Double Auctions 6

6.1 Introduction

In the auctions analyzed to this point, we have assumed that there is only one seller and many bidders, in which the seller acts as a monopolist (**forward auction**). However, the main characteristic of a **double auction** is that multiple buyers and sellers interact; it is a **two-sided auction**.

In a double auction the buyers submit *bids* for the amounts that they are willing to pay, whereas the sellers make offers, *asks*, to indicate the prices at which they are willing to sell. This auction model has been used for more than 100 years to exchange items such as stocks, bonds, and agricultural products. There are a significant number of different double-auction models, with both one round and multiple rounds. In the following sections, we will comment on several of these models, but we recommend reviewing the work of Friedman and Rust [31] to complement this study.

6.2 Sealed-Bid Double Auction

In a **sealed-bid double auction**, both the buyers and the sellers indicate, in a single round, the price at which they are willing to buy or sell an item.[1] The bid made by buyer i is represented by b_i, which is the maximum price for which the buyer is willing to buy the item. The ask made by seller k is represented by o_k, which is

[1]The **periodic double auction** is a sealed-bid auction in which potential buyers and sellers have one period of time to submit their bids or asks. Then the auctioneer determines the items to be exchanged and a new auction starts. The auctioneer establishes a new period in which buyers and sellers can submit bids for the next auction.

A. Mochón, Y. Sáez, *Understanding Auctions*, Springer Texts in Business and Economics, DOI 10.1007/978-3-319-08813-6_6

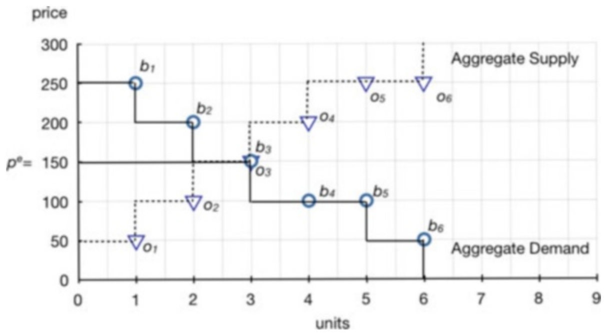

Fig. 6.1 Sealed-bid double auction

the minimum amount for which the seller is willing to sell the item.[2] The aggregate demand is obtained by rearranging the buyers' bids from the highest to the lowest. The aggregate supply is obtained by rearranging the sellers' asks from the lowest to the highest. The intersection between the aggregate supply and demand determines the amount that is bought and sold, as well as the **equilibrium price**, p^e.

To better understand how this type of auction works, let us examine the following example. There are six buyers who are willing to pay the following prices per item: $b_1 = 250$ euros, $b_2 = 200$ euros, $b_3 = 150$ euros, $b_4 = 100$ euros, $b_5 = 100$ euros, and $b_6 = 50$ euros. In addition, there are six sellers who are not willing to sell an item for less than the following amounts: $o_1 = 50$ euros, $o_2 = 100$ euros, $o_3 = 150$ euros, $o_4 = 200$ euros, $o_5 = 250$ euros, and $o_6 = 250$ euros. Figure 6.1 shows the aggregate supply and demand, in which we can observe that the demand equals supply when the price is equal to $p^e = 150$ euros. The winning bids are: $b_1^* = 250$ euros, $b_2^* = 200$ euros, and $b_3^* = 150$ euros; the winning offers are: $o_1^* = 50$ euros, $o_2^* = 100$ euros and $o_3^* = 150$ euros. Given the bids and asks that have been made, the first three sellers will effectively sell their items, and the first three buyers obtain one item each.

After determining the participants who will finally buy or sell an item, the next step is to calculate the prices that the buyers will pay and the revenue that the sellers will obtain. These amounts depend on the rules established by the auctioneer, that is, the **clearing house**. The rules that are most frequently used are uniform-price and discriminatory pricing rule.

[2]In a double auction, the bidders may also bid on or the sellers may offer multiple items, but to simplify the analysis, we will assume that each participant bids on and offers, at the most, one item.

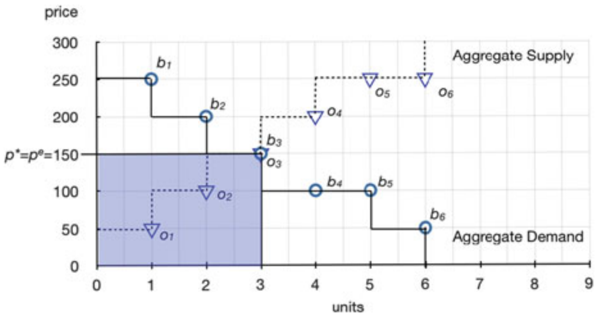

Fig. 6.2 Sealed-bid double auction with uniform-price rule

6.2.1 Sealed-Bid Double Auction with Uniform-Price Rule

If the **uniform-price rule** is established, all buyers will have to pay the same price for the acquired items, a price that is equal to the equilibrium price, that is, the price for which the aggregate demand equals the aggregate supply $p^e = p^*$. Continuing with the previous example and under this pricing rule, the winning buyers will pay: $P_1^* = P_2^* = P_3^* = 150$ euros. Each seller will also obtain a revenue of 150 euros. These payments are represented by the shaded area in Fig. 6.2.

Aggregate supply and demand may be equal not for a single equilibrium price but rather for an interval, in which case a rule must be applied to set the exchange price. The k**-double auction**, which is discussed later in this chapter, solves this problem.

6.2.2 Sealed-Bid Double Auction with Discriminatory Pricing Rule

If the seller sets the **discriminatory pricing rule** each bidder pays a different price for the items that he obtains, which will depend on the established rule. One option is to set the exchange price based on the bids made by the buyers (*pay-buyers' price*), so it is a *pay-your-bid* auction. With this rule the aggregate payment of all bidders is equal to the sum of the winning bids: $P^* = \Sigma_{i \in W} b_i^*$, where W is the set of winning bidders.

In the example of the previous section, the total amount that the buyers would pay is equal to $P^* = 250 + 200 + 150 = 600$ euros that corresponds to the shaded area in Fig. 6.3. The first buyer pays $P_1^* = 250$ euros, the second one $P_2^* = 200$ euros, and the third one $P_3^* = 150$ euros.

However, the sale price may also depend on the offers made by the sellers (*pay-sellers' price*). The total payment of all winning bidders with this rule is equal to the sum of the winning offers: $P^* = \Sigma_{k \in W} o_k^*$, where W is the set of winning sellers.

The total amount that the buyers would pay in this example is equal to $P^* = 50 + 100 + 150 = 300$ euros, represented by the shaded area in Fig. 6.4. So the

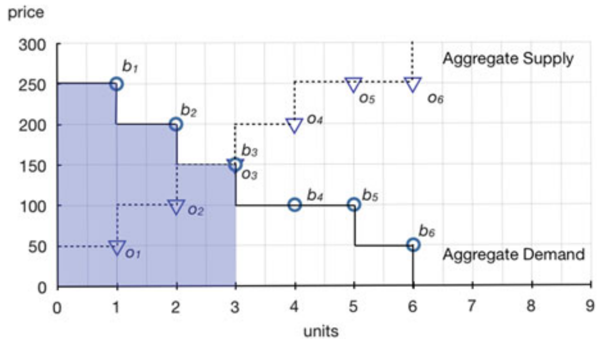

Fig. 6.3 Sealed-bid double auction with discriminatory pricing rule: pay-buyers' price

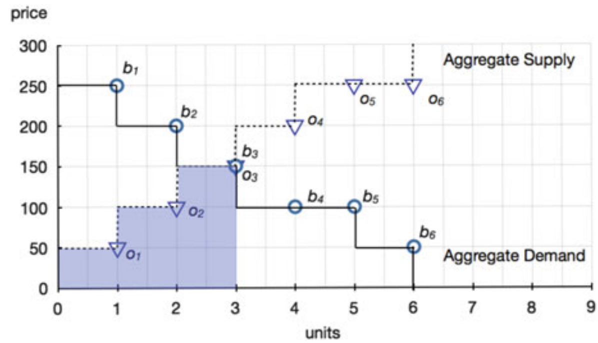

Fig. 6.4 Sealed-bid double auction with discriminatory pricing rule: pay-sellers' price

sellers' revenue is equal to $R_1^* = 50$ euros for the first one, $R_2^* = 100$ euros for the second one, and $R_3^* = 150$ euros for the third one.

The total payments done by all winning bidders using these two rules (pay-buyers' price and pay-sellers' price) are two extreme options but we could also consider an intermediate price-rule using the following equation:

$$P^* = \Sigma_{i,k \in W}(\alpha b_i^* + (1-\alpha)o_k^*), \tag{6.1}$$

where W is the set of winning buyers/sellers and α is a number between zero and one that shows the influence or bias of the buyer/seller at the time the exchange price is determined. For any value of α, no buyer pays more than what he would have bid, and no seller sells for less than what he would have offered.

Continuing with the previous example, the bids of the winning bidders are: $b_1^* = 250$ euros, $b_2^* = 200$ euros, and $b_3^* = 150$ euros. The winning asks are the following: $o_1^* = 50$ euros, $o_2^* = 100$ euros, and $o_3^* = 150$ euros. If, for example, $\alpha = 0.8$, then the aggregate payment done by the winning bidders is equal to $P^* = (0.8 \times 250) + (0.8 \times 200) + (0.8 \times 150) + (0.2 \times 50) + (0.2 \times 100) + (0.2 \times 150) = 540$ euros. Instead, if $\alpha = 0.2$, then $P^* = 360$ euros.

We can observe that, as the value of α increases, the total payment approaches the value with the pay-buyers' price rule. Conversely, as α approaches zero, the value is closer to the one obtained with the pay-sellers' price rule.

6.2.3 k-Double Auction

In a sealed-bid double auction (one round) in which the uniform-price rule is set, the supply and the demand may be equal for a range of prices instead of for a fixed price: $b^*_{\text{min}} \geq p^{\text{e}} \geq o^*_{\text{max}}$, where b^*_{min} is the lowest bid made by a buyer that exceeds an offer and o^*_{max} is the highest offer made by a seller that is below a bid. Under these circumstances, it is necessary to establish a rule to select the exchange price (p^*), which should be included in this interval. A solution that is frequently used is to draw on the **k-double auction**, as described by Satterthwaite and Williams [73]. In this model, the exchange price is calculated using the following equation:

$$p^* = kb^*_{\text{min}} + (1 - k)o^*_{\text{max}}, \tag{6.2}$$

where k is a number between zero and one. Upon applying this rule, no buyer pays more than what he has bid, and no seller sells for less than his ask.

To understand this auction, let us analyze the following example. In a sealed-bid double auction, the potential buyers submit the following bids: $b_1 = 250$ euros, $b_2 = 200$ euros, $b_3 = 150$ euros, $b_4 = 100$ euros, $b_5 = 100$ euros, and $b_6 = 50$ euros. The potential sellers make the following offers: $o_1 = 50$ euros, $o_2 = 100$ euros, $o_3 = 100$ euros, $o_4 = 150$ euros, $o_5 = 250$ euros, and $o_6 = 250$ euros. The aggregate supply and demand are represented in Fig. 6.5. As shown, the supply and the demand are equal for the interval of $b^*_{\text{min}} = 150 \geq p^{\text{e}} \geq 100 = o^*_{\text{max}}$.

If we use a k-double auction to set the price paid, the price will depend on the value of the parameter k. The sale price (which will have to be included in the interval) is calculated by the following equation:

$$p^* = k150 + (1 - k)100. \tag{6.3}$$

If $k = 0.5$, the exchange price is equal to $p^* = 125$ euros. If $k = 0.8$, the exchange price is equal to $p^* = 140$ euros. If $k = 0.2$, the exchange price is equal to $p^* = 110$ euros.

In this example, the buyers who effectively buy an item are the first, second, and third bidders. If we set a value of $k = 0.5$, the final payment that each of them

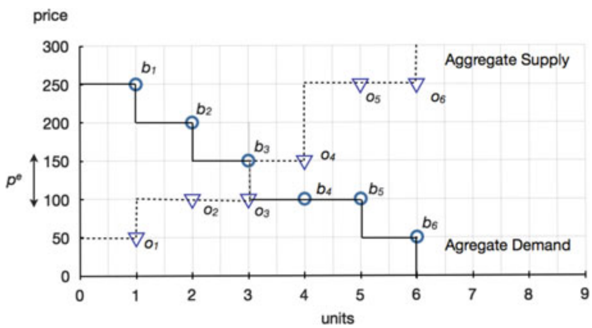

Fig. 6.5 k-double auction

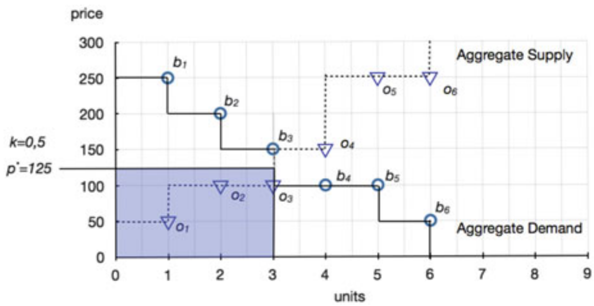

Fig. 6.6 k-double auction for $k = 0.5$

makes is equal to $P_1^* = P_2^* = P_3^* = 125$ euros. The sellers who finally sell the items that they were offering are the first three sellers, who obtain revenues equal to $R_1^* = R_2^* = R_3^* = 125$ euros for $k = 0.5$. The shaded area in Fig. 6.6 represents the payments made by the buyers under this assumption.

6.3 Continuous Double Auction

The main characteristic of a sealed-bid double auction is that all of the participants submit their bids and offers in a single round, and the clearing house establishes all of the items to be exchanged and the sale prices. However, a **continuous double auction** (**CDA**) allows items to be bought and sold over the course of multiple rounds. According to rules that are determined, different types of CDAs can be established [31]. Below, we comment on some of these formats.

6.3.1 Synchronized CDAs

A **synchronized continuous double auction**[3] is composed of r rounds, which are divided into s periods. In each period, there is a fixed number of t steps with alternate steps in which the participants make bids (buyers) and asks (sellers) and steps in which buy and sell orders are placed. Each step t begins with a bid/ask step (BA step), in which, simultaneously, the buyers indicate the prices that they are willing to pay per unit, and the sellers indicate the prices at which they are willing to sell. $b_i^{s,t}$ represents the amount that bidder i is willing to pay for an item in step t in the period s, and $o_k^{s,t}$ represents the amount for which seller k is willing to sell an item in step t of period s. Among all of the bids and asks made in step t of period s of round r, the bidder who has the *highest outstanding bid* ($b_{\max}^{s,t}$) and the seller who has the *lowest outstanding ask* ($o_{\min}^{s,t}$) are determined. These participants pass on to the buy/sell step.

[3]Developed by the Santa Fe Institute [67].

Table 6.1 Synchronized continuous double auction

Round $r = 0$								
Period $s = 0$		Bids			Asks			Price
t	Step	$b_1^{0,t}$	$b_2^{0,t}$	$b_3^{0,t}$	$o_1^{0,t}$	$o_2^{0,t}$	$o_3^{0,t}$	$p^{*0,t}$
0	BA	5	10	7	20	25	28	
	BS		No		No			
1	BA	7	17	10	15	23	20	
	BS		Ok		Ok			17
Period $s = 1$		Bids			Asks			Price
t	Step	$b_1^{1,t}$	$b_2^{1,t}$	$b_3^{1,t}$	$o_1^{1,t}$	$o_2^{1,t}$	$o_3^{1,t}$	$p^{*1,t}$
0	BA	14	15	16	18	19	20	
	BS			Ok	Ok			18
1	BA	10	8	7	16	15	13	
	BS	No				No		

In the buy/sell step (BS step), both the buyers and the sellers may accept the bids or asks and the transaction is carried out. However, the participants may choose not to accept the bids or asks, and this round may end with no exchange. In the next round ($t + 1$), the BA step begins again, which later leads to another BS step.[4]

After multiple steps and periods, the participants learn about their rivals' values and their bidding strategies. In this auction model, participants must answer three key questions: (1) how much to bid or ask, (2) when to submit a bid or make an offer, (3) and when to accept a bid or an ask. Each buyer pays a different price for the units he gets, which is equal to the exchange price corresponding to period s of BS step t in which the buyer accepts the offer and the seller accepts the bid, $p^{*s,t}$.[5]

To better understand this auction, we analyze the following example, in which there is a single round ($r = 0$), composed of two periods, which include two BA and BS steps. Table 6.1 shows both the bids and the asks made in each step by the three buyers and the three sellers involved.

The auction begins with the first BA step ($t = 0$) in the first period ($s = 0$) of the first round ($r = 0$) in which buyers and sellers submit their bids and asks. The highest bid in this BA step is $b_{\max}^{0,0} = 10$ euros, submitted by the second buyer, and the minimum ask is $o_{\min}^{0,0} = 20$ euros, demanded by the first seller. These two participants go on to the BS step ($t = 0$); however, neither of them accepts the bid/ask of the other, and therefore, no transaction takes place.

The BA step $t = 1$ of period $s = 0$ then begins, in which the highest outstanding bid and the lowest outstanding ask are $b_{\max}^{0,1} = 17$ euros and $o_{\min}^{0,1} = 15$ euros, made

[4]The main characteristic of this CDA is the time discretization, in which the BA steps alternate with the BS steps. In auctions in which time is considered continuously, the fastest agents may have certain advantages. The speed of an agent may be affected by delays in the communication system or in the processors. However, with time discretization and with a sufficient time interval for each phase, it can be ensured that all of the participants have the same opportunities.

[5]McCabe et al. [50] proposed a CDA with a uniform-price rule, the uniform-price CDA.

by the second buyer and the first seller, respectively. These participants go on to the $t = 1$ BS step, and in this case, there is a transaction because the bid is greater than the ask, and both accept the exchange. Therefore, the second buyer buys one unit from the first seller, with a sale price equal to $p^{*0,1} = 17$ euros.

After this phase, the first period ends, and the second period ($s = 1$) begins with the BA step $t = 0$. In this step, the third buyer makes the highest bid, $b^{1,0}_{\max} = 16$ euros, and the first seller makes the lowest ask, $o^{1,0}_{\min} = 18$ euros. These participants go on to the BS step, in which, despite the bid being lower than the ask, both participants accept the transaction. Therefore, the third buyer pays $p^{*1,0} = 18$ euros to the first seller for one item.

Then, in BA step $t = 1$ takes place, in which $b^{1,1}_{\max} = 10$ euros and $o^{1,1}_{\min} = 13$ euros. The participants who have made this bid and ask go on to the BS step, but no transaction takes place because neither of them accepts the bid/ask of the other.

After this phase, the auction ends. The second buyer has won an item for which he has paid $P^*_2 = 17$ euros and the third buyer has won an item for $P^*_3 = 18$ euros. The first seller has sold two items, obtaining a total revenue of $R^*_1 = 17 + 18 = 35$ euros.

6.3.2 Double Dutch Auction

The **double Dutch auction**, developed by the University of Arizona [41, 49], operates as follows. An opening high price is established for the buyers (p^s_b) that progressively decreases in multiple rounds (t_b) until a buyer stops the process and is awarded an item. Similarly, for the sellers, an opening price is established (p^s_o), which is considered low and which increases from round to round (t_o) until a seller stops increasing the price and agrees to sell an item. The process continues on both sides until the prices intersect. At this point, all of the items that have been awarded during the process are exchanged. The final price that the buyers must pay is the overlapping price from both processes, that is, the price of the final round for both the buyers and the sellers: $p^{T_b}_b = p^{T_o}_o = p^*$, in which T_b is the last buyers' round and T_o is the last sellers' round.[6]

The next example illustrates a double Dutch auction in which $\hat{b}_i$ represents the intention of buyer i to bid, and $\hat{o}_k$ shows the intention of seller k to ask. There are five buyers willing to bid for an item at the following prices: $\hat{b}_1 = 40$ euros, $\hat{b}_2 = 35$ euros, $\hat{b}_3 = 25$ euros, $\hat{b}_4 = 10$ euros, and $\hat{b}_5 = 5$ euros. In addition, there are five sellers who are not willing to sell an item for less than the following prices: $\hat{o}_1 = 10$ euros, $\hat{o}_2 = 20$ euros, $\hat{o}_3 = 25$ euros, $\hat{o}_4 = 35$ euros, and $\hat{o}_5 = 40$ euros. Knowing these intentions to bid and ask and setting bid a increment/decrement at five euros, the evolution of the auction is shown in Table 6.2.

[6]It may happen that the bid of the last round on the buyers' side not to be equal to the offer of the last round on the sellers' side. Hence, the clearing house will have to set a selling price which will not be higher than the last bid nor lower than the last offer.

Table 6.2 Double Dutch auction

Buyers' rounds	p_b^{tb}	b_1^{tb}	b_2^{tb}	b_3^{tb}	b_4^{tb}	b_5^{tb}
$t_b = 0$	$p_b^0 = 55$	0	0	0	0	0
$t_b = 1$	$p_b^1 = 50$	0	0	0	0	0
$t_b = 2$	$p_b^2 = 45$	0	0	0	0	0
$t_b = 3$	$p_b^3 = 40$	1	0	0	0	0
Sellers' rounds	p_o^{to}	o_1^{to}	o_2^{to}	o_3^{to}	o_4^{to}	o_5^{to}
$t_o = 0$	$p_o^0 = 0$	0	0	0	0	0
$t_o = 1$	$p_o^1 = 5$	0	0	0	0	0
$t_o = 2$	$p_o^2 = 10$	1	0	0	0	0
Buyers' rounds	p_b^{tb}	b_1^{tb}	b_2^{tb}	b_3^{tb}	b_4^{tb}	b_5^{tb}
$t_b = 4$	$p_b^4 = 35$	0	1	0	0	0
Sellers' rounds	p_o^{to}	o_1^{to}	o_2^{to}	o_3^{to}	o_4^{to}	o_5^{to}
$t_o = 3$	$p_o^3 = 15$	0	0	0	0	0
$t_o = 4$	$p_o^4 = 20$	0	1	0	0	0
Buyers' rounds	p_b^{tb}	b_1^{tb}	b_2^{tb}	b_3^{tb}	b_4^{tb}	b_5^{tb}
$t_b = 5$	$p_b^5 = 30$	0	0	0	0	0
$t_b = 6$	$p_b^6 = 25$	0	0	1	0	0
Sellers' rounds	p_o^{to}	o_1^{to}	o_2^{to}	o_3^{to}	o_4^{to}	o_5^{to}
$t_o = 5$	$p_o^5 = 25$	0	0	1	0	0
Intention to bid/ask	i/k	1	2	3	4	5
$\hat{b}_i$		40	35	25	10	5
$\hat{o}_k$		10	20	25	35	40

The auction begins with a high opening price for the buyers, $p_b^s = 55$ euros. At this amount, no buyer makes a bid, which means that the price decreases until round $t_b = 3$, when the price is equal to $p_b^3 = 40$ euros, and the first bidder bids. At this point, the price stops decreasing, and the process turns to the sellers. The opening price is $p_o^s = 0$ euros, and no seller is willing to sell an item for this amount. The price successively increases until, in round $t_o = 2$, the price is equal to $p_o^2 = 10$ euros and there is a seller willing to exchange an item. The price stops increasing, and we return to buyers' rounds, in which the price again decreases. In the next buyers' round ($t_b = 4$), the second bidder bids at $p_b^4 = 35$ euros. Then, we return to the sellers' round to determine the next seller who is willing to exchange an item. In round $t_o = 3$, the price is equal to $p_o^3 = 15$ euros, and there is no seller who is interested. However, in the next round, the price is equal to $p_o^4 = 20$ euros, and another seller decides to offer his item at this amount. Now there are two items awarded, but because the last price on the buyers' side is greater than the last price on the sellers' side ($p_b^4 = 35 > 20 = p_o^4$), the auction continues. The buyers' price decreases in the next round, but because no bidder bids, the price continues to decrease. In round $t_b = 6$, the price is equal to $p_b^6 = 25$ euros and another bidder wants to buy an item, so the buyers' rounds stop again. The sellers' price increases in the next round to $p_o^5 = 25$ euros, and at this amount, another seller is willing to sell. In this round, the last buyers' price is equal to the last sellers' price, which marks the end of the auction, at $p_b^6 = p_o^5 = 25$ euros.

The final outcome of the auction is that the three first buyers each win an item in different rounds, but all of them pay the same price, which is the intersection price between the buyers and the sellers: $p^* = 25$ euros. The last two bidders, who are only willing to buy an item for $b_4 = 10$ euros and $b_5 = 5$ euros, respectively, do not manage to acquire any items. On the sellers' side, the three first sellers finally sell their items at a price of $p^* = 25$ euros. The two final sellers do not manage to award their items.

6.4 Variables Used in This Chapter

In this chapter, we use the following variables:

- $I = (1, 2, \ldots, N)$: Bidders.
- $J = (1, 2, \ldots, M)$: Identical items.
- $V_i = (v_{i,1}, v_{i,2}, \ldots, v_{i,M})$: Vector of bidder i's values for each item.
- $v_{i,j}$: Value of bidder i for the item j.
- b_i: Bidder i's bid for one item in a double auction.
- o_k: Seller k's ask for one item in a double auction.
- $b_i^{s,t}$: Bidder i's bid for one item in round t of period s of a synchronized CDA.
- $o_k^{s,t}$: Seller k's ask for one item in round t of period s of a synchronized CDA.
- $b_{\max}^{s,t}$: Highest outstanding bid in round t of period s of a synchronized CDA.
- $o_{\min}^{s,t}$: Lowest outstanding ask in round t of period s of a synchronized CDA.
- $\hat{b}_i$: Bidder i's intention to bid for one item in a double Dutch auction.
- $\hat{o}_k$: Seller k's intention to ask for one item in a double Dutch auction.
- $b_{\min}^*$: Lowest bid that matches an offer in a double Dutch auction.
- $o_{\max}^*$: Highest offer that matches a bid in a double Dutch auction.
- α: Parameter between 0 and 1, which determines the influence or bias of the buyer to set the final price in a double auction.
- $b_i^{t_b}$: Bidder i's bid in round t_b (buyers' rounds) in a double Dutch auction.
- $o_k^{t_o}$: Seller k's ask in round t_o (sellers' rounds) in a double Dutch auction.
- P^*: Payment done by all bidders for the items won.
- P_i^*: Bidder i's payment for the items won.
- p^{e}: Equilibrium price in a double auction.
- p^*: Selling price in a double auction.
- $p^{*s,t}$: Selling price of period s in round t of a synchronized CDA.
- p_b^s: Starting bid in a double Dutch auction.
- p_o^s: Starting ask in a double Dutch auction.
- $p_b^{t_b}$: Bidding price in round t_b in a double Dutch auction.
- $p_o^{t_o}$: Offering price in round t_o in a double Dutch auction.

- t_b: Round t on the buyers' side in a double Dutch auction.
- t_o: Round t on the sellers' side in a double Dutch auction.
- Γ_i^*: Income of bidder i from the acquired items.
- Π_i^*: Surplus of bidder i from the acquired items.
- R^*: Seller's revenue.

6.5 Exercises

1. In a sealed-bid double auction, the buyers submit the following bids in a single round: $b_1 = 25$ euros, $b_2 = 25$ euros, $b_3 = 25$ euros, $b_4 = 20$ euros, $b_5 = 20$ euros, and $b_6 = 10$ euros. In addition, the sellers make the following offers: $o_1 = 5$ euros, $o_2 = 5$ euros, $o_3 = 10$ euros, $o_4 = 15$ euros, $o_5 = 20$ euros, and $o_6 = 25$ euros. Given this information, solve the problems below.
   (a) Graphically represent the aggregate supply and demand.
   (b) Indicate which participants buy and sell an item.
   (c) Determine the final payments that the buyers will make if an uniform-price rule is established. Graphically illustrate the total payments of all the buyers.
   (d) Determine the final payments that the buyers will make if a discriminatory pricing rule with the pay-buyer's price is set. Show this result graphically.
   (e) Determine the final payments that the buyers will make if a discriminatory pricing rule with the pay-sellers' price is set. Show this result graphically.

6.6 Solutions to Exercises

1. The results obtained after performing a sealed-bid double auction are described in the following sections.
   (a) Figure 6.7 shows the aggregate supply and demand based on the bids and the offers made by the buyers and the sellers.

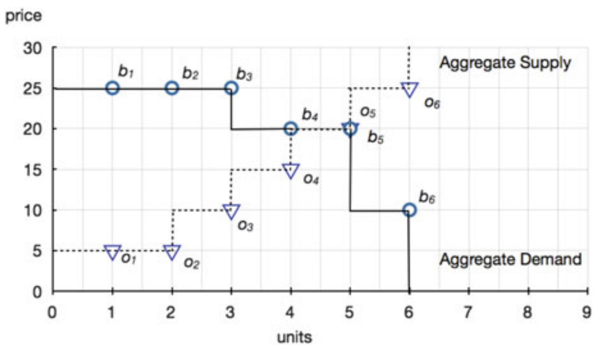

Fig. 6.7 Sealed-bid double auction

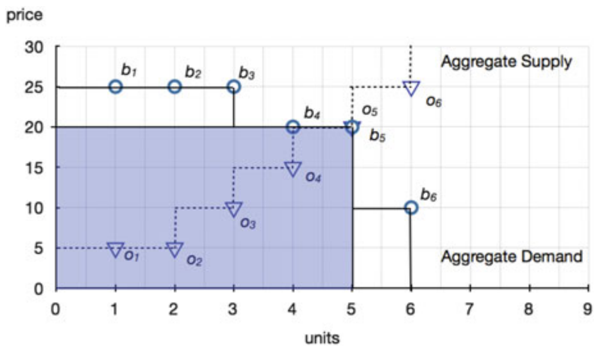

Fig. 6.8 Sealed-bid double auction with uniform-price rule

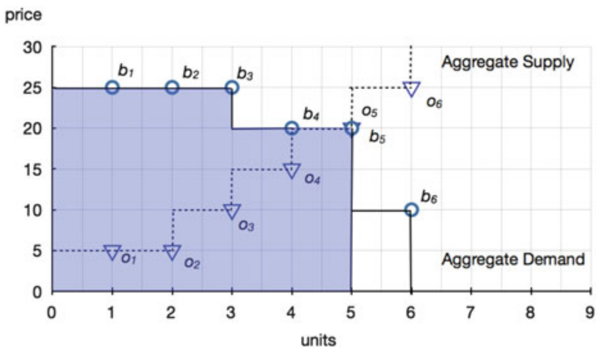

Fig. 6.9 Sealed-bid double auction with discriminatory pricing rule: pay-buyers' price

(b) Out of the six bidders, all except the last one (b_6) end up buying an item. Similarly, out of the six sellers, all except one (o_6) exchange the item that they were offering.
(c) If the selling price is established according to the uniform-pricing rule, all of the buyers pay the same amount for the acquired items. According to this rule, the exchange price is that for which the aggregate demand is equal to the aggregate supply, in this example, $p^* = p^e = 20$ euros. The total payment of all bidders is equal to $20 \times 5 = 100$ euros, represented by the shaded area in Fig. 6.8.
(d) If, instead of a uniform-price, a discriminatory price with the pay-buyers' price is set, the total amount that the buyers will pay will be equal to $(25 \times 3) + (20 \times 2) = 115$ euros. This amount is represented by the shaded area in Fig. 6.9.

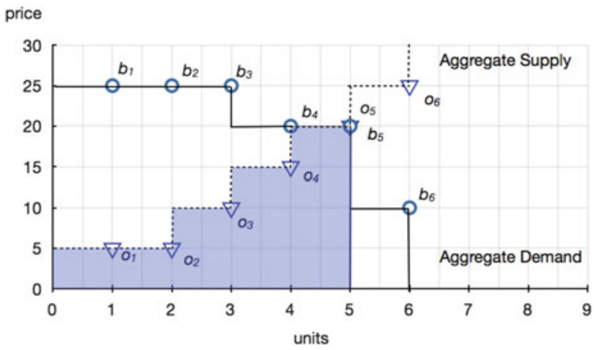

Fig. 6.10 Sealed-bid double auction with discriminatory pricing rule: pay-sellers' price

(e) If a discriminatory price with the pay-sellers' price is set, the total amount that the buyers will pay is equal to $(5 \times 2) + 10 + 15 + 20 = 55$ euros, which is represented by the shaded area in Fig. 6.10.