

# Multi-scale Level Set Method for Medical Image Segmentation without Re-initialization

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**Abstract.** This paper presents a novel level set method to segment medical image with intensity inhomogeneity (IIH). The multi-scale segmentation idea is incorporated and a new penalty energy term is proposed to eliminate the time-consuming re-initialization procedure. Firstly, the circular window is used to define the local region so as to approximate the image as well as IIH. Then, multi-scale statistical analysis is performed on intensities of local circular regions center in each pixel. The multi-scale energy term can be constructed by fitting multi-scale approximation of inhomogeneity-free image in a piecewise constant way. In addition, a new penalty energy term is constructed to enforce level set function to maintain a signed distance function near the zero level set. Finally, the multi-scale segmentation is performed by minimizing the total energy functional. The experiments on medical images with IIH have demonstrated the efficiency and robustness of the proposed method.

**Keywords:** intensity inhomogeneity, level set method, multi-scale segmentation, penalty energy term, re-initialization.

## 1 Introduction

Medical Image segmentation is usually formulated as a minimization problem where the predefined energy functional specifies the segmentation criterion and the unknown variables describe the object contours. The most representative method within this context is the level set method (LSM) whose popularity and success is due to its ability to deal with topological changes (contour splitting or merging) without additional functions. Besides, extensive numerical solutions based on Hamilton-Jacobi equations can provide the stable contour evolution for LSM.

Generally, the existing level set methods can be classified into edge-based methods and region-based methods. Edge-based methods [1-4] are efficient for segmenting object with edge defined by gradient. However, they are quite sensitive to the initial

conditions and often suffer from serious boundary leakage problems at weak edge. Region-based methods [5-7] have a better performance for image with weak object boundaries and are less sensitive to initial conditions. However, they usually fail to segment images with intensity inhomogeneity (IIH). Recently, local region-based methods have been proposed [8-10] which assume that the intensities are homogeneous in local regions. By fitting the image in terms of local regions rather than global region, they have advantage to segment image with IIH. Huang et al. [11-16] proposed designing neural network for image recognition and segmentation. However, the scale of local region is generally fixed in the existing local region-based methods, which may produce failed segmentation for medical image with severe IIH. To solve this problem, multi-scale segmentation idea can be introduced.

In practical implementation, level set function (LSF) is initially represented by a signed distance function (SDF) to keep numerical stability and accuracy of LSM. During the level set evolution, LSF often becomes very flat or steep near zero level set, which in turn affect the numerical stability. Therefore, a remedy procedure called re-initialization is applied periodically to enforce the degraded LSF being an SDF. However, it is hard to build a trade-off between speed (re-initialization is particularly time-consuming) and accuracy (LSM will develop irregularities if without re-initialization). Recently, Li et al [2, 17] proposed constraining the LSF to preserve an SDF during contour evolution and hence re-initialization can be efficiently avoided.

In this paper, we propose a new level set method which incorporates the multi-scale segmentation idea. Besides, a new penalty energy term is constructed to make our method be completely free of re-initialization. Here, we utilize circular window to define a local region so as to approximate the image as well as IIH. Then, multi-scale statistical analysis is performed on intensities of local circular regions center in each pixel. The multi-scale energy term is constructed by fitting multi-scale approximation of inhomogeneity-free image in a piecewise constant way. To avoid re-initialization, we propose a new double-well potential and construct the penalty energy term which can maintain the signed distance property of LSF near zero level set. Finally, the multi-scale segmentation is performed by minimizing the overall energy functional.

The rest of this paper is organized as follows: The detail of the proposed method is presented in Section 2. In Section 3, we provide the experimental results on several medical images with IIH. Finally, the conclusive remark is included in Section 4.

## 2 Proposed Method

### (a) Intensity Inhomogeneity

The intensity inhomogeneity (IIH) is frequently encountered in medical images. It is a systematic intensity change on both object and background which are originally homogeneous. The presence of IIH can greatly degrade the medical image segmentation performance since the intensities vary significantly for the pixels within the same class of tissue and overlap between the pixels belonging to the different

classes of tissues. Generally, IIIH can be regarded as a multiplicative component of image and is independent of noise. So, the image with IIIH can be modeled as follows:

$$I(x) = b(x)J(x) + n(x), \quad (1)$$

where  $I$  is the given image and  $J$  is the inhomogeneity-free image which is hypothetically piecewise constant.  $b$  denotes the IIIH which often manifests itself as a smooth spatially varying function.  $n$  is noise which can be approximated by a zero-mean Gaussian distribution. To simplify the computation, the noise can be ignored:

$$I(x) = b(x)J(x). \quad (2)$$

The emergence of IIIH in medical image is attributed to a number of reasons. Many of IIIH arise from the non-uniform artificial illumination which usually presents circular scattered shape. Here, we illustrated three examples of IIIH in Fig.1. It can be easily observed that they are slowly changing in circular scattered shape. Inspired by this observation, we use circular shape to define the local region rather than square shape in traditional methods. By approximating the image with local circular regions, more precise intensity information can be used to guide the contour evolution.

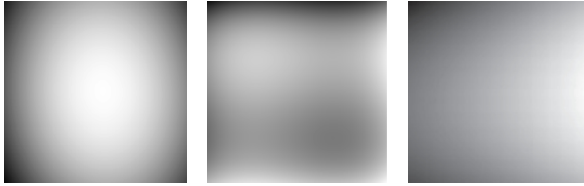


Fig. 1. Illustrations of three examples of IIIH

### (b) Multi-scale Energy Term

Generally, IIIH influences the intensity distribution of non-boundary pixels (low-frequency part), whereas for that of boundary pixels (high-frequency part), the influence is relatively small. By reducing the value of low-frequency components, we can make the variation of  $b$  less significant. So, we need to perform the local statistical analysis based on filtering technology, which implies a separation of the low-frequency IIIH from the higher frequencies of image structures. In this paper, the Homomorphic Unsharp Masking (HUM) method [18] is adopted:

$$J'(x) = I(x) / b(x) = I(x)C_N / LPF(x), \quad (3)$$

where  $J'$  is an approximation of inhomogeneity-free image  $J$ .  $LPF$  means low-pass filtering and  $C_N$  is a normalized constant to preserve the mean intensity of  $J'$ .

Among the low-pass filtering methods, mean filtering is used in our method due to its particularly simple analytical form. As mentioned above, fixing radius for all local circular regions is unreasonable. Thus, we consider introducing the multi-scale segmentation idea and constructing the multi-scale mean filtering as follows:

$$LPF_r(x) = \frac{1}{k} \sum_{y \in R_r} I(y), \quad R_r : \{y : \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2} \leq r\}, \quad r = 1..m, \quad (4)$$

where  $LPF_r(x)$  is the mean filtering at scale of  $r$ .  $R_r$  denotes the local circular region with radius also being  $r$ .  $k$  denotes the number of pixels belonging to  $R_r$  and  $m$  is the number of scales. Accordingly, HUM in (3) can be reformulated as follows:

$$J'_r(x) = I(x)C_{N,r} / LPF_r(x), \quad r = 1..m, \quad (5)$$

where  $J'_r$  denotes the approximation of inhomogeneity-free image  $J$  at scale of  $r$ . The normalized constant  $C_{N,r}$  is the average intensity of  $LPF_r(x)$ . Then, the mean of  $J'_r(x)$  is computed as the multi-scale approximation of inhomogeneity-free image  $J$ :

$$\bar{J}(x) = \frac{1}{m} \sum_{r=1}^m J'_r(x). \quad (6)$$

By reducing the low-frequency components in a multi-scale way, the intensity contrast between object boundary and background in  $\bar{J}$  can be significantly increased. The separation of object boundary and background may be performed even in the image with severe IIIH. However,  $\bar{J}$  is still hard to segment since the substantial IIIH still remains. Hence, the level set segmentation should be performed and the multi-scale energy term can be constructed in a piecewise constant way:

$$E_m^D(c_1, c_2, C) = \int_{inside(C)} |\bar{J}(x) - c_1|^2 dx + \int_{outside(C)} |\bar{J}(x) - c_2|^2 dx, \quad (7)$$

where  $c_1$  and  $c_2$  are intensity averages of  $\bar{J}$  inside and outside evolving contour  $C$ .

### (c) Regularization Energy Term

To avoid the re-initialization problem, Li et al [17] proposed constructing the penalty energy term based on a double-well potential function with two minimum points as  $s=0$  and  $s=1$ . It can efficiently maintain the signed distance property of LSF near the zero level set and keep LSF as a constant at locations far away from the zero level set. This paper takes the idea of [17] a few steps further by constructing the

following double-well potential function based on polynomial instead of trigonometric function.

$$P_2(s) = \begin{cases} \frac{1}{2}s^2(s-1)^2 + \frac{1}{2}s^3(s-1)^3, & \text{if } s \leq 1 \\ \frac{1}{2}(s-1)^2, & \text{if } s > 1 \end{cases}, \quad (8)$$

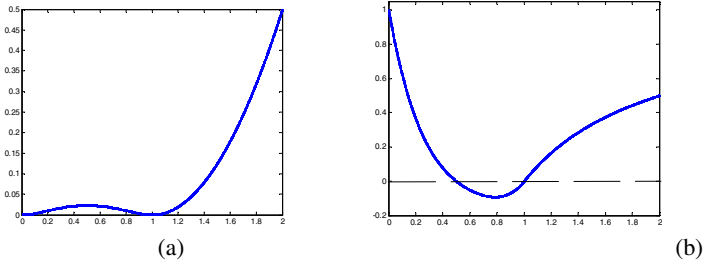
$P_2(s)$  has the same property with the double-well potential function proposed in [17] but has a less computation complexity due to the usage of polynomial. Thus, our penalty energy term can be constructed as follows:

$$R(\phi) = \int_{\Omega} P_2(|\nabla \phi(x)|) dx, \quad (9)$$

The gradient flow of (9) can be described as follows:

$$\frac{\partial \phi}{\partial t} = \text{div}(d(|\nabla \phi|) \nabla \phi), \quad (10)$$

where  $\text{div}$  denotes the divergence operator and  $d(s)$  is defined by  $P_2'(s)/s$ . Here, we gave the illustrations of  $P_2(s)$  and  $d(s)$  in Fig. 2.



**Fig. 2.** Illustration of  $P_2(s)$  and  $d(s)$ . (a) Illustration of  $P_2(s)$ . (b) Illustration of  $d(s)$ .

It can be seen from Fig.2 that  $d(s)$  satisfies the following relationship:

$$|d(s)| < 1, \quad s \in [0, \infty] \quad \text{and} \quad \lim_{s \rightarrow 0} d(s) = \lim_{s \rightarrow \infty} d(s) = 1, \quad (11)$$

Regarding (10) as a diffusion equation with the diffusion rate being  $d(|\nabla \phi|)$ , we can analyze the effect of our penalty energy term as follows:

1. If  $|\nabla \phi| > 1$ ,  $d(|\nabla \phi|)$  is positive and diffusion is forward so as to decrease  $|\nabla \phi|$  to 1;
2. If  $0.5 < |\nabla \phi| < 1$ ,  $d(|\nabla \phi|)$  is negative and diffusion is backward so as to increase  $|\nabla \phi|$  to 1;

3. If  $|\nabla\phi| < 0.5$ ,  $d(|\nabla\phi|)$  is positive and diffusion is forward so as to decrease  $|\nabla\phi|$  to 0.

Our penalty energy term will make LSF preserve an SDF near the zero level set and be a constant at locations far away from the zero level set. Hence, the re-initialization can be efficiently avoided. In addition, the frequently used length energy term  $L(\phi)$  should also be included to control the smoothness of evolving contour. Thus, the regularization energy term  $E^R$  of the proposed method consists of two parts:

$$E^R(\phi) = \mu \cdot L(\phi) + R(\phi) = \mu \cdot \int_{\Omega} \delta(\phi(x)) |\nabla\phi(x)| dx + \int_{\Omega} P_2(|\nabla\phi(x)|) dx. \quad (12)$$

where  $\mu$  controls the length penalization effect and  $\delta$  is the Dirac delta function.

#### (d) Level Set Formulation

By introducing the penalty energy term in (9), the binary step function can be utilized as the initial LSF:

$$\phi_0(x) = \begin{cases} 1, & x \text{ is inside initial contour } C_0 \\ -1, & \text{otherwise} \end{cases}. \quad (13)$$

Implicitly representing the evolving contour  $C$  by the zero level set of the LSF  $\phi$ , the overall energy functional of the proposed method can be described as follows:

$$\begin{aligned} E_m(c_1, c_2, \phi) &= E_m^D(c_1, c_2, \phi) + E^R(\phi) \\ &= \int_{\Omega} |\bar{J}(x) - c_1|^2 H_{\varepsilon}(\phi(x)) dx + \int_{\Omega} |\bar{J}(x) - c_2|^2 (1 - H_{\varepsilon}(\phi(x))) dx \\ &\quad + \mu \cdot \int_{\Omega} \delta_{\varepsilon}(\phi(x)) |\nabla\phi(x)| dx + \int_{\Omega} P_2(|\nabla\phi(x)|) dx, \end{aligned} \quad (14)$$

where  $H_{\varepsilon}(z)$  is the smoothed approximation of Heaviside function and  $\delta_{\varepsilon}(z)$  is the regularized approximation of Dirac delta function.

$$H_{\varepsilon}(z) = \frac{1}{2} \left[ 1 + \frac{2}{\pi} \arctan \left| \frac{z}{\varepsilon} \right| \right], \quad \delta_{\varepsilon}(z) = \frac{1}{\pi} \cdot \frac{\varepsilon}{\varepsilon^2 + z^2}. \quad (15)$$

Fixing  $\phi$ , we minimize (14) with respect to  $c_1(\phi)$  and  $c_2(\phi)$ . Then,  $c_1$  and  $c_2$  can be computed by calculus of variations as follows:

$$c_1(\phi) = \frac{\int_{\Omega} \bar{J}(x) H_{\varepsilon}(\phi(x)) dx}{\int_{\Omega} H_{\varepsilon}(\phi(x)) dx}, \quad c_2(\phi) = \frac{\int_{\Omega} \bar{J}(x) (1 - H_{\varepsilon}(\phi(x))) dx}{\int_{\Omega} (1 - H_{\varepsilon}(\phi(x))) dx}. \quad (16)$$

Keeping  $c_1$  and  $c_2$  fixed and minimizing  $E(c_1, c_2, \phi)$  with respect to  $\phi$ , we can deduce the associated gradient flow equation for  $\phi$ :

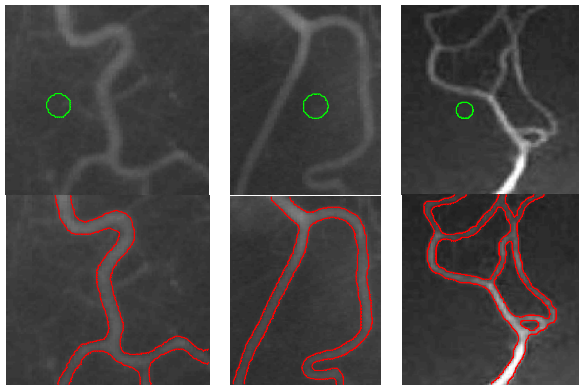
$$\frac{\partial \phi}{\partial t} = \delta_\varepsilon(\phi) \{ (\bar{J}(x) - c_2)^2 - (\bar{J}(x) - c_1)^2 \} + \mu \operatorname{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) + \operatorname{div} (d(|\nabla \phi|) \nabla \phi) . \quad (17)$$

To solve the above equation, the finite difference scheme is used in this paper.

### 3 Experimental Results

In this section, we demonstrated the experiments of the proposed method on several medical images with IIH. The proposed method was implemented by Matlab R2010a on a computer with Intel Core 2 Duo 2.2GHz CPU, 8G RAM. We used the same parameters, i.e.  $\Delta t = 0.1$ ,  $\varepsilon = 1$ ,  $m = 32$ ,  $\mu = 0.01 \times 255^2$  for all experiments.

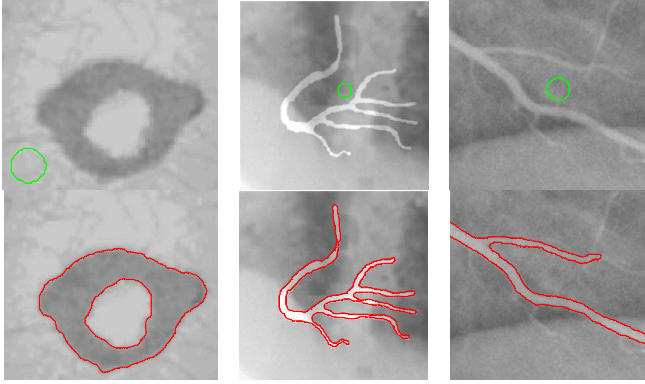
Firstly, we used the images with slight IIH to test our method (as shown in Fig.3). The first row shows three vessel images which have been regarded as the benchmark images to test the performance of local region methods. To show the good ability of our method, we placed the initial contours (green circles) near the vessels rather than on the vessels (as shown in the first row). The final segmentation results of our method are shown as the red curves in the second row. The experimental records show that the evolving contours successfully arrived at each vessel boundaries at the 25<sup>th</sup> iteration, 34<sup>th</sup> iteration and 28<sup>th</sup> iteration.



**Fig. 3.** Segmentation for medical images with slight IIH by using our method. The first row: Initial contours. The second row: Final segmentation results.

Next, we shall validate the performance of our method on segmenting images with severe IIH. In Fig.4, we provided three medical images where severe IIH appears due to the low imaging quality and inhomogeneity of reception coil sensitivity. The initial contours were still placed near the target objects as shown in the first row of Fig.4.

The second row shows that our method has achieved successful segmentation on all three images despite the presence of severe IIH. The iteration numbers for three segmentations were 36, 66 and 89, respectively.



**Fig. 4.** Segmentation for medical images with severe IIH by using our method. The first row: Initial contours. The second row: Final segmentation results.

## 4 Conclusions

By introducing multi-scale segmentation idea, a novel and efficient level set method is proposed for segmenting medical images with IIH. Here, we utilize circular window to define local region so as to approximate the image as well as IIH. Then, multi-scale statistical analysis is performed on intensities of local circular regions center in each pixel. The multi-scale energy term can be constructed by fitting the multi-scale approximation of inhomogeneity-free image in a piecewise constant way. To avoid the time-consuming re-initialization procedure, we propose a new penalty energy term to maintain the signed distance property of LSF near the zero level set. Finally, the multi-scale segmentation is performed by minimization of the overall energy functional. The experiments have demonstrated that our method is efficient and robust for segmenting medical images with slight or severe IIH.

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## References

1. Caselles, V., Kimmel, R., Sapiro, G.: Geodesic Active Contours. *Int. J. Comput. Vision* 22(1), 61–79 (1997)
2. Li, C., Xu, C., Gui, C., Fox, M.D.: Level Set Formulation without Re-initialization: A New Variational Formulation. In: *Proc. CVPR*, vol. 1, pp. 430–436 (2005)
3. Wang, X., Huang, D.: A Novel Density-Based Clustering Framework by Using Level Set Method. *IEEE Trans. Knowl. Data Eng.* 21(11), 1515–1531 (2009)
4. Gao, X., Wang, B., Tao, D., Li, X.: A Relay Level Set Method for Automatic Image Segmentation. *IEEE Trans. Syst., Man, Cybern. B, Cybernetics* 41(2), 518–525 (2011)
5. Chan, T.F., Vese, L.A.: Active Contours without Edges. *IEEE Trans. Image Process.* 10(2), 266–277 (2001)
6. Paragios, N., Deriche, R.: Geodesic Active Regions and Level Set Methods for Supervised Texture Segmentation. *Int. J. Comput. Vision* 46(4), 223–247 (2002)
7. Gao, S., Bui, T.D.: Image Segmentation and Selective Smoothing by Using Mumford-Shah Model. *IEEE Trans. Image Process.* 14(10), 1537–1549 (2005)
8. Li, C., Kao, C., Gore, J.C., Ding, Z.: Minimization of Region-Scalable Fitting Energy for Image Segmentation. *IEEE Trans. Image Process.* 17, 1940–1949 (2008)
9. Wang, X., Huang, D., Xu, H.: An Efficient Local Chan-Vese Model for Image Segmentation. *Pattern Recognition* 43(3), 603–618 (2010)
10. Sun, K., Chen, Z., Jiang, S.: Local Morphology Fitting Active Contour for Automatic Vascular Segmentation. *IEEE Trans. Biomed. Eng.* 59(2), 464–473 (2012)
11. Huang, D., Du, J.: A Constructive Hybrid Structure Optimization Methodology for Radial Basis Probabilistic Neural Networks. *IEEE Trans. Neural Networks* 19(12), 2099–2115 (2008)
12. Huang, D.: Radial Basis Probabilistic Neural Networks: Model and Application. *Int. J. Pattern Recognit. Artificial Intell.* 13(7), 1083–1101 (1999)
13. Huang, D., Chi, Z., Siu, W.C.: A Case Study for Constrained Learning Neural Root Finders. *Applied Mathematics and Computation* 165(3), 699–718 (2005)
14. Huang, D., Horace, H.S., Ip, C.Z.: A Neural Root Finder of Polynomials Based on Root Moments. *Neural Computation* 16(8), 1721–1762 (2004)
15. Huang, D.: A Constructive Approach for Finding Arbitrary Roots of Polynomials by Neural Networks. *IEEE Trans. on Neural Networks* 15(2), 477–491 (2004)
16. Zhao, Z., Huang, D., Sun, B.: Human Face Recognition Based on Multiple Features Using Neural Networks Committee. *Pattern Recognition Letters* 25(12), 1351–1358 (2004)
17. Li, C., Xu, C., Gui, C., Fox, M.D.: Distance Regularized Level Set Evolution and its Application to Image Segmentation. *IEEE Trans. Image Process.* 19(12), 3243–3254 (2010)
18. Brinkmann, B.H., Manduca, A., Robb, R.A.: Optimized Homomorphic Unsharp Masking for MR Grayscale Inhomogeneity Correction. *IEEE Trans. Med.* 17(2), 161–171 (1998)