Application of Reliability Growth Model in Step-Down Stress Accelerated Storage Test

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Abstract Step-down accelerated storage test is equalled to reliability growth, a method of step-down stress accelerated storage test based Army Materiel System Analysis Activity (AMSAA) model is proposed, according to the step-down stress, the mean time between failures (MTBF) under normal stress can be obtained, by using the AMSAA model. First, through the product's cumulative failure time, cumulative failure numbers and based on AMSAA model, the product's instantaneous MTBF is shown. Then the step-down stress is divided into n ladders, and there is only one fault under each stress, the joint probability density function is shown by the cumulative failure time of each step-down stress, and the parameters of the AMSAA model are estimated by the maximum likelihood estimation, then the point estimation of the product's instantaneous MTBF is got. By choosing instantaneous MTBF of certain accelerated stresses and combined respective physical accelerated model, the product's storage lifetime is estimated. Finally, a case study is performed using this method. The effectiveness of this method is shown, for the point estimation of each parameter is little different. Thus it provides a new evaluation method for step-down accelerated storage test.

Keywords Step-down stress · Reliability growth · AMSAA model · MTBF · Physical accelerated model

1 Introduction

As for missile products, it also has an important characteristic besides use function, that is, it needs the long-term storage before using, therefore carrying out storage test of missiles has a strategic significance. The assessment of storage reliability is very difficult under normal stress level; however, the problem can be solved well by

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the accelerated life testing (ALT). Based on the differences of the stress loaded, ALT can be divided into as: constant stress accelerated life testing, step-stress accelerated life testing, and progressive stress accelerated life testing. At present, among the accelerated life testing, the application of constant stress accelerated life testing is comparatively mature. But tests at constant,the required samples are comparatively large and the testing time is also relatively long [1], on the contrary, step-stress accelerated life testing conquers these problems, and gets more preference in the engineering. During step-stress accelerated life testing, there is little fault under initial low stress, thus the initial stress and test time are not easily controlled.

Considered above problems, Zhang and Chen [2, 3] proposed a method of step-down accelerated life testing, and studied the effectiveness by theoretical model, Monte Carlo simulations, and comparative experiments. Wang and Zhang [4] proposed a new optimizing design method of step-down accelerated life testing based on Monte Carlo simulations. Tan [5] built the data conversion formula of step-down accelerated life testing by accelerated model, and then made improvement for the three step method of step-down test. Xu [6] compared the effectiveness between step-down test and step-stress test by Monte Carlo simulations, and obtained the condition when the efficiency of the step-down test was superior to that of the step-stress test.

According to the thought of reliability growth, this paper puts forward a method of step-down accelerated storage testing based on Army Materiel System Analysis Activity (AMSAA) model, which divides step-down stress into n ladders; sets there is only one failure under each stress; describes the joint probability density function by the cumulative failure time of each step-down stress; and estimates the parameters of the AMSAA model by the maximum likelihood estimation to obtain the point estimation of the product's instantaneous MTBF. We combine instantaneous MTBF of certain accelerated stresses with relative physical model to estimate the storage lifetime of products. This paper selects temperature as accelerated storage stress and Arrhenius model as accelerated storage model.

2 AMSAA Model in the Step-Down Accelerated Storage Testing

2.1 *Calculate the Instantaneous MTBF of Products by AMSAA Model*

When we exert step-down stress on products, at the beginning, there will be a lot of failures, with the reduction of stress level, the fault will gradually decrease. Obviously, more fault information can be obtained by the step-down accelerated stress testing. We can use Reliability Enhancement Test (RET) to ascertain the highest stress level which can load on products [7], as long as the initial stress is proper, the failure mechanism will not be changed. In this experiment, the cumulative number of failures is a stochastic process, which exactly corresponds with the theory of AMSAA model; According to AMSAA model, the cumulative number of failures is an inhomogeneous Poisson distribution.

When we performance the step-down accelerated storage testing for repairable electronic products, according to the relation between the cumulative number of failures and the accumulated accelerated storage time, the instantaneous failure rate of products is given by [8–11]:

$$\lambda(t) = abt^{b-1} \tag{1}$$

where a > 0 is a scale parameter, b > 0 is a shape parameter. The instantaneous MTBF of products is the inverse of *instantaneous* failure rate, which is shown as follows:

$$M(t) = \frac{1}{\lambda(t)} = \frac{1}{abt^{b-1}} = \left[abt^{b-1}\right]^{-1} \tag{2}$$

2.2 Statistical Analysis of AMSAA Model

Due to AMSAA model is established based on rigid stochastic process theory, in the step-down accelerated storage lifetime testing, when we use AMSAA model to estimate the storage lifetime of products, there exists a series of statistical analysis methods, that is, the censored time and the censored data, and this paper chooses the censored data.

Suppose that the number of failures at each stress condition is $N_{S_1}, N_{S_2}, \ldots, N_{S_n}$, and its corresponding failure time is $t_{S_1}, t_{S_2}, \ldots, t_{S_n}$. Then we can get the accumulated failure is $t_j = \sum_{i=1}^{j} t_{S_i}$, and the accumulated number of failures is $N(t_j) = \sum_{i=1}^{j} N_{S_i}$.

Hence, the likelihood function of the data of the censored data can be obtained by:

$$\mathrm{f}(\mathrm{t}_{\mathrm{S}_1}, \mathrm{t}_{\mathrm{S}_2}, \ldots, \mathrm{t}_{\mathrm{S}_n}) = (\mathrm{ab})^{\mathrm{N}(\mathrm{t}_n)} \mathrm{e}^{-\mathrm{at}_n^{\mathrm{b}}} \times \prod_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{t}_{\mathrm{j}}^{\mathrm{b}-1} \tag{3}$$

Then in the AMSAA the maximum likelihood estimation of (a, b) is given by:

$$\widehat{\mathrm{b}} = \frac{\mathrm{N}(\mathrm{t}_\mathrm{n})}{\sum_{\mathrm{j}=1}^{\mathrm{n}} \ln \frac{\mathrm{t}_\mathrm{n}}{\mathrm{t}_\mathrm{j}}} \tag{4}$$

$$\hat{a} = \frac{\mathrm{N}(\mathrm{t}_\mathrm{n})}{\mathrm{t}_\mathrm{n}^{\hat{b}}} \tag{5}$$

Thus when the step-down stress is down to relative accelerated stress, the instantaneous MTBFi can be obtained, and the maximum likelihood estimation of MTBF is given by:

$$\hat{\theta}(t_j) = \left[\hat{a}\hat{b}t_j^{\hat{b}-1}\right]^{-1} \tag{6}$$

3 The Computation of Storage Lifetime by Accelerated Model

This paper selects temperature as accelerated storage stress, thus the accelerated model is Arrhenius model, which can be written as:

$$\xi = Ae^{E_a/kT} \tag{7}$$

where ξ denotes some life characteristic, A is a constant, k denotes Boltzmann's constant, as $8.617 \times 10^{-5} eV/K, E_a$ denotes activation energy. From Eq. (7), with the increment of temperature, the life characteristic decreases exponentially. By taking the natural logarithm of both sides of Eq. (7), we can get:

$$\ln \xi = d + e/T \tag{8}$$

Based on Eq. (6), the instantaneous MTBF can be obtained at each temperature stress, in the step-down accelerated storage lifetime, we can choose some step-down accelerated stress, then combine the step-down accelerated stress with corresponding instantaneous MTBF$_\text{i}$, and get several groups of value of temperature and lifetime. After that, the least-squares estimator (LSE) $\hat{d}, \hat{e}$ of d, e can be obtained based on Eq. (8), and the storage lifetime under normal stress can be obtained.

4 An Illustrative Example

Assuming that assembly products' lifetime is an exponential distribution, $\theta = 2,000$h, temperature as the accelerated stress, the step-down stress accelerated test can be considered as reliability growth test, which the first step data T$_1$, that is the highest stress which is called the initial value of growth test, can be ascertained based on RET. This paper assumes that T$_1$ = 85 °C, when there exists only one failure, the test will be turned to the next temperature ladder, we take 2.5 °C as a ladder. Based on the above conditions, the simulation failure data is listed in Table 1.

Based on the Eqs. (4) and (5), the MLE $\hat{a}$ and $\hat{b}$ can be computed, then we can count the instantaneous MTBFi respectively from the highest stress condition 85 °C to accelerated stress condition 35, 40, 45, 50 °C, which are listed in Table 2.

Table 1 The cumulative failure data of assemblies

N	N (t_j)	T_i (k)	T_j (h)
1	1	358	4.4
2	2	355.5	10.7
3	3	353	16.4
4	4	350.5	22.5
5	5	348	33.9
6	6	345.5	46.4
7	7	343	59.6
8	8	340.5	79
9	9	338	94
10	10	335.5	132.9
11	11	333	165.8
12	12	330.5	211.2
13	13	328	297.2
14	14	325.5	369.1
15	15	323	473.8
16	16	320.5	607.7
17	17	318	866.2
18	18	315.5	1127.3
19	19	313	1393.6
20	20	310.5	1743.2
21	21	308	2553.8

Table 2 The MTBF of the different temperatures

Stress (T_i)	35 °C	40 °C	45 °C	50 °C
a_i	1.58	1.183	1.51	0.9032
b_i	0.3233	0.3835	0.3891	0.4447
t_j	2553.8	1393.0	866.2	473.8
$MTBF_i$	566 h	192 h	106.4 h	76.9 h

From the Table 2 we can get, when the highest stress growths to the different stress, the difference of each parameter is small. Thus it illustrates that the growth rate is considered as constant in the step-down stress reliability growth test, that is, we use the thought of reliability growth to carry out the step-down stress accelerated storage test is feasible.

Based on the Table 2, the relation between the instantaneous $MTBF_i$ under accelerated stress and corresponding accelerated stress is shown in the Fig. 1.

Based on the Eq. (8), the LSE of *d, e* can be obtained, that is, $\hat{d} = -36.85$, $\hat{e} = 13250$. Then the storage lifetime under normal stress can be extrapolated.

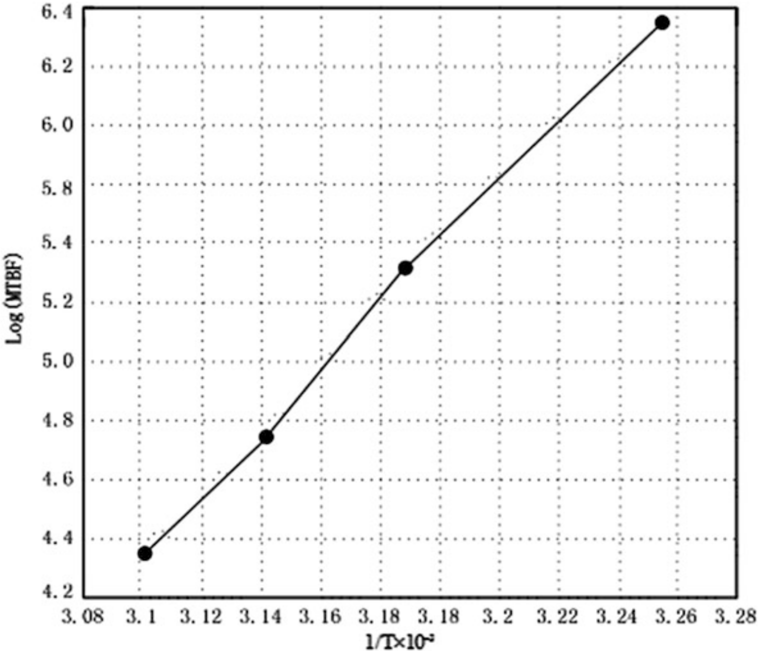

Fig. 1 The fitting chart between life and stress

$$\theta_0 = \exp(-36.85 + 13250/(273 + 25)) = 1998.2\text{h} \tag{9}$$

The result is nearly the same with real MTBF 2000 h of the assembly products.

5 Conclusion

This paper presents a method which uses the reliability growth model in the step-down stress accelerated storage test, which avoids the trouble of data conversion. Then we compute an example, the result of parameter estimation illustrates that, when the highest stress growths to different stress, the growth rate is constant, which correspond with the thought of reliability growth. Finally, the storage lifetime of the assembly products can be obtained by accelerated model, which is nearly the same with real MTBF. Thus it provides a new evaluation method for step-down accelerated storage test.

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