Cellular Automata for Crowd Dynamics

Georgios Ch. Sirakoulis

Department of Electrical and Computer Engineering Democritus University of Thrace 67100 Xanthi, Greece gsirak@ee.duth.gr http://gsirak.ee.duth.gr

Abstract. Cellular Automata (CA) as bio-inspired parallel computational models of self-reproducing organisms can capture the essential features of systems where global behavior arises from the collective effect of simple components which interact locally. In this aspect, CAs have been considered as a fine candidate to model pedestrian behavior and crowd dynamics in a fine manner. In specific, for crowd modeling, the CA models show evidence of a macroscopic nature with microscopic extensions, i.e. they provide adequate details in the description of human behavior and interaction, whilst they retain the computational cost at low levels. In this paper several CA models for crowd evacuation taking into consideration different modeling principles, like potential fields techniques, obstacle avoidance, follow-the-leader principles, grouping theory, etc. will be presented in an attempt to accomplish efficient crowd evacuation simulation. Moreover, an integrated system based on CAs that operates as an anticipative crowd management tool in cases of medium density crowd evacuation for indoor and outdoor environments is also shown, and its results different real world cases and different environments prove its efficiency. Finally, robot guided evacuation with the help of CAs is also presented. Quite recently, an evacuation system was proposed, based on an accurate CA model capable of assessing the human behavior during emergency situations takes advantage of the simulation output to provide sufficient information to a mobile robotic guide, which in turn guides people towards a less congestive exit at a time.

Keywords: Cellular Automata, Crowd Dynamics, Modeling, Simulation, Hardware Implementation.

1 Introduction

When we are at a major sporting event or traveling on public transport or shopping around in shopping precincts, our safety and comfort depend crucially on our fellow crowd members and on the design and operation of the facility we are in. Consequently, the need for realistic and efficient in case of emergency crowd dynamics modeling approaches is of utter importance. As a result, pedestrian dynamics have been reported following a great variety of approaching methods, such as Cellular Automata (CA)-based [1], lattice-gas and social

[©] Springer International Publishing Switzerland 2014

force models [2], fluid-dynamic [3] and agent-based [4], and methods related to game theory [5]. All approaches can be qualitatively distinguished, focusing on different characteristics that each of them dominantly display. In general, crowd movement simulation models can also be categorized into macroscopic and microscopic ones. In some models, pedestrians are ideally considered as homogeneous individuals, whereas in others, they are treated as heterogeneous groups with different features (e.g., gender, age, psychology). There are methods, where collective phenomena emerge from the complex interactions among individuals (self-organizing effects), thus describing pedestrian dynamics in a microscopic scale. Other methods treat crowd as a whole, modeling pedestrian dynamics on a macroscopic scale. There are models discrete in space and time and others spatial-temporally continuous.

Moreover, crowd movement could be defined as a non linear problem with many factors affecting it. A system of Partial Differential Equations (PDEs) could effectively approach it. The result is a system of PDEs very difficult to handle, which would also be demanding in terms of computer power, complexity and computation time. CA can act as an alternative to PDEs [12]. In CA based approach, the space under study is presented as a unified grid of cells with local attributes, which are generated by a set of rules that describe the behavior of the individuals. The state of each cell is defined according to the rules, the state of the cell at the previous time step and the current state of neighboring cells. Consequently, literature reports a variety of CA-based models investigating crowd behavior under different circumstances [6,7].

In this paper, some CA models for the simulation of crowd dynamics are going to be presented. In specific, a CA model based on electrostatic-induced potential, inspired by the Coulomb force as motion-driving mechanism, calculates the Euclidean distance between the destination (source) and the pedestrian (test charge), allowing smoother change of direction. Introducing an electric field approach, charges of different magnitude represent main or internal exits as well as obstacles and walls [8,9,11]. A somehow different CA model also applies an efficient method to overcome obstacles. Based on the generation of a virtual field along obstacles, a pedestrian moves along the axis of the obstacle towards the direction that the field increases its values, leading her/him to avoid the obstacle effectively [13]. Moreover, a bio-inspired CA-based model for crowd dynamics where the driving mechanism emanates from the characteristic collective behavior of biological organisms (e.g. school of fishes, flock of birds, etc.) is also presented [14,15]. The adoption of a CA approach enhanced with memory capacity allows the development of a microscopically-induced model with macroscopical characteristics. Due to the fact that in terms of simplicity, regularity, ease of mask generation, silicon-area utilization, and locality of interconnections [10,11], CA are the most promising computational architecture, the CA models can be easily implemented in Field Programmable Gate Array (FPGA) Circuits leading to support decision system for monitoring crowd dynamics in real-time, providing valuable near optimum management of crowd services. In this direction, an anticipative crowd management system preventing clogging in exits during

pedestrian evacuation processes based on some of the proposed CA models is also presented. Finally, robot guided evacuation is presented [16]; namely, an evacuation system based on an accurate CA model and capable of assessing the human behavior during emergency situations takes advantage of the simulation output to provide sufficient information to a mobile robotic guide, which in turn guides people towards a less congestive exit at a time.

2 CA Models for Crowd Dynamics

CA Basics: A CA consists of a regular uniform n-dimensional lattice (or array), usually of infinite extent. At each site of the lattice (cell), a physical quantity takes on values. The value of this physical quantity over all the cells is the global state of the CA, whereas the value of this quantity at each site is its local state. A CA is characterized by five properties:

- 1. the number of spatial dimensions (n);
- 2. the width of each side of the array (w). w_j is the width of the j^{th} side of the array, where j = 1, 2, 3, ..., n;
- 3. the width of the neighborhood of the cell (r);
- 4. the states of the CA cells;
- 5. the CA rule, which is an arbitrary function F.

The state of a cell, at time step (t + 1), is computed according to F, a function of the state of this cell at time step (t) and the states of the cells in its neighborhood at time step (t). For a 2-d CA, two neighborhoods are often considered: Von Neumann, which consists of a central cell and its four geographical neighbors north, west, south and east; and the Moore neighborhood contains, in addition, second nearest neighbors northeast, northwest, southeast and southwest, i.e. nine cells. In most practical applications, when simulating a CA rule, it is impossible to deal with an infinite lattice. The system must be finite and have boundaries, resulting to various types of boundary conditions such as periodic (or cyclic), fixed, adiabatic or reflection.

CA model based on electrostatic-induced potential fields: A 2-d CA model based on electrostatic-induced potential fields was introduced to simulate efficiently crowd dynamics in the process of developing an active guiding system of crowd during evacuation. Certain attributes of crowd behavior, such as collective effects, blockings in front of exits as well as random to coherent motion due to a common purpose have been successfully incorporated in the model. Motion mechanism is based on an virtual potential field generated by electric charges at selected positions that attract pedestrians towards exit point or repel them from obstacles and walls. Assuming that each bounded area that includes an exit corresponds to an independent level then coupling among different fields is avoided. Efficient updating rules demonstrate global behavioral patterns that distinctly characterize mass egress. Each pedestrian is represented by a test charge, with such a small magnitude that when placing it at a point has a negligible affect on the field around the point (Fig. 1). Furthermore,

the direction towards each of pedestrians should move is precisely determined. The model calculates the exact Euclidean distance between the destination (source) and the pedestrian (test charge); hence it achieves advanced estimation of crowd behavior. It is a field similar to an electrostatic one, described by the equation below, though it has some differences in order to be applicable in pedestrian motion.

$$\vec{F}(\vec{r}) = \frac{q}{4\pi\epsilon_o} \sum_{i=1}^{N} \frac{Q_i(\vec{r} - \vec{r_i})}{|\vec{r} - \vec{r_i}|^3} = \frac{q}{4\pi\epsilon_o} \sum_{i=1}^{N} \frac{Q_i}{R_i^2} \hat{R}_i = q\vec{E}(\vec{r})$$
(1)

In Eq. 1, Q_i and $\vec{r_i}$ are the magnitude and the position of the i^{th} charge, respectively, $\hat{R_i}$ is the unit vector in the direction of $\vec{R_i} = \vec{r} - \vec{r_i}$, i.e. a vector pointing from charge Q_i to charge q and R_i is the magnitude of $\vec{R_i}$, i.e. the distance between charges Q_i and q and \vec{E} the corresponding electric field. The unit vector in space is expressed, in Cartesian notation, as a linear combination of i = [1;0] and j = [0;1] and the values of its scalar components are equal to the cosine of the angle formed by the unit vector with the respective basis vector [8]. The force is attracting when generated by charges located at exits and repulsive when generated by charges that represent obstacles or walls [9]. The distance calculated is the exact Euclidean distance, thus introducing increased precision in the model as far as the direction of pedestrian concerns [11].

In the CA grid every cell covers an extent of approximately $40 \times 40 \ cm^2$ [17]. Each cell corresponds to the fixed area that a person could occupy [1]. CA cells obtain discrete values, thus indicating their status; either free or occupied. During each time step, the algorithm aims at the definition of the direction that an individual should move towards to reach the closest exit trying to occupy one of the eight possible states of its closest neighborhood (Moore). More explicitly, according to the attracting force from the exit of the room and the repelling forces from obstacles and walls, the coordinates of the next cell-target are calculated. In case that the cell-target is free or it has not been defined as target from another pedestrian, then the initial pedestrian moves towards. Otherwise, the pedestrian searches for a neighboring cell equidistant from the exit with the initial target-cell. In case that the target-cell is an exit, it is checked whether this is free or not. Only if the exit is free does the pedestrian move towards it. The convergence of the resultant force plays significant role on the driving mechanism of the model. In fact, the convergence of the resultant force upon a test charge towards the point that the closest source is located defines the direction of movement of each pedestrian, which is towards the closest exit (as shown in Fig. 1(c)). More details about the mathematical calculations of the convergence of the resultant force can be found in [11].

CA model for automated obstacle avoidance: A distinct feature of the model is an automated, computationally fast and efficient method to enables obstacle avoidance based on the effect of a virtual field generated near obstacles [13]. Inside the field, a pedestrian moves towards the direction of greater field values. Following that direction a pedestrian is enabled to overcome efficiently



Fig. 1. (a) The effect of a test charge, which represents a pedestrian, on the field around the point and (b) the corresponding effect of an ordinary charge (right). (c) Graphical solution of the case of the convergence of the resultant force upon a test charge.

even complex obstacles. Specifically, in the general case, obstacle field values are increasing forming a parabola, which is described by the following equation:

$$F(x) = \frac{1}{2p} (x - x_o)^2, \ p > 0 \forall \in \{ [x_{wl}, x_{wr}] \cap (x_A - x_{wl} \neq 0 \cup x_{wr} - x_B \neq 0) \}$$
(2)

$$x_o = \frac{x_B - x_A}{2} \tag{3}$$

In eq. 2, p corresponds to the parameter of the parabola, which also defines the distance between the two branches of the graphical representation of the function. In fact, as $\frac{1}{2p} \rightarrow 0$, then the width of the parabola increases. Moreover, $(x_A, y_A), (x_B, y_B)$ represent the coordinates of the edges of the obstacle, whereas, x_{wl}, x_{wr} correspond respectively to the very left and very right x-axis coordinate of the walls. Equation 3 defines x_o , which corresponds to the x-axis coordinate of the middle point of the obstacle. In case that the obstacle is bonded to a wall, then the field is generated according to the common coordinate of the obstacle and the wall, as described by eqs. 4 and 5:

$$x_A - x_{wl} = 0 \Rightarrow F(x) = \frac{1}{2p} \left(x - x_{wl} \right)^2 \tag{4}$$

$$x_{wl} - x_B = 0 \Rightarrow F(x) = \frac{1}{2p} \left(x - x_{wr} \right)^2 \tag{5}$$

The length of the obstacle is given by:

$$L_{obstacle} = x_B - x_A \tag{6}$$

whereas the length of the area between walls is given by:

$$m = x_{wr} - x_{wl} \tag{7}$$

In the case of complex obstacles the corresponding field is generated by the superposition of fields that correspond to fundamental obstacles. Fig. 2 clarifies the effect of the auto-defined obstacle field to the direction of pedestrian movement, in correspondence to the location and the shape of the obstacle. It should be mentioned that above mathematical presentation takes into account geometrically shaped obstacles, however with slight modification can be successfully applied to arbitrary shaped obstacles as well. More details can be found in [13]. Hardware Implementation of CA models: It should be also noticed that the aforementioned CA model is orientated as a real-time processing module of an embedded system that could prevent clogging in exits under emergency conditions. More specifically, the initialization process could be originated along with a detecting and tracking algorithm supported by cameras and the automatic response of the processor provides the location of pedestrians around escape points. Consequently, the realization of the model becomes a rational additional step. Moreover, in terms of circuit design and layout, ease of mask generation, silicon-area utilization and maximization of achievable clock speed CA are perhaps the computational structures best suited for a fully parallel hardware realization [10,13]. In contrast to the serial computers, the implementation of the model is motivated by parallelism, an inherent feature of CA that contributes to further acceleration of the model's operation. The hardware implementation of the presented model is based on FPGA logic (for example the schematic design of the closest exit tracker can be seen in Fig 3. The dedicated processor could be used as a real-time processing module of an embedded system, dedicated to surveillance that responds fast under crowd evacuation emergency conditions. More techical details about CA design and the target FPGA in [11].

Follow-the-Leader CA Model: Furthermore, a CA-based computational model has been developed that simulates the movement of a crowd formed by individuals, following some of the basic principles of flocking. The driving mechanism of the model is based on the acceptance that each member of the crowd moves independently. Whenever possible, a group of individuals approaches the



Fig. 2. (a) The graphical representation of the obstacle field in case that the obstacle lays between walls. The values of the field increase in the direction from left to right for half the length of the obstacle and the vice versa for the other half. Thus, the pedestrian is enabled to move following the one direction or the other, as indicated by arrows. (b) The response of the obstacle field, in case that the exit is closer to the left edge of the obstacle $(X_A = X_{wl})$. (c) The case of a vertical obstacle and the corresponding field. (d) The case of a complex obstacle. The final field is generated by the superposition of field cases (b) and (c).

closest exit following the shortest route. Thus, each member is supposed to have a complete knowledge of the space topology and acts completely rational. The model has been developed to simulate crowd movement both in 2-d and 3-d, according to flocking principles and incorporating the Follow-the-Leader technique [14]. Following nature's practice, the model allows dynamical transitions of the role of the leader among the members of the group. Particularly, in case that a member of the group appears in front of the leader also following the same direction of movement, then a leader's role transition occurs. The member in front becomes the leader, whereas the leader turns into a simple member. An individual follows the leader until it reaches the target, e.g. the exit. Furthermore, the model enables the creation of different groups in the crowd, assigning to each group a leader and the corresponding members. It favors the dynamic grouping rather than the static one.



Fig. 3. Structure of a cell of the CA grid for Euclidean based calculations

As far as the members of the group concern, their movement is defined by the same set of rules as the one that defines the movement of individuals towards an exit. From a mathematical point of view, the direction of movement of the individuals relies on a potential field. It derives from the negative gradient of a function that involves the distance (Manhattan) of each point of the area from the position of the leader. In the case of two-dimensions, the function f(x, y) is defined as follows:

$$f(x,y) = abs \left(x - x_o\right) + abs \left(y - y_o\right) \tag{8}$$

where (x_o, y_o) correspond to the coordinates of the leader.

The corresponding gradient of the function f(x, y) is defined as:

$$-\nabla f(x,y) = -\left(\frac{\partial f(x,y)}{\partial x}\vec{i} + \frac{\partial f(x,y)}{\partial y}\vec{j}\right)$$
(9a)

$$-\nabla f(x,y) = -\left(\frac{\partial abs(x-x_o)}{\partial x}\vec{i} + \frac{\partial abs(y-y_o)}{\partial y}\vec{j}\right)$$
(9b)

It can be thought as a collection of vectors pointing in the direction of decreasing values of f(x, y).

Spatially, the whole process is divided in eight subsections, i.e. for the case that the leader moves i) downwards $(y > y_o)$, ii) upwards $(y < y_o)$, iii) to the left $(x > x_o)$, iv) to the right $(x < x_o)$, v) downwards and to the right $(y > y_o)$ and $(x < x_o)$, vi) downwards and to the left $(y > y_o)$ and $(x > x_o)$, vii) upwards and to the left $(y < y_o)$ and $(x > x_o)$ and viii) upwards and to the right $(y < y_o)$ and $(x < x_o)$, respectively. For instance, in the case that the leader moves downwards and to the right (i.e. case (v)), the corresponding potential field that is derived from equations 9, is depicted in Figure 4. The adoption of sectors is based on the simple fact that even in real life bounded areas are divided to



Fig. 4. The corresponding potential field in the case that the leader is positioned at $(x_o = 2, y_o = -2)$ and moving downwards and to the right. (a) All individuals within blue–arrows area follow the leader (red spot). Four sectors in 3–D (b) Gates placed at the center of the internal sides of the sectors.

multiple sub-areas with their own formation and their own exits. Each sub-areas shares the same properties with the total area, thus enabling the use of the property of the superposition. Hence, the scalability of the method is reassured, allowing its application in more complicated areas. The movement of the leaders through sections takes place as follows: depending on the direction of the leaders motion, each section is supplied with two gates that the leaders use to enter the section. The leaders move towards the exits following the same rules that the members of the flock use to follow the leader.

The 3-d CA is defined in a cubic space, the dimensions of which are variable, taking into consideration that each cell needs three coordinates (i, j, k) to be properly defined. The neighborhood of each cell is shaped by its 26 closest cells, whereas there are four (4) sectors that divide the space in four rectangular parallelepipeds (Fig. 4a). In case that we wish to test the behavior of the model in 3-d dimensions, the following scheme takes place; the leaders pass through the sectors following the 1-2-3-4-1 sequence for the clockwise direction or 4-3-2-1-4 for the anti-clockwise one. Adopting similar logic as in two-dimensions, the gate that influences one sector lies inside the following sector. The gates are placed at the center of the internal sides of the sectors (Fig. 4b).

Different simulation processes were taken into account in order to verify the response of the model and investigate its efficiency [14]. Particularly, these various simulation scenarios demonstrated distinct features of crowd movement such as flocking, increasing crowd density in turnings and crowd movement deceleration as self-organized groups try to pass obstacles, transition from a random to a coordinated motion, etc. Please also check [14],[15] for further analytical presentation of the under study simulation scenarios and the corresponding results. *Anticipative crowd management tool based on CA model:* An integrated system that operates as an anticipative crowd management tool in cases of medium density crowd evacuation was also developed based on CA models [12]. Preliminary real data evaluation processes indicate that it responds fast in order to prevent clogging in exits under emergency conditions. The system consists of

three modules; the detecting and tracking algorithm, the CA model of possible route estimation and the sound and optical signals. The initialization process is originated from the detecting and tracking algorithm, which is supported by cameras. The automatic response of the algorithm provides the location of pedestrians around escape points at any time, thus providing instant initialization data to the model of possible route estimation. However, its role is not confined exclusively for initialization purposes. Instead, it also operates as a control and rectifying mechanism, by checking and correcting periodically the response of the CA dynamic model originated from electrostatic-induced potential fields. The response of the route estimation model is compared to the output of the tracking algorithm. In cases of large differences, the model is re-initialized according to the current conditions of the tracking algorithm. Finally, sound and optical signals enable the system to redirect pedestrians, enhancing its effectiveness and efficiency [12]. System operation is developed in four successive stages, setting out with the detection and tracking of pedestrians that enable dynamic initialization and continuing with the estimation of their possible route for the very near future. Then, among all possible exit points, the most suitable is proposed as an alternative, triggering the activation of appropriate guiding signals, sound and optical. The criterion of suitability is the distance of the congested exit from



Fig. 5. (a) Initialization of the pedestrian movement model outdoor. (c) The transition from the first stage of the anticipative system, i.e. the detection algorithm to the second one, i.e. the crowd movement model. Red-dotted areas correspond to areas of interest in front of exits. (b) Two successive frames displaying response of individuals during alarm activation in a teaching room. In frame (b), people move towards exit A, not having reached the area of surveillance yet. Alarm is activated in frame (d).



Fig. 6. The proposed robot guided evacuation system's architecture

an alternative one. Hence, the closest free exit is preferred. A few paradigms of the system process for outdoor and indoor study cases are depicted in Fig. 5. CA based Robot Crowd Evacuation: Recently, a robot guided evacuation was proposed, to the best of our knowledge, for the first time in literature [16]. The proposed framework relies on the well established CA simulation models, while it employs a real world evacuation implementation assisted by a mobile robot. More specifically, the implementation of a CA model capable of assessing the humans' behavior during evacuation occasions has been presented. Then, an evacuation framework based on an assistant robot that deploys in emergency situations is exhibited. The main attribute of the introduced method is the coexistence of a discrete CA simulation model and a real wold continuous implementation combined with the development and usage of a custommade robotic platform. Thus, the method exploits both the computational speed of the discrete simulation and the added value of a real robotic implementation. Additionally, the entire evacuation algorithm is accompanied by a custommade assistant robot which attracts a group of evacuees from a congestive exit and redirects them towards to a less crowded one. The proposed evacuation framework has been evaluated on real world conditions and exhibited remarkable performance in terms of speed during the evacuation proving: a) the credibility of the CA simulation modeling and b) the necessity of an intelligent mobile aid during the evacuation procedure.

3 Conclusions

CA have been proven quite efficient to model successfully crowd dynamics. In this paper, several CA models and corresponding systems for crowd dynamics were briefly presented taking into consideration different modeling principles, like potential fields techniques, obstacle avoidance, follow the leader principles, grouping theory, etc. Moreover, due to their inherent parallelism CA, some of these models have been implemented in hardware and have been considered as basis of an anticipation crowd management system which is able of preventing clogging in exits during crowd evacuation processes. Finally, robot guided evacuation was presented, based on an accurate CA model capable of assessing the human behavior during emergency situations takes advantage of the simulation output to provide sufficient information to a mobile robotic guide, which in turn guides people towards a less congestive exit at a time.

References

- Burstedde, C., Klauck, K., Schadschneider, A., Zittartz, J.: Simulation of pedestrian dynamics using a two-dimensional cellular automaton. Physica A 295, 507–525 (2001)
- Helbing, D., Farkas, I., Vicsek, T.: Simulating dynamical features of escape panic. Nature 407, 487–490 (2000)
- Goldstone, R.L., Janssen, M.A.: Computational models of collective behavior. Trends in Cognitive Sciences 9(9), 424–430 (2005)
- 4. Bonabeau, E.: Agent-based modeling: Methods and techniques for simulating human systems. PNAS 99(3), 7280–7287 (2002)
- Lo, S.M., Huang, H.C., Wang, P., Yuen, K.K.: A game theory based exit selection model for evacuation. Fire Safety Journal 41, 364–369 (2006)
- Nishinari, K., Sugawara, K., Kazama, T., Schadschneider, A., Chowdhury, D.: Modelling of self-driven particles: foraging ants and pedestrians. Physica A 372, 132–141 (2006)
- Georgoudas, I.G., Sirakoulis, G.C., Andreadis, I.: A simulation tool for modelling pedestrian dynamics during evacuation of large areas. In: Maglogiannis, I., Karpouzis, K., Bramer, M. (eds.) AIAI 2006. IFIP AICT, vol. 204, pp. 618–626. Springer, Heidelberg (2006)
- Georgoudas, I.G., Kyriakos, P., Sirakoulis, G.C., Andreadis, I.: A Cellular Automaton Evacuation Model Based on Electric and Potential Fields Technique. In: First International Conference on Evacuation Modeling (RISE 2008), September 23-25, Delft, The Netherlands (2009)
- Georgoudas, I.G., Sirakoulis, G.C., Andreadis, I.T.: Hardware implementation of a Crowd Evacuation Model based on Cellular Automata. In: PED 2008, pp. 451–463 (2008)
- Mardiris, V., Sirakoulis, G.C., Mizas, C., Karafyllidis, I., Thanailakis, A.: A CAD system for modeling and Simulation of Computer Networks using Cellular Automata. IEEE Transactions on Systems, Man and Cybernetics, Part C 38(2), 253–264 (2008)
- Georgoudas, I.G., Kyriakos, P., Sirakoulis, G.C., Andreadis, I.: An FPGA Implemented Cellular Automaton Crowd Evacuation Model Inspired by the Electrostatic-Induced Potential Fields. Microprocessors and Microsystems 34(7-8), 285–300 (2010)
- Georgoudas, I.G., Sirakoulis, G.C., Andreadis, I.: An Anticipative Crowd Management System Preventing Clogging in Exits during Pedestrian Evacuation Processes. IEEE Systems 5(1), 129–141 (2011)
- Georgoudas, I.G., Koltsidas, G., Sirakoulis, G.C., Andreadis, I.T.: A Cellular Automaton Model for Crowd Evacuation and its Auto-Defined Obstacle Avoidance Attribute. In: Bandini, S., Manzoni, S., Umeo, H., Vizzari, G. (eds.) ACRI 2010. LNCS, vol. 6350, pp. 455–464. Springer, Heidelberg (2010)
- Vihas, C., Georgoudas, I.G., Sirakoulis, G.C.: Follow-the-Leader Cellular Automata based Model Directing Crowd Movement. In: Sirakoulis, G.C., Bandini, S. (eds.) ACRI 2012. LNCS, vol. 7495, pp. 752–762. Springer, Heidelberg (2012)
- Vihas, C., Georgoudas, I., Sirakoulis, G.C.: Cellular Automata incorporating Follow the Leader Principles to Model Crowd Dynamics. Journal of Cellular Automata 8(5-6), 333–346 (2013)
- Boukas, E., Kostavelis, I., Gasteratos, A., Sirakoulis, G.C.: Robot Guided Crowd Evacuation. Accepted for publication in IEEE Transactions on Automation Science and Engineering (2014)