An Interval Programming Approach for an Operational Transportation Planning Problem

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Abstract. This paper deals with an interval programming approach for an operational transportation problem, arising in a typical agricultural cooperative during the crop harvest time. More specifically, an interval programming model with uncertain coefficients occurred in the righthand side and the objective function is developed for a single-period multi-trip planning of a heterogeneous fleet of vehicles, while satisfying the stochastic seed storage requests, represented as interval numbers. The proposed single-period interval programming model is conceived and implemented for a real life agricultural cooperative case study.

Keywords: interval linear programming, single-period multi-trip transportation planning problem, OR in agriculture.

1 Introduction

Gathering the harvest is considered one of the most crucial activity in agricultural environment for both cooperatives and individual farmers, in terms of the high costs involved and the vulnerability to weather conditions. Logistics and transportation activities constitute an inherent and primordial component of the agricultural cooperative system [2,5].

In most cases, farmers deliver their harvest to the nearest storage facility. For the sake of proximity storage availability, the cooperative forwards received cereals from buffer silos to expedition ones. Hence, new crop quantities are received as time progresses, which leads require planning the transfer and transportation of stored seed products.

Satisfaction of farmers' grain storage requests and high level reception service represent the main priority of the agricultural cooperatives during the harvest season. Quantities to be received at each storage facility are ordinarily unknown. In this regard, the case study cooperative has only an approximate information based-on prediction of the daily farmers' crop delivery quantities, represented as interval numbers. The purpose of this paper is to present a single-period multi-trip transportation planning application, modelled as a linear programming model with interval objective function and right-hand side constraints. It was motivated by a real case study encountered at a typical French agricultural cooperative.

The remainder of this paper is structured as follows: in the next section, a review of literature related to the interval programming research and applications is provided. The problem statement and modelling is presented in Section 3. After that, in Section 4, the solution method is exposed. In Section 5, several computational experiments are reported and discussed. Finally, in Section 6, overall remarks are drawn and topics for future research are outlined.

2 Literature Review

The conjecture of many real life problems presupposes miscellaneous types of uncertainties ([14],[23]). Classical mathematical programming models, nonetheless, only cope with deterministic values of problem input data. With the requirement of tackling non-deterministic data, appropriate techniques have been developed to suit various purposes and for different features of the stochastic data representation: probabilistic, possibilistic, and/or interval formats.

For decision-making problems considering uncertainty, the stochastic linear programming models touch effectively upon various random data with known probability distributions [12,22]. On this topic, the simple recourse model (a type of two-stage or multi-stage stochastic programming) consists in optimizing the expected objective function subject to some expected constraints [20]. Integrated chance-constrained programming is another approach in which solutions are feasible for a discrete set of probabilistic scenarios and all soft constraints are satisfied simultaneously with a given confidence level [8,18]. In turn, dependent chance-constrained programming pertains to maximizing some chance functions of events defined on stochastic sets in a complex uncertain decision system [16,15]. In fuzzy linear programming, the constraints and the objective function are regarded as fuzzy sets and their membership functions are assumed to be known.

Nevertheless, it turns out to be often difficult to specify a relevant membership function or an appropriate probability distribution in an stochastic environment [21,11]. In the last two decades, many theoretical studies have been focused on solving interval linear programming problems, in which the bounds on the uncertain data variation are required, without insisting on their probability distributions or membership functions. The interval analysis method was pioneered by Moore in 1959 as a tool for automatic control of the errors in a computed results [24].

Interval programming models where only the coefficients of the objective function are random and represented in interval format, are studied in [10,1,3,13]. Related to this matter, min-max regret optimization approach is usually employed, where different criteria that can be optimized, are distinguished: worst-case, bestcase, worst-case relative regret, worst-case absolute regret and maximum regret criteria, respectively. In this context two kinds of optimality (i.e. possible/weak and necessary/strong optimality) are defined.

Another stream of recent literature considers the interval left-hand side linear programming. In the work designed by [6], the authors have incorporated stochastic coefficients with multivariate normal distribution within an interval parameter linear programming context. On the other hand, when randomness, represented as interval numbers, appears in right-hand side constraints, only few results have already been obtained. The difficulty resides in the fact that the set of feasible solutions in not exactly known. In [9] the authors have investigated the complexity of two optimization versions, which correspond to the worst and best optimum solutions, when each right-hand side coefficient is defined as an interval number.

However, to the best of our knowledge and as remarked by [17], there are only few results on the issue of optimal solutions for a general interval linear programming, where the vector of the objective function, the coefficient matrix and the right-hand side are all interval matrices.

On the other hand, interval linear programming seems to be a sound approach to face uncertainty issues that are met in real life applications. Nonetheless, it is worth pointing out that a very few number of papers can be found in which interval linear programming applications are developed, notwithstanding its possible wide usage for modelling and solving real world problems [6,7].

3 Problem Statement and Modelling

Let us consider an agricultural cooperative specialized in multi-seed production and commercialization. The cooperative involves several hundred of farmers, for whom it provides consulting, drying, storage, transportation and many other customer services.

Once the cereals have reached their physiological maturity, they are harvested and carefully forwarded towards storage facilities (also named *silos*), designed especially for this purpose. Many cooperatives use two types of silos: *expedition silos* E, used for a long time period storage, and *buffer silos* B, which serve as proximity facilities at the time of harvest. Due to limited storage capacity of buffer silos, an inventory control level and a daily grain transfer to expedition silos are organized during whole harvest season. This ensures the buffer silos availability, which contributes to increase the level of reception and storage services.

Heterogeneous vehicle fleet K, previously dimensioned, is dedicated to empty the buffer silos b ($b \in B$), whose cells c ($c \in C_b$) are quickly filling up as harvest time progresses. In order to maintain buffer silos sufficiently empty, a regular (single-period) cereal transfer (delivery) is organized from buffer silos to expedition ones.

More precisely, in each period of time (day) a multi-trip planning is performed in order to guarantee a sufficient silo capacity for receiving the quantities of different varieties v expected to be delivered in the following periods of time. All of the above transfer activities are realized by seeking to minimize the exceeded quantities to be received in the next p periods of time, that cannot by adequately stored with respect to the current buffer silo b stock level of each cell c, s_{cbv} .

Table 1. Modelling notation

Parameters:	
$v,v \in V$	crop variety index
$b,b\in B$	buffer silo index
$e,e\in E$	expedition silo index
$k, k \in K$	vehicle index
$c, c \in C_b$	silo cell index
$p,p\in P$	a short time horizon
Deterministic Data:	
s_{cbv}	stock level of variety v in the cell c of buffer silo b at the beginning of
	the planning period
u_{cb}	capacity of cell c of buffer silo b
h_{cbv}	1, if the cell c is allowed to stock the variety v and 0, otherwise
$r_{v'v''}$	1, if varieties v' and v'' are compatible and 0, otherwise
g_k	capacity of vehicle k
t_{eb}	travel time between the silos e and b
t_l	loading time
t_u	unloading time
T	daily working time
M	big value, e.g. greater than or equal to u_{cb}
Stochastic Data:	
q_{bv}^{\pm}	quantity of variety v expected to be delivered to buffer silo b during the
	period of time ahead
\tilde{q}_{bv}^{\pm}	quantity of variety v expected to be delivered to buffer silo b in p fol-
	lowing periods of time
Variables:	
y_{cb}	quantity of cell c of buffer silo b to be transferred
z_{cbv}	1, if the cell c of buffer silo b contains the variety v
w_{cbv}	available capacity for stocking the variety v of the cell c of buffer silo b
x_{cbe}^k	number of round trips between the silos b and e for emptying the cell c
	of silo b
f_{vb}	exceeded quantity of variety v at silo b, in terms of the \tilde{q}_{bv}^{\pm} against the
	current silo stock level

The daily quantities to be received at each storage facility are unknown. In this sense, the case study cooperative has only an approximate information based-on statistical analysis and prediction of the daily farmers' crop delivery quantities, valued as interval numbers, whilst considering the meteorological repercussion on the progress and achievement of the gathering activity and the farmers' delivery behaviour. Therefore, let us denote by q_{bv}^{\pm} the quantity of variety v to be delivered

to the buffer silo b in the next period of time. The uncertain data \tilde{q}_{bv}^{\pm} are defined to represent the forecasted quantity interval of the variety v to be received by the buffer silo b in p following periods of time.

Before proceeding to the problem modelling, let us consider moreover the following problem assumptions:

- Each vehicle can transport per trip goods belonging only to one buffer cell.
- The vehicles start from the first expedition silo (depot point) foreseen in their respective daily multi-trip planning. Respectively, they return to the last expedition silo (ending point) due in its daily multi-trip planning.
- Speed of vehicle is given and fixed. No traffic jam is considered.
- The total working time of each vehicle is limited to T per day.
- Non-Euclidean distance is considered between any two transporting points. Thus, the travel time t_{eb} , from silos e to b, is not equal to t_{be} , from b to e.
- For all buffer and expedition silos, the loading t_l and unloading t_u times are given and fixed.

The decision integer variables x_{cbe}^k denote the number of round trips made by the vehicle k between the buffer silo b and the expedition silo e, for emptying a quantity y_{cb} from the cell c of buffer silo b. By the same token, the decision variables w_{cbv} represent the available capacity of silo c of buffer silo b for receiving the variety v, while respecting their compatibility h_{cv} and the total cell capacity u_{cb} . The data h_{cv} are defined to take the value 1, if the cell c is allowed to stock the variety v and 0, otherwise. In this manner, the cereal allotment pursuing and the variety-cell management are considered. Additionally, an inter-varietal compatibility $r_{v'v''}$ must also be taken into account for a suitable seed nature allotment and traceability. The data $r_{v'v''}$ take the value 1, if varieties v' and v'' are compatible and the value 0, otherwise. Two varieties are considered compatible, if they can be mixed and stored in the same cell.

The decision positive variables f_{vb} express the exceeded quantities of each variety v, in terms of expected quantity \tilde{q}_{bv}^{\pm} to be delivered in the following p periods of time to the buffer silo b, with reference to the total available silo storage capacity of variety v, $\sum_{c \in C_h} w_{cbv}$.

In order to guarantee an appropriate storage service, buffer silos must be emptied in such a way to minimize the exceeded quantities at each buffer silos in the following p periods of time, in terms of each expected variety to be delivered and its associated quantity. Subsequently, by considering the defined decision variables and data parameters introduced above (see Table 1), a linear programming model with interval right-hand sides and objective function (1)-(15) is formalized hereafter:

$$\min \quad \sum_{v \in V} \sum_{b \in B} f_{vb} \tag{1}$$

subject to:

$$f_{vb} \ge \tilde{q}_{bv}^{\pm} - \sum_{c \in C_b} w_{cbv} \qquad \qquad \forall v, \forall b \qquad (2)$$

$$\sum_{c \in C_b} w_{cbv} \ge q_{bv}^{\pm} \qquad \qquad \forall v, \forall b \qquad (3)$$

$$u_{cb} - \sum_{v \in V} s_{cbv} + y_{cb} = \sum_{v \in V} w_{cbv} \qquad \forall b, \forall c \qquad (4)$$

$$w_{cbv} \le M \cdot h_{cbv} \qquad \forall v, \forall b, \forall c \qquad (5)$$

$$1 + w_{cbv} > z_{cbv} \qquad \forall v, \forall b, \forall c \qquad (6)$$

$$w_{cbv} \le M \cdot z_{cbv} \qquad \qquad \forall v, \forall b, \forall c \qquad (7)$$

$$z_{cbv'} + z_{cbv''} \le r_{v'v''} + 1 \qquad \qquad \forall v', v''(v' \neq v''), \forall b, \forall c \qquad (8)$$

$$\sum_{k \in K} \sum_{e \in E} x_{cbe}^k \cdot g_k \ge y_{cb} \qquad \qquad \forall c, \forall b \qquad (9)$$

$$\sum_{c \in C_b} \sum_{b \in B} \sum_{e \in E} (t_{eb} + t_{be} + t_l + t_u) \cdot x_{cbe}^k \le T \qquad \forall k \qquad (10)$$

$$\begin{aligned} w_{cbv} &\geq 0 & \forall c, \forall b, \forall v & (11) \\ z_{cbv} &\in \{0, 1\} & \forall c, \forall b, \forall v & (12) \end{aligned}$$

$$y_{cb} \ge 0 \qquad \qquad \forall b, \forall c, \forall k \qquad (13)$$
$$x_{cbe}^{k} \in \mathbb{N} \qquad \qquad \forall e, \forall b, \forall c, \forall k \qquad (14)$$

$$f_{vb} \ge 0 \qquad \qquad \forall v, \forall b \qquad (15)$$

Exceeded quantity of each variety at each buffer silo is calculated by the constraints (2), in terms of the expected quantity to be received in the following p periods of time against the current silo stock level. Constraints (3) ensure an available silo capacity for stocking the quantity for each seed variety foreseen to be delivered in the following time period. Stock equilibrium constraints for each silo cell are expressed by (4). Constraints (5) verify if the cells c of buffer silo b is allowed to stock the varieties v. Constraints (6) and (7) impose the binary variable z_{cbv} to take the value 1, if the cell c of buffer silos b is reserved to stock the variety v in the next period of time. This is prescribed for respecting the inter-varietal compatibility in each cell c of buffer silo b, required by constraints (8). In order to guarantee a sufficient available capacity of the buffer silo, the constraints (9) trigger a seed transfer from buffer silos b to expedition ones e_{i} which is performed by using a heterogeneous vehicle fleet K. Constraints (10)confine to T the total working time of each vehicle k. The objective function (1) seeks to minimize the exceeded quantity expected to be received in the following p periods of time against the current silo stock level.

In the interval programming model (1)-(15), uncertainty, represented by intervals, concerns both the objective function and the right-hand side constraints. Hence, the set of feasible solutions is not exactly known and any solution may be not feasible for all interval right-hand side constraints. Correspondingly, classical min-max optimization criteria cannot be directly employed [9].

4 Solution Methods

An interval programming model (1)-(15) with interval coefficients, simultaneously occurring in the objective function and right-hand side constraints are considered. In this context, the aim consists of determining the best possible optimum and the worst one over all possible configurations, which correspond to an assignment of plausible values for each of the model uncertain parameters.

As far as uncertainty on objective function coefficients is regarded, two criteria are classically considered: the worst case criterion and the best case one. Let Xbe the set of (1)-(15) problem feasible solutions. Given $x \in X$, the configuration to be considered is the one that corresponds to the worst (best) for this solution. In this sense, the value of x, noted $f_{worst}(x)$ ($f_{best}(x)$) is defined as presented below:

$$f_{\text{worst}}(x) = \max_{\tilde{q}^- \le \tilde{q} \le \tilde{q}^+} \sum_{v \in V} \sum_{b \in B} f_{vb}$$
(16)

$$f_{\text{best}}(x) = \min_{\tilde{q}^- \le \tilde{q} \le \tilde{q}^+} \sum_{v \in V} \sum_{b \in B} f_{vb}$$
(17)

where $\tilde{q}^{\pm} = (\tilde{q}_{bv}^{\pm})_{b \in B, v \in V}$.

The problem is to determine the solution $x_{\text{worst}}(x_{\text{best}})$, which minimizes $f_{\text{worst}}(x)$ and $f_{\text{best}}(x)$ respectively, as follows:

$$f_{\text{worst}}(x_{\text{worst}}) = \min_{x \in X} f_{\text{worst}}(x)$$
(18)

$$f_{\text{best}}(x_{\text{best}}) = \min_{x \in X} f_{\text{best}}(x) \tag{19}$$

On the other hand, classical criteria cannot be directly applied when uncertainty concerns right-hand side constraints. Denote by P^q the program (1)-(15), where q varies in the interval $q^- \leq q \leq q^+$, $q^{\pm} = (q_{bv}^{\pm})_{b \in B, v \in V}$. In the context of linear programs with interval right-hand sides, the objective of the best (worst) optimal solution problem is to determine the minimum (maximum) value $\vartheta(P^q)$ of the optimal solution of P^q , when q varies in the interval $[q^-, q^+]$. Let us formalize the best optimal solution problem (BEST) and the worst optimal solution problem (WORST), hereafter:

$$BEST: \begin{cases} \min \vartheta(P^q) \\ \text{s.t } q^- \le q \le q^+ \end{cases}$$
(20)

WORST :
$$\begin{cases} \max \vartheta(P^q) \\ \text{s.t } q^- \le q \le q^+ \end{cases}$$
(21)

Let X^{BEST} (X^{WORST}) be the set of optimal solutions of BEST (WORST). In the light of the above mentioned approaches, four cases are studied in this paper, for handling the interval programming model (1)-(15):

$$f_{\text{worst}}^{\text{WORST}} = \min_{x \in X^{\text{WORST}}} f_{\text{worst}}(x)$$
(22)

$$f_{\text{worst}}^{\text{BEST}} = \min_{x \in X^{\text{BEST}}} f_{\text{worst}}(x)$$
(23)

$$f_{\text{best}}^{\text{WORST}} = \min_{x \in X^{\text{WORST}}} f_{\text{best}}(x)$$
 (24)

$$f_{\text{best}}^{\text{BEST}} = \min_{x \in X^{\text{BEST}}} f_{\text{best}}(x) \tag{25}$$

Criteria (22), (23), (24) and (25) allow to provide the best possible optimum and the worst one over all possible configurations in order to reveal a kind of robustness information, by handling both uncertainty on objective function and right-hand sides. As [19] stated, the range of the objective function between the best and the worst optimum values provides an overview of the risk involved, which can be reflected by specifying the values of the uncertain coefficients.

5 Computational Results

The model (1)-(15) was conceived to better organize the buffer silos emptying during the harvest season for an Agricultural Cooperative Society, situated in the region of Arcis-sur-Aube (France). More precisely, this is a grain and oilseed agricultural cooperative, for which an appropriate buffer silos emptying policy and multi-trip transportation planning is necessary to ensure a high level seed reception and storage services.

The interval programming model (1)-(15) has been implemented by using the C++ language. Criteria (22), (23), (24) and (25) have been used and the corresponding models have been solved by using ILOG CPLEX (version 12.6.0) optimization software, for instances with up to 30 varieties, 3 expedition silos, 11 buffer silos and between 10 and 15 cells per buffer silo (whose capacities vary between 200 and 800 tonnes per cell). Computational experiments have been carried out on an Intel(R) Core(TM) i7-2720QM CPU 2.20GHz workstation.

Commonly, the harvest lasts about one month. During this period, the expected quantities to be received by each case study silo were estimated based-on cooperative predictive modelling of the farmers' crop delivery behaviour and climate forecasting data, derived from nearby weather stations with an acceptable reliability level. Due to a small gap between varieties' ripeness dates and to a high farmers' gathering yield, the value of p was empirically fixed to 3.

In what follows, let us examine the figures Fig. 1 et Fig. 2, which illustrate the output results corresponding to the best possible optimum and the worst optimum over all possible configurations for a time horizon of 7 harvest days. More specifically, the figure Fig. 1 reports the optimal values for (22) and (23), as well as, the figure Fig. 2 provides the optimal value of (24) and (25), respectively.

Representative gaps between the objective values of approaches (22)-(23) and (24)-(25) corresponding to the periods 2, 3 and 6, suggest about eventual considerable buffer storage unavailability. It could be due to the fact that a significant range of varieties is expected to be delivered. For preventing unsuitable seed nature allotment or quality degradation, the cooperative should rent supplementary vehicle during the respective periods of time. Contrarily, the multi-trip



planning solutions provided by (22) could be realised during the periods 1, 4 and 5, since negligible objective values and gaps are recorded for these periods.

As computational results pointed out, approaches (22), (23), (24), (25) help to handle efficiently the inventory control and multi-trip transportation planning problem by presenting good alternative solutions. They offer a pertinent decision support by taking into account weather and farmers' delivery uncertainties.

6 Conclusions and Topics for Future Research

This paper presents an interval programming model for a single-period multiple trip transportation planning problem, for purpose of maintaining available cooperative buffer silos during the harvest season. Best and worst optimum criteria, prescribed to deal with uncertainty on objective function, have been considered for both best and worst optimal solution problems, which address uncertainty on right-hand side coefficients.

Future research would be dedicated to tackle and study other approaches of problem robustness (e.g. maximum regret criterion, etc.). Moreover, other problem formulations would be also tested to deal with the problem considered in this paper (e.g. composing the rented fleet of vehicles, whilst ensuring the buffer silos availability, etc.).

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