Building Fuzzy Robust Regression Model Based on Granularity and Possibility Distribution

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Abstract The characteristic of the fuzzy regression model is to enwrap all the given samples. The fuzzy regression model enables us to take the possibility interval for a granular instead of a single numerical value. This granular provides the wider treatment for us to human-centered understanding of the latent system. Such a granule or interval of fuzzy regression model is created by considering how far a sample is from the central values. That means when samples are widely scattered the size of a granular or an interval of the fuzzy model is widened. That is, the fuzziness of the fuzzy regression model is decided by the range of sample distribution. Therefore, outliers make the fuzzy regression model distorted. This chapter describes the model building of fuzzy robust regression from the perspective of granularity by removing improper data based on genetic algorithm. Moreover, let us build the fuzzy regression model that places the largest grade on the central point of scattering samples.

Keywords Fuzzy regression model · Robustness · Granularity · Possibility distribution · Human centered understanding · Outliers · Genetic algorithm

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W. Pedrycz and S.-M. Chen (eds.), *Information Granularity, Big Data, and Computational Intelligence*, Studies in Big Data 8,
DOI: 10.1007/978-3-319-08254-7_12,

1 Introduction

A fuzzy regression model is categorized into two types. The first type is an interval model based on the possibility concept and the second type is a non-interval model based on a least squares method. An interval model has been proposed by Tanaka [16, 17, 19] and a non-interval model has been proposed by Diamond [2, 3].

Tanaka et al. have proposed three models as an interval fuzzy regression model, a possibility model, a necessity model and a conjunction model. A difference in these three models is an inclusion relation between estimates $\mathbf{Y}$ of a fuzzy regression model and fuzzy interval data $\mathbf{y}$. The relation between a possibility model and observed data is written as $\mathbf{Y} \supseteq \mathbf{y}$, otherwise, the relation of a necessity model is $\mathbf{Y} \subseteq \mathbf{y}$, and the relation of a conjunction model is $\mathbf{Y} \cap \mathbf{y} \neq \phi$, respectively. An interval model is based on the concept of possibility theory, and this possibility model is known as the major model of a fuzzy regression model. A possibility model and a necessity model are referred to as an upper regression model and a lower regression model, respectively [5].

There are some ways to obtain an interval fuzzy regression model such as a least squares method [1, 4, 8, 12] and a linear programming (LP). This chapter, an interval model obtained by LP.

The objective of an interval fuzzy regression model is to describe a possibility of analyzed system, and to minimize a vagueness of a model in order to make us interpret a system with least bias. However, a possibility model describes a possibility of a system by enclosing data, a vagueness of a model is made bigger and a shape is made strained easily. Therefore, many possibility models have been proposed, which describe an essence of analyzed target although it is rough. These models have two approaches. One is to use an exponential possibility distribution which proposed by Tanaka et al. [5, 18]. And the other is to control the relation between a model and data, Ishibuchi and Tanaka [10], Yabuuchi and Watada [23, 26, 27] and so on have proposed.

In this chapter, the model which describes an essential possibility by controlling the relation between a model and data is focused on.

We have proposed two models. This chapter describes model building of fuzzy robust regression model with granule data or interval data by removing improper data based on genetic algorithm [14, 27]. Let us call this fuzzy robust model as the first model.

And, let us build the fuzzy regression model that places the largest grade on the central point of scattering samples [28–31] as the other model. This model is the second type model of fuzzy robust regression. Observed data are not real numbers as granular possibilities by the second type model. For this reason, the second type model has a less ill effect of irregular data than other models.

Granular concept enables us to understand a main essential target system [21] and to deal any data such as linguistic data [24].

The remaining is organized as follows: In Sect. 2, two conventional fuzzy regression models are introduced. In Sect. 3, an Asian environment, a relation between Asian economy and environment, are analyzed by a first type model of our fuzzy robust regression. In Sect. 4, a Japanese major rivers, and a relation between Asian economy and environment, is analyzed by a second type model of our fuzzy robust regression.

2 Fuzzy Regression Model

A fuzzy regression model is categorized into two types, a least squares model and an interval model. Therefore, in this section, two conventional fuzzy regression models, a least squares model by P. Diamond and an interval model by H. Tanaka, are introduced.

At first, a least squares fuzzy regression model by Diamond is illustrated. Then, an interval fuzzy regression model by Tanaka is introduced, this interval model is focused on this chapter.

2.1 Least Squares Fuzzy Regression Model

There are fuzzy regression models, a least squares model such as statistical regression model and an interval model to describe a possibility of analyzed target. A least squares fuzzy regression model has been proposed in order to give error estimates in the form of residuals by Diamond [2, 3]. Therefore, this model is focused on a linear least squares estimation for vague data.

Observed data $(\mathbf{x}_i, \mathbf{y}_i), (i = 1, 2, \ldots, n)$ are triangular fuzzy numbers, where p dimensional explanatory variables $\mathbf{x}_i = \left(\mathbf{x}_i^L, \mathbf{x}_i^C, \mathbf{x}_i^U\right)$ and a dimensional response variable $\mathbf{y}_i = \left(y_i^L, y_i^C, y_i^U\right)$ are used. Here, L, C, and U denote the lower limit, the center, and the upper limit of a fuzzy number in this paper, respectively. We assume $\mathbf{x}$ are random variables and $\mathbf{x}_i^L \geq 0$. Although a least squares fuzzy regression model by P. Diamond has p dimensional real-valued coefficients vector $\mathbf{b}$, outputs vector $\mathbf{Y}$ are triangular fuzzy numbers because $\mathbf{x}$ are triangular fuzzy numbers.

Then, parameters $\mathbf{b}$, which give a minimal distance between $\mathbf{y}$ and $\mathbf{Y}$, are regression coefficients of the best fit model.

That is, the coefficient of a least squares fuzzy regression model is obtained by minimizing the Eq. (1).

$$\sum_{i=1}^{n}\left\{\left(y_i^C - Y_i^C\right)^2 + \left(y_i^L - Y_i^L\right)^2 + \left(y_i^U - Y_i^U\right)^2\right\} \tag{1}$$

2.2 Interval Fuzzy Regression Model

As observed data should embody possibilities that a considered system has, the measured data can be interpreted as the possibilities of the system. Therefore, a fuzzy regression model is built in terms of the possibility and evaluates all observed values as possibilities that the system should contain. In other words, the fuzzy regression model aims to be built so that it could contain all observed data in the estimated fuzzy numbers resulted from the model. Therefore, this model is applied to many applications [11, 14, 20, 22].

The fuzzy regression equation is written as in the following:

$$\begin{aligned} Y_j &= A_1 x_{1j} + \cdots + A_p x_{pj} = \mathbf{A}\mathbf{x}_j, \\ x_{1j} &= 1; j = 1, 2, \ldots, n, \end{aligned} \tag{2}$$

where each regression coefficient A_i is a symmetric triangular fuzzy number $A_i = (a_i, c_i)$ with center a_i and width c_i. In the Eq. (2), x_j denotes the jth data. A various type of a fuzzy coefficient is used, a symmetric triangular shaped fuzzy number is used as fuzzy coefficients in this chapter.

According the extension principle,

$$Y_j = \mathbf{A}\mathbf{x}_j = (\mathbf{a}, \mathbf{c})\mathbf{x}_j = \left(\mathbf{a}\mathbf{x}_j, \mathbf{c}\left|\mathbf{x}_j\right|\right), \tag{3}$$

where $\left|\mathbf{x}_j\right| = \left[\left|x_{1j}\right|, \left|x_{2j}\right|, \ldots, \left|x_{pj}\right|\right]$.

The output of fuzzy regression Eq. (2), whose coefficients are fuzzy numbers, results in a fuzzy number.

The regression model with fuzzy coefficients can be expressed with center $\mathbf{a}\mathbf{x}_j$ and width $\mathbf{c}\left|\mathbf{x}_j\right|$. When sample $\left(\mathbf{y}_j, \mathbf{x}_j\right)(j = 1, 2, \ldots, n)$ with center y_j and width d_j is given as fuzzy number $\mathbf{y}_j = \left(y_j, d_j\right)$, the inclusion relation between the model and the data should be hold as follows:

$$\begin{aligned} \mathbf{a}\mathbf{x}_j + \mathbf{c}\left|\mathbf{x}_j\right| &\geq \mathbf{y}_j, \\ \mathbf{a}\mathbf{x}_j - \mathbf{c}\left|\mathbf{x}_j\right| &\leq \mathbf{y}_j, \end{aligned} \tag{4}$$

In other words, the possibilistic regression model is built to contain all data in the model. When the width of the model is large, the expression of its regression equation is vague. It is better and more convenient to obtain a clear and rigid expression. Therefore, the width of the regression should be minimized as removing the vagueness of the model as possible. The fuzzy regression model is formulated to minimize its width under constraints (4). This problem results in a linear programming.

Using the notations of observed data $\left(\mathbf{y}_j, \mathbf{x}_j\right)$,

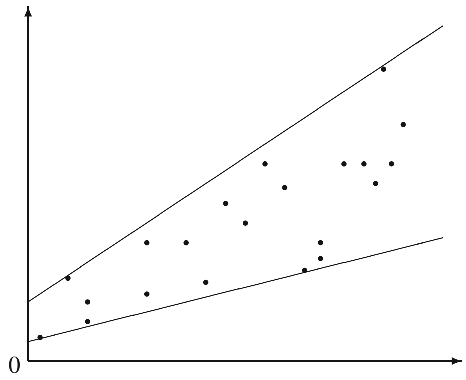

Fig. 1 Fuzzy regression model

$$\mathbf{y}_j = (y_j, d_j), \mathbf{x}_j = [x_{1j}, x_{2j}, \ldots, x_{pj}] \, (j = 1, 2, \ldots, n),$$

fuzzy coefficients $(\mathbf{a}, \mathbf{c})$ of the regression model can be mathematically written in the following LP problem:

$$\begin{array}{ll} \text{minimize} & \sum_{j=1}^{n} c|x_j| \\ \text{subject to} & \\ & \mathbf{a}\mathbf{x}_j + \mathbf{c}|\mathbf{x}_j| \geq \mathbf{y}_j \\ & \mathbf{a}\mathbf{x}_j - \mathbf{c}|\mathbf{x}_j| \leq \mathbf{y}_j \\ & \mathbf{c} \geq \mathbf{0} \, (j = 1, 2, \ldots, n). \end{array} \tag{5}$$

Solving the LP problem mentioned above, we have the possibilistic regression shown in Fig. 1. Relation (4) between the model and the data is held as shown in Fig. 1. This fuzzy regression contains all data in its width and results in expressing all possibilities that data embody and the considered system should have. It is possible in the formulation of the fuzzy regression model to treat non-fuzzy data with no width by setting width d to 0 in the above equations.

3 First Type Model of Fuzzy Robust Regression

An interval fuzzy regression model describes a possibility of analyzed system, and the model is built so as to minimize its ambiguity, as mentioned above. However, a vagueness of a model is made bigger and a shape is made strained easily. A fuzzy robust regression model is one of interval fuzzy regression models, takes into account the distortion problem of a model shape, and aims to describe an essence of analyzed target.

It is possible to consider the data is granular, and is observed as fuzzy numbers or real numbers. Therefore, regardless of the state of the data, a fuzzy regression model is intended to describe accurately the possibility of the system. Our fuzzy robust regression model has two approaches to describe an essence of analyzed target. The first approach employs the concept of a conjunction model to treat granule data distorting a model shape, this model is the first type model of fuzzy robust regression. The second approach employs a maximization the total possibility grade from a model and data, this model is the second type model of fuzzy robust regression.

3.1 Formulation of First Type Model

Irregular values are often given in real world problems. These irregular values come from such causes as the errors of observation methods, the miss-reading of the observed values, the miss-behavior of observation instruments, and so on. Such data distort the expression of the possibility that the latent system should have. Therefore, the influence of the irregular data on the system must be removed in building a regression model or be controlled.

The concept of distance is employed to build a fuzzy robust regression model and to remove the influence of irregular data in building the model.

Let us consider the fuzzy regression shown in Fig. 1 that expresses properly the possibilities of the considered system. When an irregular sample denoted as $\odot$ happens to be mixed in the data, model (2) is distorted largely from the proper figure of the possibility that the considered system should have, as shown in Fig. 2. As illustrated in figures, such irregular data influence on the fuzzy regression model very much. Furthermore, we should discriminate between the error and the possibility included in the data to build a possibility model [26]. This is an approach to build a fuzzy robust regression model by granule data or fuzzy data.

We evaluate both the error distance of data from the model and the fuzziness derived from the system separately in terms of the concept of distance. That is, in building the model we minimize not only the fuzziness included in the model but also the error distance of the samples from the model. This fuzzy regression of data with error is formulated to minimize not only its fuzzy width of the model but also the distance of irregular samples from the model.

As a result, the error between jth sample and the possibilistic model, that is, the distance, r_j, between jth sample and the possibilistic model can be written as follows:

$$r_j = \begin{cases} y_j - Y_j^U; & Y_j^U \leq y_j \\ 0; & Y_j^L \leq y_j \leq Y_j^U \\ Y_j^L - y_j; & y_j \leq Y_j^L \end{cases} \tag{6}$$

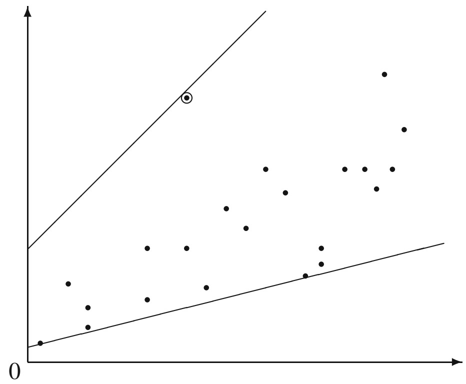

Fig. 2 Fuzzy regression model based on data with outlier⊙

where Y_j^U and Y_j^L denote the upper and lower boundaries of the model, respectively. Let us define the evaluation function of the fuzzy robust regression model using a distance function (6) as follows:

$$J = \sum_{j=1}^{n} r_j + K \sum_{i=1}^{p} c_i, \tag{7}$$

where constant K in the evaluation function is a positive real value that will be decided as a parameter how much to weigh the possibility included in error data.

Let us note that the better term of the above mentioned evaluation function can not be written only in $\sum_{i=1}^{p} c_i$, but also in several expressions and we should define it according to real problems or its objective as same as in the conventional fuzzy regression model.

Parameter K can define the property of the model and has the following meaning. When K is taken as a small value, the model results in the conventional fuzzy regression model because it emphasizes on minimizing the error distance against the model rather than the width of the model. On the other hand, when K is taken as a large value, the model results in the fuzzy regression model without error data because it emphasizes on minimizing the width of the model rather than the error distance against the model. When K is taken as a sufficiently large value the model comes in as a similar model to a statistical one. Using this parameter K it is possible to reflect in model building the knowledge which decision makers or analyzers have obtained from experience of model building.

We cannot tell which data are normal or which are error data, irregular samples or outliers. The fuzzy robust regression analysis discriminates whether data are normal or irregular based on the concept of distance.

The problem to obtain the best fuzzy regression model without the influence of error data in the possible combination of error data results in a combinatorial optimization problem. A genetic algorithm is employed to solve this combinatorial optimization problem.

As mentioned above, the approach to handle data with error was discussed. However, when errors including in data are viewed as a fuzziness, a vagueness and so on, this approach us to handle granule data or fuzzy data.

3.2 Elimination of Irregular Data

When n samples are given, it should be required to calculate 2^n times LP problems in constructing the above mentioned robust model. Generally, it requires the huge numbers of combinatorial calculations to obtain a fuzzy regression model. Nevertheless, the number of feasible solutions is very limited comparing with the number of these calculations 2^n.

All given samples should be treated in the sense of possibility in the fuzzy regression model. In other words, all the given samples should be included in the possibility that the fuzzy regression model expresses. This means that the shape of the fuzzy regression model is determined by the samples at the marginal boundaries of the model. On the other hand, samples that are distributed in the inside and central portion do not have any influence on determining the shape of the model. Therefore, in building the fuzzy robust regression model, out of samples that are near on the marginal boundaries, we can find irregular samples which should not be interpreted in terms of the possibility of system. If we can eliminate such irregular samples, the fuzzy robust model can be effectively and efficiently built. In this paper we employ a hyperelliptic function in order to detect irregular samples that might be on or near to the marginal boundaries.

If we can cluster some portion of samples that should be included in the possibility of the system using a hyperelliptic function, the combinatorial calculations can be reduced into the combination of the remained samples that are not included in the cluster. When h samples are selected out of the total n samples using the hyperelliptic function, the combinatorial calculation can be reduced from 2^n to 2^h.

3.3 Analysis of Asian Economy and Environment

It is widely known that an expansion of economic activity brings about an increase in population and energy consumption, and the increase in population and energy consumption results in an environmental change such as a large amount of air pollutant which influences the environment. Since vital economic activity is pursued in Asian region, Asian region is worried about that her energy consumption causes in an environmental change.

In this section, we analyze the relation between economic activity and environmental change in the Asian region. Population, GDP and an amount of primary energy consumption are employed for denoting economic activity, NO_x, SO_x and CO_2 are employed for describing environmental influence.

A primary energy consumption used to consist of commercial energies as coal, oil, gas and electric, but we also include plant energy in the primary energy consumption because many developing countries always depend on plant energy that is non-commercial energy in quit large amount.

SO_x and NO_x relate to global acidification and CO_2 relates to the greenhouse effect. NO_x relates to the formation of photochemical smog whose main component is ozone, ozone brings about destruction of plants and the greenhouse effect. Therefore, we employ NO_x, SO_x and CO_2 as objective variables in analyzing the environment.

As being analyzed in this section seem not to have large error. On the conventional fuzzy regression model, the width of the fuzzy regression is large. Therefore, we employ our first type model of fuzzy robust regression [11, 20, 23, 26, 27] to analyze this environmental change without influence of some countries.

Data [13] are expressed in terms of a natural logarithm and are observed on 1975.

As inputs, population is denoted by X_1, GDP by X_2 and a primary energy consumption by X_3, as outputs, estimations of NO_x by Y_{NO_x}, SO_x by Y_{SO_x} and CO_2 by Y_{CO_2}, respectively.

In this section, our purpose is to analyze the possibility of the relation between economic activity and environment by our first type model of fuzzy robust regression. We intend to analyze the tendency in this paper, so we do not consider collinearity. In this analysis, we employ a regression equation as follows:

$$Y = A_0 + A_1X_1 + A_2X_2 + A_3X_3.$$

At first, let us confirm statistical regression models based on a least squares method, denoting estimations of NO_x by $Y^{LS}_{NO_x}$, SO_x by $Y^{LS}_{SO_x}$ and CO_2 by $Y^{LS}_{CO_2}$, respectively. Then, these models are as follows:

$$\begin{aligned} Y^{LS}_{NO_x} &= 2.539 - 0.162X_1 - 0.176X_2 + 1.287X_3, \\ Y^{LS}_{SO_x} &= 2.362 - 0.500X_1 - 0.183X_2 + 1.875X_3, \\ Y^{LS}_{CO_2} &= 6.101 - 0.466X_1 + 0.150X_2 + 1.450X_3. \end{aligned}$$

These partial regression coefficients of three models makes us be understood increasing in a primary energy consumption, X_3, leads to an increase of environmental factors, NO_x, SO_x and CO_2, and a primary energy consumption is the most influential environmental factor than other factors. However, population, X_1, and environmental factors are inversely related. In addition, growth of GDP, X_2, leads decrease of NO_x and SO_x, the increase in CO_2.

Therefore, it was confirmed that it is possible to suppress the deterioration of the environment by reducing a primary energy consumption.

Next, conventional fuzzy regression models are obtained as follows:

$$
\begin{aligned}
Y^{CM}_{NO_x} &= (2.810, 0.11) + (0, 0.309)X_1 + (0, 0)X_2 + (0.922, 0)X_3,\\
Y^{CM}_{SO_x} &= (2.799, 0) + (0, 1.064)X_1 + (0, 0)X_2 + (1.136, 0)X_3,\\
Y^{CM}_{CO_2} &= (6.657, 0) + (0, 0.956)X_1 + (0.236, 0)X_2 + (0.171, 0)X_3.
\end{aligned}
$$

As well as statistical regression model, in fuzzy regression models, primary energy consumption has a positive coefficients. However, the impact of primary energy consumption is high, the other coefficients has a small value. A coefficient of GDP is 0, in NO_x model and SO_x model. Therefore, we can interpret these fuzzy regression models does not be describing the relation between the Asian environment change and an economic activity. The system possibility between the Asian environment and an economic activity is distorted in the conventional fuzzy regression model. Data used in this analysis are a real-valued, it can be considered a point in the granular sample has been observed. For data that distorting the model, as in the conjunction model, our first type fuzzy regression model describes the portion of the possibility data have.

Eight countries, which might distort a conventional fuzzy regression model, should be eliminated by a hyperellipse as distorting the possibility of the system. We analyzed objective system by using a distance function (6) to these eight countries and enclosing the other countries. The hyperellipse of NO_x is as follows:

$$
\begin{aligned}
&23.389(Y_{NO_x} - 4.589)^2 + 7.591(Y_{NO_x} - 4.589)(X_1 - 3.171) + \cdots\\
&\quad - 17.116(X_2 - 2.697)(X_3 - 2.362) + 44.296(X_3 - 2.362)^2 = 3.545,
\end{aligned}
$$

the hyperellipse of SO_x is as follows:

$$
\begin{aligned}
&1.322(Y_{SO_x} - 4.712)^2 + 1.322(Y_{SO_x} - 4.712)(X_1 - 3.171) + \cdots\\
&\quad - 7.399(X_2 - 2.697)(X_3 - 2.362) + 10.193(X_3 - 2.362)^2 = 3.226,
\end{aligned}
$$

and the hyperellipse of CO_2 is as follows:

$$
\begin{aligned}
&3.183(Y_{CO_2} - 8.451)^2 + 2.969(Y_{CO_2} - 8.451)(X_1 - 3.171) + \cdots\\
&\quad - 5.105(X_2 - 2.697)(X_3 - 2.362) + 12.234(X_3 - 2.362)^2 = 3.203.
\end{aligned}
$$

On the first type model of fuzzy robust regression, the number of combinations which eight countries are treated by possibility concept or distance concept is 2^8, genetic algorithm is employed to search a fuzzy robust regression model over all combination. We set population size to 200, crossover rate to 70 %, mutations rate to 1 % and production rate to 90 % as parameters of the genetic algorithm. In Table 1, a search rate of SO_x model ($K = 1$) and CO_2 model ($K = 1$) are low, and each rate are 36 % and 46 %. The circumstance of terminations is shown in

Table 1 Searching results by genetic algorithm

	K	Average of terminate generation	Average of solution's fitness	Average of generates	Terminal rate of optimum solution
NO_x	1	18	0.228	838	92
	100	18	8.298	854	100
SO_x	1	19	0.717	928	36
	100	25	26.934	903	94
CO_2	1	22	0.739	955	46
	100	19	12.581	954	90

Table 2 Searching rate of SO_x and CO_2 (K = 1) by genetic algorithm

Model	SO_x		CO_2	
	Fitness	Terminate rate	Fitness	Terminate rate
Optimum	0.717176	36	0.738750	46
Second optimum	0.717177	56	0.738752	90
Third optimum	0.717178	98	0.738753	100

Table 2. It should be noted that the optimum model could not be surely obtained because the genetic algorithm searches a model randomly based on probability. Therefore, we search a model 50 times under the same condition. The rate of obtaining the optimum model in the case SO_x and CO_2 ($K = 1$) are lower than the rate in the case SO_x and CO_2 ($K = 100$), but the second optimum model or the third optimum model in the case SO_x and CO_2 ($K = 1$) are gotten with high probability. The optimum, the second optimum and the third optimum model in the case SO_x and CO_2 ($K = 1$) have small deference between each fitness and model, and we can regard each model as the same model. It is not a serious problem that the rate of obtaining the optimum model is low because the difference between the optimum model and the obtained model of SO_x ($K = 1$) and CO_2 ($K = 1$) is very small. The optimum model is shown in Eqs. (9), (11) and (13).

$$\begin{aligned} Y_{NO_x}^{K=1} &= (2.717, 0.107) + (-0.382, 0.082)X_1 \\ &\quad + (-0.290, 0.040)X_2 + (1.543, 0)X_3 \end{aligned} \tag{8}$$

$$\begin{aligned} Y_{NO_x}^{K=100} &= (2.830, 0) + (-0.363, 0.071)X_1 \\ &\quad + (-0.214, 0)X_2 + (1.475, 0)X_3 \end{aligned} \tag{9}$$

NO_x model $Y_{NO_x}^{K=1}$ with $K = 1$, (8), has a small width. Coefficient of X_1 and X_2 has a small width, and these width are 0.082 and 0.040.

NO_x model does not have a large difference between the cases $K = 1$ and $K = 100$. This is common to the models of NO_x, SO_x and CO_2, the Asian population is scattered large. In comparison between the statistical model and the optimum model (9), there is a sharpness in the coefficients of $Y_{NO_x}^{K=100}$, a population and GDP have a large degree of inverse proportion to NO_x. In addition this, a primary energy consumption increase lead to a large increase in NO_x observation.

$$\begin{aligned} Y_{SO_x}^{K=1} &= (2.537, 0) + (-1.470, 0.269)X_1 \\ &\quad + (-0.583, 0.448)X_2 + (3.530, 0)X_3 \end{aligned} \tag{10}$$

$$\begin{aligned} Y_{SO_x}^{K=100} &= (1.979, 0) + (-0.578, 0.209)X_1 \\ &\quad + (0.164, 0.000)X_2 + (1.936, 0)X_3 \end{aligned} \tag{11}$$

On Eqs. (10) ($K = 1$) and (11) ($K = 100$), the coefficient of primary energy consumption (X_3) of SO_x model is a positive value with no width and the Eq. (10) is about one third of the Eq. (11). However the coefficient of X_2 ($K = 100$) is a positive value with almost no width and about one-third in the absolute value of it.

The difference between the model $Y_{SO_x}^{K=100}$ and the statistical model, is that the GDP is turned to SO_x observed loss in the model $Y_{SO_x}^{K=100}$. This can be understood subjectively.

$$\begin{aligned} Y_{CO_2}^{K=1} &= (6.042, 0.394) + (-1.101, 0.264)X_1 \\ &\quad + (0.196, 0.081)X_2 + (2.162, 0)X_3 \end{aligned} \tag{12}$$

$$\begin{aligned} Y_{CO_2}^{K=100} &= (5.960, 0) + (-0.403, 0.087)X_1 \\ &\quad + (0.349, 0)X_2 + (1.202, 0)X_3 \end{aligned} \tag{13}$$

CO_2 model has a large different between $K = 1$ model $Y_{CO_2}^{K=1}$ (12) and $K = 100$ model $Y_{CO_2}^{K=100}$ (13). $K = 100$ model has small width of a coefficient of X_1 and X_2. Several countries have a different tendency on X_1 and X_2. The coefficient of X_1 is about one third of Eq. (12), the center of the coefficient of X_2 is about half of Eq. (12), respectively.

Difference between the model $Y_{CO_x}^{K=100}$ and the statistical model, is that the GDP is to double the amount of CO_2 observed in the model $Y_{CO_2}^{K=100}$. This can be understood subjectively, too.

This fact, the proposed model is describing the essence of the system, realize assent subjective. In the context of an economic activity and the environment, eight countries distinctive rather than have been removed at the time of model building, which was used only some of the features.

As the result of analyzing, eight countries distinctive as follows:

Japan: Japan employs a measure for the reduction of the emission of air pollutants, and the Environmental Agency monitors the amount of air pollutant. Each of NO_x, SO_x and CO_2 are the smallest amount of the emission in Asian countries.

Indonesia: A number of farm workers is a half of all workers and agricultural product shares a quarter of GDP. A crude oil, a natural gas and a petrochemical share three fourth of the export. The amount of the emission of SO_x is smaller than another countries.

Taiwan: Economy is mainly industrial to manufacture industrial products such as electric devices. The amount of the emission of NO_x is smaller than another Asian countries.

Vietnam: Since Vietnam War has ended when data are observed, Vietnam can not make own supply. A fossil fuel is mainly used, and the amount of the emission of CO_2 is larger than another Asian countries.

Singapore: The main industry is to make a refined product from an imported crude oil. The amount of the emission of NO_x is smaller than another Asian country and CO_2 are larger.

Nepal: Nepal is an agricultural country, and 90 % of working force are a farm worker. The amount of the emission of NO_x, SO_x and CO_2 are smaller than another Asian countries.

Mongolia: Mongolia is an agricultural country based on pasturage. Mongolia is remarkably developing an industry. The amount of the emission of SO_x is smaller, NO_x and CO_2 are larger than another Asian countries.

Brunei: The export is crude petroleum and natural gas. The amount of the emission of NO_x and CO_x are smaller than another Asian countries.

We have discussed about eight countries that locate marginal of possibility area in case of $K = 100$ as follows. Small amount of the emission of NO_x is observed in Japan, Taiwan, Singapore, Nepal and Brunei, on the other hand, large amount of the emission of NO_x is observed in Mongolia. Small amount of the emission of SO_x is observed in Japan, Indonesia, Nepal and Mongolia. Small amount of the emission of CO_2 is observed in Japan, Nepal and Brunei, Large amount of the emission of CO_2 is observed in Vietnam, Singapore and Mongolia. Japan is enforcing the Environmental Pollution Prevention Act, Nepal is an agricultural country and has small amount of the emission of pollutant because Nepal does not have a lively industry makes pollutants (NO_x, SO_x and CO_2). Since Mongolia is an agricultural country, which mainly based on pasturage and is developing industry, the result of analysis is like as above. The main industry in Singapore is to make a refined product from an imported crude oil. Therefore, large amount of the emission of NO_x is observed in Singapore.

In Asian region, in the case that GDP and consumption of primary energy are large value, the amount of emission of CO_2 and SO_x is large scale. On the other hand, in the case that GDP are small value and consumption of primary energy are large value, the amount of emission of NO_x is large scale. Moreover, the larger population is, the smaller the amount of the emission of CO_2, SO_x and NO_x becomes. Since coefficients of primary energy consumption have a large value than another coefficients, we can understand that primary energy consumption and emission of pollutants relate strongly with each other.

As a result of economic activity, GDP is expressed. In developing country, GDP has concerned with manufacture industry, and manufacture industry uses much energy that is mainly primary energy. By consuming primary energy, air pollutants are made. A country which economic activity is developing increase in population, people mainly use plant energy in developing country. Therefore, a growth of GDP brings an increase of population, and an increase of population brings a large amount of the emission of air pollutants.

In this analysis, we employed population, GDP and primary energy consumption as input parameters to analyze the economic activity and the environment of Asian region.

In this analysis, we can have understood the tendency of Asian country and the relation between economic activity and environment by our first type model of fuzzy robust regression.

4 Second Type Model of Fuzzy Robust Regression

In the first type model of fuzzy robust regression, the concept of a conjunction model to treat granule data distorting a model shape and the possibility concept are employing the other data in order to describe an essence of analyzed target. The sum of distance between a first type model and observed values is minimized in the concept of a conjunction model, the ambiguity of the model is minimized in the possibility concept.

The second type model of fuzzy robust regression maximize the total possibility grade from a model and data in order to describe the possibility distribution of analyzed target by the model. For this reason, the center of the second type model coincides with the center of the possibility distribution.

It is possible to obtain the model without an inclusion relation between data and model. Model without using an inclusion relationship may describe the essence of the system. Moreover, this model less sensitive to outliers, and a shape of this model is less distortion.

4.1 Formulation of Second Type Model

In a interval fuzzy regression model, the possibility is represented by an interval so that the interval includes the whole data observed from the focal system [17, 19]. It is most characteristic that samples influence and distort the shape of the model, if samples are separated far from the center of data [10, 26].

On the other hand, the pivotal role of the center position of the system is emphasized in building a possibilistic regression model instead of employing an interval to describe the possibility of a focal system. Tanaka and Guo [19] employ exponential possibility distribution to build a model, while Inuiguchi et al. [9], Tajima [15] and Yabuuchi and Watada [28–31] are working independently on coinciding between the centers of possibility distribution and the center of a possibilistic regression model.

Yabuuchi and Watada proposed the model to describe a system possibility using the center of a fuzzy regression model. The proposed model fits intuitive understanding because it makes the model center and the system center coincide.

As mentioned above, our second type model of fuzzy robust regression is built by maximizing possibility grades summation which derived from estimates of the model and data. In other words, the model is built in order to illustrate the possibility distribution. Therefore, our second type model can be built by granule data or fuzzy data, the model treat data as granule data or fuzzy data.

The possibility grade $\mu(y_i, \mathbf{x}_i)$ of $(y_i, \mathbf{x}_i)$ is defined using the center Yi^C and the width W_i of the model.

$$\mu(y_i, \mathbf{x}_i) = 1 - \frac{\left|Y_i^C - y_i\right|}{W_i}$$

Let us calculate the total sum, Z_1, of possibility grades of the fuzzy regression model as follows:

$$Z_1 = \sum_{i=1}^{n} \mu(y_i, \mathbf{x}_i) = \sum_{i=1}^{n} \left(1 - \frac{\left|Y_i^C - y_i\right|}{W_i}\right). \tag{14}$$

In this part, the model is built by maximizing Z_1 defined in Eq. (14), which is the total sum of possibility grades in the objective function of a fuzzy regression model [6, 7]. It is explicit from Eq. (14) that the width, W_i, of the model gets larger if one maximizes Z_1. Therefore, the following function is defined to minimize the sum, Z_2, of the vagueness values W_i of the model:

$$Z_2 = \sum_{i=1}^{n} W_i.$$

Thus, the model is reduced to a bi-objective linear programming problem employing these Z_1 and Z_2 as a multi-objective function.

But, it is easy to build a method to solve the problem using the following weighted sum of the two objective functions as in Eq. (15):

$$\left.\begin{array}{ll} \text{maximize} & Z_3 = \alpha Z_1 - (1 - \alpha) Z_2 \\ \text{subject to} & Y_i^L \le y_i \le Y_i^U (i = 1, 2, \ldots, n) \end{array}\right\}. \tag{15}$$

where α is a weight parameter, $0 \le \alpha \le 1$. It is possible to control the shape of the model by changing parameter α of the objective function. Therefore, the value of parameter α is selected heuristically and empirically by a decision maker.

4.2 Analysis of Japanese Main Rivers

A river is created by an erosion effect of precipitation, flowing water, and so on. In addition, topographical factors such as a diastrophism, a tectonic activity, and a fault also build up a river. For this reason, a scale of river is illustrated by a basin area and a drift distance. It is easy to understand that a river with a short drift distance has a wider width and a deeper depth in case of being swollen with a big volume of water. Also, a river may be narrower and shallower in case of a low velocity and a low volume of the water flow.

Let us analyze the basin areas Y and the drift distances X of major rivers in Japan [25] by a regression model.

At beginning, a statistical regression model Y^{LS} based on a least squares method is confirmed as following:

$$Y^{LS} = -0.12 + 1.61X.$$

Here, the basin areas Y and the drift distances X are used a value obtained by logarithmic transformation and these takes a large value. Therefore, it is not problem that the constant term has a negative value.

We employ three models which are the fuzzy regression model proposed by Tanaka and Watada [17] and Tanaka and Guo [19], the fuzzy regression model paying a special attention to the center of the model proposed by Tajima [15], and our second type model of fuzzy robust regression. Therefore, the model is defined as follows:

$$\mathbf{Y} = \left(Y^C, Y^L, Y^U\right) = (\mathbf{a}_0, \mathbf{c}_0, \mathbf{d}_0) + (\mathbf{a}_1, \mathbf{c}_1, \mathbf{d}_1)X,$$

Let us denote fuzzy regression models proposed by Tanaka et al., Tajima, and us by Y^{CM}, Y^{CF}, and Y^{GR}, respectively. These are written as the following three models:

$$\begin{aligned}
Y^{CM} &= (1.58, 0, 0) + (1.27, 0.17, 0.46)X, \\
Y^{CF} &= (-0.42, 0.2, 2.52) + (1.60, 0.15, 0)X, \\
Y^{GR} &= (-0.42, 0, 0) + (1.60, 0.15, 0.58)X,
\end{aligned}$$

where X and Y denote the drift distance and the basin area, respectively, and the parameter takes the value $\alpha = 0.05$ for the proposed model. Figures 3 and 4, 5 show Y^{CM}, Y^{CF}, and Y^{GR}, respectively. The shape of the models depends on the objective function. In this paper, the fuzziness of the Tanaka's model and our model is the sum of fuzzy coefficients $\mathbf{c} + \mathbf{d}$, which shows the width of the model. In addition this, under the influence of the Yodogawa, the statistical model has a little larger constant term, the regression model is slightly unnatural.

Fig. 3 Y^{CM} on a scale of rivers in Japan

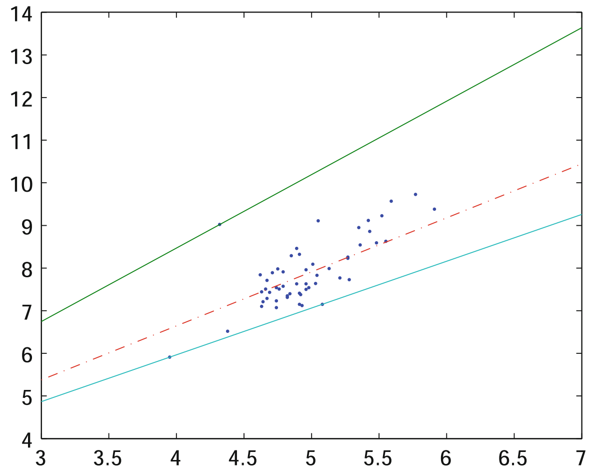

Fig. 4 Y^{CF} on a scale of rivers in Japan

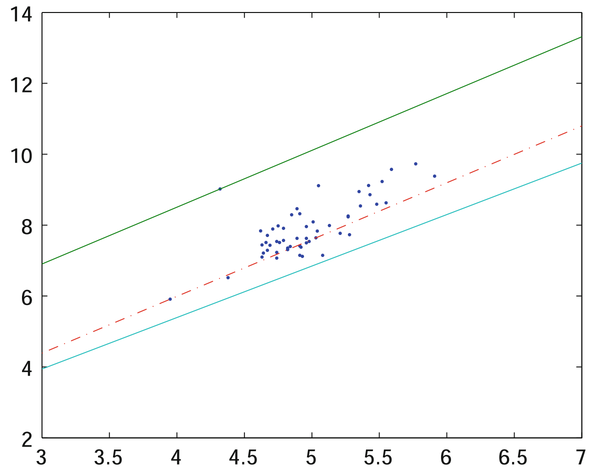

Fig. 5 Y^{GR} on a scale of rivers in Japan

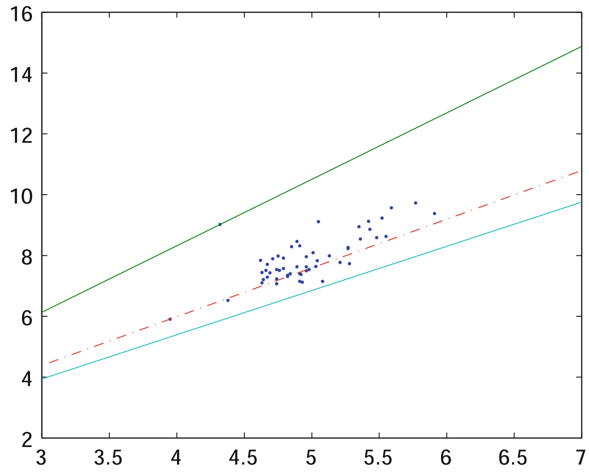

Table 3 Feature of each model

	Y^{CM}	Y^{CF}	Y^{GR}
Sum of all grades	36.904	42.704	43.755
Fuzziness	0.626	2.667	0.731
Residual error	–	344.407	–

The model Y^{CF} can be formulated as follows:

$$\begin{aligned} &\text{minimize} \quad \sum_{i=1}^{n}\left\{(\mathbf{a}\mathbf{x}_i - \mathbf{c}|\mathbf{x}_i| - y_i)^2 + (\mathbf{a}\mathbf{x}_i + \mathbf{d}|\mathbf{x}_i| - y_i)^2\right\} \\ &\text{maximize} \quad \sum_{i=i}^{n}\mu(y_i, \mathbf{x}_i) \\ &\text{subject to} \quad Y_i^L \leq y_i \leq Y_i^U (i = 1, 2, \ldots, n). \end{aligned}$$

Table 3 shows the characteristics of these models. The width of vagueness in Table 3 is the summation of fuzzy coefficient $\mathbf{c} + \mathbf{d}$. In the Tajima's model Y^{CF}, the objective function is defined by using the squared distance between the model and the observed value. This value is also shown in Table 3. These values show the characteristics of these models.

Let us discuss the rivers in Japan. The Yodogawa in Osaka is different from other rivers in Japan in the relation between the basin area Y and the drift distance X. The Yodogawa has a short drift distance X but it has a large basin area Y. Therefore, the Yodogawa is placed far from the group of other rivers. Lake Biwa is the water source of the Yodogawa, the Yodogawa carry water to Osaka Bay. The distance between Osaka Bay and Lake Biwa is short, relatively large tributaries are joined with the Yodogawa. Therefore, the three model shapes look distorted.

The conventional fuzzy regression model Y^{CM} describes the system's possibility by minimizing the fuzziness included in the model (Fig. 3). Nevertheless, the center of Y^{CM} is different from the center of possibility distribution because it is influenced by the Yodogawa, and the slope of the regression is small. As Table 3 illustrates, the fuzziness of the model is the lowest among the three models. The proposed model Y^{GR} shows the second lowest fuzziness behind Y^{CM}.

Let us study the central position of the model. The Tajima's model and the proposed model maximizing the total summation of possibility grades show the main trend of the system, even though their graph's slope is somewhat small.

As the Tajima's model can minimize the distance from the samples to the upper and lower boundaries of the model, the right side of the graph shows narrower width because the number of samples is larger. Therefore, the model shows an unnatural possibility.

The proposed model has a large width of possibility because of the influence of the Yodogawa. But Table 3 shows that among the three models, the proposed model naturally describes the system's possibility and the graph's slope. Although this model is built by maximizing the sum of the possibility grade from the model and data, the inclusion constraints might not fit the possibility distribution and the model.

When the inclusion constraints related to the Yodogawa are removed as the model includes all the samples, the following model is obtained:

$$Y^{GR'} = (-0.98, 0, 0) + (1.73, 0.18, 0.44)X.$$

Figure 6 shows this model. The sum of the possibility grades is 42.196. The fuzziness is 0.622. By means of removing the constraints on the proposed model, the sum of the possibility grades becomes smaller. This occurs because the width of the model becomes narrower. The model's construction for mitigating the influence of outlier samples is discussed below.

4.3 Model Removed Influences of Outliers

In the previous section, we illustrated removing the inclusion constraints in order to alleviate the influence of outlier samples. The center of the model is lower than the center of the data distribution in Fig. 6. The outlier, the Yodogawa, is located above the center. In this case, the outlier samples force the center of possibility distribution to move to the opposite side.

Let us discuss this phenomenon using a simple example as shown in Fig. 7. When we construct the membership function using samples including an outlier $\odot$, Fig. 7a is transformed into Fig. 7b.

The center of the model moves to the right in the majority group of samples, that is, the opposite from the outlier sample. This makes the sum of membership grades larger than the initial state.

When we intend to build a model so that the total sum of the possibility grades is maximized, the formation of the model is distorted. As a fuzzy regression model defined so as to include all samples in the model and minimize the total vagueness of the model, then the total sum of possibility grades becomes larger as the width gets wider. The reason is because the form of the membership function is defined so as to set $\mu(a) = 0$ for the outlier sample $\odot$.

In order to mitigate the distortion of the membership function at the outlier sample $\odot$, the formulation of the model is changed as is shown in Fig. 7c.

When the membership function is generated, it is possible to alleviate the influence of outlier samples by a parallel shifting of the graph of the obtained membership function by the step $\mu'(a) = -\beta$ after solving the membership function which has the maximum membership value $1 + \beta$. In other words, when considering the possibility influenced by the outlier sample $\odot$ to build the membership function, the values [0, 1] are used in the membership function. That is, the outlier sample $\odot$ is placed outside of the possibility distribution because of the nature of outlier samples.

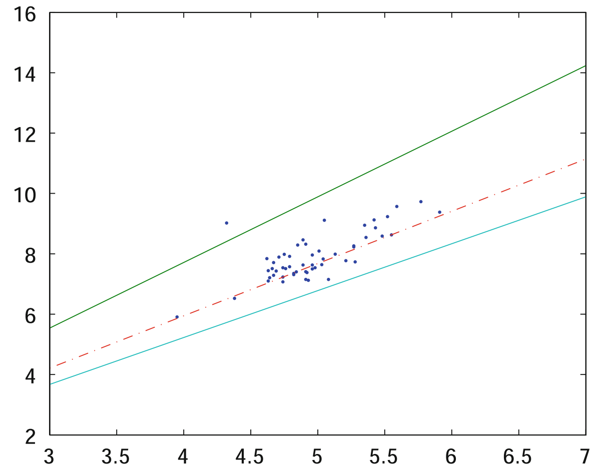

Fig. 6 $Y^{GR'}$ on a scale of rivers in Japan

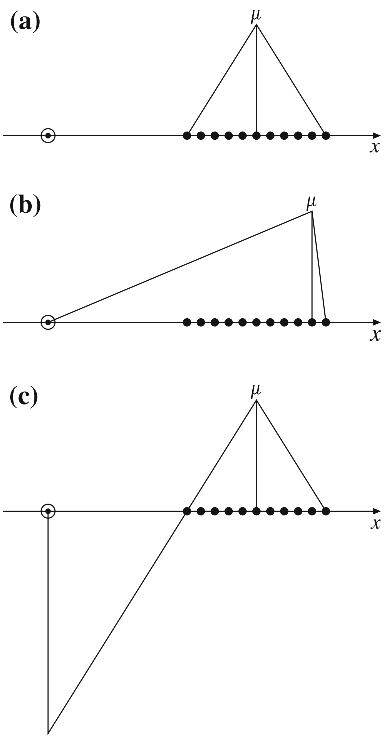

Fig. 7 Outlier mixed in data. **a** Outlier ⊙ is not included in data. **b** Outlier ⊙ is included in data. **c** Removed influences of outlier ⊙

The slope of the straight line in the previous section was smaller than the possibility of the system. Let us consider Fig. 8. The ellipsoid in Fig. 8 denotes the data distribution. When outlier samples are included at left side of the upper

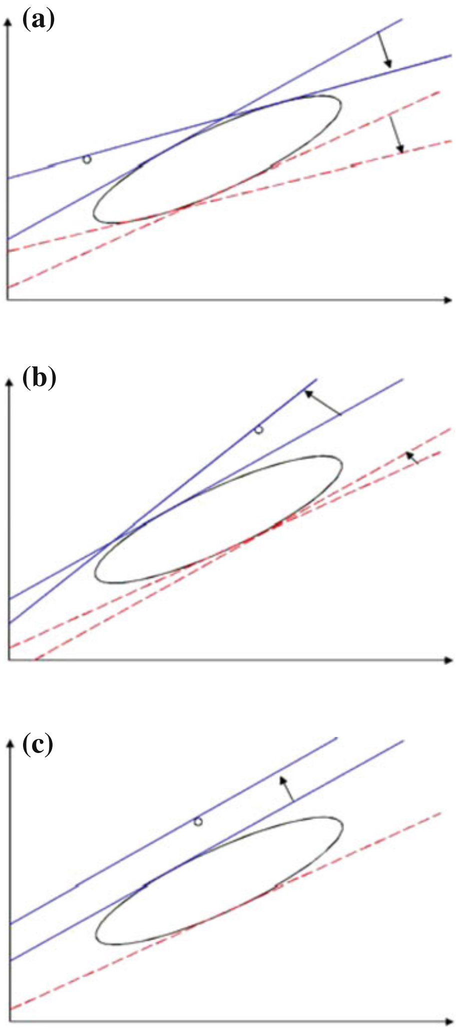

Fig. 8 Location of outlier. **a** Outlier is located in the *upper left portion* of the data distribution. **b** Outlier is located in the *upper right portion* of the data distribution. **c** Outlier is directly above the data distribution

ellipsoid, the fuzzy regression model rotates clockwise and the slope becomes smaller as shown by the arrow in Fig. 8a.

On the other hand, when outlier samples are included at the right side of the upper ellipsoid, the fuzzy regression model rotates counterclockwise and the slope becomes larger, as shown by the arrow in Fig. 8b.

Based upon the above discussion of the fuzzy regression model leads to the conclusion that influence of outlier samples on the maximum of the total sum of possibility grades can be summarized as follows:

- As the grade becomes larger, the width of the model becomes wider.
- The center of the model moves to the opposite of the outlier sample's position.

- The slope of the model's graph is influenced by outlier samples.

When the fuzzy regression model is built by maximizing the possibility grade, its objective is to describe the possibility of the latent system. That means that the emphasis is on the description of the system possibility rather than the inclusion of all the samples in the model. Therefore, it will be allowed to remove the inclusion constraints between sample y and model Y as follows:

$$Y^L \leq y \leq Y^U.$$

When the model is constructed, outlier samples greatly influence fuzzy coefficients as shown in Fig. 8c. Heuristically, we can employ the following objective function under the setting $|\mu(a)| = \beta$.

$$\text{maximize}\, Z_3 = \alpha \sum_{i=1}^{n} |\mu(y_i, \mathbf{x}_i) - \beta| - (1-\alpha)\gamma \sum_{i=1}^{n} W_i.$$

where α is the parameter to decide the selection between the maximization of possibility grade and the minimization of the vagueness of the model. γ is the parameter to tune the difference between the total sum of possibility grades and the vagueness of the model.

Let us consider an example to illustrate the proposed model. Set $\alpha = 0.1$, $\gamma = 2$ and compare cases $\beta = 0.3$ and $\beta = 0.7$. Fuzzy regression models $Y^{GR}_{\beta=0.3}$ and $Y^{GR}_{\beta=0.7}$ are obtained as follows:

$$Y^{GR}_{\beta=0.3} = (-0.98, 0, 0.0) + (1.73, 0.16, 0.45)X,$$
$$Y^{GR}_{\beta=0.7} = (-1.38, 0, 0.0) + (1.80, 0.07, 0.36)X.$$

Parameter α used in the model decides which portion is emphasized between the maximization of the total sum of the possibility grades and the minimization of the total vagueness of the model. When α is set to larger value, the vagueness of the obtained model becomes larger. In the numerical example, the setting of $\alpha = 0.1$ and $\gamma = 2$ makes an obtained model well-balanced.

As β corresponds to $\alpha-$cut, setting β to a larger value makes the width of the model smaller. Figures 9 and 10 depict $Y^{GR}_{\beta=0.3}$ and $Y^{GR}_{\beta=0.7}$, respectively. Parameter β adjusts the width of the model (Table 4). $\beta = 0.3$ shows that the model has too large vagueness. On the other hand, $\beta = 0.7$ shows that non-outlier rivers are placed outside of the possibility distribution. Then, we employed $\beta = 0.5$ to obtain the following model:

$$Y^{GR}_{\beta=0.5} = (-0.98, 0, 0.00) + (1.73, 0.15, 0.39)X.$$

Even this model still shows some influence of the Yodogawa, and the possibility distribution of samples is appropriately expressed (Fig. 11).

Fig. 9 Improved model 1 ($\beta = 0.3$)

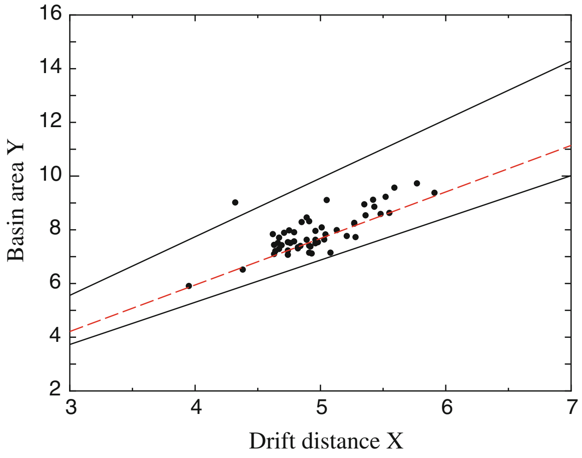

Fig. 10 Improved model 2 ($\beta = 0.7$)

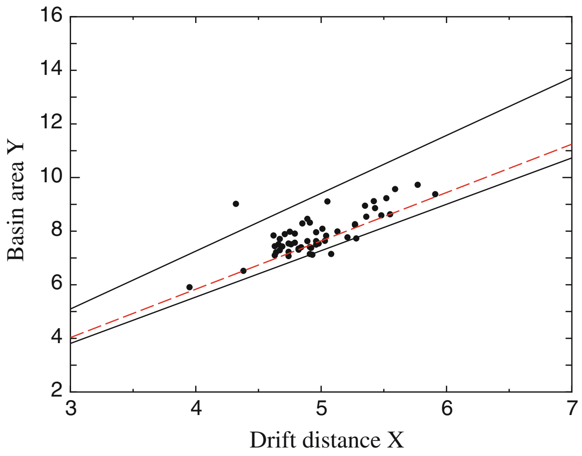

Table 4 Features of improved model

	Improved model	
	1	2
β	0.3	0.7
Sum of all grades	42.531	38.960
Value of Obj. Func.	2.826	2.585

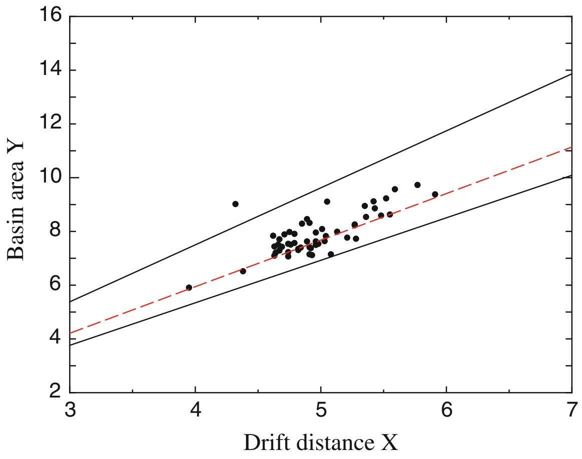

Fig. 11 Optimum model ($\alpha = 0.1$, $\beta = 0.5$)

5 Conclusions

In this chapter, we proposed two type models of fuzzy robust regression. The first type model of fuzzy robust regression treats with granule data or fuzzy data after removal of ill effects of extraordinary data by genetic algorithm. The distance concept is an easy-to-use and feasible way for data including a vagueness, this was confirmed by the first type model of fuzzy robust regression.

The relation between Asian economy and environment was analyzed by the first type model. Relation between Asian economy and environment was explained by the first type model. In this analysis, employing the distance between a conventional fuzzy regression model and slightly different character countries, the relation between Asian economy and environment was explained by the first type model.

The second type model of fuzzy robust regression illustrates the possibility of the target system by its triangular membership function. This second model is built in order to get the maximum degree of coincidence between a second type model and a possibility distribution. Therefore, this second type model is not able to handle only real-valued data but also granule data or fuzzy data.

The features of Japanese major river was analyzed by the second type model, the basin area and the drift distance was used. The center, the upper limit and the lower limit of the system was reveals by the second type model. And the feature of Japanese major river was explained. Since Yodogawa has short drift distances relative to large basin area, the possibility limits was spread. However, the problem was solved by the approach described above. This, the feature of Japanese major river became clear.

Finally, we can conclude our fuzzy robust regression models are able to describe a target possibility by granule data or fuzzy data.

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