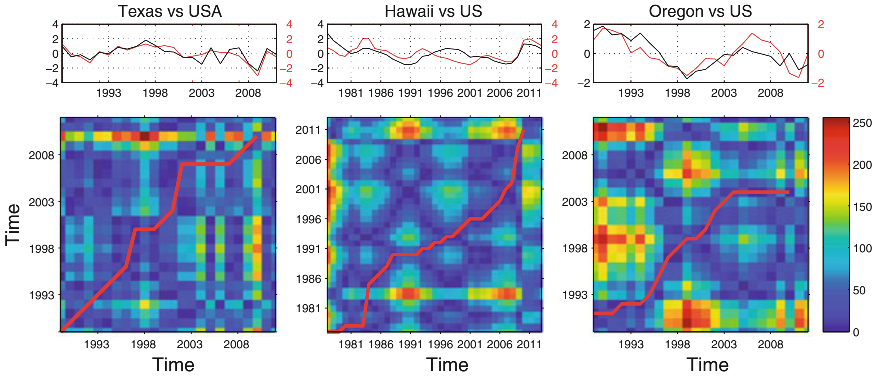

Fig. 1 (*Left*) Cross recurrence plot of Real GDP growth for Texas versus US. (*Middle*) Cross recurrence plot of GDPPI inflation for Oregon versus US. (*Right*) Cross recurrence plot of the unemployment rate for Hawaii versus US

version in Fig. 1(left). These data series are annual (that is all that is available by State in the US), so that there are only 25 observations available for the analysis. In the upper part of the figure the data is reproduced, and in the lower part of the figure the color scale denotes the distance between the two series with red denoting a small distance up to blue areas which denote large distances. Any diagonal lines in the plot indicate the synchronous dynamics in both series with a diagonal going up from the lower left to the upper right being the "line of identity" (LOI). In other words if there was just a red line going diagonally through the plot this would indicate identical series. If an unthresholded plot is used, the line of synchronous movements, otherwise termed the LOS, can be plotted using a search algorithm (see [15, 17]) to detect the best path through the plot. The LOS thus indicates coincident synchronicity and it is apparent that this is fairly consistent in the first part of the period in question in Fig. 1(left). Synchronicity though appears to break down in the late part of the cycle leading up to the 2001 recession, but once the recession hits (observations 13–15 in red in the upper figure), synchronicity is restored. In the growth period of the last business cycle, there doesn't appear to be a great amount of synchronization in growth until the most recent downturn hits in 2008, and then for the remainder of the period in question the two series are highly synchronous and convergent again. This is also shown by a return of the LOS to the leading diagonal in the cross recurrence plot. These results are consistent with the finding that synchronous behaviour in economic growth nearly always occurs during the recessionary period of the business cycle, but occurs much less frequently during the growth phase of the business cycle.[2]

Further examples are given for different US States for the other variables used in this study. Figure 1(middle) shows the cross recurrence plot for Oregon for GSPPI (Gross State Product Price Index) inflation measure against the US GDPPI (GDP Price Index) inflation measure. This particular State was chosen because of its relative

[2] Crowley and Schultz [19] have termed this phenomena "intermittent synchronicity".

asynchronous dynamic with the US. Here the LOS is much less obvious in the cross recurrence plot, and the algorithm that chooses the LOS doesn't appear to detect any consistent synchronicity beyond roughly 2004 onwards.

Lastly, we produce the cross recurrence plot for Hawaii against the US for the unemployment rate in Fig. 1(right). Hawaii does not seem to have been particularly synchronous with the rest of the US until the most recent business cycle from 2002 to present. Nevertheless there does seem to have been a variable lag synchronous effect at work throughout the 1990 s and this is detected by the algorithm generating the LOS. Note as well that the movements in the two unemployment rates are similar from 2003 onwards, but the LOS cannot jump in the CRP so that it reflects this high degree of concurrent synchronization.

So in the context of employing traditional cross recurrence plots to analyze synchronicity in economics, there are several issues, and these are:

(a) that economic series are stochastic in nature, so that synchronicity might be detected only at certain points in time;
(b) that the cross recurrence plot LOS measure is only designed to detect similar dynamic movements in variables due to time scaling, rather than similar patterns of directional dynamics in stochastic time series (see [15]); and
(c) that cross recurrence plots also incorporate an analysis of convergence, which is not the focus here.

Although recurrence quantification analysis does account for (a) above, we wish to focus in solely on the degree of synchronicity, so we wish to abstract from (c) and place the spotlight on (b), so as to concern ourselves only with directional dynamics.

2.2 A Small Sample Measure of Synchronization Based on Dynamic dissimilarity

We next introduce a measure of synchronization based on a dynamic dissimilarity measure (DDM), by focusing on the similarity of the dynamics by taking the distance measure between the cumulative sum of any two series, and seeing how this varies through time within an epoch analysis framework.

Each time series is first transformed into a stationary growth rate (e.g. by log first differencing real GDP to obtain economic growth rates) or stationary source variables are used (such as unemployment rates), and then a cumulative summation variable of this stationary variable is created:

$$X_i = \sum_{j=1}^{i} (\log x_j - \log x_{j-1}) \tag{4}$$

We refer to these modified time-series, X_i, as cumulative unsigned summation (CUS) series. Distance matrices, $D_{i,j}$ for each CUS series are then created using the standard

Euclidean distance metric as described in [8] and this operation is identical to (Eq. 1). To evaluate the dissimilarity between two time series, we then perform an epoch (moving window) analysis with a three sample window incremented one sample at a time, where in the bivariate case $D1_{i,j}$ denotes the epoch window for X_i containing NN values of $D_{i,j}$ for the epoch window of size $N \times N$. For each epoch the DDM is computed by taking the difference between the paired values in the epochs from each time series, which for the bivariate case we denote as $D1_{i,j}$ and $D2_{i,j}$:

$$E_{i,j} = \left| D1_{i,j} - D2_{i,j} \right| \tag{5}$$

where $E_{i,j}$ represents the differenced epoch window for the first series etc., and i, j are the time points in a particular epoch. Note that for example in the case where $N = 3$: (i) the dynamics included in the comparison can range over 4 periods, as each point in itself can represent a change in the distance matrix; (ii) the $E_{i,j}$ matrix incorporates both lead and lag dynamics as it includes off-diagonal elements as well; and (iii) that the range in values for $E_{i,j}$ is from 0 to $\max\{D1_{i,j}, D2_{i,j}\}$. A value of $E_{i,j} = 0$ clearly denotes complete synchronization between the two series.

Finally we take the average value of the components of $E_{i,j}$:

$$DDM = \frac{\sum_{i,j=1}^{N} E_{i,j}}{N^2} \tag{6}$$

to obtain a DDM which represents the total dissimilarity between $D1$ and $D2$ for a particular epoch. This process can be done for a single variable against another variable (as is shown above) to create a synchronicity-proxy or can be repeated for each possible pair of time series so as to create a "super" dissimilarity matrix for all variables by epoch. In the latter case, the dissimilarity matrix at each time step is then averaged to estimate the total dissimilarity between members of the set for a particular temporal window. The final product is then a one dimensional time series representing the synchronization in dynamic between members of a set with smaller values indicating greater synchronicity.

Once the absolute differences have been evaluated for a set of variables they can also be plotted to show the "within-group" average level of dissimilarity between all the members of the group.

To summarize then, the methodology is as follows:

(i) Cumulate all the (signed or unsigned) series;
(ii) Form a distance matrix $D_{i,j}$ for the cumulative series by calculating the distance of every point from every other point, then squaring, sum and square root;
(iii) Now form an epoch window over the set of cumulative distance measures $D_{i,j}$ which we label as matrix $D1_{i,j}$;
(iv) Now subtract the matrix $D2_{i,j}$ from the equivalent matrix $D1_{i,j}$ to form another matrix, $E_{i,j}$;
(v) Average the values of $E_{i,j}$ to obtain a dynamic dissimilarity/synchronicity measure between the two series.

Although the method described above is similar to the approach described in [18] for finding optimal lag or lead structures, the present method is not concerned with lead or lag structures but is solely concerned with using the general approach to construct a non-parametric measure of synchronicity. This DDM described here was first applied by [19] to EU data to show how signed macroeconomic synchronicity between European Union member states is intermittent, and in this paper we use an unsigned (Euclidean distance) measure as a means of assessing synchronicity in small samples.

3 Application to US State Macroeconomic Data

3.1 Background

Given the longevity of the US monetary union, which is partly due to the fact that the monetary union was part of the political union that took place at the same time in 1776, one would expect a high degree of economic convergence between its constituent parts. The reason for this expectation is that policies enacted at the federal level, most notably fiscal and monetary policy, should have provided a common component which could be found across all 50 states.[3]

Of course being part of an economic and monetary union could also generate specific industry dynamics which give rise to agglomeration effects, and hence faster growth in a specific location (for example technology in relation to Silicon Valley in California, or banking and securities in relation to New York), but if these location effects are spread fairly evenly across the country, then these effects will likely not overpower the impact of federal policies at the State level. At the same time, similar regional characteristics might come into play here as certain industries (such as agricultural industries) might dominate regionally, giving a higher degree of regional co-movement.

This clearly merits some exploration, given that the US has long been regarded as an optimal currency area (OCA).[4] But why is this the case? Obviously the fiscal record of each US State government is not a significant factor as it would be in the European Union, as most US States have enacted balanced budget amendments so relatively little debt is issued compared with Gross State Product (GSP) (California is a recent exception to this, as was New York State back in the 1980s). Several papers

[3] Of course fiscal policy enacted by Congress can be aimed at a particular set of States (for example disaster relief after a hurricane), or its impact might incidentally give greater benefits to a specific state (for example defense spending in relation to the Californian economy). Similarly monetary policy that benefits financial institutions might have a greater impact on those regions of the country that have a greater concentration of financial industry (such as New York and Illinois).

[4] There is little research as to the nature of the US monetary union in terms of its macroeconomic characteristics. This is partly due to the severe data limitations on availability of State macroeconomic data.

have established that the US can be regarded as an optimal currency area not only because of the convergence in many macroeconomic measures, but also because of the perceived synchronization between most US States and macroeconomic measures for the country as a whole (see [20] for an example in relation to globalization and in particular an unpublished paper by [21]). Of course the major policy measures are taken at the Federal level, not only by Congress through US fiscal policy but also there is the Federal Reserve (Fed) monetary policy. Not only this, but also if there are regional shocks which depress certain States, there is a relatively high degree of labor mobility due to a high degree of linguistic and cultural homogeneity. Thus most economists view the major criteria for being a single currency area as largely met in the US (usually for counter-factual reasons given the longevity of the arrangement), and therefore because of the common monetary policy one might expect a high correlation of business cycle variable movements between the participants of the US monetary union. This is the basic thesis for the application presented in this application, which explores the nature of the co-movement in growth and other business cycle variables.

3.2 Business Cycles and OCA Theory

The standard tool used in economic literature to evaluate the adequacy of a monetary union is the OCA theory, originated by [22, 23], with refinements by [24] and [25]. The OCA theory compares the benefits and costs to countries participating in a currency area. Benefits include lower transaction costs, price stabilization, improved efficiency of resource allocation, and increased access to product, factor, and financial markets. The main cost, however, is the country's loss of sovereignty to maintain national monetary and exchange rate policies. Both costs and benefits depend on the nature of exogenous shocks affecting potential member countries and the speed with which they adjust to them. The costs tend to be lower (higher) if shocks are symmetric (asymmetric) and market mechanisms are quick (slow) to restore equilibrium after the shock. Nonetheless, the existence of heterogeneities across countries does not necessarily imply that monetary integration cannot be achieved. This follows from the endogeneity argument—originally proposed by [26], which suggests that countries become more similar when they form a monetary union.

The synchronicity in movement of economic growth rates is economically important for 2 underlying reasons:

1. the more globalized the world becomes, the more likely that trade and financial flows will cause greater "synchronization" in growth rates between countries—known in the literature as the "international business cycle"; and
2. for collections of administrative entities that use the same currency (such as the US dollar, the Canadian dollar and the euro area member states of the European Union), similar movements in economic growth rates can either indicate

(i) *ex-ante* the suitability for adopting the same monetary policy (the optimal currency area (OCA) theory[5]); or

(ii) *ex-post*, the fact that monetary policy has been a factor in making these countries have similar patterns of growth (the endogenous OCA theory).

There has long been recognition of the propagation phenomenon of business cycles between countries (the main mechnanisms being trade and capital flows). The main indicator of this propagation is the synchronicity of turning points in business cycles (noted by [27, 28] in the real business cycle literature) but what is not recognized here is that the economic growth dynamic between these turning points (usually the recessions or peaks of business cycles) can be radically different between countries or in this case, US States. This observation has given rise to the notion and study of growth cycles in the context of the dynamic of economic growth between these turning points (see [29, 30]). From an empirical perspective there have been some efforts to empirically extract cycles for measurement and comparison across countries using frequency domain techniques (see [31–33]) but only limited research has been conducted in this area.

In the US, as the US dollar has been the adopted currency of the US for so long (despite the private printing of notes in the 19th century), according to the theory it should clearly be an OCA *ex-post*, and indeed many studies have shows that the majority of US States do exhibit high correlations in growth dynamics, but some research has indicated that the geographic extremes of the country (Hawaii, Alaska and Florida in particular) do exhibit some independent growth dynamics. Little research has been done in this vein in terms of analysis of US State business cycle synchronicity using measures other than conventional correlation measures.

Only in the last decade has the question been asked as to whether increased-break business cycle synchronization is driven more by global or regional factors, and whether this has changed over time. Research by [34] first noted that cyclical convergence was much more a global rather than a regional phenomenon, but more recently, using spectral analysis [35] showed that the convergence at lower frequencies was due to common cycles, in other words globalization. In the latter study though [35] only used the US, UK and the euro area to assess this, so this could have been due to anomalies associated with the UK situation rather than being a general result. [20] provides strong evidence in support of the conventional wisdom that rising global integration over time, through either trade or foreign direct investment flows, raises a state economy's business cycle correlation with the world economy. Interestingly openness to trade and investment promotes greater business cycle synchronization within regional US economies than with the rest of the world. If our results mirror those found by [20], then we should expect to find that there is a trend to greater synchronization between US State data over time.

To summarize, we are assessing whether the similarity in business cycle variables (economic growth, inflation and unemployment) changes over time, and whether the variance in synchronicity between States and the US aggregate has changed through time.

[5] The original and seminal contribution here was made by [22].

3.3 Data

There is very little purely macroeconomic data available by US State, but we select three variables directly related to the business cycle, namely:

3.3.1 Economic Growth

Here we measure economic growth at time t, as g_t, by taking the real Gross State Product (GSP) at time t, y_t, and transforming it by taking natural log first differences as follows:

$$g_t = \ln(y_t) - \ln(y_{t-1}) \tag{7}$$

Unfortunately this dataset is only available from 1987, so once log first differences are taken, the data runs from 1988 to 2011, giving 23 datapoints. The data is sourced from the Bureau of Economic Analysis (BEA). Data was available by State and for the US as a whole.

A sample of these economic growth rates is plotted. Figure 2 shows the data for the States in one particular region—notably the BEA Southwest region, comprising Arizona, New Mexico, Texas and Oklahoma.

Figure 2 (left) shows that even with specific geographical regions of the US, growth rates at any point in time can be quite different, but that in general the turning points in the business cycle (most notably the 2008–2010 US recession) are synchronized. Even so, it is clear that Arizona had a much deeper recession than Texas did, which was likely due to the fallout from the housing market collapse, where Arizona had much more highly inflated prices than Texas.

Synchronization in growth rates might be expected to be much less coordinated in more geographically dispersed parts of the country though, most notably in Alaska and Hawaii, so in Fig. 2 (right) we plot the growth in real GSP for Alaska, Hawaii,

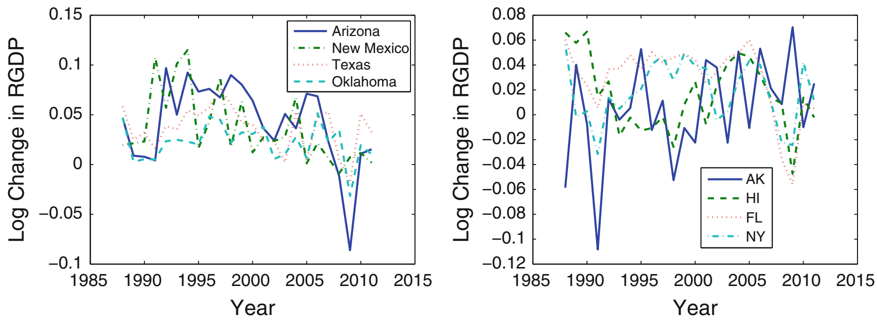

Fig. 2 (*Left*) Real GDP growth rate for South West US States. (*Right*) Geographically dispersed US State GSP growth

Florida and New York (New York is selected to represent a highly synchronized US state).

Interestingly Alaska appears not to have been affected by the recent economic downturn until several years later, and to a much smaller extent whilst Hawaii appears to have been affected with a lag, but as severely as Florida. In the growth period there appears to be little synchronization in growth during this period with large 2 year cycles in Alaska, and smaller growth cycles in other States.

As economic growth is considered to be an important measure of the health of the economy, we also reproduce the figures from the BEA which show the latest levels of growth in individual states in Appendix A.

3.3.2 Inflation

Here this is proxied by the GSP deflator, as a Consumer Price Index (CPI) is only available for urban areas, and so does not cover all States, and even when there is a major urban center in a specific State, it might not reflect prices for the whole of a larger State. Once again the natural log first difference is taken (to create the equivalent of an inflation rate), and also the data is similarly sourced from the BEA[6] and contains just 23 datapoints. This dataset had to be derived from BEA data on real GSP and nominal GSP.

Figure 3 (left) shows the data for the Southwestern US States, and Fig. 3 (right) shows the data for the Far Western US States. In Fig. 3(left) the inflation data for New Mexico, Texas and Oklahoma are fairly synchronized in the last growth phase after the 2001 recession, and move very closely together into a deflationary period during the recent downturn from 2007–2009, before rebounding in 2010. Arizona however does not appear to follow the trend of the other 3 States—although inflation slows in the late part of the last decade, there is actually a slight pickup in inflation going into the recession, followed by a fall in inflation as there is a pickup in inflation elsewhere in the region. In the Far West region of the US, Fig. 3 (right) shows that the States concerned exhibit a very narrow range of inflation rates in the early part of the period, but the range of inflation rates widens out in the late 1990s with inflation highest in Nevada and lowest in Oregon. There appears to have been greater synchronicity in inflation during the most recent downturn, although Oregon appears to have experienced lower inflation rates than the other 3 States throughout the period.

3.3.3 Unemployment

This is taken as the usual definition of the unemployment rate, i.e., the number of unemployed divided by the labor force. This is available from 1976 onwards, both

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[6] Two series had to be spliced together to create this series. Details are available from the author on request.

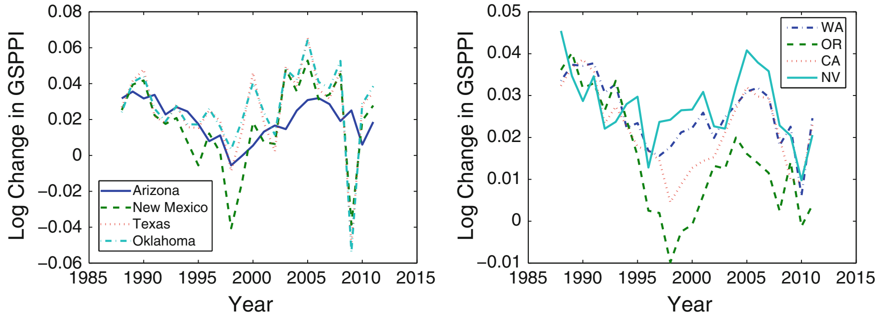

Fig. 3 (*Left*) GSPPI inflation for Southwestern US States. (*Right*) GSPPI inflation for Far West US States

monthly and annually from the Bureau of Labor Statistics. Two versions of this series are used in the analysis—one that uses the full data span available and the other which uses only the data from 1988 to 2011 so as to be comparable with the economic growth and inflation variables detailed above.

Unemployment is usually viewed as a lagging indicator when referencing the business cycle, and in Fig. 4 (left) for the Southwestern US States, this appears to be very much the case. Unemployment magnitudes are, however, different accross the region with unemployment rates peaking at around 7 % in Oklahoma but at over 10 % in Arizona. One of the worst parts of the country to be hit by large increases in unemployment rates was the Great Lakes region, and the relevant unemployment rates are shown in Fig. 4 (right). Perhaps the most interesting aspect of this figure is the fact that for all these US States the unemployment rates were higher in the early 1980s recession than in the most recent economic downturn. Once again, the movement in the unemployment rates is fairly well synchronized across all the States.

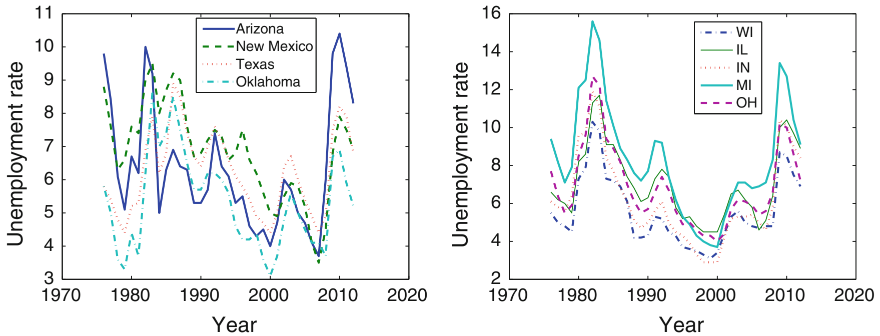

Fig. 4 (*Left*) Unemployment rate for Southwestern US States. (*Right*) Unemployment rates for the Great Lakes US States

3.4 Results

Results here are presented for each US State vs the US aggregate. An alternative approach which is currently being explored is to look at intra-State synchronicity and then average these over all States or a collection of States (e.g. for a specific geographical BEA region).

3.4.1 Economic Growth

In this section we review the results of applying the technique above to US state economic growth data, which we plot by BEA region. As shown in appendix A there are 8 BEA regions. As some US states are extremely large relative to the rest of the US, we construct aggregates for each State for real GDP such that the aggregate represents [US real GDP—State i GDP] and then conduct the dissimilarity exercise on the State vs this aggregate. For purposes of statistical testing, we use the 95 % confidence limit on the actual distribution of States.

In Figs. 5 and 6 we display the results by State collected by BEA region In Fig. 5 (left) it is noticeable that Oregon and Hawaii track into the shaded (significant) area in the figure denoting significant asynchronicity with the US as a whole. Another feature of all these figures relating to real GDP growth is the fact that most of the US states clearly had similar dynamics going through the last major economic downturn in 2008 (this represents the window centred on 2008, so covers 2007–2009), but that there are some notable exceptions. So for example in 2008, in Fig. 5e North Dakota appears to have been an exception, and did not experience the same recessionary dynamics that the rest of the country faced, and also Virginia in Fig. 6b and Wyoming in Fig. 5f appear to have faced somewhat different growth dynamics during the recession. Overall, for the period as a whole there appears to have been a slight increase in synchronicity in growth (as measured by the fall in dissimilarity), which mirrors the results of [20].

In Fig. 7 (left) both the mean dissimilarity and the standard deviation of the DDM are plotted. The results clearly confirm the increase in synchronicity documented earlier, and a small fall in standard deviation.

The kernel estimate of the PDF is given in Fig. 7 (right). The 95 % confidence interval under the null hypothesis of similar dynamics is shown in the figure and is at 0.2102.

As a robustness check on the qualitative results obtained above, we repeat the exercise in Appendix B for personal income over a longer time period and using quarterly data. The qualitative results are similar although not identical, but that is not surprising as personal income is a nominal variable and also it is only one component of GDP.

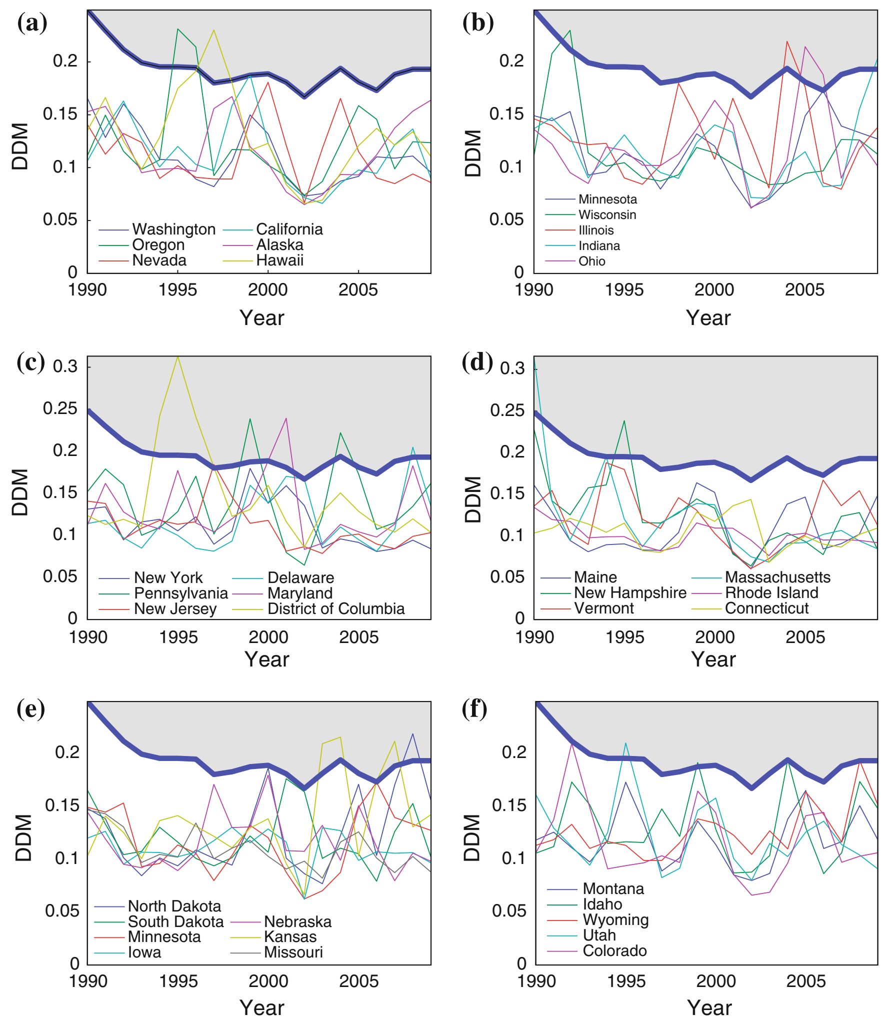

Fig. 5 Real GSP growth of **a** Far West State. **b** Great Lakes State. **c** Mid East State. **d** New England State. **e** Plains State. **f** Rocky Mountain State versus US national aggregate

3.4.2 Inflation

In this section we repeat the exercise conducted above but instead for the growth in the real GSP deflator, which is derived from simple calculations using published estimates of state real GSP and state nominal GSP.

Figures 8 and 9 show the dissimilarity plots for the real GDP deflator variable. Here the pattern is a little different, with a clear divergence in GDP deflator growth rates in the 2007–2009 downturn, so noticeably at the 2008 mid-point. Clearly the

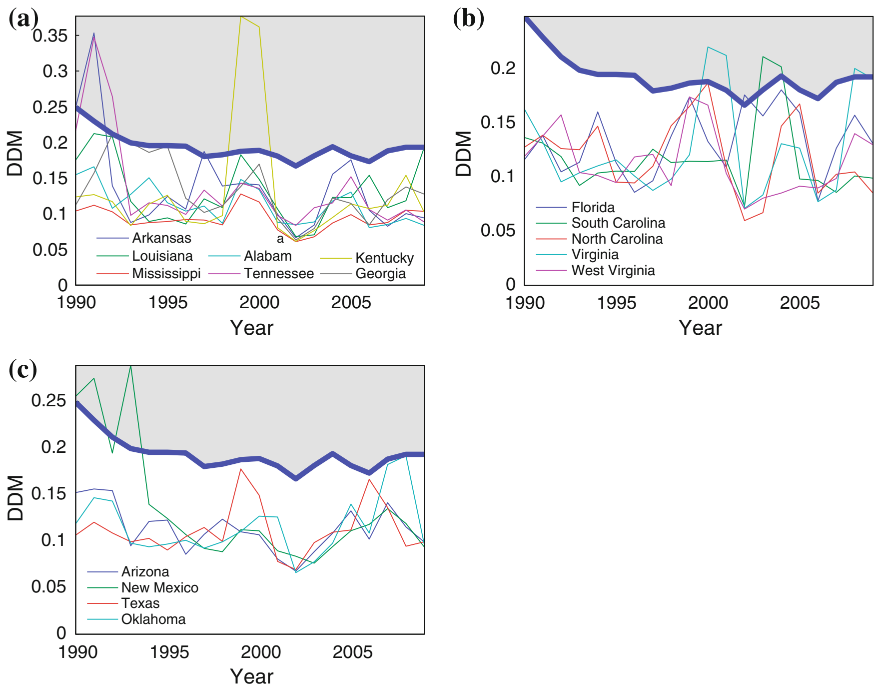

Fig. 6 Real GSP growth of **a** South East State. **b** South East State. **c** South West State versus US national aggregate

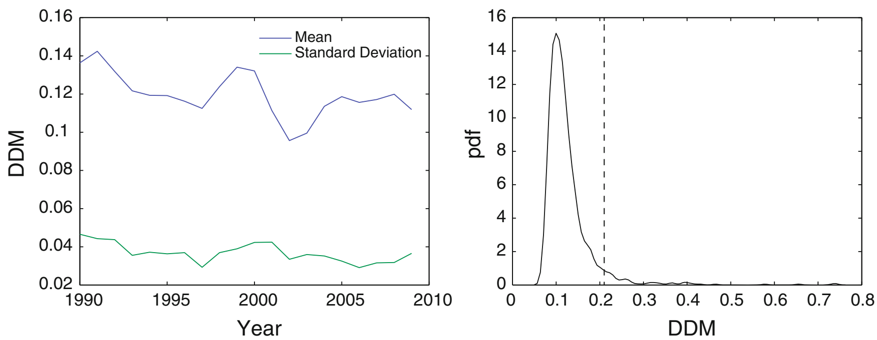

Fig. 7 (*Left*) Mean and standard deviation of dissimilarity measure for real GSP growth versus US aggregate. (*Right*) Kernel density estimation for US real GDP growth synchronization measure

distribution of different inflation dynamics widens out on entering the recession but is more convergent coming out of the recession in 2009. Oregon, Delaware, Ohio and West Virginia appear to have had quite different dynamics from other states during the last recession. The general trend in synchronicity though is definitely towards

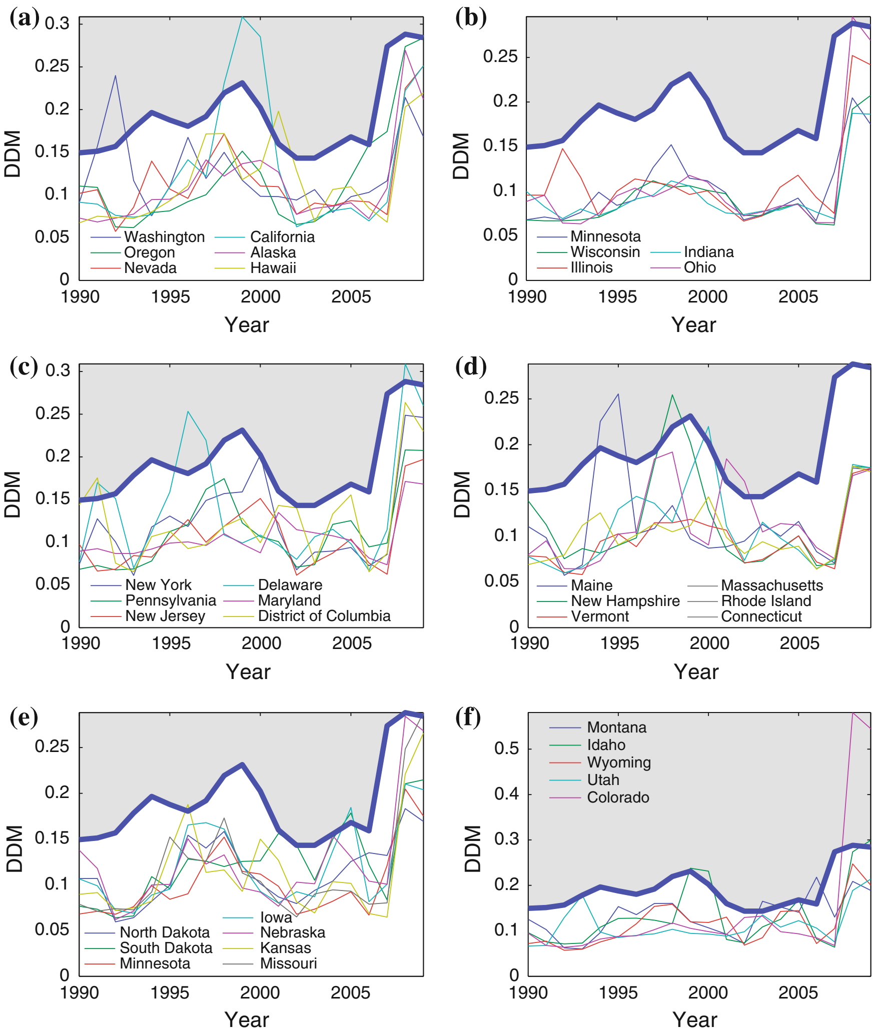

Fig. 8 Real GSP deflator growth of **a** Far West State. **b** Great Lakes State. **c** Mid East State. **d** New England State. **e** Plains State. **f** Rocky Mountain State versus US national aggregate

less synchronization during the last downturn and recovery and likely is related to the differing experiences of States to the housing boom and bust before and during the last recession.

The mean and standard deviation of the DDM are plotted by year in Fig. 10 (left). There has been a clear decrease in synchronization after 2000, and in fact greater variation in levels of synchronicity since 1995. This greater dispersion could reflect a widening gap between the States which contain major urban areas, and those which

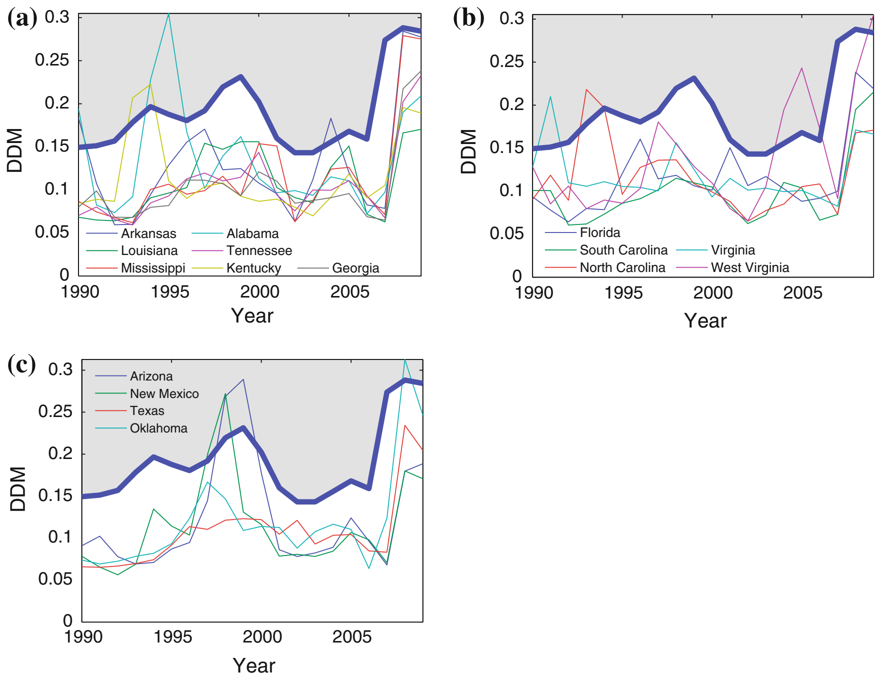

Fig. 9 Real GSP deflator growth of **a** South East State. **b** South East State. **c** South West State versus US national aggregate

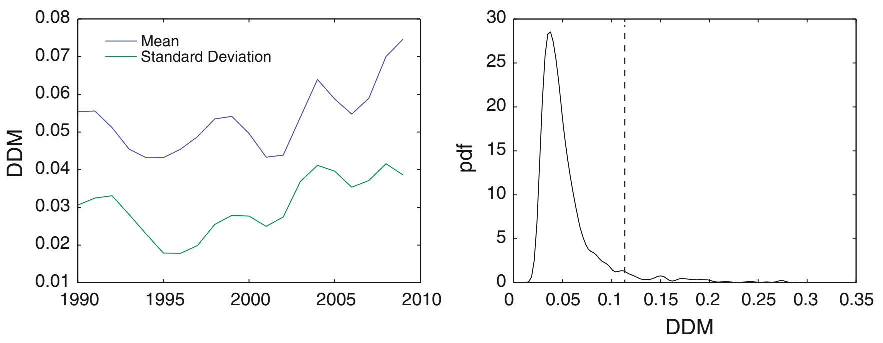

Fig. 10 (*Left*) Mean and standard deviation of dissimilarity measure for inflation vs US aggregate. (*Right*) Kernel density estimation for US GSPPI growth synchronization measure

are still mostly rural, in terms of their housing markets and also in terms of a general widening in the cost of living between these two types of States.

The estimate of the kernel density function is provided in Fig. 10 (right). The 95 % confidence interval under the null hypothesis of similar dynamics is shown in the figure and is at 0.1137, indicating a greater degree of similarity in inflation dynamics between US States than for economic growth.

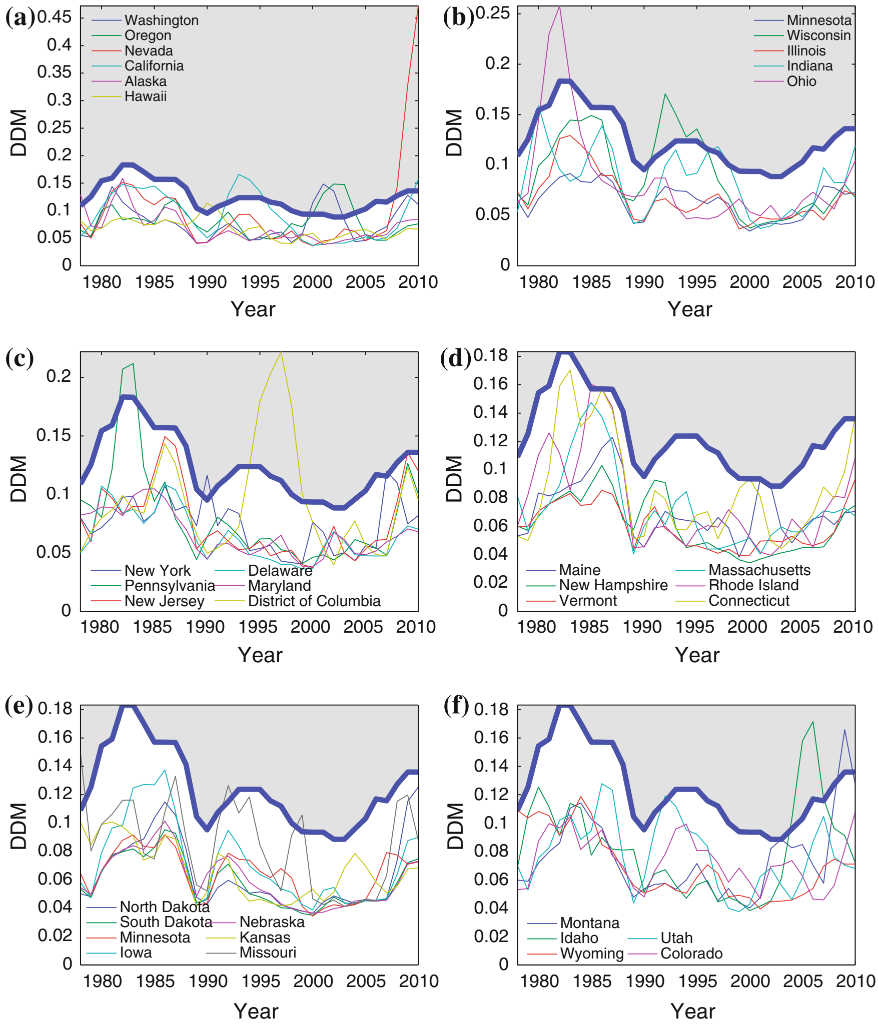

Fig. 11 Unemployment rates of **a** Far West State. **b** Great Lakes State. **c** Mid East State. **d** New England State. **e** Plains State. **f** Rocky Mountain State versus US national aggregate

3.4.3 Unemployment

Here we use unemployment rates as defined by the Bureau of Labor Statistics (BLS) as the basis for creating dissimilarity indices by State.

In Figs. 11 and 12 we present the same synchronization exercise for the State unemployment rates against the US rate. Here the dynamics of unemployment across the states have been quite similar, in the sense of there being a falling degree of synchronicity going into 3 of the 4 past recessions, but a rising degree of synchronicity coming out of these recessions (with the notable exception of the 2001 recession).

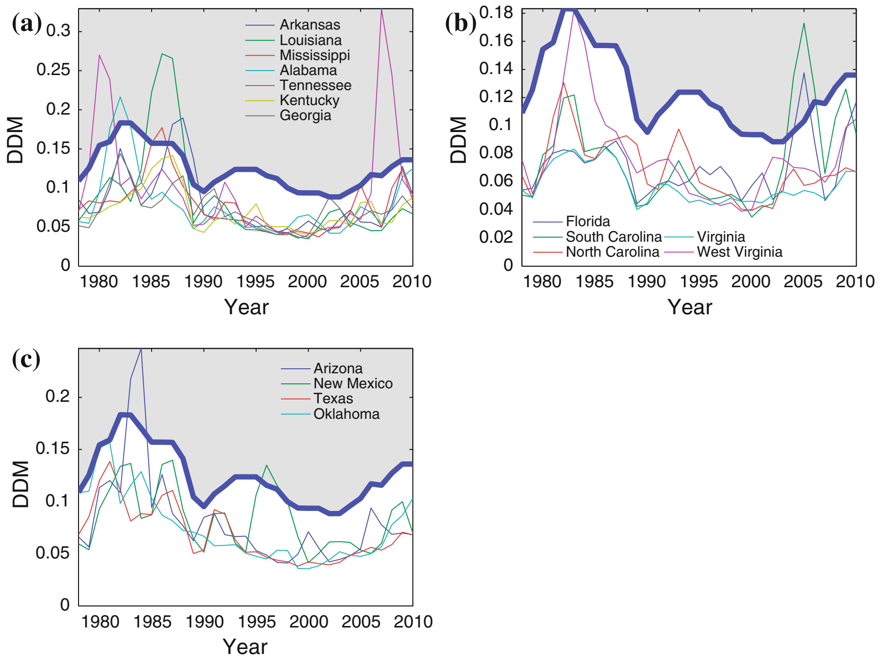

Fig. 12 Unemployment rates of **a** South East State. **b** South East State. **c** South West State versus US national aggregate

What is noticeable here though is that the general trend was, up until the last recession, an increase in synchronicity of unemployment rates. Also it is noteworthy that the spike in dissimilarity during the last downturn in economic growth was not as high as for the recession of 1982–1983. In Fig. 13 (left) the mean and standard deviation of the DDM are now plotted, and show a decrease in synchronicity since 2000, with

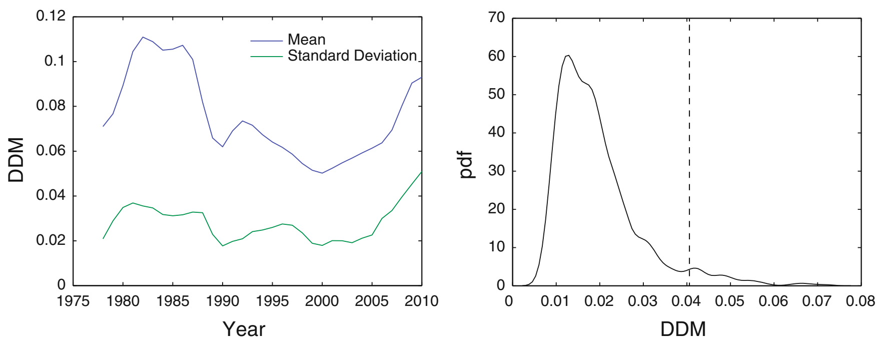

Fig. 13 (*Left*) Mean and standard deviation of dissimilarity measure for unemployment rate versus US aggregate. (*Right*) Kernel density estimation for US unemployment rate synchronization measure

the mean dissimilarity rising during the 2000s and then accelerating upwards going into the last recession. The standard deviation plot seems to have been on a slow decline up until 2005, when the dispersion of unemployment rate synchronicities appears to have widened. This is an unexpected result, as the popular perception is that unemployment generally in the US was highly synchronized between US States during the economic boom of the mid-2000s.

Once again, the kernel density estimate for the pdf of the dissimilarity measure is shown in Fig. 13 (right) and the 95 % confidence interval under the null hypothesis of similar dynamics is shown in the figure and is at 0.0406, which signifies much greater synchronization than for either economic growth or inflation. As the unemployment rate is the ratio of two large stock variables, and economic growth is the rate of change of a flow variable, this greater dynamic conformity with the unemployment rate is to be expected.

4 Conclusions

The main purpose of this paper is to present a measure of time series synchronicity, that is particularly suited to small samples, and is derived from the recurrence plot approach. The measure is non-parametric, is not dependent on stationarity of data and is fully flexible in terms of encompassing specified lead and lag dynamics. The measure was successfully applied to the evaluation of synchronicity in an example of macroeconomic data for US States compared with the US aggregate. In the latter case we have used the measure as a means of detecting whether synchronicity in macroeconomic variables measured at the US State level occurs consistently through time, and whether there are certain US states that are less synchronous with the US on aggregate than others.

Our findings are that the measure shows the time-varying level of synchronicity for a small dataset consisting of US State macroeconomic variables. In the latter case, although regions are fairly well grouped together in terms of similar economic dynamics, there are some notable differences within regions, whereby states like Alaska, which is geographically part of the Far West region, nevertheless seems to have a rather different business cycle from the rest of the country. The results indicate that synchronicity has been on upward trend for real GDP growth and on a downward trend for the unemployment rate, but that there has been little change in synchronicity of inflation rates. Our results also confirmed the long-standing finding that growth synchronicity among the US states seems to be at its highest when entering a recession, and least similar when exiting a recession.

In terms of future research, one of the uses of this methodology that would be informative in the sphere of economics[7] would be to test whether being in a monetary union causes greater macroeconomic synchronicity in business cycle variables—this would require data on similar variables for other monetary unions and then data from

[7] We acknowledge one of the reviewers of this paper for suggesting this approach.

a collection of countries that are not part of monetary unions to act as a control group. In terms of the methodology itself, future research could also employ varying epoch window sizes (with more temporally disaggregated data) to test if the results are robust to such changes, and if so whether an optimal window size could be derived.

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Appendix A: BEA regions

Here we duplicate the most recent results from the BEA for real GSP growth. In Fig. 14 the real GDP by State is given for 2012. As can be seen, the 8 BEA regions do not correspond to the 12 Federal Reserve districts,

Figure 15 also shows the economic growth dynamic following the last economic downturn by BEA region. Clearly the Southwest has had the largest rebound, and the New England area has had only a moderate rebound since the last recession.

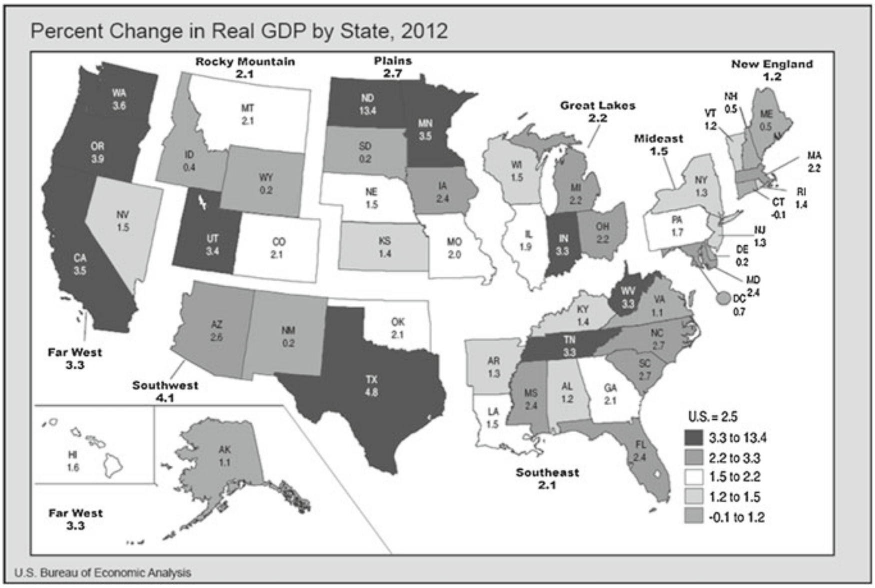


Fig. 14 Real GSP growth by US State for 2012

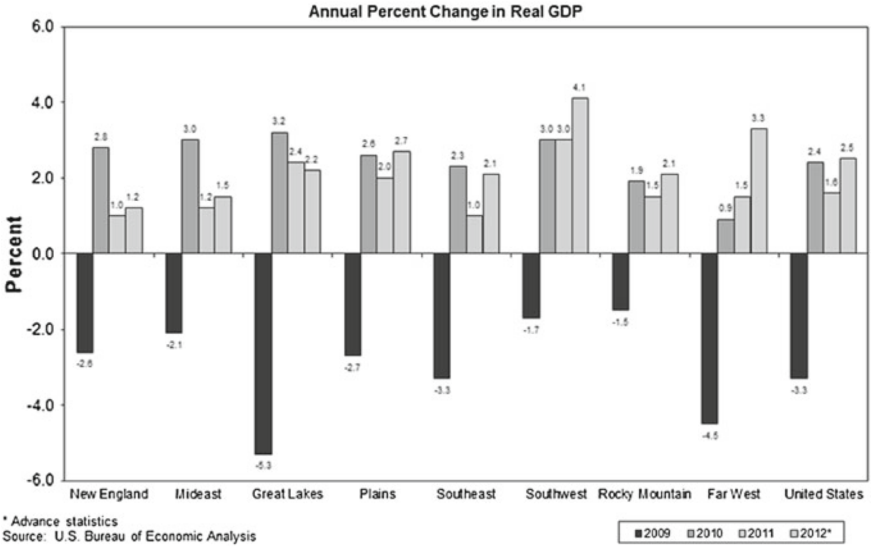
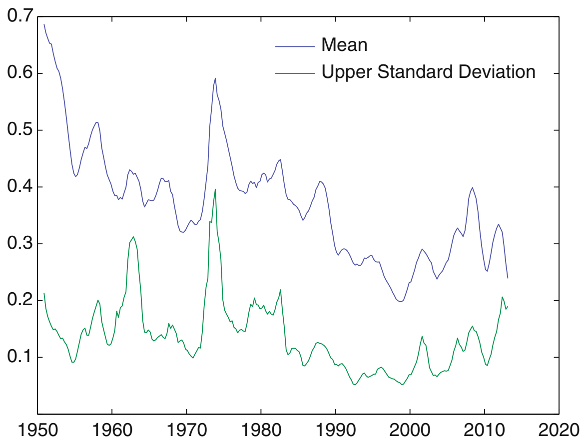


Fig. 15 Economic growth dynamic by BEA region

Fig. 16 Mean and standard deviation of dissimilarity measure for personal income versus US aggregate



Appendix B: Personal Income

Here, as a robustness check for our results as applied to US State macroeconomic data, we report the results of an identical exercise, except with personal income (sourced from the BEA) by State which contains quarterly data from 1950. We use a 12 quarter epoch window, which is equivalent to the 3 year window used with the annual data for real GDP growth. Figure 16 shows the mean and standard deviations for the dissimilarity metric. Once again, the trend in the mean is downwards, with an uptick on both entering and exiting the last recession—this is a difference with the real GDP measure, which only showed an increase on exiting the recession. On exiting the recession, the standard deviation also increases.

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