# **Density-Based Local Outlier Detection on Uncertain Data***-*

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**Abstract.** Outlier detection is one of the key problems in the data mining area which can reveal rare phenomena and behaviors. In this paper, we will examine the problem of density-based local outlier detection on uncertain data sets described by some discrete instances. We propose a new density-based local outlier concept based on uncertain data. In order to quickly detect outliers, an algorithm is proposed that does not require the unfolding of all possible worlds. The performance of our method is verified through a number of simulation experiments. The experimental results show that our method is an effective way to solve the problem of density-based local outlier detection on uncertain data.

# **1 Introduction**

Uncertainty is inherent to many important applications, such as location-based services (LBS), sensor monitoring and radio-frequency identification (RFID) [1,9]. In these applications, outlier detection often is essential when analyzing uncertain data. In many real-world applications, determining whether the object is an outlier, not only its distance to neighbors is considered, but also the density of surrounding neighbors should be considered. As such density-based outlier detection can consider local information [4]. incertain data. In many rear-world applications, determined the distance to neighbors is considered.<br>
density of surrounding neighbors should be considered.<br>
outlier detection can consider local information [4].<br>
Uncertai

Uncertain objects as referred to in this paper, by default, is in a  $d$ -dimensional numerical tuple. Given two uncertain instances  $\tilde{u}_i$  and  $\tilde{u}_j$ , the distance between density of surrounding neigotilier detection can consid<br>
Uncertain objects as refer<br>
numerical tuple. Given two<br>
these is denoted by  $d(\tilde{u}_i, \tilde{u}_j)$ these is denoted by  $d(\tilde{u}_i, \tilde{u}_j)$ . In this paper, we employ the Euclidian distance metric, but the developed techniques can be easily extended to other distance metrics.

**Uncertain Objects.** The uncertain object set U consists of  $\{u_1, u_2, \dots, u_i, \dots\}$  $u<sub>u</sub>$ . Each uncertain object  $u<sub>i</sub>$  has d dimensions. In this paper, we focus on the discrete case, an uncertain object  $u_i$  consists of a set of m instances  $\{u_i^1, \ldots, u_i^N\}$  $u_i^2, \dots, u_i^j, \dots, u_i^m$  w[her](#page-4-0)e  $1 \leq j \leq m$ ,  $p(u_i^j)$   $(0 \leq p(u_i^j) \leq 1)$  denotes the probability of instance  $u_i^j$  occurring.

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**Definition 1.** *Distance sum of**k***-neighbors: We assume that instance**  $\tilde{u}_j$  **is i** *K*. Cao et al.<br> **Definition 1. Distance sum of** *k***-neighbors:** We assum<br> *k*-neighbor of instance  $\widetilde{u_i}$  in a possible world W, and  $n_k(\widetilde{u_i})$ *i* in a possible world *W*, and  $n_k(\tilde{u}_i)$  is a k-neighbor set<br>
denote the distance sum of k-neighbors of instance  $\tilde{u}_i$ <br> *n*<br>
dis $_k(\tilde{u}_i) = \sum$  dis( $\tilde{u}_j, \tilde{u}_i$ ) *of k*. Cao et al.<br> **Definition 1.** *Distance* sum of *k*-neighbors: We assume that instance  $\widetilde{u}_j$  *i*, *k*-neighbor of instance  $\widetilde{u}_i$  in a possible world W, and  $n_k(\widetilde{u}_i)$  is a *k*-neighbor se of  $\widetilde{u$ of  $\tilde{u}_i$  in W. Let  $dis_k(\tilde{u}_i)$  denote the distance sum of k-neighbors of instance  $\tilde{u}_i$ *in possible world* W*, then i*) of *k*-neight<br>possible words the distance of  $\hat{z}_i$ <br> $\hat{z}_i$  =  $\sum$ 

$$
dis_k(\widetilde{u_i}) = \sum_{\widetilde{u_j} \in n_k(\widetilde{u_i})} dis(\widetilde{u_j}, \widetilde{u_i})
$$

 $dis_k(\widetilde{u_i}) = \sum_{\widetilde{u_j} \in n_k(\widetilde{u_i})} dis(\widetilde{u_j}, \widetilde{u_i})$ <br> **Definition 2.** *Density of an instance: Let instances*  $\widetilde{u_j}$  *and*  $\widetilde{u_i}$  *be entities finally distribution 2. Density of an instance: Let instances*  $\widetilde{u}_j$  and  $\widetilde{u}_i$  be entities from different uncertain objects, i.e.  $j \neq i$ . If  $\widetilde{u}_j$  is a k-neighbor of  $\widetilde{u}_i$  in possible **Definition 2. Density of an in**<br>from different uncertain objects, i.e<br>world W, the density of instance  $\widetilde{u_i}$ *world* W, the density of instance  $\tilde{u}_i$  is defined as **y** of an instance: Let<br>  $\overline{u}$  objects, i.e.  $j \neq i$ . If  $\widetilde{u}_j$ <br>  $f$  instance  $\widetilde{u}_i$  is defined as<br>  $den(\widetilde{u}_i) = \frac{k}{\sum_{W \in \mathbb{W}} dis_k(\widetilde{u}_i)}$ 

$$
den(\widetilde{u_i}) = \frac{k}{\sum_{W \in \mathbb{W}} dis_k(\widetilde{u_i}) P(W)}
$$

 $den(\widetilde{u_i}) = \frac{k}{\sum_{W \in \mathbb{W}} dis_k(\widetilde{u_i})P(W)}$ <br> **Definition 3.** *k*-neighbor set of an instance: Let  $\widetilde{u_i}$  denote an instance of  $den(\widetilde{u}_i) = \frac{\partial}{\partial w_i}$ <br> **Definition 3.** *k*-neighbor set of an instance: Let  $\widetilde{u}_i$  denote an instance of increasing object  $u_i$ ,  $(u_i \in U)$ .  $n_k(\widetilde{u}_i)$  denotes the *k*-neighbor set of instance  $\widetilde{u}_i$ **i**<br> **i** *instance:* Let  $\widetilde{u}_i$  denote an instance:<br> *in possible world* W. Let  $N_k(\widetilde{u}_i)$  denote the k-neighbor set of instance<br> *in possible world* W. Let  $N_k(\widetilde{u}_i)$  denote the k-neighbor set of instance  $\widetilde$ *in possible world* W. Let  $N_k(\widetilde{u}_i)$  denote the k-neighbor set of instance  $\widetilde{u}_i$  in all *possible worlds, then*  $\begin{aligned} \textit{set of an instance} \ \hat{n}_k(\widetilde{u}_i) \ \textit{denotes } th(\widetilde{u}_i) \ \textit{denote the } k\text{-}ne \ \hat{N}_k(\widetilde{u}_i) = \bigcup \ n_k(\widetilde{u}_i) \end{aligned}$ 

$$
N_k(\widetilde{u_i}) = \bigcup_{W \in \mathbb{W}} n_k(\widetilde{u_i})
$$

**Definition 4.** *Local Outlier Factor of an instance: Given that instance*  $p$ c $\mathbf{D}$  $\widetilde{u}$  $N_k(\widetilde{u_i}) = \bigcup_{W \in \mathbb{W}} n_k(\widetilde{u_i})$ <br> **Definition 4. Local Outlier Factor of an instance:** Given that instance<br> *i* is a k-neighbor of instance  $\widetilde{u_i}$ , den( $\widetilde{u_j}$ ) denotes the density of  $\widetilde{u_j}$ , LOF( $\widetilde{u_i}$  $N_i$ <br> **Definition 4. Local Outlier**<br>  $\widetilde{u}_j$  is a k-neighbor of instance<br>
denotes the LOF of instance  $\widetilde{u}_i$ *denotes the LOF of instance*  $\tilde{u_i}$ . *L* Outlier Factor of an infinitance  $\tilde{u}_i$ , den $(\tilde{u}_j)$  denotes<br>instance  $\tilde{u}_i$ .<br> $LOF(\tilde{u}_i) = \frac{\sum_{\widetilde{u}_j \in n_k(\widetilde{u}_i)} den(\widetilde{u}_j)}{\sum_{j \in \mathcal{U}_i} dom(\widetilde{u}_j)}$  $\frac{u e}{\widetilde{u_j}}$  $\widetilde{\mu_{j}}$ ) denote<br> $\widetilde{\mu_{i}}$ ) denote<br> $k \times den(\widetilde{u_{i}})$ 

$$
LOF(\widetilde{u_i}) = \frac{\sum_{\widetilde{u_j} \in n_k(\widetilde{u_i})} den(\widetilde{u_j})P(W)}{k \times den(\widetilde{u_i})}
$$
  
Definition 5. *Local Outline Factor of an uncertain object:* Let  $\widetilde{u_i}$  denote

 $LOF(\widetilde{u_i}) = \frac{\sum_{i,j} a_i \overline{a_i u_j}}{k \times den(\widetilde{u_i})}$ <br> **Definition 5.** Local Outlier Factor of an uncertain object: Let  $\widetilde{u_i}$  denotes any one instance of object  $u_i$ ,  $P(\widetilde{u_i})$  denotes the probability of  $\widetilde{u_i}$ , and any one instance of object  $u_i$ ,  $P(\tilde{u_i})$  denotes the probability of  $\tilde{u_i}$ , and  $LOF(\tilde{u_i})$ **Definition 5.** Local Outlier Factor of an unce<br>any one instance of object  $u_i$ ,  $P(\tilde{u_i})$  denotes the prodenotes the LOF of  $\tilde{u_i}$ . LOF $(u_i)$  is then defined as **Dutlier Factor**<br>
ect  $u_i$ ,  $P(\tilde{u_i})$  der<br>  $LOF(u_i)$  is the<br>  $LOF(u_i) = \sum$ f an ur<br>*defined*<br>*LOF*( $\widetilde{u_i}$  $\begin{array}{l} \textit{ncert} \ \textit{proba} \ \textit{l as} \ \end{array}$ 

$$
LOF(u_i) \text{ is then defined as}
$$

$$
LOF(u_i) = \sum_{\widetilde{u_i} \in u_i} LOF(\widetilde{u_i})P(\widetilde{u_i})
$$

**Definition 6.** *Density-based local outlier: When the uncertain objects are sorted in descending order based on their Local Outlier Factor, then the top-*n *uncertain objects are the density-based local outliers of the uncertain data set.*

## **2 Algorithms**

In this section, we present an algorithm for Density-based Local Outlier detection on Uncertain data  $(UDLO)$ , which transforms the outlier definition into a probability problem. Density based outliers can be calculated based on the definition in a naive way by finding all k-neighbors in all possible worlds. This solution however is impractical as the number of possible worlds grows exponentially with the number of instances. To overcome this, we propose an exact algorithm to compute the density of an instance. definition in a naive was<br>
solution however is importially with the numbral<br>
algorithm to compute the<br> **Theorem 1.** Let  $N_k(\tilde{u})$ by finding all *k*-neighbors in all possible worlds. This<br>practical as the number of possible worlds grows expo-<br>ber of instances. To overcome this, we propose an exact<br>he density of an instance.<br> $\tilde{u}_i$  *denote the k-n* solution however is impractical as the number of possible worl<br>nentially with the number of instances. To overcome this, we pr<br>algorithm to compute the density of an instance.<br>**Theorem 1.** Let  $N_k(\tilde{u}_i)$  denote the k-ne

*of an uncertain object are all in the k-neighbor set of instance*  $\tilde{u_i}$ , then object is *a complete k*-neighbor object of instance.<br> **Theorem 1.** Let  $N_k(\tilde{u}_i)$  denote the k-neighbor set of instance  $\tilde{u}_i$ . If all instances of an uncertain object are all in the k-neighbor set of instance  $\tilde{u}_i$ , then **Theorem 1.** Let  $N_k$ <br>of an uncertain object<br>a complete k-neighbo<br>objects of instance  $\widetilde{u}_i$ *objects of instance*  $\tilde{u}_i$  *equals* k. The maximal distance from instance to its com-**Theorem 1.** Let  $N_k(\tilde{u}_i)$  denote the k-neighbor set of instance  $\tilde{u}_i$ . If all instance of an uncertain object are all in the k-neighbor set of instance  $\tilde{u}_i$ , then object is a complete k-neighbor object of ins plete k-neighbor object is denoted by  $dis_{c}(\tilde{u}_{i})$ . The k-neighbor set of instance  $\tilde{u}_{i}$ *consists of instance*  $\tilde{u}_i$  *are integral in the k-neighbor set of instance*  $\tilde{u}_i$ , then object a complete k-neighbor object of instance  $\tilde{u}_i$ . The number of complete k-neighbor objects of instance  $\tilde{u}_i$  eq consists of the instances whose distance to instance  $\tilde{u}_i$  is not larger than  $dis_c(\tilde{u}_i)$ . *objects of instance*  $\tilde{u}_i$  equals  $k$ . The maximal distance from instance to its complete  $k$ -neighbor object is denoted by  $dis_{\epsilon}(\tilde{u}_i)$ . The  $k$ -neighbor set of instance  $\tilde{u}_i$  consists of the instances whose

*is*  $t_i$  *in list* L.  $P(S_{t_i}, \kappa)$  *denotes the probability that there are*  $\kappa$  *instances existing in* <sup>S</sup>*<sup>t</sup><sup>i</sup> .* <sup>P</sup>(S*<sup>t</sup><sup>i</sup>* , 0) = <sup>P</sup>(S*<sup>t</sup>i−*<sup>1</sup> , 0)(1 <sup>−</sup> <sup>P</sup>(t*i*)) = *<sup>i</sup> <sup>j</sup>*=1(1 <sup>−</sup> <sup>P</sup>(t*i*))*, and* <sup>P</sup>(S*<sup>t</sup><sup>i</sup>* , κ) =  $P(S_{t_{i-1}}, \kappa - 1)P(t_i) + P(S_{t_{i-1}}, \kappa)(1 - P(t_i))$ **Theorem 2.** In the basic case, for  $1 \le i, j \le |N_k(u_j)|$ <br>is  $t_i$  in list L.  $P(S_{t_i}, \kappa)$  denotes the probability that the<br>in  $S_{t_i}$ .  $P(S_{t_i}, 0) = P(S_{t_{i-1}}, 0)(1 - P(t_i)) = \prod_{j=1}^i (1 - P(S_{t_{i-1}}, \kappa - 1)P(t_i) + P(S_{t_{i-1}}, \kappa)(1 - P(t_i))$ <br>**Theore** *ta t<sub>i</sub> i m tast L*.  $P(S_{t_i}, \kappa)$  *denotes the probability the*  $in S_{t_i}$ .  $P(S_{t_i}, 0) = P(S_{t_{i-1}}, 0)(1 - P(t_i)) = \prod_{j=1}^{i} P(S_{t_{i-1}}, \kappa - 1)P(t_i) + P(S_{t_{i-1}}, \kappa)(1 - P(t_i))$ <br> **Theorem 3.** We assume that the instances  $\tilde{u}_j$ 

*ihe instances*  $\tilde{u}_j$  *and*  $\tilde{u}_i$  *are from different uncer<sup>j</sup>* ) *denotes the probability, then*  $\begin{aligned} &\textit{in}\,\,S_{t_i}.\,\,P(S_{t_i},0)=P(S_{t_i})\ &\textit{P}(S_{t_{i-1}},\kappa-1)P(t_i)+P(\ &\textbf{Theorem}\,\, \textbf{3.}\,\,\,We\,\,assum\,\, \textit{tain\,\,objects},\,\, \widetilde{u_j}\,\,\textit{is a $k$-not}\,\, \textit{the density of instance}\,\, \widetilde{u_j} \end{aligned}$  $\begin{aligned} &\textit{ssume that the instances } \widetilde{u_j} \textit{ and } \widetilde{u_i} \textit{ are} \\ &\textit{l.k-neighbour} \textit{if} \textit{ is calculated as follows:} \\ &\textit{den}(\widetilde{u_i}) = \frac{k}{\sum_{\widetilde{u_j} \in N_k(\widetilde{u_i})} \textit{dis}(\widetilde{u_j}, \widetilde{u_i}) P_k(\widetilde{u_j})} \end{aligned}$ 

the density of instance 
$$
\tilde{u}_i
$$
 is calculated as follows:  
\n
$$
den(\tilde{u}_i) = \frac{k}{\sum_{\widetilde{u}_j \in N_k(\widetilde{u}_i)} dis(\widetilde{u}_j, \widetilde{u}_i) P_k(\widetilde{u}_j)}
$$
\n**Theorem 4.** If we assume that instance  $\tilde{u}_j$  is a k-neighbor of  $\tilde{u}_i$ , then  $P_k(\tilde{u}_j)$ 

 $den(\widetilde{u_i}) = \frac{\widetilde{h_i}}{\sum_{\widetilde{u_j} \in N_k(\widetilde{u_i})} dis(\widetilde{u_j}, \widetilde{u_i}) P_k(\widetilde{u_j})}$ <br> **Theorem 4.** If we assume that instance  $\widetilde{u_j}$  is a k-neighbor of  $\widetilde{u_i}$ , then  $P_k(\widetilde{u_j})$ <br> *is its k-neighbor probability.*  $den(\widetilde{u_i})$  $i$ <sup>*k*</sup> *u<sub>j</sub> i***s** *a*  $\kappa$ -heightor bj *u<sub>i</sub>*, then  $\Gamma_k(u_j)$ <br>*i den*( $\widetilde{u_j}$ ) are the densities of  $\widetilde{u_i}$  and  $\widetilde{u_j}$ ,<br> $\frac{k(\widetilde{u_i})}{k}$  *den*( $\widetilde{u_j}$ ) $P_k(\widetilde{u_j})$ *then the* LOF *is given by:* assume that instance  $\widetilde{u}_j$  is a k-neigh<br>bability. den( $\widetilde{u}_i$ ) and den( $\widetilde{u}_j$ ) are the<br>en by:<br> $LOF(\widetilde{u}_i) = \frac{\sum_{\widetilde{u}_j \in N_k(\widetilde{u}_i)} den(\widetilde{u}_j) P_k(\widetilde{u}_j)}{N}$ *u*-

$$
LOF(\widetilde{u_i}) = \frac{\sum_{\widetilde{u_j} \in N_k(\widetilde{u_i})} den(\widetilde{u_j}) P_k(\widetilde{u_j})}{k \times den(\widetilde{u_i})}
$$

# **3 Experiments**

We conducted several experiments on two real data sets and a synthetic data set to examine the efficiency and accuracy. In the remainder of this paper, these algorithms will be referred to as follows: outlier detection algorithm (denoted by UDLO). For comparison, we implemented the outlier detection algorithms (denoted by  $BULOF$  and  $ULOF$ ) which are proposed in [7].

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#### **3.1 Efficiency**

Efficiency is an important term frequently used in outlier detection studies. Figure 1 shows the results on the two different datasets. As expected UDLO performs better than  $ULOF$ . Parameter n varies from 20 to 100, the running time increases as n increases. The running time of  $ULOF$  is much higher than  $UDLO$ algorithm.



**Fig. 1.** Running time vs. Data size



**Fig. 2.** Running time vs. *<sup>n</sup>*

## **3.2 Accuracy**

In this section, we give the experimental results on accuracy, as shown in Figure 3. Since of the outliers lie in the center of a cluster, it is hard for the ULOF algori[th](#page-4-1)m to pick out this kind of objects from the entire dataset. The UDLO algorithm adheres more [st](#page-4-2)rictly to the outlier definition, and therefore the accuracy of UDLO algorithm is higher than that of ULOF. As expected, UDLO ca[n](#page-4-3) deliver the best results on all datasets.

## **4 Related Work**

Aggarwal, C.C. *et al.* [2] were the first to investigate the problem of outlier detection on uncertain data. Wang *et al.*[8] focused on distance-based outlier detection on uncertain data, in which each data is affiliated with a confidence value. Jiang *et al.* [6] started with a comprehensive model considering both uncertain objects and their instances. In Ref.[3] they attempted to find outliers by building a global classifier. However, it is difficult to build a clear boundary

<span id="page-4-0"></span>

**Fig. 3.** Accuracy vs. *<sup>k</sup>*

between normal data and abnormal data. Fan [5] proposed density-based topk outlier detection algorithm on uncertain objects. In their work, due to the distance between two objects is approximation, the density of object can not be accurate, so that affect the detection results. In Liu *et al.* [7], the authors proposed a signed outlier detection algorithm based on local information (local density and local uncertainty level) on uncertain data.

# <span id="page-4-1"></span>**5 Conclusions**

<span id="page-4-4"></span><span id="page-4-3"></span>There are many important applications that require outlier detection on uncertain data. These applications always require that outlier are identified based on local information. We first derived an algorithm, which can effectively detect outliers without unfolding all possible worlds.

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