A Dynamic System Matching Technique-An Analytical Study

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1 Introduction

Consider the design and production of an analog system where the desired design is defined by a differential equation which defines a desired relationship between an input signal and an output signal. Production systems will deviate from the desired design due to manufacturing errors in the coefficients of the differential equations describing the production systems. These errors contribute to the errors in the desired system input-output relationship. In an earlier paper, [2], the authors presented a method for combining the measurements from many nominally equal micro-gyroscopes using a technique based on a design technique from electronics called dynamic element matching, see [1]. This technique essentially transforms the system output noise caused by manufacturing errors into an additive (almost) white noise in the system output. This 'spreading of the spectrum' reduces the noise power in the pass band of the analog system. Additional reduction in the effect of the noise can be attained by appropriately filtering the system output. In a more recent paper [3], this technique was generalized by applying dynamic element matching to analog systems. Since the method deals with systems rather than elements, it was called a *dynamic system matching technique* (DSMT). The DSMT proposed in that paper, generates the system output by randomly switching between the outputs of several, nominally identical, production systems. \A heuristic analysis in that paper indicated that the DSMT is effective in reducing the effects of the random coefficient variations in a system output. In a more recent paper [4], a simulation study of the DSMT was developed which, along with some additional analysis, provided further validation of the DSMT. In this paper, a detailed analysis of the noise in the output signal of a DSMT-based system is presented. The results of this study provide not only a validation of the effectiveness of the DSMT, but also provide formulae which can be used to aid in the design of a DSMT system.

2 A Dynamic System Matching Technique

Assume that a system design can be represented by a ***nominal differential equation***

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_1\frac{dy}{dt} + a_0 y = z \qquad (1)$$

where the n coefficients, $a_0, a_1, \cdots, a_{n-1}, a_n$, are called the ***nominal coefficients*** and the equation represents a ***nominal system***. Now, consider a production version of the system which is defined by the nominal differential equation with the same nominal coefficients except that the coefficient a_0 is replaced by a perturbed coefficient, $a_0 + \Delta a_0$, where the perturbation Δa_0 is a random variable with the first and second order statistics, $E[\Delta a_0] = 0$ and $\mathrm{Var}[\Delta a_0] = \sigma^2_{a_0}$. This system will be called a ***real system***. The input signal to the real system is the same as for the nominal system and the real system output signal is perturbed from $y(t)$ to $y(t) + \Delta y(t)$; therefore, the real system is defined by the differential equation:

$$\frac{d^n(y+\Delta y)}{dt^n} + a_{n-1}\frac{d^{n-1}(y+\Delta y)}{dt^{n-1}} + \cdots + a_1\frac{d(y+\Delta y)}{dt} + (a_0+\Delta a_0)(y+\Delta y) = z$$

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Assume that Δa_0 is small enough that the second order term $\Delta a_0 \Delta y$ is negligible and can be ignored. Under this assumption, the differential equation defining the random perturbation $\Delta y(t)$ is linear and given by:

$$\frac{d^n \Delta y}{dt^n} + a_{n-1}\frac{d^{n-1}\Delta y}{dt^{n-1}} + \cdots + a_1 \frac{d\Delta y}{dt} + a_0 \Delta y = -\Delta a_0 y \quad (2)$$

Now consider a set of N real systems with a common input $z(t)$ applied to each system. The real systems will differ from each other in that the random coefficient perturbations will define a set of N independent identically distributed random variables, $\Delta a_0^i,\ i = 1, 2, \cdots, N$. The output signals of the real systems are combined through a switching circuit in a DSMT structure. The switching circuit output signal at a specific time t is the output signal $y(t)$ of one of the N systems which has been randomly selected by the switching circuit. The switching circuit output continues to be the output of that system over a fixed interval of time, T, the ***switching circuit period***, and then it is switched to the output of a different system which has been randomly selected by the switching circuit. A different system is randomly selected for each successive switching circuit period *ad infinitum*. Note that if each system were a ***nominal system***, each of their outputs would be equal and, assuming perfect switching, the output of the switching circuit would equal the output of the nominal system. Because the random perturbations are independent, the noise in the sequence of switched outputs formed by the switching circuit is an 'almost white' sequence. The time correlation in the sequence is due to the fact that the number of different random perturbations is finite and each perturbation will, with non-zero probability, be chosen more than once by the switch. Apparently, the sequence tends to become 'whiter' as N increases.

The linear perturbation $\Delta y(t)$ can be written in the form of a convolution integral

$$\Delta y(t) = (-\Delta a_0)\int_{\tau=-\infty}^{\tau=t} h(t-\tau)y(\tau)d\tau$$

where $h(t)$ is the unit impulse response of the nominal system defined by Equation (1). In the Fourier transform domain, the linear random perturbation is given by $\Delta Y(j\omega) = (-\Delta a_0)H*(j\omega)Z(j\omega)$ where $\Delta Y(j\omega)$ is the Fourier transform of $\Delta y(t)$, $Z(j\omega)$ is the Fourier transform of $z(t)$, and $H*(j\omega) = |H(j\omega)|^2$ where $H(j\omega)$ is the Fourier transform of $h(t)$.

Note: To simplify the mathematical development in this paper, but without diminishing the value of the results, only one of the parameters defining the real systems will be considered to be random. For the more complex development involving several, or all, of the parameters being random, the same basic development can be used for each parameter and, assuming independence of the random variations, results for the case of all parameters being random can be generated by combining the results for the individual parameter variations.

3 The Unfiltered Output Noise in a DSMT System

Consider the set of independent random output perturbations (output noise) $\Delta y^1(t), \Delta y^2(t), \cdots, \Delta y^N(t)$ generated by the set of N random coefficient variations, $\Delta a_0^i,\ i = 1, 2, \cdots, N$, associated with the N real systems each of which is driven by the same input signal $z(t)$ and each of which is defined by Equation (2). Now, consider a time sequence of pulses $p_k(t)$, $-\infty < k < \infty$ where the pulse $p_k(t)$ has width T, has unity amplitude for $kT \le t < (k+1)T$, and is zero elsewhere. Let $\Delta y^{i_k}(t)$ represent that one of the linear perturbations that is chosen randomly by the switching circuit during the k^{th} switching period. The noise in the output of the switching circuit of the DSMT can then be written as:

$$M(t) = \sum_k \Delta y^{i_k}(t) p_k(t)$$

Combining the outputs of Equations (1) and (2) the perturbation $\Delta y^{i_k}(t)$ can be written as

$$\Delta y^{i_k}(t) = \left(-\Delta a_0^{i_k}\right)\int_{\theta=0}^{\theta=\infty} h^*(\theta) z(t-\theta)d\theta$$

where $h^*(\theta) = \int_{\tau=0}^{\tau=\theta} h(\theta-\tau)h(\tau)d\tau$ Then

$$N(t) = \sum_k \left(-\Delta a_0^{i_k}\right)\int_{\theta=0}^{\theta=\infty} h^*(\theta)z(t-\theta)d\theta\, p_k(t)$$
$$= \left[\int_{\theta=0}^{\theta=\infty} h^*(\theta)z(t-\theta)d\theta\right]\cdot\left[\sum_k \left(-\Delta a_0^{i_k}\right)p_k(t)\right] = N_1(t)\cdot N_2(t)$$

Assuming that $z(t)$ and $\Delta a_0^i,\ i = 1, 2, \cdots, N$ are statistically independent, then $M_1(t)$ and $M_2(t)$ are independent random processes. $M(t)$ is the ***output noise*** in a DSMT system.

4 Power Spectral Density and Total Power in the Output Noise

The autocorrelation function of $M(t)$ can be written as:

$$\begin{aligned}R_M(\tau) &= E[M(t)M(t+\tau)]\\ &= E[M_1(t)M_1(t+\tau)]\cdot E[M_2(t)M_2(t+\tau)]\\ &= R_{M_1}(\tau)\cdot R_{M_2}(\tau)\end{aligned}$$

The power spectral density (PSD) of $M(t)$ is given by:

$$\begin{aligned}S_M(\omega) &= \int_{\tau=-\infty}^{\tau=\infty} e^{-j\omega\tau}R_{M_1}(\tau)R_{M_2}(\tau)d\tau\\ &= \frac{1}{2\pi}\int_{v=-\infty}^{v=\infty} S_{M_1}(\omega-v)S_{M_2}(v)dv\end{aligned}$$

where $S_{M_1}(\omega)$ is the PSD of $M_1(t)$ and $S_{M_2}(\omega)$ is the PSD of $M_2(t)$. Because white noise excites all frequencies of the system equally; in this general development, it is assumed that $z(t)$ is white noise with the mean and the autocorrelation function:

$$E[z(t)] = 0 \quad\text{and}\quad R_z(\tau) = K_z\delta(\tau)$$

Under this assumption, the autocorrelation function of $M_1(t)$ is given by:

$$R_{M_1}(\tau) = E[M_1(t)M_1(t+\tau)] = K_z\int_{\theta=0}^{\theta=\infty} h^*(\theta)h^*(\tau+\theta)d\theta$$

and the PSD of $M_1(t)$ is given by:

$$S_{M_1}(\omega) = K_z|H(j\omega)|^4 - \infty < \omega < \infty$$

Assuming that the nominal system has a low-pass frequency response function with bandwidth ω_s, it is approximated by an idealized low-pass frequency response function $H(j\omega) = u(\omega+\omega_s)u(\omega_s-\omega)e^{-j\omega} - \infty < \omega < \infty$

where $u(\cdot)$ is a unit step function. Using this idealized frequency response function, the PSD of $M_1(t)$ is given by:

$$S_{M_1}(\omega) = K_z u(\omega+\omega_s)u(\omega_s-\omega) \quad -\infty < \omega < \infty$$

and the PSD of $M(t)$ can be written:

$$S_M(\omega) = \frac{K_z}{2\pi}\int_{v=\omega-\omega_s}^{v=\omega+\omega_s} S_{M_2}(v)dv \quad -\infty < \omega < \infty$$

The mean and variance of the noise component $M_2(t)$ are:

$$E[M_2(t)] = 0, \quad \mathrm{Var}[M_2(t)] = \sigma^2_{a_o}, \quad -\infty < t < \infty$$

The autocorrelation function of $M_2(t)$ is developed as follows:

$$R_{M_2}(t,\tau) = E[M_2(t)M_2(\tau)]$$

For $t,\tau\in[kT,(k+1)T)$, $M_2(t) = M_2(\tau) = -\Delta a_0^{i_k}$, and

$$R_{M_2}(t,\tau) = E\left[\left(-\Delta a_0^{i_k}\right)^2\right] = \sigma^2_{a_0}$$

For $t \in [jT, (j+1)T)$, $\tau \in [kT, (k+1)T)$ and $j \neq k$, either $M_2(t) = M_2(\tau) = \Delta a_0^{i_k}$ with probably $\frac{1}{N}$ or $M_2(t) \neq M_2(\tau)$ with probability $\frac{N-1}{N}$; therefore for $j \neq k$

$$\begin{aligned}R_{M_2}(t,\tau) &= E[M_2(t)M_2(\tau)\mid M_2(t)=M_2(\tau)]\cdot \Pr[M_2(t)=M_2(\tau)]\\ &\quad + E[M_2(t)M_2(\tau)\mid M_2(t)\neq M_2(\tau)]\cdot \Pr[M_2(t)\neq M_2(\tau)]\\ &= \frac{\sigma^2_{a_0}}{N} + E[M_2(t)M_2(\tau)\mid M_2(t)\neq M_2(\tau)]\cdot\frac{N-1}{N}\end{aligned}$$

If $j \neq k$ and $M_2(t) \neq M_2(\tau)$, then $E[M_2(t)M_2(\tau)|M_2(t) \neq M_2(\tau)] = 0$ and

$$R_{M_2}(t,\tau) = \frac{\sigma^2_{a_0}}{N}$$

Then, for all t and τ, the autocorrelation function can be written as

$$R_{M_2}(t,\tau) = \left(\frac{N-1}{N}\right)\cdot\sigma^2_{a_0}\cdot\delta(k,j) + \frac{\sigma^2_{a_0}}{N}$$

where $\delta(k, j)$ is a Kronecker delta, that is,

$$\delta(k,j) = \begin{cases} 1 \ \text{for}\ t,\tau\in[kT,(k+1)\tau) \\ 0 \ \ t\in[jT,(j+1)T),\ \tau\in[kT,(k+1)T)\ \ k\neq j\end{cases}$$

The power spectral density of $M_2(t)$ is given by

$$\begin{aligned}S_{M_2}(\omega) &= \int_{-\infty}^{\infty} e^{-j\omega\theta}\left(\frac{N-1}{N}\right)\cdot\sigma^2_{a_0}\cdot\delta(k,j)d\theta\\ &\quad + \int_{-\infty}^{\infty} e^{-j\omega\theta}\frac{\sigma^2_{a_0}}{N}d\theta - \infty < \omega < \infty.\end{aligned}$$

The first term in $S_{M_2}(\omega)$ is given by

$$\int_{-\infty}^{\infty} e^{-j\omega\theta}\left(\frac{N-1}{N}\right)\cdot\sigma_{a_0}^2\cdot\delta(k,j)\,d\theta$$
$$=\left(\frac{N-1}{N}\right)\cdot\sigma_{a_0}^2\cdot\int_{-\infty}^{\infty} e^{-j\omega\theta}\delta(k,j)\,d\theta$$

The integral $\int_{-\infty}^{\infty} e^{-j\omega\theta}\delta(k,j)\,d\theta$ is integrated as follows.

$$\int_{-\infty}^{\infty} e^{-j\omega\theta}\delta(k,j)\,d\theta=\sqrt{2}T\left[\int_{0}^{\frac{\sqrt{2}T}{2}} e^{-j\omega\theta}d\theta+\int_{-\frac{\sqrt{2}T}{2}}^{0} e^{-j\omega\theta}d\theta\right]$$
$$-2\left[\int_{0}^{\frac{\sqrt{2}T}{2}}\theta e^{-j\omega\theta}d\theta-\int_{-\frac{\sqrt{2}T}{2}}^{0}\theta e^{-j\omega\theta}d\theta\right]=T^2\mathrm{sinc}^2\omega\frac{\sqrt{2}T}{4}$$
$$=4\left[\frac{1}{\omega^2}\left(1-\cos\frac{\omega T}{\sqrt{2}}\right)\right]$$

where $\mathrm{sinc}\,x=\frac{\sin x}{x}$. The second term in $S_{M_2}(\omega)$ is the Fourier transform of the constant $\frac{\sigma_{a_0}^2}{N}$ and is given by $\int_{-\infty}^{\infty} e^{-j\omega\theta}\frac{\sigma_{a_0}^2}{N}d\theta=2\pi\left(\frac{\sigma_{a_0}^2}{N}\right)\delta(\omega)$. This term indicates that the noise component $M_2(t)$ contains a random DC offset. Finally then the power spectral density (PSD) of the noise component $M_2(t)$ is given by:

$$S_{M_2}(\omega)=4\left(\frac{N-1}{N}\right)\cdot\sigma_{a_0}^2\cdot\left[\frac{1}{\omega^2}\left(1-\cos\frac{T\omega}{\sqrt{2}}\right)\right]$$
$$+2\pi\left(\frac{\sigma_{a_0}^2}{N}\right)\delta(\omega)\quad-\infty<\omega<\infty$$

and the PSD of $M(t)$ is given by:

$$S_M(\omega)=K_z\left(\frac{2}{\pi}\right)\left(\frac{N-1}{N}\right)\sigma_{a_0}^2\cdot\int_{v=\omega-\omega_s}^{v=\omega+\omega_s}\frac{1}{v^2}\left[1-\cos\frac{Tv}{\sqrt{2}}\right]dv$$
$$+K_z\left(\frac{\sigma_{a_0}^2}{N}\right)\int_{v=\omega-\omega_s}^{v=\omega+\omega_s}\delta(v)dv\quad-\infty<\omega<\infty \tag{3}$$

Using this expression, the total power in the unfiltered noise of a DSMT system can be determined by the following double integral:

$$P_M=\int_{\omega=-\infty}^{\omega=\infty}S_M(\omega)d\omega=K_z\left(\frac{2}{\pi}\right)\left(\frac{N-1}{N}\right)\cdot$$
$$\sigma_{a_0}^2\int_{\omega=-\infty}^{\omega=\infty}\int_{v=\omega-\omega_s}^{v=\omega+\omega_s}\frac{1}{v^2}\left[1-\cos\frac{Tv}{\sqrt{2}}\right]dv\,d\omega$$
$$+K_z\left(\frac{\sigma_{a_0}^2}{N}\right)\int_{\omega=-\infty}^{\omega=\infty}\int_{v=\omega-\omega_s}^{v=\omega+\omega_s}\delta(v)dvd\omega$$

Interchanging the order of integrations, this can be rewritten:

$$P_M=K_z\left(\frac{2}{\pi}\right)\left(\frac{N-1}{N}\right)\cdot\sigma_{a_0}^2\int_{v=-\infty}^{v=\infty}\frac{1}{v^2}\left[1-\cos\frac{Tv}{\sqrt{2}}\right]\left[\int_{\omega=v-\omega_s}^{\omega=v+\omega_s}d\omega\right]dv$$
$$+K_z\left(\frac{\sigma_{a_0}^2}{N}\right)\int_{v=-\infty}^{v=\infty}\delta(v)\left[\int_{\omega=v-\omega_s}^{\omega=v+\omega_s}d\omega\right]dv$$

Carrying out the integration of the ω variable first, results in:

$$P_M=K_z\cdot 2\omega_s\cdot\left(\frac{2}{\pi}\right)\left(\frac{N-1}{N}\right)$$
$$\cdot\sigma_{a_0}^2\int_{v=-\infty}^{v=\infty}\frac{1}{v^2}\left[1-\cos\frac{Tv}{\sqrt{2}}\right]dv+K_z\cdot 2\omega_s\cdot\left(\frac{\sigma_{a_0}^2}{N}\right)$$

Because of the double pole at the origin, the integral in P_M must be evaluated using the contour integral:

$$\oint\frac{1}{z^2}\left(1-e^{i\left(\frac{T}{\sqrt{2}}z\right)}\right)dz=0$$

where the contour is the infinite semicircle enclosing the upper half of the complex plane, an analytic region. Performing the integration generates the result:

$$\int_{\omega=-\infty}^{\omega=\infty}\frac{1}{v^2}\left(1-\cos\frac{Tv}{\sqrt{2}}\right)dv$$
$$=\lim_{R\to\infty,\,\varepsilon\to 0}2\int_{v=\varepsilon}^{v=R}\frac{1}{v^2}\left(1-\cos\frac{Tv}{\sqrt{2}}\right)dv$$
$$=\lim_{\varepsilon\to 0}\left[-\int_{\theta=\pi}^{\theta=0}\frac{1}{\left(\varepsilon e^{i\theta}\right)^2}\left(1-e^{i\left(\frac{T}{\sqrt{2}}\varepsilon e^{i\theta}\right)}\right)i\varepsilon e^{i\theta}d\theta\right]=\frac{T}{\sqrt{2}}\pi$$

Finally, then the total power in the unfiltered noise of a DSMT system is given by:

$$P_M = K_z \cdot 2\omega_s \cdot \sqrt{2} \cdot \left(\frac{N-1}{N}\right) \cdot \sigma_{a_0}^2 \cdot T + K_z \cdot 2\omega_s \cdot \left(\frac{\sigma_{a_0}^2}{N}\right)$$

Note that the parameters $K_z, \omega_s,$ and $\sigma_{a_0}^2$ are fixed by the original system design and are not involved in the design of the DSMT system. Note, also, that the second term in P_N represents the power in a random DC bias and is dependent only on N.

5 Total Power in the Filtered Output Noise

Optimally, the output signal of the DSMT system is filtered by an ideal, unity gain, low-pass filter whose pass band is equal to the system pass band, in which case, the total power remaining in the noise signal, after filtering, is given by:

$$P_f = \int_{\omega=-\omega_s}^{\omega=\omega_s} S_N(\omega) d\omega = K_z \left(\frac{2}{\pi}\right)\left(\frac{N-1}{N}\right)$$

$$\cdot \sigma_{a_0}^2 \int_{\omega=-\omega_s}^{\omega=\omega_s} d\omega \int_{v=\omega-\omega_s}^{v=\omega+\omega_s} \frac{1}{v^2}\left[1 - \cos\frac{Tv}{\sqrt{2}}\right] dv$$

$$+ K_z \left(\frac{\sigma_{a_0}^2}{N}\right) \int_{\omega=-\omega_s}^{\omega=\omega_s} \int_{v=\omega-\omega_s}^{v=\omega+\omega_s} \delta(v) dv d\omega$$

Interchanging the order of the integrations, the filtered noise power can be written as:

$$P_f = K_z \left(\frac{4}{\pi}\right)\left(\frac{N-1}{N}\right) \cdot \sigma_{a_0}^2 \cdot \left\{ \left(-1 + \cos\left(\sqrt{2}T\omega_s\right)\right) + \left(\sqrt{2}T\omega_s\right) \int_{x=0}^{x=\left(\sqrt{2}T\omega_s\right)} \frac{\sin x}{x} dx - 2 \int_{x=0}^{x=\left(\frac{\sqrt{2}T\omega_s}{2}\right)} \sin x \frac{\sin x}{x} dx \right\}$$
$$+ K_z \cdot 2\omega_s \cdot \left(\frac{\sigma_{a_0}^2}{N}\right)$$

Note that the second term, the DC term, has not been changed by the filter and is controlled by N and that the first term is controlled by N and $\sqrt{2}T\omega_s$; therefore, an obvious design methodology is to first choose N to reduce the effect of the DC random bias and then choose $\sqrt{2}T\omega_s$ to reduce the effect of the non-DC noise. In examining the effect of filtering on the output noise, the DC random bias term will be ignored. The non-DC total power terms for the unfiltered and filtered noise powers are defined as:

$$P_M' = K_z \cdot 2\omega_s \cdot \sqrt{2} \cdot \left(\frac{N-1}{N}\right) \cdot \sigma_{a_0}^2 \cdot T$$

$$P_f' = K_z \left(\frac{4}{\pi}\right)\left(\frac{N-1}{N}\right) \cdot \sigma_{a_0}^2 \cdot \left\{ \left(-1 + \cos\left(\sqrt{2}T\omega_s\right)\right) + \left(\sqrt{2}T\omega_s\right) \int_{x=0}^{x=\left(\sqrt{2}T\omega_s\right)} \frac{\sin x}{x} dx - 2 \int_{x=0}^{x=\left(\frac{\sqrt{2}T\omega_s}{2}\right)} \sin x \frac{\sin x}{x} dx \right\}$$

6 A Measure of the Effectiveness of the DSMT

A measure of the effectiveness of using a DSMT system without filtering and with filtering is how the signal-to-noise ratio changes when these methods are used. The total power in the output signal $y(t)$ of the idealized nominal system with a white noise input is easily seen to be $P_y = K_z \cdot 2\omega_s$. If a real system is chosen at random, the total power in the noise due to the random parameter perturbation Δa_0 is easily shown to be $P_{\Delta y} = K_z \cdot 2\omega_s \cdot \sigma_{a_0}^2$. The signal-to-noise ratio is given by:

$$\frac{P_y}{P_{\Delta y}} = \frac{K_z \cdot 2\omega_s}{K_z \cdot 2\omega_s \cdot \sigma_{a_0}^2} = \frac{1}{\sigma_{a_0}^2}$$

The total power in the unfiltered DSMT noise signal is:

$$P_M = K_z \cdot 2\omega_s \cdot \sqrt{2} \cdot \left(\frac{N-1}{N}\right) \cdot \sigma_{a_0}^2 \cdot T + K_z \cdot 2\omega_s \cdot \left(\frac{\sigma_{a_0}^2}{N}\right)$$

Apparently, the DSMT reduces the random DC bias by a factor of N and, examining the PSD in Equation (3), the DSMT spreads the rest of the noise over the infinite spectrum. Comparing the total noises of a single real system and an unfiltered DSMT system, it is seen that

$$P_M = \left[\frac{\sqrt{2}(N-1)T + 1}{N}\right] P_{\Delta y}$$

which implies that if $T < \frac{1}{\sqrt{2}}$, the noise power in the unfiltered DSMT system will be less than the noise power in a single real system. To examine how filtering further reduces the output noise power, the partial noise terms P_M' and P_f' are compared by examining the ratio:

$$\frac{P_f'}{P_M'} = \frac{\left(-1+\cos\left(\sqrt{2}T\omega_s\right)\right)+\left(\sqrt{2}T\omega_s\right)\int\limits_{x=0}^{x=\left(\sqrt{2}T\omega_s\right)}\frac{\sin x}{x}dx}{\left(\sqrt{2}T\omega_s\right)\cdot\frac{\pi}{2}} \times \frac{-2\int\limits_{x=0}^{x=\frac{\left(\sqrt{2}T\omega_s\right)}{2}}\sin x\frac{\sin x}{x}dx}{\left(\sqrt{2}T\omega_s\right)\cdot\frac{\pi}{2}}$$

As an example, let $\sqrt{2}T\omega_s = \frac{1}{10}$, then $\frac{P_f'}{P_N'} \cong \frac{\sqrt{2}T\omega_s}{\pi} \cong 0.0318$ which represents an approximate 15 db decrease in the non-DC component of the system noise. If the switching frequency is denoted f and if the maximum frequency in the system output signal is denoted f_s, then:

$$f = \frac{1}{T} = 10\sqrt{2}\omega_s = \sqrt{2}\cdot 20\pi\cdot f_s \cong 88.876\cdot f_s$$

Thus, a switching frequency of about 90 times the maximum frequency in the output signal, reduces the power in the non-DC component of the DSMT noise by 15 db.

7 Conclusions

In a series of earlier papers, the authors introduced the concept of a dynamic system matching technique (DSMT), as a generalization the dynamic element matching technique which is used in the design of electronic systems. In those papers, heuristic analyses and a simulation were used to argue that the DSMT can be used to reduce the effects of noise in a system output due to manufacturing errors. In this paper, a DSMT technique was developed for an idealized nominal system with a white noise input. A detailed analysis of the noise in the output of that DSMT system was generated which allows a comparison of the noise powers in a non-DSMT system, an unfiltered DSMT system and a filtered DSMT system. In particular, detailed analytical expressions for the power spectral densities and the total powers of the noises in the unfiltered and filtered outputs of a DSMT system were developed. These results show conclusively that for the idealized system with a white noise input, the DSMT reduces the noise power and that filtering the DSMT output further reduces the noise power.

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