

# Steady Lateral Flow in Sloping Soils: Which is the Effect of the Conductivity at Saturation Profile?

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## Abstract

The effect of the shape of the decreasing conductivity at saturation profile with depth on the steady lateral flow was investigated. The Zaslavsky transformation provided a synthetic, graphically based, approach which resumes the infiltration thresholds to characterize a perched water table and its thickness. It moreover allowed to easily compare the effect of different conductivity at saturation profiles on the capability of subsurface runoff production of the slope. As a case study, a reconstructed layered soil was described by means of two different conductivity profiles. The effect of the two models on the flow field is presented and discussed.

## 386.1 Introduction

Perched water tables in the upper soil layers are important shallow landslides triggering mechanisms and they are seat for preferential subsurface and hypodermic flow which contributes to the formation of the quick runoff. Therefore an accurate description of the subsurface soil–water dynamics can lead to important information both on the slope stability (Eichenberger et al. 2011; Tsai et al. 2008) and on the runoff production mechanism as well (Ruiz-Villanueva et al. 2011). A key role is played by the profile of conductivity at saturation, which can be multilayered as in pyroclastic slopes (Greco et al. 2013) or simply layered as in young mountain soils, but in most of the cases is such that a capillary barrier with poor retention capability lays below a less conductive layer. In these cases a perched water table can onset in the

impervious layer, both above and below delimited by a water table, with a positive pressure head in within (Barontini et al. 2012). In this paper, an application of the classical Zaslavsky transformation (Zaslavsky 1964) is introduced in order to graphically summarize some key properties of the perched water tables. The transformation allows to compare how different models of the soil conductivity profile lead to different estimate of the properties of the perched water tables, their thickness and their attitude to the lateral subsurface runoff. Finally an application to a reconstructed soil column with decreasing conductivity with depth is presented as a case study.

## 386.2 Uniform Flow in Perched Water Tables

Let us consider the central branch of a uniform sloping soil, with horizontal projection much longer than a characteristic transverse dimension (Fig. 386.1). Let the soil be undecomposable and be seat of a Darcian, steady and incompressible flow. The continuity equation and the Darcy law are written as:

$$\nabla^{\star} \cdot \mathbf{q} = 0 \quad , \quad \mathbf{q} = -\mathbf{K} \cdot \nabla^{\star} \Phi. \quad (386.1)$$

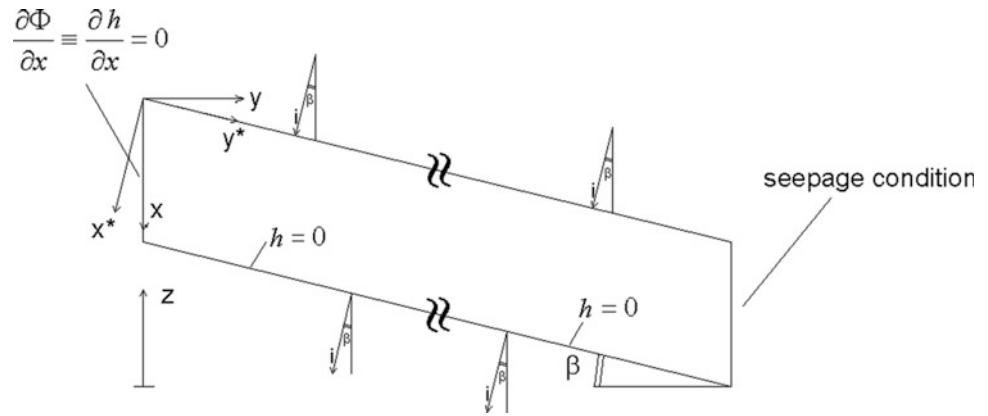
In Eq. (386.1)  $\mathbf{q}(x^{\star}, y^{\star}) = (q_{x^{\star}}, q_{y^{\star}})$  [LT<sup>-1</sup>] is the Darcian velocity field,  $\mathbf{K}(x^{\star}, y^{\star})$  [LT<sup>-1</sup>] is the hydraulic conductivity tensor,  $\Phi(x^{\star}, y^{\star})$  [L] is the generalised

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**Fig. 386.1** Sketch of the investigated soil domain



piezometric potential and  $\nabla^{\star}$  is the pseudovector in the intrinsic, transverse and lateral, coordinates  $(x^{\star}, y^{\star})$ . In usual conditions it is  $\Phi = h + z$ , where  $h$  is the tensiometer–pressure potential and  $z$  is the gravimetric one, with  $z$  vertical and positive upward.

We assume that the non-homogeneity of the soil hydraulic properties is fully described by a monotonic decrease along  $x^{\star}$  of the transverse hydraulic conductivity at soil saturation (Barontini et al. 2007). Let it be uniform the ratio  $r \geq 1$ , said anisotropy factor, between the lateral conductivity, along  $y^{\star}$ , and the transverse conductivity, along  $x^{\star}$  (the latter is usually lower due to the soil horization processes). The conductivity tensor  $\mathbf{K}$  is therefore diagonal with components  $K_{x^{\star}} = K_{s,o}k(h)f(x^{\star})$  and  $K_{y^{\star}} = K_{s,o}rk(h)f(x^{\star})$ , where  $K_{s,o} = K_s(0, y^{\star})$  is the hydraulic conductivity at saturation at the soil surface;  $k(h)$  is the relative conductivity, equal to 1 if  $h \geq 0$ , and monotonically increasing from 0 to 1 for  $h \leq 0$ ;  $f(x^{\star})$  is a monotonically decreasing function, equal to 1 if  $x^{\star} = 0$ . Say  $i$  a uniform infiltration rate at  $x^{\star} = 0$  and, for the sake of continuity of  $\Phi$  with the underlying soil layer, let  $h(x_f^{\star}) = 0$ , with  $x_f^{\star}$  the thickness of the soil layer. According to (Zaslavsky and Sinai 1981) a balance between infiltration and drainage is reached in a central branch of the soil domain so that uniform flow conditions take place, unless for some upstream– and downstream–boundary effects.

This allows to uncouple the Darcy law (386.1) and to solve it for the transverse and lateral components separately. It can be proved (Barontini et al. 2012) that the infiltration threshold  $i_f(\beta)$  for a perched water to onset, depending on the soil slope  $\beta$ , is given by:

$$i_f(\beta) = K_s(x_f^{\star}) \cos \beta. \tag{386.2}$$

The threshold  $i_f(\beta)$  decreases if  $\beta$  increases as the gravitational potential is less effective at sustaining the transverse flow. As  $i > i_f(\beta)$  a perched water table onsets above  $x_f^{\star}$ . Say  $x_s^{\star}$  the upper boundary of the perched water table, i.e.

the soil depth at which  $h$  is null again. The thickness of the perched water table  $\Delta x^{\star} = x_f^{\star} - x_s^{\star}$  is equal to the soil depth above  $x_f^{\star}$  which can sustain the flux by a merely gravitational gradient. The (transverse) Darcy law can be rewritten in an integral form as:

$$i = K_{s,eq}^{[\Delta x^{\star}]} \cos \beta. \tag{386.3}$$

In (386.3) the equivalent conductivity at saturation  $K_{s,eq}^{[\Delta x^{\star}]}$  was defined as:

$$K_{s,eq}^{[\Delta x^{\star}]} = \Delta x^{\star} \left[ \int_{x_s^{\star}}^{x_f^{\star}} \frac{dx'}{K_s(x')} \right]^{-1}. \tag{386.4}$$

The maximum thickness of the perched water is therefore reached in correspondence of the infiltration rate  $i^{\star}(\beta)$  which is able to lead the soil to waterlogging, i.e. when  $\Delta x^{\star} = x_f^{\star} - 0$ .

### 386.3 A Reanalysis by Means of the Zaslavsky Transformation

The classical Zaslavsky transformation (Zaslavsky 1964) is now introduced:

$$dX^{\star} = \frac{x^{\star}}{K_s(x^{\star})}. \tag{386.5}$$

It allows to symbolically map an unhomogeneous soil in a uniform one (with unitary conductivity), in which the saturated flow is characterised by the same gradients if it takes place over corresponding paths. Say  $X_s^{\star}$  and  $X_f^{\star}$  the corresponding values of  $x_s^{\star}$  and  $x_f^{\star}$  in the transformed soil, respectively, and say  $\Delta X^{\star} = X_f^{\star} - X_s^{\star}$  the corresponding

thickness of the saturated layer. The transformed thickness  $\Delta X^*$  is given by:

$$\Delta X^* = \int_{x_s^*}^{x_f^*} \frac{dx'}{K_s(x')} \tag{386.6}$$

Comparing (386.4) with (386.6) it is recognized that soil layers, that are characterised by the same equivalent conductivity at saturation over the same thickness  $\Delta x^*$ , show the same transformed thickness  $\Delta X^*$  as well:

$$K_{s,eq}^{[\Delta x^*]} = \frac{\Delta x^*}{\Delta X^*} \tag{386.7}$$

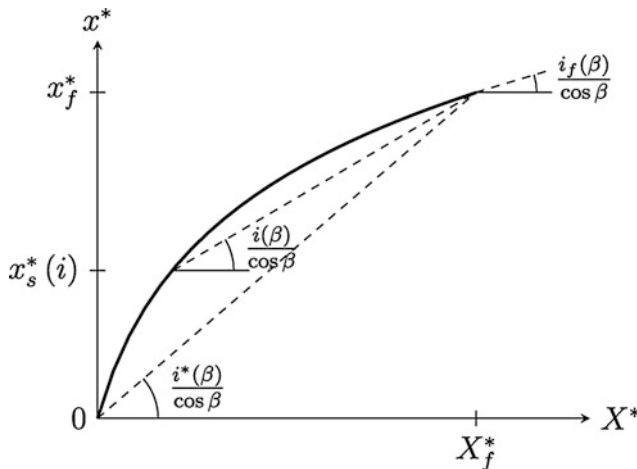
The Zaslavsky transformation  $X^*(x^*)$  is strongly related to some properties of the perched water table, as presented in Fig. 386.2. Taking in fact the limit of (386.7) for  $x_s^* \rightarrow x_f^*$ , from (386.2) one gets:

$$\left. \frac{dx^*}{dX^*} \right|_{x_f^*} = \lim_{x_s^* \rightarrow x_f^*} K_{s,eq}^{[\Delta x^*]} = K_s(x_f) \equiv \frac{i_f(\beta)}{\cos \beta}, \tag{386.8}$$

i.e. the slope of  $X^*(x^*)$  at  $x_f^*$  defines the threshold  $i_f(\beta)$  for a perched water table to onset. Moreover for  $i > i_f(\beta)$ , from (3) it follows that:

$$\frac{\Delta x^*}{\Delta X^*} = \frac{i}{\cos \beta}, \tag{386.9}$$

i.e. the thickness  $\Delta x^*$  of the perched water table is read on the  $x^*$ -axis once it is drawn the chord with slope  $i/\cos \beta$ , starting from the point  $(x_f^*, X_f^*)$ . As a consequence, the infiltration threshold  $i^*(\beta)$ , which leads the soil to

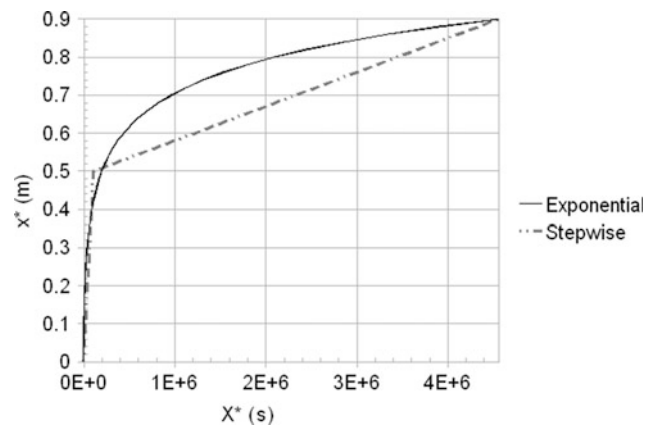


**Fig. 386.2** Transformed soil depth  $X^*(x^*)$  and some properties of the perched water table, as defined by (386.2, 386.3)

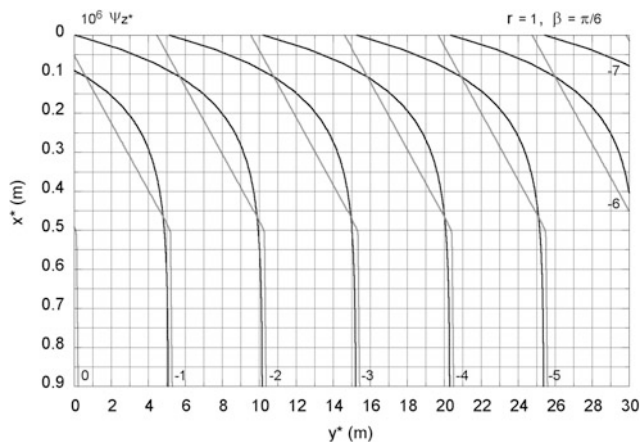
waterlogging and is provided by (386.3) with  $\Delta x^* = x_f^* - 0$ , is obtained from the slope of the chord starting at  $(0, 0)$  and ending at  $(x_f^*, X_f^*)$ .

### 386.4 Results and Discussion

As a consequence of the uniform flow condition, a uniform subsurface lateral flow takes place and it is equal to  $q_{y^*} = K_{s,o} k(h) r f(x^*) \sin \beta$ . The total discharge (per unitary-widthness slope) is given by its integration over the whole soil thickness  $x_f^*$ . Anyway, as the relative conductivity function  $k(h)$  rapidly decreases as  $h < 0$ , the meaningful contribution to the integration is given by the flow within the perched water table, in agreement with the common hydrological practice of considering the subsurface flow to take place only within the saturated layer, if any exists. Therefore a meaningful lateral flow onsets only if the infiltration rate  $i$  is greater than the threshold  $i_f(\beta)$  for a perched water table to onset. Once  $i > i_f(\beta)$ , the lateral flow is strongly conditioned by the thickness of the perched water table, and by the shape of the function  $X^*(x^*)$ , as a consequence of (386.9). In order to stress this conclusion, let us assume as a case study that a 0.9 m-thick soil layer with equivalent (transverse) conductivity at saturation  $1.97E - 7 \text{ ms}^{-1}$  is described either by means of a soil with exponentially decreasing  $K_s(x^*)$  or by means of two uniform soil layers, 0.5 and 0.4 m-thick from above, respectively. In Fig. 386.3 the function  $X^*(x^*)$  is represented for the two approximations. As a consequence of the different soil description, the slope  $x^*/X^*$  at  $x_f^*$  is in this case greater for the stepwise layered soil than for the gradually layered one. This means that the first would experience perched water tables onset, and lateral flow as well, for greater infiltration



**Fig. 386.3** Transformed soil depth  $X^*$  as a function of the shape of the profile of conductivity at saturation  $K_s(x^*)$



**Fig. 386.4** Flow patterns in a layered soil with decreasing conductivity at saturation with depth: comparison between an exponential model (*black lines*) and a stepwise model (*grey lines*)

rates than the latter. For  $i \geq i_f(\beta)$  the thickness of the perched water table and the magnitude of the lateral flow gradually increase in the gradually layered soil, while sharply increase in the stepwise layered one, as all the bottom layer is saturated at  $i = i_f(\beta)$ . In Fig. 386.4 the flow patterns, determined as isolines of the Lagrange stream function of the flow field, are presented for the waterlogging condition of the case study soil, for both the approximations, and for a slope  $\beta = \pi/6$  and  $r = 1$ . In both cases a drop infiltrating at the soil surface is laterally deflected due to the soil slope but follows sensitively transverse patterns and crosses the whole soil layer within a finite lateral displacement. The shape of the patterns is strongly characterised by the shape of  $K_s(x^*)$ , being greater the lateral displacement where greater is  $K_s$ .

### 386.5 Conclusions

An application of the Zaslavsky transformation was presented to describe some properties of the perched water tables in a layered soil with decreasing conductivity with

depth. When a perched water table onsets, the corresponding infiltration rate is given by the slope of a chord cutting on the mapping function  $X^*(x^*)$  a segment whose projection on the  $x^*$ -axis is the thickness of the perched water table itself. As different descriptions of the soil layering  $K_s(x^*)$  provide different functions  $X^*(x^*)$ , and as the lateral runoff is strongly related to the thickness of the saturated layer, the analysis allows to graphically discuss the sensitivity of the lateral runoff estimate with respect to the shape of  $K_s(x^*)$ , for different infiltration rates in steady conditions.

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