Particle Swarm Optimizations for Multi-type Vehicle Routing Problem with Time Windows

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Abstract. This paper presents a variant of vehicle routing problem with time windows (VRPTW) named multi-type vehicle routing problem with time windows (MT-VRPTW), which considers both multiple types of the vehicle and the uncertain number of vehicles of various types. As a consequence, the different combinations of multi-type vehicle will lead to diverse results, which should be evaluated by its own fitness function. In order to solve the proposed MT-VRPTW problem, six variants of particle swarm optimization (PSO) are used. The 2N dimensions encoding method is adopted to express the particle (N represents the number of demand point). In the simulation studies, the performances of the six PSO variants are compared and the obtained results are analyzed.

Keywords: Particle swarm optimization (PSO), Vehicle routing problem (VRP), Time windows.

1 Introduction

The vehicle routing problem (VRP) has been proposed and discussed for several decades. Considering different factors, researchers study this problem with various perspectives. In 1984, Solomon [1] designed a sequential insertion heuristics algorithm for vehicle routing and scheduling problems with time window constraints. In [2], Desrochers and Solomon introduced a branch-and-bound algorithm to get an excellent lower bound for the integer set partitioning formulation.

Compared with traditional optimization technique for the VRP, evolutionary computation has aroused increasing concerns in recent years. For instance, Prins [3] developed an effective evolutionary algorithm (EA) to tackle the VRP by spacing a smaller population of distinct solution. From another perspective, Ombuki and Hanshar [4] represented the VRP as a multiple objective problem and used Pareto ranking technique in genetic algorithm to obtain a set of equally valid solutions.

As one of effective swarm intelligence, particle swarm optimization (PSO) achieves a excellent effect when employed to solve some NP-hard problems such as

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traveling salesman problem (TSP) and knapsack problem. Similarly, many researchers implement PSO to solve the VRP. Chen and Yang [5] combined discrete PSO with simulated annealing (SA) algorithm to address capacitated vehicle routing problem. Considering multi-type vehicle and backhauls, Belmecheri et al. [6] introduced PSO with local search to solve vehicle routing problem with time windows (VRPTW).

Since the depot usually assigns a truck to meet a plurality of customer demands, single type vehicle cannot respond to the requirements flexibly. Therefore, we discuss a case of multi-type vehicle routing problem with time windows (MT-VRPTW) in this paper, which also considers a factor about the uncertain number of multi-type vehicles. Moreover, six PSO variants are employed to optimize this problem on account of the following aspects. First of all, we select global PSO (GPSO) with $w = 0.4$ [7] and GPSO with linearly decreasing w from 0.9 to 0.4 [7] as the fundamental variants altering the parameter. Secondly, three representative variants constructing neighborhood structure are expected to retain diversity of the individuals as well in this case, including unified particle swarm optimizer (UPSO) [8], fully informed particle swarm (FIPS) [9], and comprehensive learning particle swarm optimizer (CLPSO) [10]. Finally, multi-swarm cooperative particle swarm optimizer (MCPSO) [11, 12] utilizing the multiple swarms approach to search for global optimum is used.

The rest of this paper is organized as follows. Section 2 emphasizes to extend a mathematical model. Section 3 introduces various PSO variants. And Section 4 presents experiment study and the results. Conclusions are given in Section 5.

2 Description of MT-VRPTW

Based on the basic model for VRPTW [13], two extra key factors are involved. On one hand, we take multiple types of the vehicle into account. On the other hand, the number of vehicles of different types is uncertain. For simplifying the problem in the model, we only suppose and list two types of the vehicle. The number of vehicles of the first type is $K1$ and another type is $K2$. The extended mathematical formulation based on Kallehauge's basic VRPTW model [13] is presented as follows.

Define variable:

$$x_{ijk} = \begin{cases} 1 & \text{if vehicle k traverses route (i,j)} \\ 0 & \text{else} \end{cases} \tag{1}$$

Objective function

$$Minimize\ Z = \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} c_k * d_{ij} * x_{ijk} + pe * \sum_{i=1}^{N} \max(ET_i - t_i, 0) + pl * \sum_{i=1}^{N} \max(t_i - LT_i, 0) \tag{2}$$

where

$$t_j = \sum x_{ijk} (t_i + s_i + d_{ij} / v) \qquad for \quad t_0 = 0, s_0 = 0 \tag{3}$$

Subject to

$$
\begin{cases}
\sum_{j=1}^{N} x_{ijk} = \sum_{j=1}^{N} x_{jik} \le 1 & for\ \ i=0,\ 1 \le k \le K & (4) \\
\sum_{j=0}^{N} \sum_{k=1}^{K} x_{ijk} = 1 & for\ \ 0 \le i \le N & (5) \\
\sum_{i=0}^{N} \sum_{k=1}^{K} x_{ijk} = 1 & for\ \ 0 \le j \le N & (6) \\
\sum_{i=0}^{N} \sum_{j=0}^{N} x_{ijk} * q_i \le Q_k & for\ \ k=1,2,\ldots,K1\ \ and\ \ i \ne j & (7) \\
\sum_{i=0}^{N} \sum_{j=0}^{N} x_{ijk} * q_i \le Q_k & for\ \ k=1,2,\ldots,K2\ \ and\ \ i \ne j & (8) \\
\sum_{j=1}^{N} \sum_{k=1}^{K} x_{jik} = \sum_{j=1}^{N} \sum_{k=1}^{K} x_{ijk} \le K & for\ \ i=0 & (9) \\
0 \le \sum_{i=0}^{N} \sum_{j=0}^{N} x_{ijk} \le N & for\ \ 1 \le k \le K,\ i \ne j & (10)
\end{cases}
$$

where let t_i denote the time when vehicle k arrives at the ith customer. And the demands of the ith customer is q_i and the corresponding unloading time is defined as s_i (s_0=0). The distance between the ith and jth customer is d_{ij} and the time window is described as $[ET_i, LT_i]$. There will be a cost *pe* incurred by waiting while a cost *pl* would be incurred for lateness. Besides, all vehicles have same velocity which is denoted as v. And we use c_1 and c_2 to represent unit freight of vehicles of different types ($c_k = c_1$ when k =1, 2, …, $K1$; $c_k = c_2$ when k =1, 2, …, $K2$).

Equation (2) is the goal of this model that minimizes the total cost. Equation (3) is used to calculate the time point. Constraint (4) means that each vehicle departs from the depot and comes back the same depot. Constraints (5) and (6) point out that each customer must be visited once and only once by exactly one vehicle. Then, constraints (7) and (8) restrict the vehicle load to the specified scope. And constraint (9) explains the number of delivery vehicles within limited scope. Finally, constraint (10) indicates that the customers served by the kth vehicle must not exceed total clients N.

3 PSO Variants for MT-VRPTW

In this section, we introduce six algorithms in detail and the key properties of those optimization algorithms are described specifically. In addition, the particle encoding scheme and design of fitness function are presented.

3.1 Description of the PSO Variants

By imitating the swarm behavior of birds flocking, Kennedy and Eberhart [14] proposed PSO in 1995. To achieve a balance between the global and local search, inertia weight *w* [7] was introduced. Subsequently, Shi and Eberhart [7] designed a linearly decreasing *w* to enhance prophase global search and anaphase local search.

In the past decade, various variants of PSO have sprung up via exploring a variety of types of topology. For instance, UPSO [8] obtained a good performance after combining the global PSO and local PSO together. Moreover, Mendes and Kennedy [9] introduced a new variant of PSO that is called the FIPS algorithm, which enables the particle to get information from all its neighbors to update the velocity. FIPS algorithm with U-ring topology will be adopted in the experiment. Firstly proposed by Liang et al. in 2006, CLPSO [10] is a good choice for tackling multimodal problems by harnessing all other particle's historical best information to update the particle velocity. Furthermore, inspired by the symbiotic interrelationships in nature, Niu et al. [11] developed MCPSO to emphasize the information exchange among multiple swarms. They took advantage of the relationship between a master swarm and several slave swarms to maintain the balance of global exploitation and local exploration.

The six PSO variants can improve the performance of algorithm in different manner, not only on the benchmark functions but in the realistic problems. With respect to the proposed MT-VRPTW, it is advisable to use these algorithms mentioned above for this complicated optimization problem and make a comparison between each other to find a better one.

3.2 Particle Encoding Scheme

Assumed that there are N customers in this distribution, 2N dimensions encoding mode [12] is adopted in this paper, that is, one of demand point corresponds with two dimensions: the first dimension denotes the marking number of vehicle and the second one is the sequence number of customers served by this vehicle. For example, we can define a particle as (N_v , N_c). Obviously, one can learn that N_v means the sequence number of vehicles and N_c represents the order of customers served by the corresponding vehicle in the N_v .

3.3 Design of Fitness Function

The penalty function is used to develop the fitness function as follows:

$$fitness = \min z + M * \max(G_k \text{-} Q_k ,0) \tag{11}$$

where G_k represents the actual vehicle loading. And Q_k is the given capacity of vehicle *k*. Most of all, *M* is set to an infinite number to restrict the vehicle load.

4 Experiments and Results

In this section, we consider a scenario of one depot to distribute goods for 14 customers. There are 8 vehicles in this depot and each vehicle is with the velocity of 60 units. The earliest departure time of each vehicle is half to seven. And the demands of customers, unloading time and time windows are shown in Table 1, respectively.

Besides, we assume that there are two types of the vehicle denoted as type A and type B. The specified vehicle load of type A and type B is 500 units and 400 units, respectively. The unit freight of type A is 3 and type B is 2.5. In addition, the waiting cost *pe* is 10 units per hour and the cost *pl* for being late is 100 units per hour. The distance between depot 0 and customer *i* (*i*=1, 2, …, 14) is shown in Table 2.

Table 1. The demands and time windows of customers

Customer	Demands	Unloading time (minutes)	Time windows	
			ET	LT
1	120	60	8:00	17:00
2	200	90	8:00	10:30
3	120	60	8:00	17:00
4	130	25	8:00	16:00
5	150	80	8:00	12:00
6	140	70	8:00	8:30
7	60	40	8:00	17:00
8	110	60	8:00	17:00
9	100	120	8:00	18:00
10	180	90	8:00	10:45
11	90	50	8:00	8:30
12	160	90	8:00	13:00
13	140	60	8:00	12:30
14	150	70	8:00	17:00

Table 2. The distance between the depot and customers

d_{ij}	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	40	106	46	36	22	41	81	52	41	48	26	95	118	87
1		0	79	25	6	23	62	78	66	39	82	48	61	113	103
2			0	61	77	85	94	60	84	68	123	124	27	68	104
3				0	21	24	48	53	46	18	73	64	50	88	82
4					0	17	56	73	59	33	76	48	61	108	97
5						0	40	67	47	25	59	41	73	104	84
6							0	48	14	30	29	66	94	84	46
7								0	34	42	74	106	73	38	44
8									0	28	40	78	87	69	38
9										0	57	65	64	79	65
10											0	68	122	106	55
11												0	109	143	112
12													0	91	115
13														0	60
14															0

Based on MATLAB 7.14, every experiment conducted in this paper was run 30 times. For fair comparison, the population size is set to 100 and the maximum iteration is set to 500 in all algorithms. Furthermore, two acceleration coefficients are set to 2.0. Owing to the uncertain number of multi-type vehicles, there are seven kinds of combinations like ($K1$, $K2$) in total for this optimization problem. That is, ($K1, K2$) can be one of (1, 7), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), and (7, 1). The mean values and variance of each algorithm for different combinations of $K1$ and $K2$ is presented in Table 3. The computing results show that MCPSO got the best mean values for each combination of $K1$ and $K2$ when compared with five other PSO variants in this case. Especially, the minimum of total cost 2413.00 is obtained by MCPSO algorithm as the combination of ($K1, K2$) is equal to (5, 3).

Table 3. The mean and variance results for different situations

(K1,K2)		CLPSO	UPSO	FIPS	GPSO w: 0.9-0.4	GPSO w=0.4	MCPSO
(1,7)	Mean	2567.66	2599.01	2612.07	2534.46	2562.02	2464.50
	Variance	68.23	47.73	39.59	72.34	97.42	50.07
(2,6)	Mean	2581.97	2568.10	2568.90	2519.06	2605.32	2447.40
	Variance	51.34	66.37	66.69	94.05	83.64	56.31
(3,5)	Mean	2579.84	2577.15	2579.41	2558.57	2529.36	2438.60
	Variance	56.92	78.95	65.25	76.90	95.01	55.17
(4,4)	Mean	2555.53	2534.93	2586.38	2546.94	2525.05	2447.60
	Variance	75.56	62.41	78.44	97.34	99.71	65.01
(5,3)	Mean	2556.43	2531.82	2604.13	2538.79	2540.76	2413.00
	Variance	70.96	68.48	64.24	112.46	136.77	44.56
(6,2)	Mean	2552.99	2519.27	2551.39	2536.02	2567.01	2433.70
	Variance	65.98	88.85	98.14	94.60	107.06	78.79
(7,1)	Mean	2534.39	2546.57	2535.71	2540.51	2568.08	2434.70
	Variance	76.45	91.13	72.29	124.35	102.68	63.03

Table 4. The vehicle route and fee

Vehicle		Route of the vehicle	Fee
Type A	1	0→6→14→7→9→0	2377.6
	2	0→12→2→3→0	
Type B	1	0→10→5→0	
	2	0→1→11→0	
	3	0→13→8→4→0	

Table 5. The time when the corresponding vehicle arrives at customers

Customer	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Time	7.2	10.0	12.3	11.9	9.2	7.2	11.0	10.3	12.4	7.3	8.6	8.1	8.5	9.1

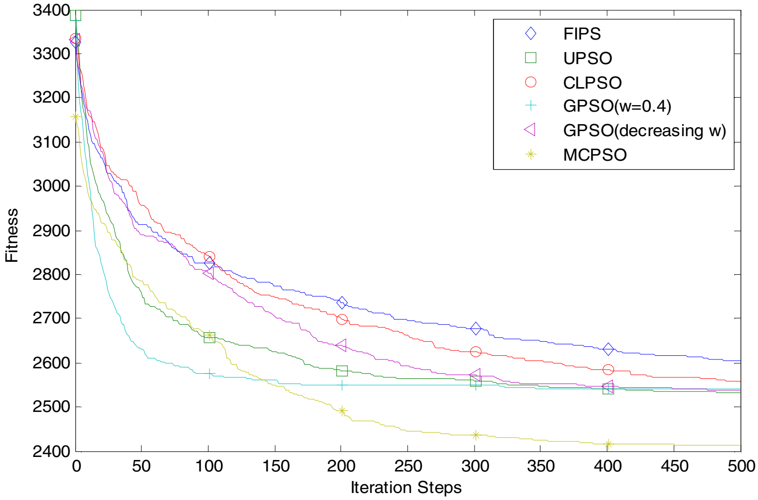

Fig. 1. The convergence graph of the six PSO variants

In particular, we discuss this situation where $K1$ is equal to 5 and $K2$ is equal to 3. And Fig. 1 presents the convergence graph for the six PSO variants. The optimal vehicle route and fee discovered by MCPSO are presented in Table 4. And the corresponding time point at which the vehicle reaches customers is showed in Table 5. From the Table 4, we can learn that there are only five vehicles assigned to serve customers actually. But, an excellent solution yielding the best fitness value can be accepted in the case by this time. This situation demonstrates a necessary optimization in the realistic vehicle allocation. In fact, there are more than vehicles needed to serve a group of customers in the depot. So it is highly significant that the depot could assign the transportation vehicle reasonably.

5 Conclusions

This paper develops an extension of VRPTW called MT-VRPTW and proposes an improved model. In the model, we consider two key realistic factors, namely multiple types of the vehicle and the uncertain number of vehicles of different types. By integrating those two aspects together, we can optimize the use of resources more efficiently in real world, especially in the arrangement of vehicles. Then, we select six representative PSO variants and apply those optimization algorithms to address a case of MT-VRPTW. According to the experiment results, we can observe that MCPSO performs better for this proposed problem with comparison to five other PSO variants.

In the future, we need to implement some other swarm intelligence algorithms [15, 16, 17] to solve this proposed problem. On the other hand, we can take more than one goal into account. And future work should focus on the multi-objective optimization of MT-VRPTW and the larger size simultaneously.

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